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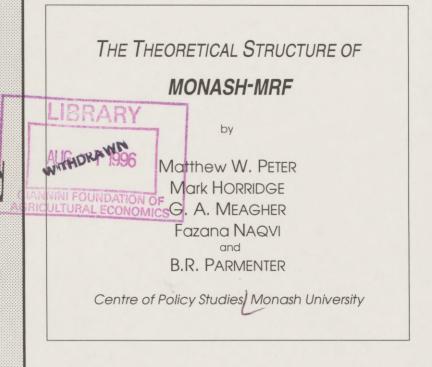
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The Centre of Policy Studies (COPS) is a research centre at Monash University devoted to quantitative analysis of issues relevant to Australian economic policy. The Impact Project is a cooperative venture between the Australian Federal Government and Monash University, La Trobe University, and the Australian National University. COPS and Impact are operating as a single unit at Monash University with the task of constructing a new economy-wide policy model to be known as *MONASH*. This initiative is supported by the Industry Commission on behalf of the Commonwealth Government, and by several other sponsors. The views expressed herein do not necessarily represent those of any sponsor or government.

ABSTRACT

This paper presents the theoretical specification of the *MONASH-MRF* model. *MONASH-MRF* is a multiregional multisectoral model of the Australian economy. Included is a complete documentation of the model's equations, variables and coefficients. The documentation is designed to allow the reader to cross-reference the equation system presented in this paper in ordinary algebra, with the computer implementation of the model in the TABLO language presented in CoPS/IMPACT *Preliminary Working Paper* No. OP-82.

Keywords: multiregional, regional modelling, CGE, regional and Federal government finances.

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The Theoretical Structure of MONASH-MRF

1. Acknowledgement and explanation

This document contains a draft version of chapter 2 from the forthcoming monograph, *MONASH-MRF: A Multiregional Multisectoral Model of the Australian Economy.* The *MONASH-MRF* project was initiated in mid 1992 with the sponsorship of the NSW and Victorian State government Treasuries. The authors thank P. B. Dixon, W. J. Harrison and K. R. Pearson for their helpful advice.

2.1. Introduction

MMRF divides the Australia economy into eight regional economies representing the six States and two Territories. There are four types of agent in the model: industries, households, governments and foreigners. In each region, there are thirteen industrial sectors. The sectors each produce a single commodity and create a single type of capital. Capital is sector and region specific. Hence, MMRF recognises 104 industrial sectors, 104 commodities and 104 types of capital. In each region there is a single household and a regional government. There is also a Federal government. Finally, there are foreigners, whose behaviour is summarised by demand curves for regional international exports and supply curves for regional international imports.

In common with the stylised multiregional model described in Chapter 1, in MMRF, regional demands and supplies of commodities are determined through optimising behaviour of agents in competitive markets. Optimising behaviour also determines demands for labour and capital. National labour supply can be determined in one of two ways. Either by demographic factors or by labour demand. National capital supply can also be determined in two ways. Either it can be specified exogenously or it can respond to rates of return. Labour and capital can cross regional borders in response to labour-market and capital-market conditions.

The specifications of supply and demand behaviour coordinated through market clearing conditions, comprise the CGE core of the model. In addition to the CGE core are blocks of equations describing: (i) regional and Federal government finances; (ii) accumulation relations between capital and investment, population and population growth, foreign debt and the foreign balance of trade, and; (iii) regional labour market settings.

Computing solutions for MMRF

MMRF is in the Johansen/ORANI class of models¹ in that its structural equations are written in linear (percentage-change) form and results are deviations from an initial solution. Underlying the linear representation of

¹ For an introduction to the Johansen/ORANI approach to CGE modelling, see Dixon, Parmenter, Powell and Wilcoxen (DPPW, 1992) Ch. 3.

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MMRF is a system of non-linear equations solved using GEMPACK. GEMPACK (see Harrison and Pearson 1994) is a suite of general purpose programs for implementing and solving general and partial equilibrium models. A percentage-change version of MMRF is specified in the TABLO syntax which is similar to ordinary algebra.² GEMPACK solves the system of nonlinear equations arising from MMRF by converting it to an Initial Value problem and then using one of the standard methods, including Euler and midpoint (see, for example, Press, Flannery, Teukolsky and Vetterling 1986), for solving such problems.

Writing down the equation system of MMRF in a linear (percentage-change) form has advantages from computational and economic standpoints. Linear systems are easy for computers to solve. This allows for the specification of detailed models, consisting of many thousands of equations, without incurring computational constraints. Further, the size of the system can be reduced by using model equations to substitute out those variables which may be of secondary importance for any given experiment. In a linear system, it is easy to rearrange the equations to obtain explicit formulae for those variables, hence the process of substitution is straightforward.

Compared to their levels counterparts, the economic intuition of the percentage-change versions of many of the model's equations is relatively transparent.³ In addition, when interpreting the results of the linear system, simple share-weighted relationships between variables can be exploited to perform back-of the-envelope calculations designed to reveal the key cause-effect relationships responsible for the results of a particular experiment.

The potential cost of using a linearised representation is the presence of linearisation error in the model's results when the perturbation from the initial solution is large. As mentioned above, GEMPACK overcomes this problem by a multistep solution procedure such a Euler or midpoint. The accuracy of a solution is a positive function of the number of steps applied. Hence, the degree of desired accuracy can be determined by the model user in the choice of the number of steps in the multistep procedure.⁴

Notational and computational conventions

In this Chapter we present the percentage-change equations of MMRF. Each MMRF equation is linear in the percentage-changes of the model's variables. We distinguish between the percentage change in a variable and its levels value by

² The TABLO version of MMRF is presented in the Appendix to Chapter 3.

³ See Horridge, Parmenter and Pearson (1993) for an example based on input demands given CES production technology.

⁴ See Harrison and Pearson (1994) for an introduction to the solution methods (including Euler) available in GEMPACK. For details on the Euler method, including the Richardson extrapolation, see Dixon, Parmenter, Sutton and Vincent (DPSV, 1982) Chs. 2 & 5, DPPW (1992) Chs. 3, and Horridge, Parmenter and Pearson (1993).

using lower-case script for percentage change and upper-case script for levels. Our definition of the percentage change in variable X is therefore written as

$$x = 100 \left(\frac{\Delta X}{X}\right)$$

In deriving the percentage-change equations from the nonlinear equations, we use three rules:

the product rule, $X = \beta YZ \Rightarrow x = y + z$, where β is a constant, the power rule, $X = \beta Y^{\alpha} \Rightarrow x = \alpha y$, where α and β are constants, and the sum rule, $X = Y + Z \Rightarrow Xx = Yy + Zz$.

As mentioned above, the MMRF results are reported as percentage deviations in the model's variables from an initial solution. With reference to the above equations, the percentage changes x, y and z represent deviations from their levels values X, Y and Z. The levels values (X, Y and Z) are solutions to the models underlying levels equations. Using the product-rule equation as an example, values of 100 for X, 10 for Y and 5 for Z represent an initial solution for a value of 2 for β . Now assume that we perturb our initial solution by increasing the values of X and Y by 3 per cent and 2 per cent respectively, i.e., we set y and z at 3 and 2. The linear representation of the product-rule equation would give a value of y of 5, with the interpretation that the initial value of X has increased by 5 per cent for a 3 per cent increase in Y and a 2 per cent increase in Z. Values of 5 for x, 3 for y and 2 for z in the corresponding percentage change equation means that the levels value of X has been perturbed from 100 to 105, Y from 10 to 10.3 and Z from 5 to 5.1.

In the above example, the reader will notice that while satisfying the percentage-change equations, updating the levels values of X, Y and Z by their percentage changes does not satisfy the levels form of the product-rule equation i.e., $105 \neq 2 \times 10.3 \times 5.1$. Given the percentage changes to Y and Z, the solution to the nonlinear equation is X = 105.06, giving a linearisation error of 0.06 (i.e., 0.06 = 105.06 - 105). GEMPACK eliminates the linearisation error by the application of a multistep procedure which exploits a positive relationship between the size of the perturbation from the initial solution and the size of the linearisation error. The principle of the Euler version of the multistep solution method can be illustrated using our above example. Instead of increasing the values of Y and Z by 3 per cent and 2 per cent, let us break the perturbation into two steps, first increasing x and y by half the desired amount, i.e., 1.5 per cent and 1.0 per cent respectively. Solving the linear equation gives a value for x of 2.5 per cent. Updating the value of X by 2.5 per cent gives an intermediate value of X of 102.5 [i.e., $100 \times (1 + 2.5/100)$]. Now apply the remainder of our desired perturbation to Y and Z. The percentage increase in y is 1.4778 per

cent (i.e., $100 \times 0.15/10.15^5$) and the percentage change in z is 0.9901 per cent (i.e., $100 \times 0.05/5.05$), giving a value for x (in our second step) of 2.4679 per cent. Updating our intermediate value of X by 2.4679 per cent, gives a final value of X of 105.045, which is close to the solution of the nonlinear equation of 105.06. We can further improve the accuracy of our solution by implementing more steps and by applying an extrapolation procedure.

In the percentage-change form of the power-rule equation, a constant α appears as a coefficient. In the percentage-change form of the sum-rule equation, the levels values of the variables appear as coefficients. By dividing by X, this last equation can be rewritten so that x is a share-weighted average of y and z. There are two main types of coefficients in the linear equation system of MMRF: (i) price elasticities and (ii) shares of levels values of variables. Two price elasticity coefficients appear in MMRF: elasticities of substitution and own-price elasticities.⁶ In the MMRF equation system, elasticities are identified by the prefix ELAST. Equations with share coefficients are typically written in the form of the sum-rule equation above. Coefficients associated with shares are levels values and therefore are written in upper-case script.

The percentage-change equation system of MMRF is given in Table 2.1. The equations of Table 2.1 are presented in standard algebraic syntax. Each equation has an identifier beginning with the prefix E_{-} . Using the equation identifiers, the reader can cross reference the equations in Table 2.1 with the equations of the annotated TABLO file in the Appendix to Chapter 3. In Table 2.1, below the identifier in brackets, the section in which the equation appears in the annotated TABLO file is listed. The annotated TABLO file of Chapter 3 is a reproduction of the computer implementation of MMRF. The model's variables are listed in Table 2.2. Descriptions of the model's coefficients appear in Table 2.3, and Table 2.4 describes the sets used in the model.

The remainder of this Chapter is devoted to the exposition of the MMRF equation system beginning, in section 2.2, with the equations of the CGE core.

⁵ Note that in our first step we have also updated the values of Y and Z, e.g., after the first step, our updated value of Y is $10.15 = 10 \times 1.5/100$.

⁶ For example, if, in the power-rule equation, X is quantity demanded and Y is the price of X, α could be interpreted as a (constant) own-price elasticity of demand.

2.2. The CGE core⁷

The CGE core is based on ORANI, a single-region model of Australia (Dixon, Parmenter, Sutton and Vincent 1982). Each regional economy in MMRF looks like an ORANI model. However, unlike the single-region ORANI model, MMRF includes interregional linkages. The transformation of ORANI into the CGE core of MMRF, in principle, follows the steps by which the stylised single-region model of Chapter 1 was transformed into the stylised multiregional model.

A schematic representation of the CGE core

Figure 2.1 is a schematic representation of the CGE core's input-output database. It reveals the basic structure of the CGE core. The columns identify the following agents:

(1) domestic producers divided into J industries in Q regions;

(2) investors divided into J industries in Q regions;

(3) a single representative household for each of the Q regions;

(4) an aggregate foreign purchaser of exports;

(5) an other demand category corresponding to Q regional governments; and

(6) an other demand category corresponding to Federal government demands in the Q regions.

The rows show the structure of the purchases made by each of the agents identified in the columns. Each of the I commodity types identified in the model can be obtained within the region, form other regions or imported from overseas. The source-specific commodities are used by industries as inputs to current production and capital formation, are consumed by households and governments and are exported. Only domestically produced goods appear in the export column. R of the domestically produced goods are used as margin services (domestic trade and transport & communication) which are required to transfer commodities from their sources to their users. Commodity taxes are payable on the purchases. As well as intermediate inputs, current production requires inputs of three categories of primary factors: labour (divided into M occupations), fixed capital and agricultural land. The other costs category covers various miscellaneous industry expenses. Each cell in the input-output table contains the name of the corresponding matrix of the values (in some base year) of flows of commodities, indirect taxes or primary factors to a group of users. For example, MAR2 is a 5-dimensional array showing the cost of the R margins services on the flows of I goods, both domestically and imported (S), to I investors in Q regions.

Figure 2.1 is suggestive of the theoretical structure required of the CGE core. The theoretical structure includes: demand equations are required for our

⁷ Section 2.2. draws on Horridge, Parmenter and Pearson (1993).

			Al	BSORPTION	MATRIX		
		1	2	3	4	5	6
		Producers	Investors	Household	Export	Regional Govt.	Federal Govt.
	Size	$\leftarrow J \times Q \rightarrow$	$\leftarrow J \times Q \rightarrow$	← Q →	← 1 →	$\leftarrow Q \rightarrow$	$\leftarrow Q \rightarrow$
Basic Flows	↑ I×S ↓	BAS1	BAS2	BAS3	BAS4	BAS5	BAS6
Margins Taxes	↑ I×S×R ↓	MARI	MAR2	MAR3	MAR4	MAR5	MAR6
	↑ I×S ↓	TAX1	TAX2	TAX3	TAX4	TAX5	TAX6
Labour	î↑ M ↓	LABR	I = Number of Commodities J = Number of Industries M = Number of Occupation Types R = Number of Commodities used as Margins S = 9: 8 × Domestic Regions plus 1 × Foreign Import				
Capital	↑ 1 ↓	CPTL					
Land	↑ 1 ↓	LAND					
Other Costs	↑ 1 ↓	OCTS					

Figure 2.1. The CGE core input-output database

six users; equations determining commodity and factor prices; market clearing equations; definitions of commodity tax rates. In common with ORANI, the equations of MMRF's CGE core can be grouped according to the following classification:

- producer's demands for produced inputs and primary factors;
- *demands for inputs to capital creation;*
- household demands;
- export demands;

- government demands;
- demands for margins;
- zero pure profits in production and distribution;
- market-clearing conditions for commodities and primary factors; and
- indirect taxes;
- Regional and national macroeconomic variables and price indices.

Naming system for variables of the CGE core

In addition to the notational conventions described above in section 2.1, the following conventions are followed (as far as possible) in naming variables of the CGE core. Names consist of a prefix, a main user number and a source dimension. The prefixes are:

 $a \Leftrightarrow$ technological change/change in preferences;

del \Leftrightarrow ordinary (rather than percentage) change;

 $f \Leftrightarrow \text{shift variable};$

nat \Leftrightarrow a national aggregate of the corresponding regional variable;

 $p \Leftrightarrow prices;$

 $x \Leftrightarrow$ quantity demanded;

- $xi \Leftrightarrow price deflator;$
- $y \Leftrightarrow investment;$
- $z \Leftrightarrow$ quantity supplied.

The main user numbers are:

- $1 \Leftrightarrow$ firms, current production;
- $2 \Leftrightarrow$ firms, capital creation;
- $3 \Leftrightarrow households;$
- $4 \Leftrightarrow$ foreign exports;
- $5 \Leftrightarrow$ regional government;
- $6 \Leftrightarrow$ Federal government

The number 0 is also used to denote basic prices and values. The source dimensions are:

 $a \Leftrightarrow all$ sources, i.e., 8 regional sources and foreign;

 $r \Leftrightarrow$ regional sources only;

 $t \Leftrightarrow$ two sources, i.e., a domestic composite source and foreign;

 $c \Leftrightarrow$ domestic composite source only;

 $o \Leftrightarrow domestic/foreign composite source only.$

The following are examples of the above notational conventions:

p1a ⇔ price of commodities (p), from all nine sources (a) to be used by firms in current production (1);

 $x2c \Leftrightarrow$ demand for domestic composite (c) commodities (x) to be used by firms for capital creation.

Variable names may also include an (optional) suffix descriptor. These are: cap \Leftrightarrow capital;

imp ⇔ imports; lab ⇔ labour; land ⇔ agricultural land; lux ⇔ linear expenditure system (supernumerary part); marg ⇔ margins; oct ⇔ other cost tickets; prim ⇔ all primary factors (land, labour or capital); sub ⇔ linear expenditure system (subsistence part); Sections 2.2.1 to 2.2.14 outline the structure of the CGE core.

2.2.1. Production: demand for inputs to the production process

MMRF recognises two broad categories of inputs: intermediate inputs and primary factors. Firms in each regional sector are assumed to choose the mix of inputs which minimises the costs of production for their level of output. They are constrained in their choice of inputs by a three-level nested production technology (Figure 2.2). At the first level, the intermediate-input bundles and the primary-factor bundles are used in fixed proportions to output. These bundles are formed at the second level. Intermediate input bundles are constant-elasticity-of-substitution (CES) combinations of international imported goods and domestic goods. The primary-factor bundle is a CES combination of labour, capital and land. At the third level, inputs of domestic goods are formed as CES combinations of goods from each of the eight regions, and the input of labour is formed as a CES combination of inputs of labour from eight different occupational categories. We describe the derivation of the input demand functions working upwards from the bottom of the tree in Figure 2.2. We begin with the intermediate-input branch.

Demands for intermediate inputs

At the bottom of the nest, industry j in region q chooses intermediate input type i from domestic region s ($X_{i,s,i,d}$) to minimise the costs

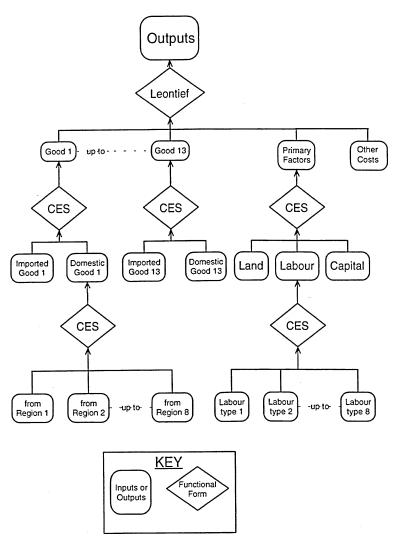
$$\sum_{s=1}^{8} P1A_{i,s,j,q} X1A_{i,s,j,q}, \qquad i,j=1,...,13 \quad q=1,...,8, \quad (2.1)$$

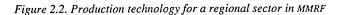
of a composite domestic bundle

$$X1C_{iig} = CES(X1A_{i1ig},...,X1A_{i8ig}),$$
 $i,j=1,...,13$ q=1,...,8, (2.2)

where the composite domestic bundle $(X1C_{i,j,q})$ is exogenous at this level of the nest. The notation CES() represents a CES function defined over the set of variables enclosed in the brackets. The CES specification means that inputs of the same commodity type produced in different regions are not perfect substitutes for one another. This is an application of the so-called Armington

Theoretical structure of MONASH-MRF





(1969 1970) specification typically imposed on the use of domestically produced commodities and foreign-imported commodities in national CGE models such as ORANI.

By solving the above problem, we generate the industries' demand equations for domestically produced intermediate inputs to production.⁸ The percentage-change form of these demand equations is given by equations E x1a1 and E p1c. The interpretation of equation E x1a1 is as follows: the commodity demand from each regional source is proportional to demand for the composite X1C_{i,i,q} and to a price term. The percentage-change form of the price term is an elasticity of substitution, $\sigma 1C_i$, multiplied by the percentage change in a price ratio representing the price from the regional source relative to the cost of the regional composite, i.e., an average price of the commodity across all regional sources. Lowering of a source-specific price, relative to the average, induces substitution in favour of that source. The percentage change in the average price, plc_{i,i,a}, is given by equation E_plc. In E_plc, the coefficient S1A_{is,ig} is the cost share in of the ith commodity from the sth regional source in the jth industry from region q's total cost of the ith commodity from all regional sources. Hence, p1c_{i,j,q} is a cost-weighted Divisia index of individual prices from the regional sources.

At the next level of the production nest, firms decide on their demands for the domestic-composite commodities and the foreign imported commodities following a pattern similar to the previous nest. Here, the firm chooses a costminimising mix of the domestic-composite commodity and the foreign imported commodity

 $P1A_{i,foreign,i,q}X1A_{i,foreign,j,q} + P1C_{i,i,q}X1C_{i,i,q}, i,j=1,...,13$ q=1,...,8, (2.3)

where the subscript 'foreign' refers to the foreign import, subject to the production function

 $X1O_{i,j,q} = CES(X1A_{i,foreign,j,q},...,X1C_{i,j,q}),$ i,j=1,...,13 q=1,...,8. (2.4)

As with the problem of choosing the domestic-composite, the Armington assumption is imposed on the domestic-composite and the foreign import by the CES specification in equation 2.4.

The solution to the problem specified by equations 2.3 and 2.4 yields the input demand functions for the domestic-composite and the foreign import represented in their percentage-change form by equations E_x1c , E_x1a2 , and E_p1o . The first two equations show, respectively, that the demands for the domestic-composite commodity (X1C_{i,j,q}) and for the foreign import

 $^{^8}$ For details on the solution of input demands given a CES production function, and the linearisation of the resulting levels equation, see Dixon, Bowles and Kendrick (1980), DPSV (1982), DPPW (1992) and Horridge, Parmenter and Pearson (1993).

 $(X1A_{i,foreign,j,q})$ are proportional to demand for the domestic-composite/foreignimport aggregate $(X1O_{i,j,q})$ and to a price term. The $X1O_{i,j,q}$ are exogenous to the producer's problem at this level of the nest. Common with the previous nest, the change form of the price term is an elasticity of substitution, $\sigma 1O_i$, multiplied by a price ratio representing the change in the price of the domesticcomposite (the $p1c_{i,j,q}$ in equation E_x1c) or of the foreign import (the $p1a_{i,foreign,j,q}$ in equation E_x1a) relative to price of the domesticcomposite/foreign-import aggregate (the $p1o_{i,j,q}$ in equations E_x1c and E_x1a). The percentage change in the price of the domesticcomposite/foreign-import aggregate, defined in equation E_p1o is again a Divisia index of the individual prices. We now turn our attention to the primary-factor branch of the input-demand tree of Figure 2.2.

Demands for primary factors

At the lowest-level nest in the primary-factor branch of the production tree in Figure 2.2, producers choose a composition of eight occupation-specific labour inputs to minimise the costs of a given composite labour aggregate input. The demand equations for labour of the various occupation types are derived from the following optimisation problem for the jth industry in the qth region.

Choose inputs of occupation-specific labour type m, $X1LABOI_{j,q,m}$, to minimise total labour cost

$$\sum_{m=1}^{8} P1LABOI_{j,q,m} X1LABOI_{j,q,m}, \qquad j=1,...,13, q=1, ,8, (2.5)$$

subject to,

$$EFFLAB_{in} = CES(X1LABOI_{i,n,m}), \quad j=1,...,13 \quad q,m=1,...,8, \quad (2.4)$$

regarding as exogenous to the problem the price paid by the jth regional industry for the each occupation-specific labour type (P1LABOI_{j,q,m}) and the regional industries' demand for the effective labour input (EFFLAB_{j,q}).

The solution to this problem, in percentage-change form, is given by equations E_x1 laboi and E_p1 lab. Equation E_x1 laboi indicates that the demand for labour type m is proportional to the demand for the effective composite labour demand and to a price term. The price term consists of an elasticity of substitution, $\sigma 1LAB_{j,q}$, multiplied by the percentage change in a price ratio representing the wage of occupation m (p1laboi_{j,q,m}) relative to the average wage for labour in industry j of region q (p1lab_{j,q}). Changes in the relative wages of the occupations induce substitution in favour of relatively cheapening occupations. The percentage change in the average wage is given by equation E_p1 lab where the coefficients $S1LABOI_{j,q,m}$ are value shares of occupation m in the total wage bill of industry j in region q. Thus, p1lab_{j,q} is a Divisia index of the p1laboi_{i,a,m}. Summing the percentage changes in occupation-specific labour demands across occupations, using the $S1LABOI_{j,q,m}$ shares, for each industry gives the percentage change in industry labour demand (labind_{j,q}) in equation E_labind.

At the next level of the primary-factor branch of the production nest, we determine the composition of demand for primary factors. Their derivation follows the same CES pattern as the previous nests. Here, total primary factor costs

$$\begin{array}{l} P1LAB_{j,q}EFFLAB_{j,q} + P1CAP_{j,q}CURCAP_{j,q} + P1LAND_{j,q}N_{j,q} \\ j = 1,...,13, \ q = 1,...8, \end{array}$$

where $P1CAP_{j,q}$ and $P1LAND_{j,q}$ are the unit costs of capital and agricultural land and $CURCAP_{j,q}$ and $N_{j,q}$ are industry's demands for capital and agricultural land, are minimised subject to the production function

$$X1PRIM_{j,q} = CES\left(\frac{EFFLAB_{j,q}}{A1LAB_{j,q}}, \frac{CURCAP_{j,q}}{A1CAP_{j,q}}, \frac{N_{j,q}}{A1LAND_{j,q}}\right)$$

j = 1,...,13, q = 1,...8,

where $X1PRIM_{j,q}$ is the industry's overall demand for primary factors. The above production function allows us to impose factor-specific technological change via the variables $A1LAB_{i,q}$, $A1CAP_{i,q}$ and $A1LAND_{i,q}$.

The solution to the problem, in percentage-change form is given by equations E_efflab, E_curcap, E_n and E_xi_fac. From these equations, we see that for a given level of technical change, industries' factor demands are proportional to overall factor demand (X1PRIM_{jq}) and a relative price term. In change form, the price term is an elasticity of substitution (σ 1FAC_{jq}) multiplied by the percentage change in a price ratio representing the unit cost of the factor relative to the overall effective cost of primary factor inputs to the jth industry in region q. Changes in the relative prices of the primary factors induce substitution in favour of relatively cheapening factors. The percentage change in the average effective cost (xi_fac_{jq}), given by equation E_xi_fac, is again a cost-weighted Divisia index of individual prices and technical changes.

Demands for primary-factor and commodity composites

We have now arrived at the topmost input-demand nest of Figure 2.2. Commodity composites, the primary-factor composite and 'other costs' are combined using a Leontief production function given by

$$Z_{j,q} = \frac{1}{A1_{j,q}} \times MIN\left(X1O_{i,j,q}, \frac{X1PRIM_{j,q}}{A1PRIM_{j,q}}, \frac{X1OCT_{j,q}}{A1OCT_{j,q}}\right)$$

i,j = 1,...,13, q = 1,...,8.

In the above production function, $Z_{j,q}$ is the output of the jth industry in region q, the $A1_{j,q}$ are Hicks-neutral technical change terms, $X10CT_{j,q}$ are the

industries' demands for 'other cost tickets'⁹ and A1OCT_{j,q} which are the industry-specific technological change associated with other cost tickets.

As a consequence of the Leontief specification of the production function, each of the three categories of inputs identified at the top level of the nest are demanded in direct proportion to $Z_{j,q}$ as indicated in equations E_x1_0 , $E_x1_prim and E_x1_oct$.

2.2.2. Demands for investment goods

Capital creators for each regional sector combine inputs to form units of capital. In choosing these inputs they cost minimise subject to technologies similar to that in Figure 2.2. Figure 2.3 shows the nesting structure for the production of new units of fixed capital. Capital is assumed to be produced with inputs of domestically produced and imported commodities. No primary factors are used directly as inputs to capital formation. The use of primary factors in capital creation is recognised through inputs of the construction commodity (service).

The model's investment equations are derived from the solutions to the investor's three-part cost-minimisation problem. At the bottom level, the total cost of domestic-commodity composites of good i $(X2C_{i,j,q})$ is minimised subject to the CES production function

$$X2C_{i,i,q} = CES(X2A_{i,1,i,q},...,X2A_{i,8,i,q})$$

 $i,j = 1,...,13$ $q = 1,...,8$,

where the $XAC_{i,1,j,q}$ are the demands by the jth industry in the qth region for the ith commodity from the sth domestic region for use in the creation of capital.

Similarly, at the second level of the nest, the total cost of the domestic/foreign-import composite $(X2O_{i,j,q})$ is minimised subject to the CES production function

$$X2O_{i,j,q} = CES(X2A_{i,foreign,j,q},...,X2C_{i,j,q}),$$

 $i,j = 1,...,13 \ q = 1,...,8,$

where the X2A_{i,foreign,i,q} are demands for the foreign imports.

The equations describing the demand for the source-specific inputs $(E_x2a1, E_x2a2, E_x2c, E_p2c \text{ and } E_p2o)$ are similar to the corresponding equations describing the demand for intermediate inputs to current production (i.e., E_x1a1, E_x1c, E_p1c and E_p2o).

At the top level of the nest, the total cost of commodity composites is minimised subject to the Leontief function

⁹ Demand for other cost tickets includes demand for working capital and production taxes.

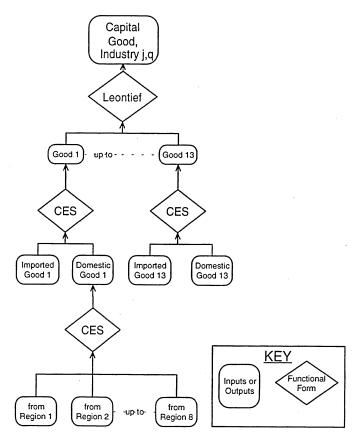


Figure 2.3. Structure of investment demand

$$Y_{j,q} = MIN\left(\frac{X2O_{i,j,q}}{A2IND_{i,j,q}}\right) \qquad i,j = 1,...,13, \ q = 1,...,8.$$
(2.5)

where the total amount of investment in each industry $(Y_{j,q})$ is exogenous to the cost-minimisation problem and the A2IND_{i,j,q} are technological-change variables in the use of inputs in capital creation. The resulting demand equations for the composite inputs to capital creation (E_x2o) correspond to the demand equations for the composite input to current production (i.e., E_x1o).

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Determination of the number of units of capital to be formed for each regional industry (i.e., determination of $Y_{j,q}$) depends on the nature of the experiment being undertaken. For comparative-static experiments, a distinction is drawn between the short run and long run. In short-run experiments (where the year of interest is one or two years after the shock to the economy), capital stocks in regional industries and national aggregate investment are exogenously determined. Aggregate investment is distributed between the regional industries of relative rates of return.

In long-run comparative-static experiments (where the year of interest is five or more years after the shock), it is assumed that the aggregate capital stock adjusts to preserve an exogenously determined economy-wide rate of return, and that the allocation of capital across regional industries adjusts to satisfy exogenously specified relationships between relative rates of return and relative capital growth. Industries' demands for investment goods is determined by exogenously specified investment/capital ratios.

MMRF can also be used to perform forecasting experiments. Here, regional industry demand for investment is determined by an assumption on the rate of growth of industry capital stock and an accumulation relation linking capital stock and investment between the forecast year and the year immediately following the forecast year.

Details of the determination of investment and capital are provided in section 2.4.1 below.

2.2.3. Household demands

Each regional household determines the optimal composition of its consumption bundle by choosing commodities to maximise a Stone-Geary utility function subject to a household budget constraint. A *Keynesian* consumption function determines regional household expenditure as a function of household disposable income.

Figure 2.4 reveals that the structure of household demand follows nearly the same nesting pattern as that of investment demand. The only difference is that commodity composites are aggregated by a Stone-Geary, rather than a Leontief function, leading to the linear expenditure system (LES).

The equations for the two lower nests (E_x3a1 , E_x3a2 , E_x3c , E_p3c and E_p3o) are similar to the corresponding equations for intermediate and investment demands.

The equations determining the commodity composition of household

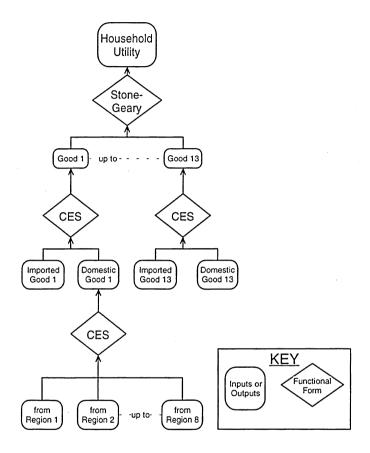


Figure 2.4. Structure of household demand

demand, which is determined by the Stone-Geary nest of the structure, differ form the CES pattern established in sections 2.2.1 and 2.2.2.¹⁰ To analyse the Stone-Geary utility function, it is helpful to divide total consumption of each commodity composite $(X3O_{i,q})$ into two components: a subsistence (or minimum) part $(X3SUB_{i,q})$ and a luxury (or supernumerary) part $(X3LUX_{i,q})$

¹⁰ For details on the derivation of demands in the LES, see Dixon, Bowles and Kendrick (1980) and Horridge, Parmenter and Pearson (1993).

$$X_{3O_{i,q}} = X_{3SUB_{i,q}} + X_{3LUX_{i,q}}, \qquad i = 1,...,13, q = 1,...,8.$$
 (2.6)

A feature of the Stone-Geary function is that only the luxury components effect per-household utility (UTILITY), which has the Cobb-Douglas form

UTILITY_q =
$$\frac{1}{\text{QHOUS}_q} \sum_{i=1}^{13} X3LUX_{i,q}^{A3LUX_{i,q}} q = 1,...,8,$$
 (2.7)

where

$$\sum_{i=1}^{13} A3LUX_{i,q} = 1 \qquad q = 1,...,8.$$

Because the Cobb-Douglas form gives rise to exogenous budget shares for spending on luxuries

$$P3O_{i,q}X3LUX_{i,q} = A3LUX_{i,q}LUXEXP_q$$
 $i = 1,...,13$ $q = 1,...,8$, (2.8)

A3LUX_{i,q} may be interpreted as the marginal budget share of total spending on luxuries (LUXEXP_q). Rearranging equation (2.8), substituting into equation (2.6) and linearising gives equation E_x30, where the subsistence component is proportional to the number of households and to a taste-change variable (a3sub_{i,q}), and ALPHA_I_{i,q} is the share of supernumerary expenditure on commodity i in total expenditure on commodity i. Equation E_utility is the percentage-change form of the utility function (2.7).

Equations E_a3sub and E_a3lux provide default settings for the tastechange variables ($a3sub_{i,q}$ and $a3lux_{i,q}$), which allow for the average budget shares to be shocked, via the $a3com_{i,q}$, in a way that preserves the pattern of expenditure elasticities.

The equations just described determine the composition of regional household demands, but do not determine total regional consumption. As mentioned, total household consumption is determined by regional household disposable income. The determination of regional household disposable income and regional total household consumption is described in section 2.xx.

2.2.4. Foreign export demands

To model export demands, commodities in MMRF are divided into two groups: the traditional exports, agriculture and mining, which comprise the bulk of exports; and the remaining, non-traditional exports. Exports account for relatively large shares in total sales of agriculture and mining, but for relatively small shares in total sales for non-traditional-export commodities.

The traditional-export commodities (X4R_{i,s}, i \in agricult,mining) are modelled as facing downwardly-sloping foreign-export demand functions

$$X4R_{i,s} = FEQ_{i} \left(\frac{P4R_{i,s}}{FEP_{i}NATFEP} \right)^{EXP_{e}ELAST_{i}} \qquad i = 1, 2, \ s = 1,...,13, \ (2.9)$$

where EXP_ELAST_i is the (constant) own-price elasticity of foreign-export demand. As EXP_ELAST_i is negative, equation (2.9) says that traditional exports are a negative function of their prices on world markets (P4R_{i,s}). The variables FEQ_i and FEP_i allow for horizontal (quantity) and vertical (price) shifts in the demand schedules. The variable NATFEP allows for an economywide vertical shift in the demand schedules. The percentage-change form of (2.9) is given by E_x4r .

In MMRF the commodity composition of aggregate non-traditional exports is exogenised by treating non-traditional exports as a Leontief aggregate (equation E_nt_x4r). Total demand is related to the average price via a constant-elasticity demand curve, similar to those for traditional exports (see equations E_aggnt_x4r and E_aggnt_p4r).

2.2.5. Government consumption demands

Equations E_x5a and E_x6a determine State government and Federal government demands (respectively) for commodities for current consumption. State government consumption can be modelled to preserve a constant ratio with State private consumption expenditure by exogenising the 'f5' variables in equation E_x5a . Likewise, Federal government consumption expenditure can be set to preserve a constant ratio with national private consumption expenditure by exogenising the 'f6' variables in equation E_x5a .

2.2.6. Demands for margins

Equations E_x1 marg, E_x2 marg, E_x3 marg, E_x4 marg, E_x5 marg and E_x6 marg give the demands, of our six users, for margins. Two margin commodities are recognised in MMRF: transport & communication and finance. These commodities, in addition to being consumed directly by the users (e.g., consumption of transport when taking holidays or commuting to work), are also consumed to facilitate trade (e.g., the use of transport to ship commodities from point of production to point of consumption). The latter type of demand for transport & communication and finance are the so-called demands for margins. The margin demand equations in MMRF indicate that the demands for margins are proportional to with the commodity flows with which the margins are associated.

2.2.7. Prices

As is typical of ORANI-style models, the price system underlying MMRF is based on two assumptions: (i) that there are no pure profits in the production or distribution of commodities, and (ii) that the price received by the producer is uniform across all customers. Also in the tradition of ORANI is presence of two types of price equations: (i) zero pure profits in current production, capital creation and importing and (ii) zero pure profits in the distribution of commodities to users. Zero pure profits in current production, capital creation and importing is imposed by setting unit prices received by producers of commodities (i.e., the commodities' basic values) equal to unit costs. Zero pure profits in the distribution of commodities is imposed by setting the prices paid by users equal to the commodities' basic value plus commodity taxes and the cost of margins.

Zero pure profits in production, capital creation and importing

Equations E_p0a and E_a impose the zero pure profits condition in current production. Given the constant returns to scale which characterise the model's production technology, equation E_p0a defines the percentage change in the price received per unit of output by industry j of region q ($p0a_{j,q}$) as a cost-weighted average of the percentage changes in effective input prices. The percentage changes in the effective input prices represent (i) the percentage change in the cost per unit of input and (ii) the percentage change in the use of the input per unit of output (i.e., the percentage change in the technology variable). These cost-share-weighted averages define percentage changes in average costs. Setting output prices equal to average costs imposes the competitive zero pure profits condition.

Equation E_pi imposes zero pure profits in capital creation. E_pi determines the percentage change in the price of new units of capital (pi $_{j,q}$) as the percentage change in the effective average cost of producing the unit.

Zero pure profits in imports of foreign-produced commodities is imposed by Equation E_p0ab. The price received by the importer for the ith commodity (P0A_{i,foreign}) is given as product of the foreign price of the import (PM_i), the exchange rate (NATPHI) and one plus the rate of tariff (the so-called power of the tariff: POWTAXM_i)¹¹.

Zero pure profits in distribution

The remaining zero-pure-profits equations relate the price paid by purchasers to the producer's price, the cost of margins and commodity taxes. Six users are recognised in MMRF and zero pure profits in the distribution of commodities to the users is imposed by the equations E_p1a , E_p2a , E_p3a , E_p4r , E_p5a and E_p6a .

2.2.8. Market-clearing equations for commodities

Equations E_mkt_clear_margins, E_mkt_clear_nonmarg and E_x0impa impose the condition that demand equals supply for domesticallyproduced margin and nonmargin commodities and imported commodities

¹¹ If the tariff rate is 20 percent, the power of tariff is 1.20. If the tariff rate is increased from 20 percent to 25 percent, the percentage change in the power of the tariff is 4, i.e., 100*(1.25-1.20)/1.20 = 4.

respectively. The output of regional industries producing margin commodities, must equal the direct demands by the model's six users and their demands for the commodity as a margin. Note that the specification of equation E_mkt_clear_margins imposes the assumption that margins are produced in the destination region, with the exception that margins on exports and commodities sold to the Federal government are produced in the source region.

The outputs of the nonmargin regional industries are equal to the direct demands of the model's six users. Import supplies are equal to the demands of the users excluding foreigners, i.e., all exports involve some domestic value added.

2.2.9. Indirect taxes

Equations E_deltax1 to E_deltax6 contain the default rules for setting sales-tax rates for producers (E_deltax1), investors (E_deltax2), households (E_deltax3), exports (E_deltax4), and government (E_deltax5 and E_deltax6). Sales taxes are treated as ad valorem on the price received by the producer, with the sales-tax variables (deltax(i), i=1,...,6) being the ordinary change in the percentage tax rate, i.e., the percentage-point change in the tax rate. Thus a value of deltax1 of 20 means the percentage tax rate on commodities used as inputs to current production increased from, say, 20 percent to 40 percent, or from, say, 24 to 44 percent.

For each user, the sales-tax equations allow for variations in tax rates across commodities, their sources and their destinations.

Equations E_taxrev1 to E_taxrev6 compute the percentage changes in regional aggregate revenue raised from indirect taxes. The bases for the regional sales taxes are the regional basic values of the corresponding commodity flows. Hence, for any component of sales tax, we can express revenue (TAXREV), in levels, as the product of the base (BAS) and the tax rate (T), i.e.,

TAXREV =
$$BAS \times T$$
.

Hence,

$$\Delta TAXREV = T\Delta BAS + BAS\Delta T$$
(2.10)

The basic value of the commodity can be written as the product the producer's price (P0) and the output (XA)

$$BAS = P0 \times XA. \tag{2.11}$$

Using equation (2.11), we can derive the form of equations $E_{taxrev1}$ to $E_{taxrev6}$ by taking the percentage change of the first two terms in (2.10) and the ordinary change in the last term of (2.10) multiplied by 100

 $TAXREV \times taxrev = TAX \times (p0 + xa) + BAS \times deltax$

where

$$taxrev = 100 \left(\frac{\Delta TAXREV}{TAXREV} \right)$$

 $TAX = BAS \times T$

$$p0 = 100 \left(\frac{\Delta P0}{P0}\right)$$

$$xa = 100 \left(\frac{\Delta XA}{XA}\right)$$

and

deltax = $100 \times \Delta T$

2.2.10. Regional incomes and expenditures

In this section, we outline the derivation of the income and expenditure components of regional gross product. We begin with the nominal income components.

Income-side aggregates of regional gross product

The income-side components of regional gross product include regional totals of factor payments, other costs and the total yield from commodity taxes. Nominal regional factor income payments are given in equations E_caprev, E_labrev and E_Indrev for payments to capital, labour and agricultural land, respectively. The regional nominal payments to other costs are given in equation E_octrev.

The derivation of the factor income and other cost regional aggregates are straightforward. Equation E_caprev, for example is derived as follows. The total value of payments to capital in region q (AGGCAP_q) is the sum of the payments of the j industries in region q (CAPITAL_{j,q}), where the industry payments are a product of the unit rental value of capital (P1CAP_{j,q}) and the number of units of capital employed (CURCAP_{i,q})

AGGCAP_q =
$$\sum_{j=1}^{13} P1CAP_{j,q}CURCAP_{j,q}$$
, q = 1,...,8. (2.12)

Equation (2.12) can be written in percentage changes as

caprev_q=
$$(1.0/AGGCAP_q)\sum_{j=1}^{13}CAPITAL_{j,q}(p1cap_{j,q} + curcap_{j,q}),$$

q = 1,...,8,

giving equation E_caprev, where the variable $caprev_q$ is the percentage change in rentals to capital in region q and has the definition,

$$caprev_{q} = 100 \left(\frac{\Delta AGGCAP_{q}}{AGGCAP_{q}} \right) \qquad q = 1,...8.$$

The regional tax-revenue aggregates are given by equations E_taxind, E_taxrev6 and E_taxrevm. E_taxrev6 has been discussed in section 2.2.9. E_taxind determines the variable taxind_q, which is the weighted average of the percentage change in the commodity-tax revenue raised from the purchases of producers, investors, households, foreign exports and the regional government. Equation E_taxrevm determines tariff revenue on imports absorbed in region q (taxrem_q). Equation E_taxrevm is similar in form to equations E_taxrev1 to E_taxrev6 discussed in section 2.2.9. However, the tax-rate term in equation E_taxrevm, powtaxm_q, refers to the percentage change in the power of the tariff (see footnote 2) rather than the percentage-point change in the tax rate (as is the tax-rate term in the commodity-tax equations of section 2.2.9).

Expenditure-side aggregates of regional gross product

For each region, MMRF contains equations determining aggregate expenditure by households, investors, regional government, the Federal government and the interregional and foreign trade balances. For each expenditure component (with the exception of the interregional trade flows), we define a quantity index and a price index and a nominal value of the aggregate. For interregional exports and imports, we define an aggregate price index and quantity index only.

As with the income-side components, each expenditure-side component is a definition. As with all definitions within the model, the defined variable and its associated equation could be deleted without affecting the rest of the model. The exception is regional household consumption expenditure (see equations E_c_a , E_cr and E_xi3). It may seem that the variable c_q is determined by the equation E_c_a . This is not the case. Nominal household consumption is determined either by a consumption function (see equation E_c_b in section xxx) or, say, by a constraint on the regional trade balance. Equation E_c_a therefore plays the role of a budget constraint on household expenditure.

The equations defining the remaining aggregate regional real expenditures, nominal expenditures and related price indices are listed below in the order: investment, regional government, Federal government, interregional exports, interregional imports, international exports and international imports. The equations defining quantities are E_ir, E_othreal5, E_othreal6, E_int_exp, E_int_imp, E_expvol and E_impvol. The equations describing price indices are E_xi2, E_xi5, E_xi6, E_psexp, E_psimp, E_xi4 and E_xim. The definitions of nominal values are given by equations E_in, E_othnom5, E_othnom6, E_export and E_imp (remembering that the model does not include explicit equations or variables defining aggregate nominal interregional trade flows).

The derivation of the quantity and price aggregates for the interregional trade flows involves an intermediate step represented by equations E_trd and E_psflo . These equations determine inter- and intra- regional nominal trade flows in basic values.¹² To determine the interregional trade flows, say for interregional exports in E_int_exp , the intraregional trade flow (the second term on the RHS of E_int_exp) is deducted from the total of inter- and intra- regional trade flows (the first term on the RHS of E_int_exp).

2.2.11. Regional wages

The equations in this section have been designed to provide flexibility in the setting of regional wages. Equation E_p laboi separates the percentage change in the wage paid by industry (p1laboi_{j,q,m}) into the percentage change in the wage received by the worker (pwagei_{j,q}) and the percentage change in the power of the payroll tax (arpi_{j,q}). Equation E_p wagei allows for the indexing of the workers' wages to the national consumer price index (natxi3, see section xxx). The 'fwage' variables in E_p wagei allow for deviations in the growth of wages relative to the growth of the national consumer price index.

Equation E_wage_diff allows flexibility in setting movements in regional wage differentials. The percentage change in the wage differential in region q (wag_diff_q) is defined as the difference the aggregate regional real wage received by workers (pwage_q - natxi3) and the aggregate real wage received by workers across all regions (natrealwage). Equation E_pwage defines the percentage change in the aggregate regional nominal wage (pwage_q) as the average (weighted across industries) of the pwagei_{j,q} and equation E natrealw defines the percentage change in the variable natrealwage.

2.2.12. Other regional factor-market definitions

The equations in this section define aggregate regional quantities and prices in the labour and capital markets.

Equations E_l, E_kt and E_z_tot define aggregate regional employment, capital use and value added respectively. E_lambda defines regional employment of each of the eight occupational skill groups.

¹² The determination in basic values reflects the convention in MMRF that all margins and commodity taxes are paid in the region which absorbs the commodity.

The remaining equations of this section define aggregate regional prices of labour and capital.

2.2.13. Other miscellaneous regional equations

Equation E_ploct allows for the indexation of the unit price of other costs to be indexed to the national consumer price index. The variable $floct_{jq}$ can be interpreted as the percentage change in the real price of other costs to industry j in region q.

Equation E_cr_shr allows for the indexing of regional real household consumption with national real household consumption in the case where the percentage change in the regional-to-national consumption variable, cr_shr_q is exogenous and set to zero. Otherwise, cr_shr_q is endogenous and regional consumption is determined elsewhere in the model (say, by the regional consumption function).

Equation E_ximp0 defines the regional duty-paid imports price index. Equation E_totdom and E_totfor define, for each region, the interregional and international terms of trade respectively.

2.2.14. National aggregates

The final set of equations in the CGE core of MMRF define economywide variables as aggregates of regional variables. As MMRF is a bottoms-up regional model, all behavioural relationships are specified at the regional level. Hence, national variables are simply add-ups of their regional counterparts.

2.3. Government finances

In this block of equations, we determine the budget deficit of regional and federal governments, aggregate regional household consumption and Gross State Products (GSP). To compute the government deficits, we prepare a summary of financial transactions (SOFT) which contain government income from various sources and expenditure on different accounts. To determine each region's aggregate household consumption, we compute regional household disposable income and define a regional consumption function. The value added in each region is determined within the CGE core of the model. Within the government finance block, are equations which split the regions' value added between private and public income. In this disaggregation process, the GSPs are also computed from the income and expenditure sides. The government finance equations are presented in five groups:

(i) value added disaggregation;

(ii) gross regional product;

(iii) miscellaneous equations.

(iv) summary of financial transactions;

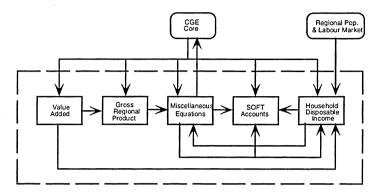
(v) household income.

Figure 2.5 illustrates the interlinkages between the five government finance equation blocks and their links with the CGE core and regional population and labour market equation block of MMRF. The activity variables and commodity taxes are determined in the CGE core.

From Figure 2.5, we see that all the equation blocks of government finances have backward links to the CGE core. The disaggregation-of-valueadded block takes expenditures by firms on factors of production from the CGE core and disaggregates them into gross returns to factors and production taxes. Regional value added is used in the determination of the income side of gross regional product, hence the link from the value-added block to the grossregional-product block in Figure 2.5. Production tax revenue also appears as a source of government income in the SOFT accounts block, which explains the link between value-added and SOFT in Figure 2.5. Factor incomes help explain household disposable income and this is recognised by the link between the value-added block and the household-disposable-income block in Figure 2.5.

The miscellaneous block in Figure 2.5 contains intermediary equations between the gross-regional-product block and the SOFT accounts, and between the household-disposable-income block and the CGE core. There are two types of equations in the miscellaneous block: (i) aggregating equations that form national macroeconomic aggregates of the expenditure and income sides of GDP by summing the corresponding regional macroeconomic variables determined in the gross-regional-product block and; (ii) mapping equations that rename variables computed in the gross-regional-product block and the CGE core for use in the SOFT accounts.

The SOFT accounts compute the regional and Federal governments' budgets. On the income side are government tax revenues, grants from the Federal government to the regional governments, interest payments and other miscellaneous revenues. Direct taxes and commodity taxes come from the gross-regional-product block via the miscellaneous block as described above. Production taxes come from the value-added block. On the expenditure side of the government budgets are purchases of goods and services, transfer payments, interest payments on debt, and for the Federal government, payments of grants to the regional governments. The purchases of goods and services come from the CGE core via the miscellaneous block. Transfer payments come from the household-disposable-income block.



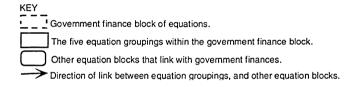


Figure 2.5. Government finance block of equations

Finally, we come to the household-disposable-income block. Within this block, household disposable income is determined as the difference between the sum of factor incomes and transfer payments, and income taxes. Figure 2.5 shows that factor incomes come from the value-added block and that unemployment benefits are determined using information from the regional population and labour market block. Household disposable income feeds back to the miscellaneous block, which contains an equation specifying the level of regional household consumption as a function of regional household disposable income. The value of regional household consumption, in turn, feeds back to the CGE core. Also, the value of transfer payments, determined in the household-disposable-income block, feedback to the SOFT accounts.

A notation for the government finance block

Following the style of the CGE core, all variable names are in lower case. However, the coefficient naming system is different. A variable name in upper case with a prefix of "C_" defines the coefficient associated with the variable. For example, the variable 'hhldy000' represents the percentage change in household disposable income. Its associated coefficient is 'C_HHLDY00' which represents the level of household disposable income.

We have given specific notation to variables of each of the blocks which form the government finance set of equations. The following are the major array names for variables and coefficients:

(i)	z_**_r	(a value-added component in gross regional production).
(ii)	dompy***	(domestic regional production, an income component);
(iii)	dompq***	(domestic regional production, an expenditure component);
(iv)	softy***	(summary of financial transaction, an income component);
(v)	softq***	(summary of financial transaction, an expenditure component) and;
(vi)	hhldy***	(a household income component).

In each array name '***' represents three or two digits component numbers.

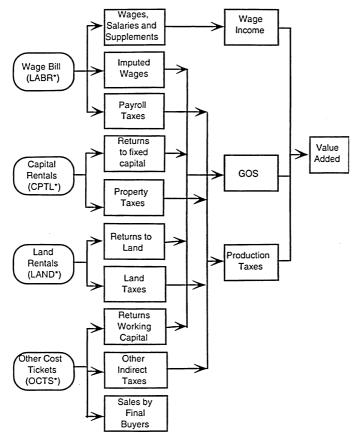
In the following sections we discuss the various blocks which constitute the government finance equations of MMRF beginning with the disaggregation of value added.

2.3.1. Disaggregation of value added

Figure 2.6 shows that the essential purpose of the disaggregation-ofvalue-added block is to disaggregate the four elements of value added determined in CGE core (i.e., the wage bill, the rental cost of capital and land, and other costs) into ten components in order to separate production taxes from payments to factors. In addition, the block prepares national values as aggregates of regional values. We now turn to the details of the value-added equation block as presented in section 2.3 of Table 2.1.

In equations E_z01_r and E_z02_r , we assume that the wages, salaries and supplements and the imputed wage bill vary in direction proportion to the pre-tax wage bill, which is determined in the CGE core. Equation E_z03_r shows that payroll taxes are determined by the pre-tax wage bill and the payroll tax rate. Equations E_z04_r and E_z05_r show that the return to fixed capital and property taxes are assumed to vary in proportion to the rental cost of capital, which is determined in the CGE core. Returns to agricultural land and land taxes are determined by equations E_z06_r and E_z07_r respectively and vary in proportion to the total rental cost of land. Returns to working capital, other indirect taxes and sales by final buyers are all assumed to vary in proportion to other costs. The relevant equations are E_z08_r , E_z09_r and E_z010_r respectively.

The values of the ten national components of value added are the sums of the corresponding regional components. The national values are calculated in the 'E $z0^{**}$ ' equations from which the 'r' suffix has been omitted.



KEY

* The name in parenthesis indicates the corresponding array name in Figure 2.1.

C Elements from the CGE Core.

Elements in the disaggregation-of-value-added block.

Figure 2.6. Components of regional value added

Equation E_{zg_r} defines regional gross operating surplus (GOS) as the sum of imputed wages and returns to fixed capital, working capital and agricultural land. Equation E_{zt_r} calculates total regional production tax revenues.

Equation E_rpr sets the payroll tax rates by industry and region which are determined by the payroll tax rate for all industries in a region and a shift in the industry and region specific payroll tax rate. A change in the payroll tax rate drives a wedge between the wage rate received by the workers and the cost to the producer of employing labour. The change in the cost of employing labour for a given change in the payroll tax rate depends on the share of the payroll taxes in total wages. Equation E_rpri adjusts the payroll tax rate to compute the wedge between the wage rate and labour employment costs. The wedge is used to define the after-tax wage rate in the CGE core.

The last equation in the value-added block, E_xisfb2, defines a regional price index for sales by final buyers.

2.3.2. Gross regional domestic product and its components

This block of equations defines the regional gross products from the income and expenditure sides using variables from the CGE core and the valueadded block.

Figure 2.7 reveals that gross regional product at market prices from the income side is sum of wage income, non-wage income and indirect tax revenues.

In section 2.3 of Table 2.1, equations $E_dompy100$, $E_dompy200$ and $E_dompy330$ show, respectively, that wage income, non-wage income and production taxes are mapped from the value-added block. In addition to production taxes, there are two other categories of indirect tax: tariffs and other commodity taxes less subsidies. Equations $E_dompy330$ and $E_dompy320$ show that these taxes are mapped from the CGE core. Total indirect taxes are defined by $E_dompy300$ as the sum of the three categories of indirect taxes.

Summing wage and non-wage income, and indirect taxes gives gross regional product from the income side (equation E_dompy000).

In addition to the defining the income-side of gross regional product, we also disaggregate factor incomes into disposable income and income tax. Disposable income is used in the household-disposable-income block and income taxes are a source of government revenue in the SOFT accounts. In equation E_dompy110, disposable wage income is defined as wage income net of PAYE taxes. In equation E_dompy120, PAYE taxes are assumed to be proportional to wage income and the PAYE tax rate. Likewise, equation E_dompy210 defines disposable non-wage income as the difference between non-wage income and income taxes. Equation E_dompy220 sets non-wage income tax proportional to non-wage income and the non-wage income tax rate.

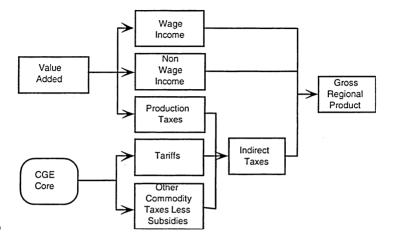


Figure 2.7. Income-side components of gross regional product

Figure 2.8 shows that gross regional product from the expenditure side is defined as the sum of domestic absorption and the interregional and international trade balances. This definition is reflected in equation E_dompq000. Domestic absorption is defined in equation E_dompq100 as the sum of private and public consumption and investment expenditures. Equations E_dompq110 to E_dompq150 reveal that the components of domestic absorption are mapped from the CGE core. Within the components of domestic absorption, the assumption is made, in equations E dompg120 and E_dompq150, that the shares of private and government investment in total investment are fixed. Equations E_dompq210 and E_dompq220 show that regional exports and imports are also taken from the CGE core. The difference between regional exports and imports forms the regional trade balance and is calculated in equation E_dompq200. Similarly the international trade variables for each region are taken from the CGE core (E dompg310 and E dompg320 for international exports and imports respectively) and are used to define the international trade balance for each region in equation E_dompq300.

2.3.3. Miscellaneous equations

The miscellaneous equation block appears in section 2.3.3 of Table 2.1. In equation E_tir, commodity taxes are mapped from the gross-regionalproduct block. The regional values are then summed in equation E_ti to form the national aggregate of commodity taxes. Similarly, gross regional products are mapped from the gross-regional-product block in equation E yn r and

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Theoretical structure of MONASH-MRF

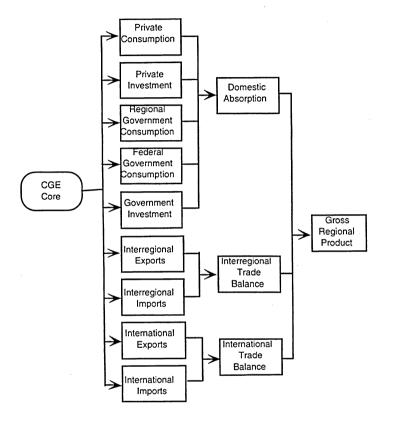


Figure 2.8 Expenditure-side components of gross regional product

then summed, in equation E_yn , to form national nominal GDP. In equation E_xiy_r , a gross-regional-product price deflator from the expenditure side, is formed using price indices taken from the CGE core. The resulting price deflator is used to define real gross regional product in equation E_yr_r .

Taking a weighted average of the gross regional product price deflators gives the price deflator for national GDP in equation E_xiy . This deflator is used to define real national GDP in equation E_yr .

National GDP at factor cost is defined in equation E_yf as the sum of the regional wage and non-wage incomes. Equation E_bstar defines the percentage-point change in the national balance of trade surplus to national GDP ratio. Note that E_bstar defines a percentage-point change in a ratio, rather than a percentage change. The underlying levels equation of E_bstar is

$$BSTAR = \frac{\sum_{q=1}^{8} DOMPQ300_{q}}{YN},$$

where the upper-case represents the levels values of the corresponding percentage-point change and percentage change variables. Taking the first difference of BSTAR and multiplying by 100 gives the percentage-point change in BSTAR (i.e., the variable bstar on the LHS of E_bstar)

$$100 \times \Delta BSTAR = bstar = \frac{\sum_{q=1}^{8} DOMPQ300_{q} dompq300_{q}}{YN} - \frac{NATB}{YN} yn,$$

where

NATB =
$$\sum_{q=1}^{8} DOMPQ300_q$$
.

Aggregate national income taxes are calculated in equation E_ty. They are the sum of regional PAYE taxes and regional taxes on non-wage income. Pre-tax national wage income is calculated in equation E_yl by summing pretax regional wage incomes. Equation E_wn defines the nominal pre-tax national wage rate as the ratio of the pre-tax wage income to national employment. The nominal post-tax national wage income is calculated in equation E_ylstar by summing the nominal post-tax regional wage incomes. The nominal post-tax national wage rate is defined in equation E_wnstar as the ratio of the nominal post-tax wage income to national employment. The real post-tax national wage rate is defined in E_wrstar by deflating the nominal post-tax national wage rate by the national CPI.

Equations E_g_rA and E_g_rB define the regional government and Federal government consumption expenditures respectively. The vector variable determined in these equations, g_r , drives government consumption expenditures in the SOFT accounts (see section 2.3.4 below).

Equations E_{ip} to $E_{ig_r_fed}$ disaggregate investment expenditure into private investment expenditure and public investment expenditure. The resulting values for public investment expenditure are used to drive government capital expenditures in the SOFT accounts (see section 2.3.4 below). Equation E_{ip} imposes the assumption that aggregate private investment expenditure moves proportionally with a weighted average of total (private and public) regional investment. Aggregate public investment expenditure is then determined as a residual in equation E_{ig} , as the difference between total aggregate total investment and aggregate private investment. Equation E_{ig_reg} imposes the assumption that investment expenditure of regional governments moves in proportion to total regional investment. Equation $E_{ig_r}fed$ determines investment expenditure of the Federal government as the difference between total public investment and the sum of regional governments' investment.

The miscellaneous equation block is completed with equation E_c_b defining the regional household consumption function and equation E_rl relating the PAYE tax rate to the tax rate on non-wage income.

2.3.4. Summary Of Financial Transactions of the regional and Federal governments: the SOFT accounts.

In this block of equations, we prepare a statement of financial transactions containing various sources of government income and expenditure. A separate statement is prepared for each regional government and the Federal government. Our accounting system is based on the *State Finance Statistics* (cat. no. 5512.0) of the Australian Bureau of Statistics (ABS).

The SOFT accounts contain equations explaining movements in the income and expenditure sides of the governments' budgets. Our exposition of this equation block starts with the income side of the accounts. Figure 2.9 depicts the income side of the SOFT accounts. There are two major categories of government income: (i) revenues and (ii) financing transactions. Government revenues are further divided into direct and indirect tax revenues, interest payments, Commonwealth grants (for regional governments) and other revenues. The categories of direct taxes are income taxes (for the Federal government), and other direct taxes. Indirect taxes consist of tariffs (for the Federal government), other commodity taxes and production taxes. Commonwealth grants are divided into grants to finance consumption expenditure and grants used to finance capital expenditure. Financing transactions capture the change in the governments' net liabilities and represent the difference between government revenue and government expenditure. If government expenditure exceeds/is less than government revenue (i.e., the government budget is in deficit/surplus), then financing transactions increase/decrease as either the governments' net borrowings increase and/or other financing transactions increase. Variations in the latter item principally consist of changes in cash and bank balances.¹³ We now turn our attention to

¹³ The reader will note that financing transactions includes a third item, increase in provisions. This is a very small item in the governments' SOFT accounts and in model simulations we hold its value fixed at zero change.

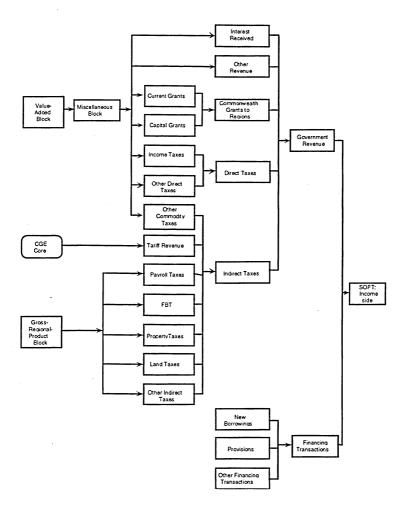


Figure 2.9. Summary of Financial Transaction (SOFT): income side

the specification of movements in the income-side components of the SOFT accounts as presented in section 2.3.4 of Table 2.1.

Equation E_softy111 shows that Federal government income tax collections are mapped from the miscellaneous block based on the corresponding variable in the gross-regional-product block. Equations E_softy112A and E_softy112B impose the assumption that regional government and Federal government collections of 'other direct taxes' move in proportion to nominal gross regional product and nominal national GDP respectively. The sum of the two above categories of direct taxes for each government is given in equation E_softy110.

Federal government tariff revenue is mapped from the CGE core to equation E_softy121. Equation E_softy122A shows that regional government collections of 'other commodity taxes' are mapped from the miscellaneous block based on the corresponding variable in the gross-regional-product block. Federal government collections of 'other commodity taxes' are calculated in equation E_softy122B as the difference between total national collection of 'other commodity taxes' and the sum of the regional governments' collections of 'other commodity taxes'. The value of the total national collection of 'other commodity taxes' is mapped from the miscellaneous block based on the corresponding variable in the gross-regional-product block.

Equation E_softy123a shows that the regional governments' collections of payroll taxes are mapped from the value-added block. In equation E_softy123b, the assumption is imposed that the Federal government's collection of fringe benefits moves in proportion with the value-added-block variable 'other indirect taxes'. Equations E_124, E_125 and E_126A show that regional governments' collections of, respectively, property taxes, land taxes and other indirect taxes, are mapped from their corresponding values in the value-added block. Equation E_softy126B ties movements in Federal governments 'other indirect taxes' to the weighted average of movements in regional governments' other indirect taxes'.

Equation E_softy120 calculates movements in the collection of aggregate indirect taxes in each region as the weighted average of movements in the regional collection of tariffs, other commodity taxes and the production taxes.

Interest received by governments¹⁴ is determined in equation $E_softy130$. We assume that interest received and interest paid by the government move in proportion with the size of the economy as measured by

¹⁴ This item includes gross interest received on bank balances, investment and advances of the government (see ABS cat. no. 1217.0)

nominal gross regional product for regional governments, and nominal national GDP for the Federal government.¹⁵

The next item of government revenue applies only to regional governments and is Commonwealth grants. Equations E_softy141 and softy142 recognise Commonwealth grants for current expenditure purposes and capital expenditure purposes respectively. For each type of grant, our default assumption is that their nominal values are proportional to the nominal value of the region's gross product. Equations E_softy141 and E_softy142 include shift variables which allow the default option to be overridden.

The final item of government revenue, 'other revenue', is described in equation E_softy150 (for regional governments) and equation E_softy150B (for the Federal government). As with Commonwealth grants, the default option is that 'other revenue' of regional governments is proportional to nominal gross regional product and for the Federal government it is proportional to national nominal GDP. Also in common with the 'grants' equations is the presence of shifters which allow the default option to be overridden.

Equation E_softy100 sums the various components of government revenue to determine total government revenue.

Equation E_softy300 identifies the budget deficit or surplus as the difference between government expenditure and government revenue. As mentioned above, financing transactions consists of three components. Equation E_softy320 allows a default option which sets the 'increase in provisions' component of financing transactions proportional to government expenditure on goods and services. A shift variable in equation E_softy320 allows the default option to be overridden. The second component of financing transactions, 'other financing transactions', is determined by equation E_softy300. The default setting is that 'other financing transactions' move in proportion with 'financing transactions'. The presence of a shift variable allows the default setting to be overridden. Government's net borrowing is determined by equation E_softy300 as the difference between 'financing transactions' and the sum of the two components, 'increase in provisions' and 'other financing transactions'.

Figure 2.10 depicts the expenditure-side of the SOFT accounts. Figure 2.10 shows that regional government expenditure consists of five broad categories: goods and services, personal benefits, subsidies, interest payments and other outlays. Figure 2.10 also shows that expenditure by the Federal government includes the same five categories, in common with the regional

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¹⁵ We would prefer to relate the change in government interest receipts and payments to changes in the level and composition of government debt. At this stage of the model's development, we lack a specification of the mechanism by which the governments may target and achieve a particular debt outcome. Our current specification of interest payments is consequently *ad hoc*.

Theoretical structure of MONASH-MRF

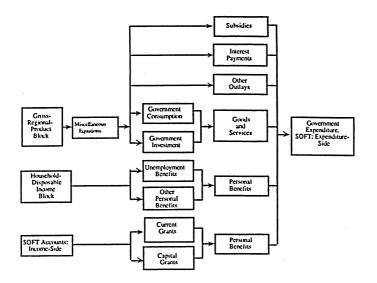


Figure 2.10. Summary of Financial Transaction (SOFT): expenditure side

governments, plus grants to the regional governments. The composition of the expenditure side of the SOFT accounts is reflected in equation E_softq000 which gives the total, for each governments' expenditures.

Government expenditure on goods and services consists of government consumption expenditure and investment expenditure (equation E_softq100). Government consumption expenditure (equation E_softq110) is mapped from the miscellaneous block based on the corresponding nominal government consumption expenditure variable in the CGE core. Likewise, government investment expenditure (equation E_softq120) is mapped from the miscellaneous block based on the corresponding nominal government investment expenditure variable in the CGE core.

Governments' expenditures on personal benefits (equation $E_softq200$) are mapped from the household-disposable-income block (see section 2.3.5 below). Personal benefit payments are divided into 'unemployment benefits' and 'other personal benefit payments'. Unemployment benefits (equation $E_softq210$) are mapped from the household-disposable-income block. This leaves 'other personal benefit payments' (equation $E_softq220$) to be determined residually as the difference between personal benefit payments and unemployment benefits.

MONASH-MRF: A multiregional model of Australia

Equations E_softq300A and E_softq300B determine payments of subsidies by regional governments and the Federal government respectively. These equations impose the assumption that the ratio of the government payment of subsidies to commodity tax revenues is constant.

Equations E_softq400A and E_softq400A define nominal interest payments by regional governments and the Federal government respectively. As described above, we assume that interest paid by the government moves in proportion with nominal gross regional product for regional governments, and nominal national GDP for the Federal government.

Expenditure by the Federal government on grants to the regions (equation $E_softq500$) consists of grants for the purposes of consumption expenditure (equation $E_softq510$) and capital expenditure (equation $E_softq520$).

The final item in the governments' expenditure-side of the SOFT accounts is 'other outlays' (equation $E_softq600$). The default setting is that 'other outlays' is proportional to total government expenditure.

There are four remaining equations in the SOFT accounts block. Three of these define various measures of the budget deficit. The remaining equation is a mapping equation for a variable used in the household-disposable-income block.

Equation E_realdefr defines the percentage change in the real budget deficit of regional governments. It is defined as the difference in the percentage change in financing transactions (a measure of the percentage change in the nominal budget deficit) and the percentage change in the regional CPI. Equation E_realdeff defines the corresponding variable for the Federal government. This equation is analogous to the regional version, where the regional variables are replaced by Federal and national variables. Equation E_dgstar calculates the percentage-point change in the ratio of government net borrowing to total outlays. The final equation, E_tod_r maps the 'other indirect tax' variable (softy112) to the variable name 'tod_r'.

2.3.5. Household disposable income

This block of equations computes the various components of regional household disposable income. Figure 2.11 outlines the composition of household disposable income. Regional household disposable income consists of four broad components: primary factor income, personal benefit payments from the governments, 'other income', and direct taxes. Equation E_hhldy000 defines regional household disposable income based on these four components.

Figure 2.11 also shows the disaggregation and sources of the components of household disposable income. Primary factor income is disaggregated into wage income and non-wage income (equation E_hhldy100). Movements in wage income and non-wage income are determined in the value-added block (equation E_hhldy110 and E_hhldy120 respectively). Personal

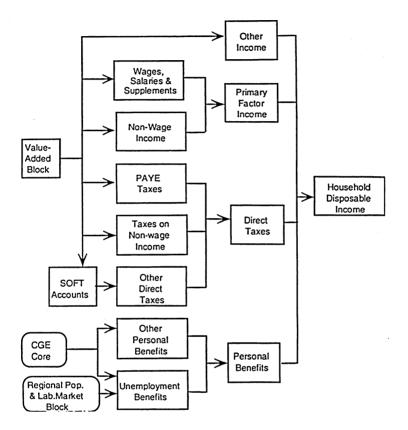


Figure 2.11. Household disposable income

benefit payments are consist of unemployment benefits and other benefit payments (equation E_hhldy200). Equation E_hhldy210 shows that unemployment benefits are indexed to the national CPI and that changes in the numbers unemployed are taken from variables determined in the regional population and labour market equation block. Other personal benefit payments are determined in equation E_hhldy220. They are indexed to the national CPI and proportional to the regional population. Equation E_hhldy300 imposes the assumption that regional household 'other net income' moves in proportion with the region's nominal gross product. In Figure 2.11, three components of direct taxes are identified: PAYE taxes, taxes on non-wage income and other direct taxes (equation E_hhldy400). Equations E_hhldy410 and E_hhldy420 show that PAYE taxes and non-wage taxes are assumed to move in proportion to wage income and non-wage income respectively. Equation E_hhldy430 shows that other direct taxes move in line with the corresponding variable taken from the SOFT accounts.

There are five remaining equations in this block. The first is a mapping equation, E_ydr, that maps regional household disposable income to a variable to be used in the consumption-function equation described above in the miscellancous-equation block (see section 2.3.3). The next equation, E_upb, aggregates regional unemployment benefits to form total unemployment benefits. This variable drives Federal government expenditure on unemployment benefits in the SOFT accounts in section 2.3.4 above. The third equation, E_pbp_r is a mapping equation that renames the regional 'personal benefits payments' for use in the SOFT accounts, where it determines movements in regional governments' payments of personal benefits. The fourth equation, E_pbpA, calculates aggregate personal benefit payments by summing the regional payments. The final equation, E_pbpB, uses the value of the aggregate payments of personal benefits determined in E_pbpA and the regional payments determined in E_pbp_r, to determine Federal government payments of personal benefits in the SOFT accounts.

2.4. Dynamics for forecasting

This block of equations facilitates medium-run and long-run forecasting experiments, and the movement between comparative-static and forecasting versions of the model. The equations link key flow variables with their associated stock variables. The dynamics of MMRF are confined to accumulation relations connecting industry capital stock with industry investment, regional population with regional natural growth in population and foreign and interregional migration, and the foreign debt with the trade balance.

Also included in this block are the comparative-static alternatives to the forecasting equations. In some cases, such as investment and capital, the comparative-static and forecasting versions of the model contain different equations. In other cases, such as the trade balance and the foreign debt, we move between comparative-static and forecasting versions by different settings of exogenous variables within a common set of equations.

Comparative statics and forecasting¹⁶

MMRF can produce either comparative-static or forecasting simulations. In comparative-static simulations, the model's equations and variables all refer implicitly to the economy at some future time period.

This interpretation is illustrated in Figure 2.12, which graphs the values of some variable, say employment, against time. The base period (period 0) level of employment is at A and B is the level which it would attain in T years time if some policy, say a tariff change, were *not* implemented. With the tariff change, employment would reach C, all other things being equal. In a comparative-static simulation, MMRF might generate the percentage change in employment 100(C-B)/B, showing how employment *in period* T would be affected by the tariff change alone.

Employment

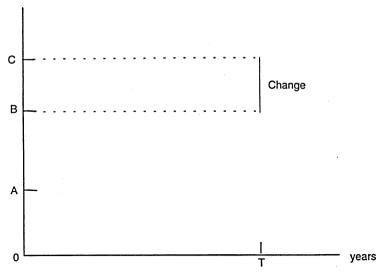


Figure 2.12. Comparative-static interpretation of results

Comparative-static simulations are usually interpreted as measuring either the short-run or long-run effects of a policy change. A distinguishing feature of short run versus long run comparative-static simulations is the treatment of industry capital. Short run simulations are characterised by the

¹⁶ This section draws on Horridge, Parmenter and Pearson (1993).

assumption of fixed industry capital stocks. That is, industry capital stocks are held at their pre-shock level. Econometric evidence suggests that a short-run equilibrium will be reached in about two years, i.e., T=2 (see Cooper, McLaren and Powell, 1985). The typical long-run assumption is that industry capital stocks will have adjusted to restore (exogenous) rates of return. This might take six years, 10 years or 20 years, i.e., T=6, 10 or 20. In either case, short run or ling run, only the assumptions about which variables are fixed and the interpretation of the results bear on the timing of changes: the model itself is atemporal.

The comparative-static interpretation of MMRF results lends itself to policy analysis. Business and government planners, however, require forecasts of industry output, prices and other variables to inform their investment decisions. The forecasting interpretation of MMRF results is shown in Figure 2.13. As before, the model generates percentage changes in its variables but in this case they are interpreted as 100(D-A)/A, comparing the values of the variables at two different time periods, period 0 and period T. In contrast to comparative-static simulations, which usually show the effect of one or a few exogenous changes, forecasting simulations normally show the effects of all exogenous changes assumed to occur over the simulation period 0 to T.

In the remainder of this section, we outline the forecasting equations of MMRF and, where applicable, their comparative-static alternatives. We begin with capital and investment.

2.4.1. Industry capital and investment

To derive the investment-capital accumulation equations, we start with the accumulation relation

$$K_{j,q,t+1} = K_{j,q,t}DEP_j + Y_{j,q,t}, \quad j=1,...,13, q=1,...,8, t=0,...,T,$$
 (2.13)

where $K_{j,q,t}$ is industry j's, in the qth region, capital stock in year t, $Y_{j,q,t}$ is industry j's, in the qth region, investment in year t and DEP_j is one minus the rate of depreciation in industry j. Note that we assume that there is no regionspecific rate of depreciation on capital. Equation 2.13 is therefore a standard investment-capital accumulation relation with a one year gestation lag between investment and capital.

On the basis of equation 2.13, we can write the accumulation equation for our forecast year, year T, as

Before we derive the linearised expression of equation 2.14, we must first address the issue of an initial solution for the equation. In common with other ORANI-style models, MMRF is solved for deviations from an initial solution. In forecasting simulations, we solve the model for perturbations from

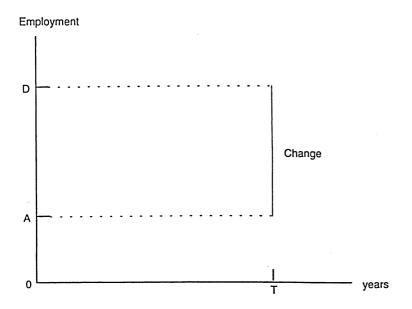


Figure 2.13. Forecasting interpretation of results

year 0 in Figure 2.13. That is, we compute the values of variables at year T, given initial values of the model's variables at year 0. For example, we solve for $K_{j,q,T+1}$, $K_{j,q,T}$ and $Y_{j,q,T}$ for initial values, $K_{j,q,0}$ and $Y_{j,q,0}$. The initial values of the model's variables must represent a solution to the model's equations. Therefore, to solve MMRF, we require a database which represents an initial solution to the model's equations. For the equations of the CGE core, the input-output data, schematically represented in Figure 2.1, provide an initial solution. Similarly, the database for the government-finance block provides an initial solution for the equations of section 2.3. A feature of the equations of sections 2.2 and 2.3 is that all their variables relate to a single time period. Hence, the database providing a solution to those equations need only contain data for a single year.

A difficulty with equation 2.14 is that it includes variables in more than one time period. This is a problem because no longer do our basecase values (which are from a single period) guarantee an initial solution. Typically, our initial solution for a variable, say X_T , is X_0 . However, unless by coincidence

$$K_{j,q,0}(DEP_j - 1) + Y_{j,q,0} = 0, \qquad j=1,...,13, q=1,...,8$$
 (2.15)

 $\{K_{j,q,0}, K_{j,q,0}, Y_{j,q,0}\}$ is not a solution for $\{K_{j,q,T+1}, K_{j,q,T}, Y_{j,q,T}\}$. We solve this problem by the purely technical device of augmenting equation 2.14 with an additional term as follows:

$$K_{j,q,T+1} = K_{j,q,T} DEP_j + Y_{j,q,T} - F[K_{j,q,0}(1 - DEP_j) - Y_{j,q,0}],$$

 $j=1,...,13, q=1,...,8$ (2.16)

where the initial value of F is minus unity so that equation 2.14 is satisfied when $K_{j,q,T+1}$ and $K_{j,q,T}$ equal $K_{j,q,0}$ and $Y_{j,q,T}$ equals $Y_{j,q,0}$. In forecasting simulations, we shock F to zero ($\Delta F=1$). In equation 2.16, { $K_{j,q,0}, K_{j,q,0}, Y_{j,q,0}, -1$ } is now a true initial solution for { $K_{j,q,T+1}, K_{j,q,T}, Y_{j,q,T}, F$ }, and the forecasting solution for equation 2.16 is { $K_{j,q,T+1}^*, K_{j,q,T}^*, Y_{j,q,T}^*, 0$ }, where the 'asterisk' denotes a forecast solution.

Taking ordinary changes in F and percentage changes in $K_{j,q,T+1}$, $K_{j,q,T}$ and $Y_{j,q,T}$, equation 2.16 becomes

$$K_{j,q,T+1}k_{j,q,T+1} = K_{j,q,T} DEP_{j}k_{j,q,T} + Y_{j,q,T}y_{j,q,T} - 100\Delta F[K_{j,q,0}(1 - DEP_{j}) - Y_{j,q,0}],$$

$$j=1,...,13, q=1,...,8, (2.17)$$

where, according to our conventions, the upper case denotes levels values of coefficients, the lower case denotes percentage changes in the model's variables and the Δ signifies an ordinary change in a variable. Notice that the presence of the last term on the RHS of equation 2.17 means that $k_{j,q,T+1}$ is the percentage deviation of the capital stock in industry j, region q, from the initial (base-year) value of its capital stock, i.e., all variables, including the T+1 variables, are deviations from their values in the same base year.

Equation 2.17 is reported in Table 2.1 as equation E_yTA . The coefficient and variable names that map equation 2.17 to equation E_yTA are as follows:

$$\begin{array}{l} K_{j,q,T+1} \Leftrightarrow VALK_{-}Tl_{j,q}; \\ K_{j,q,T} \Leftrightarrow VALK_{-}Tj_{j,q}; \\ DEP_{j} \Leftrightarrow DEP_{j}; \\ Y_{j,q,T} \Leftrightarrow INVEST_{j,q}; \\ K_{j,q,0} \Leftrightarrow VALK_{-}0j_{j,q}; \\ Y_{j,q,0} \Leftrightarrow INVEST_{-}0j_{j,q}; \\ k_{j,q,T+1} \Leftrightarrow curcap_{-}tl_{j,q}; \\ k_{j,q,T} \Leftrightarrow curcap_{j,q}; \\ y_{j,q,T} \Leftrightarrow y_{j,q} \text{ and}; \\ \Delta F \Leftrightarrow delkfudge. \end{array}$$

The next equation of this section, E_curcapTIA, sets the growth in the capital stock between the forecast year (year T) and the year following (year

T+1) equal to the average annual growth rate over the forecast period, i.e., the average between year 0 and year T. Using the notation of equations 2.14 to 2.17,

$$K_{j,q,T+1}/K_{j,q,T} = (K_{j,q,T}/K_{j,q,0})^{\frac{1}{T}},$$
 j=1,...,13, q=1,...,8. (2.18)

Remembering that $K_{j,q,0}$ is a constant and rearranging, the percentage change form of equation 2.18 is

$$k_{j,q,T+1} = (\frac{1}{T} - 1) k_{j,q,T},$$
 j=1,...,13, q=1,...,8, (2.19)

where the mapping given above for equations 2.17 and E_yTA also apply for equations 2.19 and E_curcapT1A, with the addition that $(\frac{1}{7}-1)$ in equation 2.19 is represented by the coefficient K_TERM in equation E_curcapT1A.

The comparative-static alternative to the above forecasting specification is given by swapping equation E_yTA for equation E_ytB and equation $E_curcapT1A$ for $E_curcapT1B$. We must remember that our interpretation of the model is different in comparative-static simulations compared to forecasting simulations. As Figures 2.11 and 2.12 suggest, in comparative-static simulations, the percentage changes in the variables are interpreted as deviations from their base values at time T, whereas in forecasting, the percentage changes in variables are deviations from their base values at time 0.

Equation E_yTB says the percentage change in an industry's capital stock in year T, is equal to the percentage change in an industry's investment in year T. Equation E_curcapT1B says the percentage change in an industry's capital stock in year T+1, is equal to the percentage change in an industry's capital stock in year T. In short-run comparative-static simulations, the typical assumption is that the industries' capital stocks are fixed at their base values. Hence, the variable curcap_{iq} is exogenous and set at zero change. In short-run comparative-static simulations, the standard closure (or choice of exogenous and endogenous variables) includes curcapia on the exogenous list and the relationship between industries' rates of return (r0ig) and the world interest rate (natr_tot), that is, the variable f_rate_xx_{jq} in equation E_f_rate_xx, on the endogenous list.¹⁷ With industries' capital stocks set at zero change, equation E_yTB ensures that the percentage change in industries' investment (y_{i,q}) are also zero, when the default option, of setting the shift variable (delf_rate_{i,q}) exogenously and at zero, is taken. The assumption is that the short-run is a time period over which, not only are capital stocks fixed, but also the industries' investment plans are fixed. The default option can be overridden by swapping

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¹⁷ For a full discussion of alternative model closures, see Chapter 4.

the industries' investment with the shift variable on the endogenous/exogenous list.

In long-run simulations, we assume that the industries' capital stocks are determined endogenously and the relationship between the industries rates of return and the world interest rate is exogenous, i.e., $curcap_{j,q}$ is endogenous and $f_{rate_xx_{j,q}}$ is exogenous. Equation E_curcapT1B ensures that the percentage increase in industries' capital stocks in year T+1 is the same as the percentage increase in capital stocks in year T.

The interpretation of the long-run specification is that sufficient time has elapsed such that the effects of a sustained shock to the economy no longer disturbs the *rate* of capital accumulation in our year of interest, year T. Note that the assumption is that the *rate* of capital accumulation, but not the *level* of the capital stock is assumed to be undisturbed in year T. Having made the assumption in equation E_curcapT1B that (using the above notation)

$$k_{jq,T+1} = k_{jq,T},$$
 j=1,...,13, q=1,...,8, (2.20)

then the percentage change form of equation 2.14 can be written as

$$K_{j,q,T+1}k_{j,q,T} = K_{j,q,T}DEP_{j}k_{j,q,T} + Y_{j,q,T}y_{j,q,T}, j=1,...,13, q=1,...,8.$$
 (2.21)

Rearranging equation 2.21 gives

$$(K_{j,q,T+1} - K_{j,q,T} DEP_j)k_{j,q,T} = Y_{j,q,T}y_{j,q,T}, \qquad j=1,...,13, q=1,...,8. (2.21)$$

From equation 2.14, we note that

$$K_{j,q,T+1} - K_{j,q,T} DEP_j = Y_{j,q,T},$$
 $j=1,...,13, q=1,...,8, (2.22)$

hence,

$$k_{j,q,T} = y_{j,q,T},$$
 $j=1,...,13, q=1,...,8, (2.23)$

or equation E_yTB in Table 2.1, section 2.4.1.

The investment and capital equations of the model are completed with the specification of industry rates of return and an equation defining aggregate economy-wide investment. These equations are common to both the forecasting and comparative-static versions of the model.

Equation E_r0 in Table 2.1, section 2.4.1, defines the percentage change in the rate of return on capital (net of depreciation) in industry j, region q. In levels this is the ratio of the rental price of capital (P1CAP_{j,q}) to the supply price (PI_{j,q}), minus the rate of depreciation. Hence, the coefficient QCOEF_{j,q} is the ratio of the gross to the net rate of return.

Equation E_f_rate_xx makes the change in the net rate of return in an industry relative to the economy-wide rate a positive function of the change in

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the industry's capital stock relative to the region-wide stock. It is interpreted as a risk-related relationship with relatively fast-/slow-growing industries requiring premia/accepting discounts on their rates of return. The parameter BETA_R_{j,q} specifies the strength of this relationship. The variable f_rate_xx_{j,q} allows exogenous shifts in the industry's rate of return and also allows us to move between long-run and short-run comparative-static simulations as described above.

The investment-capital equations conclude with equation E_naty which defines changes in the economy-wide industry investment as the weighted average of the changes in regional industry investment.

2.4.2. Accumulation of national foreign debt¹⁸

This section contains equations modelling the nation's foreign debt. They relate the debt to accumulated balance-of-trade deficits. Analogous to equation 2.13 above, we have

$$DEBT_{i+1} = DEBT_i(R_WORLD) + B_i, \qquad (2.24)$$

where $DEBT_t$ is the debt at year t, B_t is the trade deficit in year t, and R_WORLD is the interest rate factor; one plus the world real interest rate, which we treat as a parameter.

In the forecast simulations, we arrive at the value of the debt in year T by accumulating, on the basis of equation 2.24 starting at year 0, leading to

DEBT_T = DEBT₀(R_WORLD)^T +
$$\sum_{t=0}^{T-1} B_t (R_WORLD)^{T-t-1}$$
. (2.25)

We also assume that the trade deficit in the time span 0-T follows a straight line path

$$B_t = B_0 + \frac{t}{T} (B_T - B_0),$$
 $t=0,...,T.$ (2.26)

Thus

$$DEBT_{T} = DEBT_{0}(R_WORLD)^{T} + \sum_{t=0}^{T-1} [B_{0} + \frac{t}{T} (B_{T} - B_{0})](R_WORLD)^{T-t-1}.$$
(2.27)

Equation 2.27 can be written as

¹⁸ This section draws on Horridge, Parmenter and Pearson (1993).

 $DEBT_{T} - DEBT_{0} = DEBT_{0}(R_WORLD^{T}-1) + B_{0}N_DEBT + (B_{T}-B_{0})M_DEBT$ (2.28)

where

$$M_{DEBT} = \sum_{t=0}^{T-1} (R_{WORLD})^{T-t-1}$$
(2.29)

and

N_DEBT =
$$\sum_{t=1}^{T} (R_WORLD)^{T-t}$$
. (2.30)

Notice that $DEBT_T$ is linearly related to B_T , and to predetermined values of $DEBT_0$ and B_0 . We treat $DEBT_T$ and B_T as variables, and $DEBT_0$, B_0 and R_WORLD as parameters. As explained above, our initial solution for $\{DEBT_T, B_T\}$ is $\{DEBT_0, B_0\}$. Analogous to the capital accumulation relation in section 2.4.1, unless

$$DEBT_0(R_WORLD^T-1) + B_0N_DEBT = 0,$$

these values for DEBT_T and B_T will not satisfy equation 2.28. The resolution of the problem is to augment equation 2.28 with an additional variable DFUDGE as follows:

$$DEBT_{T} - DEBT_{0} = [DEBT_{0}(R_WORLD'-1) + B_{0}N_DEBT]DFUDGE + (B_{T}-B_{0})M_DEBT.$$
(2.31)

We choose the initial value of DFUDGE to be 0 so that equation 2.31 is satisfied when DEBT_T = DEBT₀ and B_T = B₀. In forecasting simulations, we shock DFUDGE to 1 (Δ DFUDGE=1). Then equation 2.31 is equivalent to equation 2.28, and our percentage-change results are consistent with equations 2.25 to 2.28 as desired.

Taking the ordinary change in equation 2.31, and writing the ordinary change in the variables in lower case with the prefix del, gives

$$deldebt_{T} = [DEBT_{0}(R_WORLD^{T}-1)+B_{0}N_DEBT]deldfudge+M_DEBTdelbt_{T}.$$
(2.32)

Equation 2.32 appears in Table 2.1, section 2.4.2 as equation E_deldebt.

Since R_WORLD is a (fixed) real foreign interest rate, DEBT, and B, are denominated in base-year foreign-currency units. However, they must be related to other variables, such as the values of exports and imports, which are

Theoretical structure of MONASH-MRF

measured in year-t Australian dollars (A). We define a coefficient, P_GLOBAL, to convert A values into base-year foreign dollars. It is given in percentage change form as natxim (see Table 2.1, section 2.2.14, equation E_natxim). This means that our 'best guess' at movements in world prices and the nominal exchange rate is given by the movements in the index of national foreign imports, which is the sum of the movements of a weighted average of the foreign prices of national imports plus the movement in the nominal exchange rate.

Equation E_delbt defines the ordinary change in the real trade deficit. The levels form of this equation is

$$BT = \frac{NATXIM \times NATIMPVOL - NATXI4 \times NATEXPVOL}{P_GLOBAL},$$
 (2.33)

where BT is the national real foreign trade balance (B_T in equations 2.27 to 2.31 above), NATXIM is the price index of national imports, NATIMPVOL is the volume of national foreign imports, NATXI4 is the price index of national exports, NATEXPVOL is the volume of national foreign exports. Taking the ordinary change in BT (delbt) in equation 2.33 and the percentage changes in the remaining variables (remembering the percentage change in P_GLOBAL is natxim) in equation 2.33 gives equation E_delbt.

The last equation of this section defines the ordinary change in the national debt/GDP ratio. In the levels, the debt/GDP ratio (DEBT_RATIO) is given as

$$DEBT_RATIO = DEBT/(NATGDPEXP/P_GLOBAL), \qquad (2.44)$$

where NATGDPEXP is national nominal GDP. Taking the ordinary change in DEBT_RATIO (deldebt_ratio) and the percentage changes in the remaining variables gives equation E_deldebt_ratio.

The above national debt accumulation equations produce comparativestatic results when the variable deldfudge in equation E_deldebt is set exogenously at zero change.

The final accumulation equation is one which relates regional population to various elements of regional population growth. We hold over the discussion of the accumulation of regional population to the next section.

2.5. Regional population and regional labour market settings

This block of equations computes regional population from natural growth, foreign migration and interregional migration. The block also includes various regional labour market relationships. For each region, the system is designed to allow for either: (i) an exogenous determination of regional population, with an endogenous determination of at least one variable of the regional labour market, chosen from regional unemployment, regional participation rates or regional wage relativities, or; (ii) an exogenous determination of all the previously mentioned variables of the regional labour market and an endogenous determination of regional migration, and hence, of regional population.

In case (i), the user can take on board the forecasts of the three population flows (natural growth, regional migration and foreign migration) from a demographic model thereby exogenously determining regional populations. For example, the ABS (cat. no. 3222.0) makes forecasts of these flows and of regional population. The labour market & migration block of equations can then be configured to determine regional labour supply from the exogenously specified regional population and given settings of regional participation rates and movements in the ratios of population to population of working age. With labour supply determined, the labour market and regional migration block will determine either interregional wage differentials, (given regional unemployment rates) or regional unemployment rates (given regional wage differentials). With given regional unemployment rates and regional labour supply, regional employment is determined as' a residual and wage differentials adjust to accommodate the labour market outcome. Fixing wage differentials determines the demand for labour so that with regional labour supply given, the model will determine regional unemployment rates as a residual.

In alternative (ii), interregional wage differentials and regional unemployment rates are exogenously specified. The labour market and regional migration block then determines regional labour supply and regional population for given settings of regional participation rates and ratios of population to population of working age.

The equations of this block have been designed with sufficient flexibility to allow variations on the two general methods described above. Importantly, the block allows for some regions to be subject to method (i) and other regions to be subject to method (ii) in the same simulation.

We begin our exposition of the equations of this section with the accumulation of regional population. In the levels, we start with the accumulation relation

$$POP_{u,t+1} = POP_{u,t} + FM_{a,t} + RM_{a,t} + G_{u,t}, \quad q=1,...,8, t=1,...,T,$$
 (2.45)

where $POP_{q,t}$ is regional population in year t, $FM_{q,t}$ is the net migration of overseas residents to region q in year t, $RM_{q,t}$ net migration of residents from other regions to region q in year t and $G_{q,t}$ is region q's natural growth in population in year t.

Accumulating on the basis of equation 2.45 over the period 0-T, we can derive a value for regional population in year T (POP_{q,T})

$$POP_{q,T} = POP_{q,0} + \sum_{t=1}^{T} (FM_{q,t} + RM_{q,t} + G_{q,t}), \qquad q=1,...,8.$$
 (2.46)

As with the accumulation of national foreign debt, in section 2.4.2 above, we make the simplifying assumption that the flow variables in equation 2.46 grow smoothly over the period 0-T, giving

$$FM_{q,t} = FM_{q,0} + \frac{1}{T}(FM_{q,T} - FM_{q,0}), \qquad q=1,...,8, t=0,...,T,$$
 (2.47)

$$RM_{q,t} = RM_{q,0} + \frac{t}{T}(RM_{q,T} - RM_{q,0}), \qquad q=1,...,8, t=0,...,T$$
 (2.48)

and

$$G_{q,t} = G_{q,0} + \frac{t}{T} (G_{q,T} - G_{q,0}),$$
 q=1,...,8, t=0,...,T. (2.49)

Substituting equations 2.47 to 2.49 into equation 2.46 and rearranging gives

$$POP_{q,T} - POP_{q,0} = T(FM_{q,0} + RM_{q,0} + G_{q,0}) + \frac{T+1}{2} [(FM_{q,T} - FM_{q,0}) + (RM_{q,T} - RM_{q,0}) + (G_{q,T} - G_{q,0})], q=1,...,8.$$
(2.50)

As with our earlier accumulation equations, we have the problem of an initial solution to equation 2.50 in that unless by chance

$$T(FM_{a,0} + RM_{a,0} + G_{a,0}) = 0,$$

our base-year values of POP, FM, RM and G do not constitute an initial solution. To resolve the problem, we adopt the technique applied to the national-debt-accumulation equation; we multiply the first term on the RHS of equation 2.50 by the variable RPFUDGE

Giving RPFUDGE an initial value of 0, and the remaining elements their baseyear values, now provides an initial solution satisfying equation 2.50. For forecasting simulations, RPFUDGE is shocked to 1, (Δ RPFUDGE=1).

Treating the zero subscripted elements and T in equation 2.50 as constants, and taking the percentage change in $POP_{q,T}$ (pop_q) and ordinary changes in $FM_{q,T}$, $RM_{q,T}$, $G_{q,T}$ and RPFUDGE (del_fm_q, del_rm_q, del_g_q and delrpfudge, respectively), gives equation E_del_rm in Table 2.1, section 2.5.1. The following list maps the coefficients from equation 2.50 to equation E_del_rm:

 $\begin{array}{l} \text{POP}_{q,T} \Leftrightarrow \text{C}_{POP}_{q};\\ \text{100T}(\text{FM}_{q,0} + \text{RM}_{q,0} + \text{G}_{q,0}) \Leftrightarrow \text{C}_{PRI}_{q};\\ \text{50}(\text{T}+1) \Leftrightarrow \text{C}_{PA2}. \end{array}$

As with the accumulation of national debt, equation E_del_rm can be implemented in comparative-static mode by assigning a value of zero to delrpfudge.

The remaining equations of this section can be grouped into the following categories: definitions; equations imposing arbitrary assumptions; equations imposing adding-up constraints and; national aggregates based on summing regional variables.

The definitional equations are E_del_labsup and E_wpop. The former equation defines the percentage-point change in the regional unemployment (del_unr_q) in terms of the percentage changes in regional labour supply $(labsup_q)$ and persons employed $(employ_q)$. The latter equation defines the percentage change in regional labour supply in terms of the percentage changes in the regional participation rate (pr_q) and the regional population of working age $(wpop_q)$.

In equation E_pop, the assumption that the regional population of working age is proportional to the regional (total) population is imposed. The default setting can be overridden by endogenising the shift variable ($f_{wpop_{i}}$).

Equation E_rm_0, allows for the imposition of either the assumption that the change in net regional migration (del_rm_q) is equal to a forecast change in regional migration (del_rm_0) , or that del_rm_q is equal to $del_rm_0_q$ plus a common (to all q) constant. We can interpret equation E_rm_0 as imposing the former assumption when $del_rm_0_q$ are set exogenously (equal to, say, an ABS forecast) and when the shift variable, $delf_rm_0$, is set exogenously at zero change. The latter assumption is imposed when all but one of the $del_rm_0_q$ are set exogenously and $delf_rm_0$ is set endogenously. The purpose of the second assumption is as follows. We may wish to believe some, *but not all*, of the ABS forecasts of net regional migration. We may wish to determine one of the region's net regional migration from economic factors within MMRF. However; we may still wish that the remaining net regional migration flows to be approximately equal to the ABS forecasts. To the extent that the region's net regional migration determined by MMRF deviates from that forecast by the ABS means that the sum of the regional net migration will not equal zero if the remaining regional net migration flows are set equal to their ABS forecasts. To overcome this problem, we distribute the positive/negative amount of net migration evenly across the regions. If it is desired that all regional net migration flows are determined by economic factors, rather than exogenously, then all elements of del_rm_0_q are set endogenously and swapped with a relevant labour market variable such as regional relative wage rates (wage_diff_q, see equation E_wage_diff, section 2.2.11 above and in Table 2.1).

Equation E_remploy_interf imposes the assumption that regional employment in wage-bill weights is proportional to regional employment in person weights by setting the percentage change in regional wage-bill weighted employment (l_q) equal to the percentage change in regional person-weighted employment (employ_q) when the shift variable f_l_q is exogenous and set to zero change. The default option can be overridden by setting f_l_q to non-zero values.

Equation E_pop_interf imposes the assumption that regional household formation is proportional to regional population by setting the percentage change in regional household formation $(qhous_q)$ equal to the percentage change in regional population (pop_q) when the shift variable f_qhous_q is exogenous and set to zero change. The default option can be overridden by setting f_qhous_q to non-zero values.

An adding-up constraint is imposed in equation E_{rm_a} addup. If the variable delf_rm is exogenous (and set to zero), then at least one of the del_rm_q must be endogenous. If all the del_rm_q are endogenous, then delf_rm must endogenously equal zero for the simulation to be valid.

The remaining equations of this section, E_delnatfm, E_delnatg, E_natlabsup, E_natemploy, and E_natunr determine national aggregate variables by summing the corresponding regional variables.

Table 2.1. The MMRF Equations

Identifier	Equation	Subscript Range	Number	Description
2.2. The C	CGE Core		· · ·	
2.2.1. Pro	duction: demand for inputs to the production process			
E_x1a1 (2.8.1)	$x la_{i,s,j,q} = x lc_{i,j,q} - \sigma lC_i(p la_{i,s,j,q} - p lc_{i,j,q})$	i∈ COM s∈ RSOU j∈ IND q€ RDES	13×8×13×8 .	Demand for goods by regional source, User 1
E_pic (2.8.1)	$plc_{i,j,q} = \sum_{s \in RSOU} SlA_{i,s,j,q} pla_{i,s,j,q}$	i∈ COM j∈ IND q∈ RDES	13×13×8	Price of domestic composite, User 1
E_x1c (2.8.1)	x1c _{i,j,q} = x1o _{i,j,q} - σ1O _i (p1c _{i,j,q} - p1o _{i,j,q})	i∈ COM j∈ IND q∈ RDES	13×13×8	Demand for domestic composite, User I
E_x1a2 (2.8.1)	$x la_{i,s,j,q} = x lo_{i,j,q} - \sigma lO_i(p la_{i,s,j,q} - p lo_{i,j,q})$	i∈COM s=foreign j∈ IND q∈ RDES	13×1×13×8	Demand for foreign imports, User 1
E_plo (2.8.1)	$PVAL1O_{i,j,q}P1o_{i,j,q} = \sum_{s \in ASOU} PVAL1A_{i,s,j,q}P1a_{i,s,j,q}$	ie COM je IND qe RDES	13×13×8	Price of domestic/foreign composite, User 1
E_x11aboi (2.8.2)	x11aboi _{j,q,m} = efflab _{j,q} - σ 1LAB _{j,q} [p11aboi _{j,q,m} - p11ab _{j,q}]	m∈ OCC j∈ IND q∈ RDES	8×13×8	Demand for labour by industry and skill group

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Identifier	Equation	Subscript Range	Number	Description
E_p11ab (2.8.2)	$LABOUR_{j,q}p11ab_{j,q} = \sum_{m \in OCC} LAB_OCC_IND_{m,j,q}p11abo_{j,q,m}$	j∈ IND q∈ RDES	13×8	Price to each industry of labour in general
E_labind (2.8.2)	$LABOUR_{j,q} abind_{j,q} = \sum_{m \in OCC} LAB_OCC_IND_{m,j,q} \times Iaboi_{j,q,m}$	j∈ IND q∈ RDES	13×8	Employment by industry
E_efflab (2.8.2)	$efflab_{j,q} = x lprim_{j,q} + allab_{j,q} - \sigma lFAC_{j,q}[pllab_{j,q} + allab_{j,q} - xi_fac_{j,q}]$	j∈ IND q∈ RDES	13×8	Industry demands for effective labour
E_curcap (2.8.2)	$curcap_{j,q} = x lprim_{j,q} + a lcap_{j,q} - \sigma lFAC_{j,q}[p lcap_{j,q} + a lcap_{j,q} - xi_{fac_{j,q}}]$	j∈ IND q∈ RDES	13×8	Industry demands for capital
E_n (2.8.2)	$n_{j,q} = x lprim_{j,q} + a lland_{j,q} - \sigma lFAC_{j,q}[p lland_{j,q} + a lland_{j,q} - xi_fac_{j,q}]$	j∈ IND q∈ RDES	13×8	Industry demands for land
E_xi_fac (2.8.2)	$\begin{aligned} TOTFACIND_{j,q}xi_{fac_{j,q}} &= LABOUR_{j,q}(pllab_{j,q} + allab_{j,q}) + CAPITAL_{j,q}(plcap_{j,q} + alcap_{j,q}) \\ &+ LAND_{j,q}(plland_{j,q} + alland_{j,q}) \end{aligned}$	j∈ IND q∈ RDES	13×8	Effective price term for factor demand equations
E_x1o (2.8.1)	$x lo_{i,j,q} = z_{j,q} + a l_{j,q}$	i∈ COM j∈ IND q∈ RDES	13×13×8	Demands for domestic/foreign composite inputs, User 1
E_x1prim (2.8.2)	$x lprim_{j,q} = z_{j,q} + al_{j,q} + alprim_{j,q}$	j∈ IND q∈ RDES	13×8	Industry demands for the primary- factor composite
E_x1oct (2.8.1)	$x \operatorname{loct}_{j,q} = z_{j,q} + a I_{j,q} + a \operatorname{loct}_{j,q}$	j∈ IND q∈ RDES	13×8	Industry demands for other cost tickets
2.2.2. Demar	nds for investment goods	•		
E_x2a1 (2.8.3)	$x^{2a_{i,s,j,q}} = x^{2c_{i,j,q}} - \sigma^{2C_{i}(p^{2a_{i,s,j,q}} - p^{2c_{i,j,q}})}$	i∈ COM s∈ RSOU j∈ IND q∈ RDES	13×8×13×8	Demand for goods by regional source, User 2

Identifier	Equation	Subscript Numb Range	per Description
E_x2a2 (2.8.3)	$x2a_{i,s,j,q} = x2o_{i,j,q} - \sigma 2O_i(p2a_{i,s,j,q} - p2o_{i,j,q})$	i∈COM 13×1×1 s=foreign j∈IND q∈RDES	3×8 Demand for foreign imports, User 2
E_x2c (2.8.3)	$x_{2c_{i,j,q}} = x_{2o_{i,j,q}} - \sigma_{2O_{i}(p_{2c_{i,j,q}} - p_{2o_{i,j,q}})}$	i∈COM 13×13× j∈IND q∈RDES	8 Demand for domestic composite, User 2
E_p2c (2.8.3)	$PVAL2T_{i,ss,j,q}p^{2}c_{i,j,q} = \sum_{s \in RSOU} PVAL2A_{i,s,j,q}p^{2}a_{i,s,j,q}$	i∈ COM 13×1×1 ss=domesti c j∈ IND q∈ RDES	3×8 Price of domestic composite, User 2
E_p2o (2.8.3	$PVAL2O_{i,j,q}p2o_{i,j,q} = \sum_{s \in ASOU} PVAL2A_{i,s,j,q}p2a_{i,s,j,q}$	i∈ COM 13×13× j∈ IND q∈ RDES	8 Price of domestic/foreign composite, User 2
E_x2o (2.8.3)	$x2o_{i,j,q} = y_{j,q} + a2ind_{j,q}$	i∈ COM 13×13× j∈ IND q∈ RDES	8 Demands for domestic/foreign composite inputs, User 2
2.2.3. House	chold demands	·	
E_x3a1 (2.8.4)	$x3a_{i,s,q} = x3c_{i,q} - \sigma 3C_i(p3a_{i,s,q} - p3c_{i,q})$	i∈ COM 13×8×8 s∈ RSOU q∈ RDES	Demand for goods by regional source, User 3
E_x3a2 (2.8.4)	$x3a_{i,s,q} = x3o_{i,q} \cdot \sigma 3O_i(p3a_{i,s,q} - p3o_{i,q})$	i∈ COM 13×1×8 s=forcign q∈ RDES	Demand for foreign imports, User 3
E_x3c (2.8.4)	$x_{i,q} = x_{i,q} \cdot \sigma_{3O_i(p_{i,q} - p_{3O_{i,q}})}$	i∈COM ^{13×8} q∈RDES	Demand for domestic composite, User 3

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Identifier	Equation		Subscript Range	Number	Description
E_p3c (2.8.4)	$PVAL3T_{i,domestic,q}p3c_{i,q} = \sum_{s \in RSOU} PVAL3A_{i,s}$	s,q ^{p3a} i,s,q	i∈COM q∈RDES	13×8	Price of domestic composite, User 3
E_p3o (2.8.4)	$PVAL3O_{i,q}p3o_{i,q} = \sum_{s \in ASOU} PVAL3A_{i,s,q}p3a_{i,s}$	s.q	i∈COM q∈RDES	13×8	Price of domestic/foreign composite, User 3
E_x3o (2.8.4)	$x3o_{i,q} = [1 - ALPHA_{i,q}][qhous_q + a3sub_{i,q}]$	+ ALPHA_l _{i,q} (luxcxp _q + a3lux _{i,q} - p3o _{i,q})	i∈ COM q∈ RDES	13×8	Household demand for domestic/foreign composite commodities
E_utility (2.8.4)	$utility_q = luxexp_q - qhous_q - \sum_{i \in COM} DELTA_{i,q}p$	30 _{i,q}	q∈ RDES	8	Change in utility disregarding taste change terms
E_a3sub (2.8.4)	$a3sub_{i,q} = a3com_{i,q} - \sum_{k \in COM} S3COM_{k,q}a3com_{k,q}a3$	s.q	i∈COM q∈RDES	13×8	Default setting for subsistence taste shifter
E_a3lux (2.8.4)	$a3lux_{i,q} = a3sub_{i,q} - \sum_{k \in COM} DELTA_{k,q}a3sub_{k,q}$	I	i∈COM q∈RDES	13×8	Default setting for luxury taste shifter
2.2.4. Foreign	export demands				
E_x4r (2.8.8)	$x4r_{i,s} - feq_i = EXP_ELAST_i[p4r_{i,s} - fep_i - nation$	5b]	i∈TEXP s∈RSOU	2×13	Traditional export demand functions
E_aggnt_x4r (2.8.8)	aggnt_x4r _s - aggnt_feq _s = EXP_ELAST _{manuf}	_{act} [aggnt_p4r _s - aggnt_fep _s - natfep]	s∈RSOU	8	Demand for the non-traditional export aggregate
E_nt_x4r (2.8.8)	$x4r_{i,s} = aggnt_x4r_s + faggnt_i_i + faggnt_s_s + fag$	aggnt_is _{is}	i∈NTEXP s∈RSOU	11×13	Non-traditional-export demand functions
E_aggnt_p4r (2.8.8)	$AGGEXPNT_{s}aggnt_p4r_{s} = \sum_{i \in NTEXP} PVAL4R_{isP}$	4r _{is} + faggnt_p4r _s	i∈NTEXP s∈RSOU	11×13	Average price of non-traditional exports
2.2.5.	Government consumption demands				

....Table 2.1. continued

Identifier	Equation	Subscript Range	Number	Description
E_x5a (2.8.9)	$x5a_{i,s,q} = cr_q + f5a_{i,s,q} + f5gen_q + natf5gen$	i∈ COM s∈ ASOU q∈ RDES	13×9×8	Regional other demands
E_x6a (2.8.10)	$x6a_{i,s,q} = natcr + f6a_{i,s,q} + f6gen_q + natf6gen$	i∈ COM s∈ ASOU q∈ RDES	13×9×8	Federal other demands
2.2.6. Dema	nds for margins			
E_x1marg (2.8.11)	xlmarg _{i,s,j,q,r} = xla _{i,s,j,q}	i∈ COM j∈ IND q∈ RDES s∈ ASOU r∈ MARG	13×13×8×9×	² Margins on sales to producers
E_x2marg (2.8.11)	$x2marg_{i,s,j,q,r} = x2a_{i,s,j,q}$	i∈ COM j∈ IND q∈ RDES s∈ ASOU r∈ MARG		² Margins on sales to capital creators
E_x3marg (2.8.11)	$x3marg_{i,s,q,r} = x3a_{i,s,q}$	i∈ COM s∈ ASOU q∈ RDES r∈ MARG	13×9×8×2	Margins on sales to household consumption
E_x4marg (2.8.11)	$x4marg_{i,s,r} = x4r_{i,s}$	i∈ COM r∈ MARG s∈ RSOU	13×2×8	Margins on exports: factory gate to port

Identifier	Equation	Subscript Range	Number	Description
E_x5marg (2.8.11)	$x5marg_{i,s,q,r} = x5a_{i,s,q}$	i∈ COM s∈ ASOU q∈ RDES r∈ MARG	13×9×8×2	Margins on sales to regional other demands
E_x6marg (2.8.11)	$x 6 marg_{i,s,q,r} = x 6 a_{i,s,q}$	i∈ COM r∈ MARG s∈ ASOU q∈ RDES	13×2×9×8	Margins on sales to federal other demands in each region
2.2.7. Prices				
E_p0a (2.8.13)	$COSTS_{j,q}\{p0a_{j,q} - a_{j,q}\} = \sum_{i \in COM} \sum_{s \in ASOU} \sum_{j,q} PVAL1A_{i,s,j,q} p1a_{i,s,j,q}$	j∈ IND q∈ RDES	13×8	Zero pure profits in current production
	+ $\sum_{m \in OCC} LAB_OCC_IND_{m,j,q} p Ilaboi_{j,q,m}$ + CAPITAL _{j,q} p I cap _{j,q} m $\in OCC$			
	+ LAND _{j,q} p11and _{j,q} + OTHCOST _{j,q} p1oct _{j,q}			
E_a (2.8.13)	$COSTS_{j,q}[a_{j,q} - a_{j,q}] = TOTFACIND_{j,q}alprim_{j,q} + LABOUR_{j,q}allab_{j,q} + CAPITAL_{j,q}alcap_{j,q} + LAND_{j,q}alland_{j,q} + OTHCOST_{j,q}alcot_{j,q}$	j∈ IND q∈ RDES	13×8	Technical change by industry - current production
E_pi (2.8.13)	$INVEST_{j,q}(pi_{j,q} - a2ind_{j,q}) = \sum_{i \in COM} \sum_{s \in ASOU} PVAL2A_{i,s,j,q}p2a_{i,s,j,q}$	j∈ IND q∈ RDES	13×8	Zero pure profits in capital creation
E_p0ab (2.8.13)	$p0a(i,"foreign") = pm_i + natphi + powtaxm_i$	i∈COM	13	Zero pure profits in importing
E_pla (2.8.6)	$\begin{aligned} PVAL1A_{i,s,j,q} P1a_{i,s,j,q} &= [BAS1_{i,s,j,q} + TAX1_{i,s,j,q}] P0a_{i,s} + BAS1_{i,s,j,q} dcltax1_{i,s,j,q} \\ &+ \sum_{r \in MARG} MAR1_{i,s,j,q,r} P0a_{q,r} \end{aligned}$	i∈ COM j∈ IND q∈ RDES s∈ ASOU	13×13×8×9	Purchasers prices - User 1

Identifier	Equation	Subscript Range	Number	Description
E_p2a (2.8.6)	$PVAL2A_{i,s,j,q}p2a_{i,s,j,q} = [BAS2_{i,s,j,q} + TAX2_{i,s,j,q}]p0a_{i,s} + BAS2_{i,s,j,q}deltax2_{i,s,j,q} + \sum_{r \in MARG} MAR2_{i,s,j,q,r}p0a_{q,r}$	i∈ COM j∈ IND q∈ RDES s∈ ASOU	13×13×8×9	Purchasers prices - User 2
E_p3a (2.8.6)	$PVAL3A_{i,s,q}p3a_{i,s,q} = [BAS3_{i,s,q} + TAX3_{i,s,q}]p0a_{i,s} + BAS3_{i,s,q}deltax3_{i,s,q}$ $+ \sum_{r \in MARG} MAR3_{i,s,q,r}p0a_{q,r}$	i∈COM q∈RDES s∈ASOU	13×8×9	Purchasers prices - User 3
E_p4r 2.8.6)	$PVAL4R_{i,s}(natphi + p4r_{i,s}) = [BAS4_{i,s} + TAX4_{i,s}]p0a_{i,s}$ $+ BAS4_{i,s}deltax4_{i,s} + \sum_{r \in MARG} MAR4_{i,s,r}p0a_{r,s}$	i∈ COM s∈ RSOU	13×8	Purchasers prices - User 4
E_p5a 2.8.6)	$PVAL5A_{i,s,q}p5a_{i,s,q} = [BAS5_{i,s,q} + TAX5_{i,s,q}]p0a_{i,s} + BAS5_{i,s,q}dcltax5_{i,s,q} + \sum_{r \in MARG} MAR5_{i,s,q,r}p0a_{s,r}$	i∈ COM q∈ RDES s∈ ASOU	13×8×9	Purchasers prices - User 5
E_p6a 2.8.6)	$PVAL6A_{i,s,q}p6a_{i,s,q} = [BAS6_{i,s,q} + TAX6_{i,s,q}]p0a_{i,s} + BAS6_{i,s,q}deltax6_{i,s,q} + \sum_{r \in MARG} MAR6_{i,s,q,r}p0a_{r,s}$	i∈ COM s∈ ASOU q∈ RDES	13×9×8	Purchasers prices - User 6

2.2.8. Market-clearing equations for commodities

Identifier	Equation		Subscript Range	Number	Description
E_mkt_clear_ margins (2.8.12)	$SALES_{r,s}z_{r,s} = \sum_{j \in IND q \in RDES} \sum_{r,s,j,q} x_{1a_{r,s,j,q}} + BAS2_{r,s,j,q} x_{2a_{r,s,j}}$	$q + \sum_{q \in RDES} BAS3_{r,s,q} x 3a_{r,s,c}$	r∈ MARG s∈ RSOU	2×8	Demand equals supply for margin commodities
(,	+ $BAS4_{r,s}x4r_{r,s}$ + $\sum_{q \in RDES} BAS5_{r,s,q}x5a_{r,s,q}$ + $\sum_{q \in RDES} BAS6$	r,s,q ^{x6a} r,s,q			· ·
	+ $\sum_{j \in IND \ i \in COM \ ss \in ASOU} \sum MARI_{i,ss,j,s,r} x Imarg_{i,ss,j,s,r} + M$	AR2 _{i,ss.j.s,r} x2marg _{i,ss.j.s,r}	. _.		
	+ $\sum_{i \in COM} \sum_{ss \in ASOU} MAR3_{i,ss,s,r} x_{3marg_{i,ss,s,r}} + \sum_{i \in COM} MAR$	⁴ i,s,r ^{x4marg} i,s,r			
	+ $\sum_{i \in COM} \sum_{ss \in ASOU} MAR5_{i,ss,s,r} x5marg_{i,ss,s,r} + \sum_{i \in COM} \sum_{ss \in ASOU} \sum_{s \in ASOU} \sum_{s \in ASOU} \sum_{s \in ASOU} \sum_{s ASOU} \sum_{s ASOU} \sum_{s ASOU} \sum_{s ASOU} \sum_{s ASOU} \sum_{s ASOU} \sum_{s ASOU} \sum_$	2011	r		
E_mkt_clear_ nomarg (2.8.12)	$SALES_{r,s}z_{r,s} = \sum_{j \in IND q \in RDES} \sum_{j \in IND q \in RDES} BAS1_{r,s,j,q}x1a_{r,s,j,q} + \sum_{j \in IND q \in RDES} \sum_{j \in IND q \in RDES} BAS2$	r,s.j,q ^{x2a} r,s,j,q	r∈ NONMAR	2×8	Demand equals supply for nonmargin commodities
(2.0.12)	+ $\sum_{q \in RDES} BAS3_{r,s,q} x 3a_{r,s,q} + BAS4_{r,s} x 4r_{r,s} + \sum_{q \in RDES} BAS5_{r,s,q} x 5a_{r,s,q}$	$q + \sum_{q \in RDES} BAS6_{r,s,q} x 6a_{r,s,q}$	G ,q s∈ RSOU		
E_x0impa (2.8.12)	$IMPORTS_{i,q} \times Oimp_{i,q} = \sum_{j \in IND} BASI_{i,foreign,j,q} \times Ia_{i,foreign,j,q} + BAS2_{i,q}$	foreign,j,q ^{x2a} i,foreign,j,q	i∈COM q∈RDES	13×8	Import volumes of commodities by region
2.2.9. Indirect	+ BAS3 _{i,foreign,q} x3a _{i,foreign,q} + BAS5 _{i,foreign,q} x5a _{i,foreign,q} +BAS laxes	⁶ i,foreign,q ^{x6a} i,foreign,q			
E_deltax 1 (2.8.5)	deltax $I_{i,s,j,q} = deltax_i + deltax Iall + deltaxsource_s + deltaxdest_q$		i∈ COM s∈ ASOU j∈ IND q∈ RDES	13×9×13×8	Tax rate on sales to User 1

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Identifier	Equation	Subscript Range	Number	Description
E_deltax2 (2.8.5)	$deltax_{i,s,j,q} = deltax_i + deltax_2all + deltaxsource_s + deltaxdest_q$	i∈ COM s∈ ASOU j∈ IND q∈ RDES	13×9×13×8	Tax rate on sales to User 2
E_deltax3 (2.8.5)	$deltax3_{i,s,q} = deltax_i + deltax3all + deltaxsource_s + deltaxdest_q$	i∈ COM s∈ ASOU q∈ RDES	13×9×8	Tax rate on sales to User 3
E_deltax4 (2.8.5)	$deltax4_{i,s} = deltax_i + deltax4ail + deltaxsource_s + deltaxdest_q$	i∈ COM s∈ RSOU q=foreign	13×8	Tax rate on sales to User 4
E_deltax5 (2.8.5)	$deltax5_{i,s,q} = deltax_i + deltax5all + deltaxsource_s + deltaxdest_q$	i∈ COM s∈ ASOU q∈ RDES	13×9×8	Tax rate on sales to User 5
E_deltax6 (2.8.5)	deltax $6_{i,s,q}$ = deltax _i + deltax6all + deltaxsource _s + deltaxdest _k	i∈ COM s∈ ASOU q∈ RDES k=federal	13×9×8	Tax rate on sales to User 6
E_taxrcv1 (2.8.7)	$AGGTAXI_{q} a xrevI_{q} = \sum_{i \in COM} \sum_{s \in ASOU} \sum_{j \in IND} TAXI_{i,s,j,q} \{pOa_{i,s} + xIa_{i,s,j,q}\} + BASI_{i,s,j,q} deltaxI_{i,s,j,q}$	q∈ RDES	8	Aggregate revenue from indirect taxes levied on flows to User 1
E_taxrev2 (2.8.7)	$AGGTAX2_{q}taxrev2_{q} = \sum_{i \in COM} \sum_{s \in ASOU} \sum_{j \in IND} TAX2_{i,s,j,q}(pOa_{i,s} + x2a_{i,s,j,q}) + BAS2_{i,s,j,q}deltax2_{i,s,j,q}$	q∈RDES	8	Aggregate revenue from indirect taxes levied on flows to User 2
E_taxrev3 (2.8.7)	$AGGTAX3_{q}taxrcv3_{q} = \sum_{i \in COM} \sum_{s \in ASOU} TAX3_{i,s,q} \{p0a_{i,s} + x3a_{i,s,q}\} + BAS3_{i,s,q} deltax3_{i,s,q}$	q∈RDES	8	Aggregate revenue from indirect taxes levied on flows to User 3
E_taxrev4 (2.8.7)	$AGGTAX4_{s}taxrev4_{s} = \sum_{i \in COM} TAX4_{i,s} \{p0a_{i,s} + x4r_{i,s}\} + BAS4_{i,s}deltax4_{i,s}$	s∈RSOU	8	Aggregate revenue from indirect taxes levied on flows to User 4

Identifier	Equation	Subscript Range	Number	Description
E_taxrev5 (2.8.7)	$AGGTAXS_{q}taxrevS_{q} = \sum_{i \in COM} \sum_{s \in ASOU} TAXS_{i,s,q} \{pOa_{i,s} + xSa_{i,s,q}\} + BASS_{i,s,q} deltaxS_{i,s,q}$	q∈RDES	8	Aggregate revenue from indirect taxes levied on flows to User 5
E_taxrev6 (2.8.7)	$AGGTAX6_{q}taxrev6_{q} = \sum_{i \in COM} \sum_{s \in ASOU} TAX6_{i,s,q} \{pOa_{i,s} + x6a_{i,s,q}\} + BAS6_{i,s,q} deltax6_{i,s,q}$	q∈ RDES	8	ggregate revenue from indirect taxes levied on flows to User 6
2.2.10. Regio	mal incomes and expenditures			
E_caprev (2.8.14)	$caprev_{q} = (1.0/AGGCAP_{q}) \sum_{j \in IND} CAPITAL_{j,q} \{p cap_{j,q} + curcap_{j,q}\}$	q∈ RDES	8	Aggregate payments to capital
E_labrev (2.8.14)	$labrev_{q} = (1.0/AGGLAB_{q}) \sum_{j \in IND} \sum_{m \in OCC} LAB_OCC_IND_{m,j,q} \{p1 aboi_{j,q,m} + x1 aboi_{j,q,m}\}$	q∈RDES	8	Aggregate payments to labour
E_Indrev (2.8.14)	$Indrev_{q} = (1.0/AGGLND_{q}) \sum_{j \in IND} LAND_{j,q} \{p land_{j,q} + n_{j,q} \}$	q∈ RDES	8	Aggregate payments to land
E_octrev (2.8.14)	octrev _q = (1.0/AGGOCT _q) $\sum_{j \in IND} OTHCOST_{j,q} \{ploct_{j,q} + xloct_{j,q}\}$	q∈ RDES	8	Aggregate other cost ticket payments
E_taxind (2.8.14)	$taxind_{q} = (1.0/AGGTAX_{q})(AGGTAXI_{q}taxrevI_{q} + AGGTAX2_{q}taxrev2_{q} + AGGTAX3_{q}taxrev3_{q} + AGGTAX5_{q}taxrev5_{q})$	q∈ RDES	8	Aggregate value of indirect taxes
E_taxrevm (2.8.14)	$AGGTAXM_{q}taxrevm_{q} = \sum_{i \in COM}^{n} TARIFF_{i,q} \{pm_{i} + natphi + x0imp_{i,q}\} + IMPORTS_{i,q}powtaxm_{i}$	q∈ RDES	8	Aggregate tariff revenue
E_c_a (2.8.14)	$AGGCON_{q}c_{q} = \sum_{i \in COM} \sum_{s \in ASOU} PVAL3A_{i,s,q} \{x3a_{i,s,q} + p3a_{i,s,q}\}$	q∈ RDES	8	Household budget constraint
E_cr (2.8.14)	$cr_q = c_q - xi3_q$	q∈RDES	8.	Real household consumption

Identifier	Equation	Subscript Range	Number	Description
E_xi3 (2.8.16)	$xi3_q = (1.0/AGGCON_q) \sum_{i \in COM} \sum_{s \in ASOU} \sum_{p \in ASOU} \sum_{s \in $	q∈ RDES	8	Consumer price index
E_ir (2.8.14)	$ir_q = (1.0/AGGINV_q) \sum INVEST_{j,q}y_{j,q}$ $j \in IND$	q∈ RDES	8	Real investment
E_othreal5 (2.8.14)	othreal $5_q = (1.0/AGGOTH S_q) \sum_{i \in COM} \sum_{s \in ASOU} \sum_{s \in asOUU} \sum_{s \in asOU} \sum_{s \in asOU} \sum_{s \in asOU} \sum_{s \in asOU} \sum_{s \in asOUU} \sum_{s \in asOUUU} \sum_{s \in asOUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUU$	q∈ RDES	8	Real regional other demands
E_othreal6 (2.8.14)	$AGGOTH6_{q}othreal6_{q} = \sum_{i \in COM} \sum_{s \in ASOU} PVAL6A_{i,s,q}x6a_{i,s,q}$	q∈ RDES	8	Real federal other demand
E_int_exp (2.8.14)	$C_XSEXP_{s}(psexp_{s} + xsexp_{s}) = \sum_{q \in RDES} C_XSFLO_{s,q}(psflo_{s,q} + xsflo_{s,q}) - C_XSFLO_{s,s}(psflo_{s,s} + xsflo_{s,s})$	s∈ RSOU	8	Interregional exports
E_int_imp (2.8.14)	$C_XSIMP_q(psimp_q + xsimp_q) = \sum_{s \in RSOU} C_XSFLO_{s,q}(psflo_{s,q} + xsflo_{s,q}) - C_XSFLO_{q,q}(psflo_{q,q} + xsflo_{q,q})$	q∈RDES	8	Interregional imports
E_expvol (2.8.14)	$expvol_q = export_q + natphi - xi4_q$	q∈ RDES	8	Export volume index
E_impvol (2.8.14)	$impvol_q = imp_q + natphi - xim_q$	q∈RDES	8	Import volume index
E_xi2 (2.8.16)	$xi2_q = (1.0/AGGINV_q) \sum_{j \in IND} \sum_{j,q} p_{ij,q}$	q∈ RDES	8	Investment price index
E_xi5 (2.8.16)	$xi5_q = (1.0/AGGOTH5_q) \sum_{i \in COM} \sum_{s \in ASOU} \sum_{s \in ASOU} \sum_{s,s,q} \sum_{i,s,q} \sum_{s,q} \sum_{s \in ASOU} \sum_{S$	q∈ RDES	8	Regional other demands price index
E_xi6 (2.8.16)	$x_{i6_q} = (1.0/AGGOTH_{6_q}) \sum_{i \in COM} \sum_{s \in ASOU} \sum_{s \in a_{i,s,q} p_{6a_{i,s,q}}} \sum_{s,q} \sum_{s \in a_{i,s,q}} \sum_{s$	q∈RDES	8	Price index for Federal other demand

Identifier	Equation	Subscript Range	Number	Description
E_psexp (2.8.16)	$C_XSEXP_{s}psexp_{s} = \sum_{q \in RDES} C_XSFLO_{s,q}psflo_{s,q} - C_XSFLO_{s,s}psflo_{s,s}$	s∈RSOU	x	Price index for interregional exports
E_psimp (2.8.16)	$C_XSIMP_qpsimp_q = \sum_{s \in RSOU} C_XSFLO_{s,q}psflo_{s,q} - C_XSFLO_{q,q}psflo_{q,q}$	q∈ RDES	8	Price index for interregional imports
E_xi4 (2.8.16)	$xi4_q$ - natphi = (1.0/AGGEXP _q) $\sum_{i \in COM} PVAL4R_{i,q}p^{4r}_{i,q}$	q∈RDES	8	Foreign exports price index
E_xim (2.8.16)	$xim_q - natphi = (1.0/AGGIMP_q)$ $\sum IMPCOST_{i,q}pm_i$ $i \in COM$	q∈ RDES	8	Foreign imports price index
E_in (2.8.14)	$in_q = ir_q + xi2_q$	q∈RDES	8	Nominal investment
E_othnom5 (2.8.14)	$othnomS_q = othrealS_q + xiS_q$	q∈ RDES	8	Nominal value of regional other demands
E_othnom6 (2.8.14)	$othnom6_q = othreal6_q + xi6_q$	q∈ RDES	8	Nominal Federal other demand
E_export (2.8.14)	$export_{q} = (1.0/AGGEXP_{q}) \sum_{i \in COM} PVAL4R_{i,q}[p4r_{i,q} + x4r_{i,q}]$	q∈ RDES	8	Foreign currency value of exports
E_imp (2.8.14)	$imp_{q} = (1.0/AGGIMP_{q}) \sum_{i \in COM} IMPCOST_{i,q}[pm_{i} + x0imp_{i,q}]$	q∈RDES	8	Foreign currency value of imports

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Identifier	Equation		Subscript Range	Number	Description
E_trd (2.8.14)	C_XSFLO _{s,q} (psflo _{s,q} + xsflo _{s,q}) = i	$\sum_{\substack{s \in COM \\ j \in IND}} \sum_{j \in IND} \sum_{$	s∈ RSOU q∈ RDES	8×8	Interregional trade flows (including diagonal term)
		$\sum_{c \in OM} \sum_{j \in IND} \sum_{i,s,j,q} (p_{0a_{i,s}} + x_{2a_{i,s},j,q})$			
		$\sum_{i,s,q} BAS3_{i,s,q}(p0a_{i,s} + x3a_{i,s,q})$			
		$\sum_{i,s,q} BAS5_{i,s,q}(p0a_{i,s} + x5a_{i,s,q})$			
E_psflo (2.8.16)	$C_XSFLO_{s,q}psflo_{s,q} = \sum_{i \in COM} \sum_{j \in IN}$	$\begin{array}{l} BAS1_{i,s,j,q} P_{a_{i,s}} + \sum \sum BAS2_{i,s,j,q} P_{a_{i,s}} \\ ND & i \in COM j \in IND \end{array}$	s∈ RSOU q∈ RDES	8×8	Price index for interregional trade flows
	+ ∑BAS3 _{i,s,q} i∈COM	$p_{0a_{i,s}} + \sum_{i \in COM} BAS5_{i,s,q} p_{0a_{i,s}}$			
2.2.11. Region	al wages				
E_pllaboi (2.8.17)	pllaboi _{j,q,m} = pwagei _{j,q} + arpri _{j,q}		j∈ IND q∈ RDES m∈ OCC	13×8×8	Payroll tax adjustment
E_pwagei (2.8.17)	$pwagei_{j,q} = natxi3 + natfwage + fwage_q + fwagei_{j,q}$		j∈ IND q∈ RDES	13×8	Flexible setting of money wages
E_wage_diff (2.8.17)	wage_diff _q = pwage _q - natxi3 - natr	ealwage	q∈ RDES	8	Regional real wage differential
E_pwage (2.8.17)	AGGLAB _q pwage _q = ∑LABOUR j∈ IND	j,q ^{nwagci} j,q	q∈ RDES	X	Regional nominal wage received by workers

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Identifier	Equation	Subscript Range	Number	Description
E_natrealw (2.8.17)	NATAGGLAB natrealwage = $\sum \sum \sum LABOUR_{j,q}(natfwage + fwage_q + fwage_{j,q})$ j \in IND \e RDES		1	National real wage: consumer
2.2.12. Other	egional factor market definitions			
E_I (2.8.14)	$I_q = (1.0/AGGLAB_q) \sum_{j \in IND} \sum_{j \in $	q∈ RDES	8	Employment: wage bill weights
E_kt (2.8.14)	$k_{i_q} = (1.0/AGGCAP_q) \sum CAPITAL_{j,q}curc_{ap_{j,q}}$ $j \in IND$	q∈ RDES	x	Usage of capital: rental weights
E_z_tot (2.8.14)	$TOTFAC_{q}z_{tot_{q}} = \sum_{j \in IND} TOTFACIND_{j,q}z_{j,q}$	q∈RDES	8	Output: value-added weights
E_lambda (2.8.19)	$LAB_OCC_{m,q} lambda_{m,q} = \sum_{j \in IND} LAB_OCC_IND_{m,j,q} \times Ilaboi_{j,q,m}$	m∈ OCC q∈ RDES	8×8	Demand for labour by occupation
E_pwage_p (2.8.17)	$AGGLAB_{q}pwage_{p_{q}} = \sum_{j \in IND} \sum_$	q∈ RDES	8	Nominal wage paid by producers
E_reg_p1cap (2.8.18)	$reg_plcap_q = caprev_q - kt_q$	q∈ RDES	8	Rental price of capital
E_realwage_w (2.8.18)	$rcalwage_w_q = pwage_q - xi3_q$	q∈ RDES	8	Real wages for workers: deflated by CPI
E_realwage_p (2.8.18)	$realwage_p_q = pwage_p_q - xiy_r_q$	q∈ RDES	8	Real wages for producers: deflated by GDP deflator
E_r0_tot (2.8.18)	$r_{10_{iq}} = (1.0/AGGCAP_q) \sum_{j \in IND} \sum_{j,q} r_{j,q}$	q∈ RDES	8	Rate of return on capital
E_xiplpk_ind (2.8.18)	$xiplpk_ind_{j,q} = p1lab_{j,q} - p1cap_{j,q}$	j∈ IND q∈ RDES	13×8	Index of relative price movements of labour & capital

Identifier	Equation	Subscript Range	Number	Description
E_xiplpk (2.8.18)	xiplpk _q = pwage_p _q - reg_plcap _q	q∈RDES	8	Index of relative price movements of labour & capital: regional aggregate
2.2.13. Other	niscellaneous regional equations			
E_ploct (2.8.1)	$ploct_{j,q} = xi3_q + floct_{j,q}$	j∈ IND q∈ RDES	13×8	Indexing of prices of other cost tickets
E_cr_shr (2.8.14)	$cr_q = natcr + cr_shr_q$	q∈RDES	8	Regional shares in national real household consumption
E_ximp0 (2.8.16)	$ximp0_q = (1.0/[AGGIMP_q + AGGTAXM_q]) \sum_{i \in COM} IMPORTS_{i,q}p0a_{i,s}$	q∈RDES s=forcign	8	Duty-paid imports price index
E_totdom (2.8.18)	$totdom_q = psexp_q - psimp_q$	q∈RDES	8	Domestic terms of trade
E_totfor (2.8.18)	$totfor_q = xi_{q} - xim_q$	q∈RDES	8	Foreign terms of trade
2.2.14. Nation	al aggregates			
E_nattaxrev1 (2.8.7)	NATAGGTAX1 nattaxrev1 = $\sum AGGTAX1_q taxrev1_q$ q $\in RDES$		1	Revenue from indirect taxes levied on flows to User 1
E_nattaxrev2 (2.8.7)	NATAGGTAX2 nattaxrev2 = $\sum AGGTAX2_q taxrev2_q$ q $\in RDES$		1	Revenue from indirect taxes levied on flows to User 2
E_nattaxrev3 (2.8.7)	NATAGGTAX3 nattaxrev3 = $\sum AGGTAX3_q taxrev3_q$ q $\in RDES$		I	Revenue from indirect taxes levied on flows to User 3
E_nattaxrev4 (2.8.7)	NATAGGTAX4 nattaxrev4 = $\sum_{s \in RSOU} AGGTAX4_{s} taxrev4_{s}$		1	Revenue from indirect taxes levied on Nows to User 4

Identifier	Equation	Subscript Range	Number	Description
E_nattaxrev5 (2.8.7)	NATAGGTAXS nattaxrevS = $\sum AGGTAXS_q taxrevS_q$ q $\in RDES$		1	Revenue from indirect taxes levied on flows to User 5
E_nattaxrev6 (2.8.7)	NATAGGTAX6 nattaxrev6 = $\sum_{q \in RDES} AGGTAX6_q taxrev6_q$		1	Revenue from indirect taxes levied on flows to User 6
E_natx0imp (2.8.12)	NATIMPORTS _i natx0imp _i = \sum IMPORTS _{i,q} x0imp _{i,q} q RDES	i∈COM	13	Import volumes
E_natlabind (2.8.14)	$NATLABOUR_{jnatlabind_{j}} = \sum LABOUR_{j,q} abbind_{j,q}$ $q \in RDES$	j∈IND	13	Employment: wage bill weights
E_natcaprev (2.8.15)	natcaprev = (1.0/NATAGGCAP) $\sum_{q \in RDES} AGGCAP_q caprev_q$		I	Payments to capital
E_natlabrev (2.8.15)	natlabrev = (1.0/NATAGGLAB) $\sum_{q \in RDES} AGGLAB_q labrev_q$		1	Payments to labour
E_natIndrev (2.8.15)	natIndrev = (1.0/NATAGGLND) $\sum_{q \in RDES} AGGLND_q Indrev_q$		I	Payments to land
E_natoctrev (2.8.15)	natoctrev = (1.0/NATAGGOCT) $\sum_{q \in RDES} AGGOCT_q octrev_q$		I	Other cost ticket payments
E_nattaxrevm (2.8.15)	nattaxrevm = (1.0/NATAGGTAXM) $\sum_{q \in RDES} AGGTAXM_q taxrevm_q$		1	Tariff revenue
E_nattaxind (2.8.15)	nattaxind = (1.0/NATAGGTAX)(NATAGGTAX1nattaxrev1 + NATAGGTAX2nattaxrev2 + NATAGGTAX3nattaxrev3 + NATAGGTAX4nattaxrev4 + NATAGGTAX5nattaxrev5 + NATAGGTAX6nattaxrev6 + NATAGGTAXMnattaxrevm)		I	Value of indirect taxes

Identifier	Equation	Subscript Range	Number	Description
E_natgdpinc (2.8.15)	natgdpinc = (1.0/NATGDPIN)(NATAGGLNDnatIndrev + NATAGGCAPnatcaprev + NATAGGLABnatlabrev + NATAGGOCTnatoctrev + NATAGGTAXnattaxind))	1	Nominal GDP from income side
E_natkt (2.8.15)	natkt = (1.0/NATAGGCAP) $\sum_{q \in RDES} AGGCAP_q kt_q$		1	Usage of capital: rental weights
E_natl (2.8.15)	natl = (1.0/NATAGGLAB) $\sum_{q \in RDES} AGGLAB_q!_q$		1	Employment: wage bill weights
E_natz_tot (2.8.15)	NATTOTFAC natz_tot = $\sum TOTFAC_q z_{tot_q}$ q $\in RDES$		I	Aggregate output: value-added weights
E_natz (2.8.15)	$NATTOTFACIND_{jnatz_{j}} = \sum_{q \in RDES} \sum_{q \in RDES} \sum_{j,q} \sum_{q \in RDES} \sum_{j,q} \sum_{q \in RDES} \sum_{j,q} \sum_{q \in RDES} \sum_{j,q} \sum_{j,q} \sum_{q \in RDES} \sum_{j,q} \sum_{j,q} \sum_{j,q} \sum_{q \in RDES} \sum_{j,q} \sum_{j,q$	j∈IND	13	Industry output: value-added weights
E_natc (2.8.15)	NATAGGCON nate = $\sum_{q \in RDES} AGGCON_q c_q$		1	Nominal household consumption
E_natcr (2.8.15)	NATAGGCON nator = $\sum_{q \in RDES} AGGCON_q cr_q$		I	Real household consumption
E_natin (2.8.15)	natin = natir + natxi2		1	Nominal investment
E_natir (2.8.15)	natir = (1.0/NATAGGINV) $\sum_{j \in IND} NATINVEST_{j}naty_{j}$		1	Real investment
E_natothnom (2.8.15)	5 natothnom5 = natothreal5 + natxi5		1	Nominal value of regional other demands
• •	6 natothnom6 = natothreal6 + natxi6		1	Nominal value of Federal other demands

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Identifier	Equation	Subscript Range	Number	Description
E_natothreal5 (2.8.15)	natcthreal5 = (1.0/NATAGGOTH5) $\sum_{q \in RDES} AGGOTH5_q othreal5_q$		ı	Real regional other demands
E_natothreal6 (2.8.15)	NATAGGOTH6 natothrcal6 = $\sum_{i \in COM} \sum_{s \in ASOU} \sum_{q \in RDES} \sum_{r,s,q} \sum_{x,s,q} \sum_{r,s,q} \sum_{s,q} \sum_{r,s,q} \sum_{s,q} \sum_{r,s,q} \sum_{s,q} \sum_{r,s,q} \sum_{s,q} \sum_{r,s,q} \sum_{s,q} \sum_{r,s,q} \sum_{s,q} \sum_{r,s,q} \sum_{r,s,q} \sum_{s,q} \sum_{r,s,q} \sum_{r,$		I	Real Federal other demands
E_natexport (2.8.15)	natexport = (1.0/NATAGGEXP) $\sum_{s \in RSOU} AGGEXP_{s}export_{s}$		1	Foreign currency value of exports
E_natexpvol (2.8.15)	natexpvol = natexport + natphi - natxi4		1	Export volume index
E_natimp (2.8.15)	natimp = (1.0/NATAGGIMP) $\sum_{i \in COM} NATIMPCOST_i[pm_i + natx0imp_i]$		1	Foreign currency value of imports
E_natimpvol (2.8.15)	natimpvol = natimp + natphi - natxim		1	Import volume index
E_natgdpexp (2.8.15)	natgdpexp = (1.0/NATGDPEX)(NATAGGCONnate + NATAGGINVnatin + NATAGGOTH5natothnom5 + NATAGGOTH6natothnom6 + NATAGGEXP(natexport + natphi) - NATAGGIMP(natimp + natphi))		1	Nominal GDP from expenditure side
E_natgdpreal (2.8.15)	natgdpreal = natgdpexp - natxigdp		I	Real GDP: expenditure side
E_natdelb (2.8.15)	1000.0100.0 natdelb = NATAGGEXPnatexport - NATAGGIMPnatimp		1	Balance of trade in billions of dollars
E_natxi3 (2.8.16)	NATAGGCON natxi3 = $\sum_{q \in RDES} AGGCON_q xi3_q$		I	Consumer price index
E_natxi2 (2.8.16)	NATAGGINV natxi2 = $\sum_{q \in RDES} AGGINV_q xi2_q$		1	Investment price index

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Identifier	Equation	Subscript Range	Number	Description
E_natxi4 (2.8.16)	natxi4 = (1.0/NATAGGEXP) $\sum_{q \in RDES} AGGEXP_q xi4_q$		1	Exports price index
E_natxi5 (2.8.16)	natxi5 = (1.0/NATAGGOTH5) $\sum_{q \in RDES} AGGOTH5_q xi5_q$		I	Price index for regional other demands
E_natxi6 (2.8.16)	NATAGGOTH6 natxi6 = $\sum \sum \sum \sum PVAL6A_{i,s,q}p6a_{i,s,q}$ ie COM se ASOU qe RDES		t	Price index for Federal other demands
E_natxigdp (2.8.16)	natxigdp = (1.0/NATGDPEX)(NATAGGCONnatxi3 + NATAGGINVnatxi2 + NATAGGOTH5natxi5 + NATAGGOTH6natxi6		1	Price index for GDP: expenditure side
	+ NATAGGEXPnatxi4 - NATAGGIMPnatxim)			4.
E_natxim (2.8.16)	natxim = (1.0/NATAGGIMP) $\sum_{q \in RDES} AGGIMP_q xim_q$		1	Imports price index
E_natximp (2.8.16)	natximp0 = (1.0/[NATAGGIMP + NATAGGTAXM]) $\sum_{i \in COM} NATIMPORTS_i p0a_{i,s}$	s=foreign	1	Duty-paid imports price index
E_nattot (2.8.16)	nattot = natxi4 - natxim		1	Terms of trade
E_natp1cap (2.8.18)	natplcap = natcaprev - natkt		1	Nominal capital rentals
E_natpwage (2.8.18)	NATAGGLAB natpwage = $\sum AGGLAB_q pwage_q$ q $\in RDES$		1	Nominal wages received by workers
E_natpwage_ (2.8.18)	P NATAGGLAB natpwage_p = \sum AGGLAB _q pwage_p _q q e RDES		I	Nominal wages paid by producers
E_natrwage_v (2.8.18)	v natrwage_w = natrealwage		1	Real wages for workers: deflated by CPI

Identifier	Equation	Subscript Range	Number	Description
E_natrwage_[(2.8.18)	p natrwage_p = natpwage_p - natxigdp		1	Real wages for producers
E_natxiplpk (2.8.18)	natxiplpk = natpwage_p - natplcap		1	National movement in relative prices of labour and capital
E_natlambda (2.8.19)	NATLAB_OCC(m)natlambda(m) = $\sum_{q \in RDES} LAB_OCC_{m,q}$ lambda _{m,q}	m∈OCC	8	Demand for labour by occupation
2.3. Governm	ent Finances			
2.3.1. Disagg	regation of value added			
E_z01_r (3.8.1)	$C_Z01_R_qz01_r_q = \sum_{j \in IND} C_Z01_I_R_{j,q}(labind_{j,q} + pwagci_{j,q})$	q∈RDES	8	Wages, salaries and supplements - regions
E_z02_r (3.8.1)	$C_Z02_R_qz02_r_q = \sum C_Z02_I_R_{j,q}(labind_{j,q} + pwage_{j,q})$ j \in IND	q∈RDES	8	Imputed wages - regions
E_z03_r (3.8.1)	$C_{Z03}R_{q}z03_{r_{q}} = \sum_{j \in IND} \sum_{j,q} (rpri_{j,q} + labind_{j,q} + pwagei_{j,q})$	q∈ RDES	8	Payroll taxes - regions
E_z03 (3.8.1)	$C_{Z03 z03} = \sum_{q \in RDES} C_{Z03}R_{q}z03_{r_{q}}$		1	Payroll taxes - national
E_z04_r (3.8.1)	$C_Z04_R_qz04_r_q = \sum_{j \in IND} C_Z04_I_R_{j,q}(curcap_{j,q} + p1cap_{j,q})$	q∈ RDES	8	Returns to fixed capital - regions
E_z05_r (3.8.1)	$C_{Z05_{q}z05_{q}} = \sum_{j \in IND} \sum_{q \in IND} \sum_{j,q} (curcap_{j,q} + p_{1}cap_{j,q})$	q∈ RDES	8	Property taxes - regions
E_z05 (3.8.1)	$C_{Z05 z05} = \sum_{q \in RDES} C_{Z05} R_{q} z_{05} r_{q}$		1	Property taxes - national

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Identifier	Equation	Subscript Range	Number	Description
E_z06_r (3.8.1)	$C_Z06_R_qz06_r_q = \sum_{j \in IND} C_Z06_I_R_{j,q}(n_{j,q} + p land_{j,q})$	q∈RDES	8	Returns to agricultural land - regions
E_z07_r (3.8.1)	$C_Z07_R_q z07_r_q = \sum_{j \in IND} \sum_{q \in I$	q∈RDES	8	Land taxes - regions
E_z07 (3.8.1)	$C_{Z07 z07} = \sum_{q \in RDES} C_{Z07_R_q z07_r_q}$		I	Land taxes - national
E_z08_r (3.8.1)	$C_{Z08}R_{q}z08_{r_{q}} = \sum_{j \in IND} \sum_{k=1}^{2} \sum_{j,q} (x_{j,q} + p_{loct_{j,q}})$	q∈RDES	8	Returns to working capital - regions
E_z09_r (3.8.1)	$C_{Z09}R_{q}z_{9}r_{q} = \sum_{j \in IND} C_{Z09}I_{R_{j,q}}(x_{10ct_{j,q}} + p_{10ct_{j,q}})$	q∈RDES	8	Other indirect taxes - regions
E_z09 (3.8.1)	$C_2 Z_{09} q^{z_{09}} = \sum_{q \in RDES} \sum_{r_q} C_2 Z_{09} R_q Z_{09} r_q$		1	Other indirect taxes - national
E_z10_r (3.8.1)	$C_Z10_R_qz10_r_q = \sum_{j \in IND} C_Z10_I_R_{j,q}(x \operatorname{loct}_{j,q} + \operatorname{ploct}_{j,q})$	q∈RDES	8	Sales by final buyers - regions
E_z10 (3.8.1)	$C_Z10 z10 = \sum_{q \in RDES} C_Z10_R_q z10_r_q$		I	Sales by final buyers - national
E_zg_r (3.8.1)	$C_ZG_R_nzg_r_n = C_Z02_R_nz02_r_n + C_Z04_R_nz04_r_n + C_Z06_R_nz06_r_n + C_Z08_R_qz08_r_q$	q∈RDES	8	Gross operating surplus - regions
E_zt_r 3.8.1)	$C_2T_R_{a}zt_{r_0} = C_203_R_{a}z03_{r_0} + C_205_R_{a}z05_{r_0} + C_207_R_{a}z07_{r_0} + C_209_R_{q}z09_{r_q}$	q∈RDES	8	Production taxes - regions
E_rpr (3.8.1)	$C_{203_{l_R_{j,q}}rpri_{j,q}} = (C_{201_{l_R_{j,q}}} + C_{202_{l_R_{j,q}}} + C_{203_{l_R_{j,q}}arpri_{j,q}})$	j∈ IND q∈ RDES	13×8	Payroll tax adjustment

Identifier	Equation	Subscript Range	Number	Description
E_rpri (3.8.1)	$rpri_{j,q} = rpr_q + frpri_{j,q}$	j∈ IND q∈ RDES	13×8	Setting of payroll tax rates
E_xisfb2 (3.8.1)	$C_{Z10}R_{q}xisfb_{q} = \sum_{j \in IND} C_{Z10}I_{R_{j,q}p1oct_{j,q}}$	q∈RDES	8	Price index : sales by final buyers
2.3.2. Gross r	gional product and its components			
E_dompy100 (3.8.2)	$dompy100_q = z01_r_q$	q∈ RDES	8	Wages, salaries and supplements
E_dompy120 (3.8.2)	dompy120 _q = dompy100 _q + rl	q∈ RDES	8	PAYE taxes
E_dompy110 (3.8.2)	$C_DOMPY100_q dompy100_q = C_DOMPY110_q dompy110_q + C_DOMPY120_q dompy120_q dompy120_q$	20 _q q∈RDES	8	Disposable wage income (residual)
E_dompy200 (3.8.2)	$dompy200_q = zg_r_q$	q∈ RDES	8	Non - wage primary factor income
E_dompy220 (3.8.2)	$dompy220_q = dompy200_q + rk$	q∈RDES	8	Taxes on non - wage primary factor income
E_dompy210 (3.8.2)	$C_{DOMPY200_{q}}dompy200_{q} = C_{DOMPY210_{q}}dompy210_{q} + C_{DOMPY220_{q}}dompy220_{q}dompy220_{q}}dompy220_{q}dompy220_$	20 _q q∈ RDES	8	Disposable non - wage primary factor income (residual)
E_dompy310 (3.8.2)	$dompy310_q = taxrevm_q$	q∈ RDES	8	Tariff revenue
E_dompy320 (3.8.2)	$\begin{split} \text{C_DOMPY320}_{q} \text{dompy320}_{q} &= \text{AGGTAX1}_{q} \text{taxrev1}_{q} + \text{AGGTAX2}_{q} \text{taxrev2}_{q} + \text{AGGTAX3}_{q} \text{taxrev} \\ &+ \text{AGGTAX4}_{q} \text{taxrev4}_{q} + \text{AGGTAX5}_{q} \text{taxrev5}_{q} + \text{AGGTAX6}_{q} \text{taxrev5}_{q} \end{split}$	v3 _q q∈ RDES	8	Other commodity taxes less subsidies
E_dompy330 (3.8.2)	$dompy330_q = zt_r_q$	q∈ RDES	x	Production taxes
E_dompy300 (3.8.2)	$\begin{split} \text{C_DOMPY300}_{q} \text{dompy300}_{q} = \text{C_DOMPY310}_{q} \text{dompy310}_{q} + \text{C_DOMPY320}_{q} \text{dompy320}_{q} \text{dompy330}_{q} \\ &+ \text{C_DOMPY330}_{q} \text{dompy330}_{q} \end{split}$	20 _q q∈RDES	8	Indirect taxes less subsidies

Identifier	Equation	Subscript Range	Number	Description
E_dompy000 (3.8.2)	$\begin{split} \textbf{C}_{DOMPY000_{q}dompy000_{q}} = \textbf{C}_{DOMPY100_{q}dompy100_{q}} + \textbf{C}_{DOMPY200_{q}domp} \\ + \textbf{C}_{DOMPY300_{q}dompy300_{q}} \end{split}$	y200 _q q∈RDES	8	GDP at market prices (income side) - regions
E_dompq110 (3.8.2)	$dompq 110_q = c_q$	q∈ RDES	8	Private consumption
E_dompq120 (3.8.2)	$dompq120_q = in_q$	q∈RDES	8	Private investment
E_dompq130 (3.8.2)	$dompq130_q = othnom5_q$	q∈RDES	8	Government consumption - regions
E_domq140 (3.8.2)	$dompq140_q = othnom6_q$	q∈RDES	8	Government consumption - federal
E_dompq150 (3.8.2)	$dompq150_q = in_q$	q∈ RDES	8	Government investment
E_dompq100 (3.8.2)	$\begin{split} \text{C_DOMPQ100}_{q} \text{dompq100}_{q} &= \text{C_DOMPQ110}_{q} \text{dompq110}_{q} + \text{C_DOMPQ120}_{q} \text{dompq120}_{q} \\ &+ \text{C_DOMPQ130}_{q} \text{dompq130}_{q} + \text{C_DOMPQ140}_{q} \text{dompq140}_{q} \\ &+ \text{C_DOMPQ150}_{q} \text{dompq150}_{q} \end{split}$	q∈RDES	8	Domestic absorption
E_dompq210 (3.8.2)	$dompq210_q = psexp_q + xsexp_q$	q∈ RDES	8	Inter - regional exports
E_dompq220 (3.8.2)	dompq220 _q = psimp _q + xsimp _q	q∈ RDES	8	Inter - regional imports
E_dompq200 (3.8.2)	$C_{DOMPQ200_{q}dompq200_{q}} = C_{DOMPQ210_{q}dompq210_{q}} - C_{DOMPQ220_{q}dompq200_{q}}$	q220 _q q∈ RDES	8	Inter - regional trade balance
E_dompq310 (3.8.2)	$dompq310_q = export_q + natphi$	q∈ RDES	8	International exports
E_dompq320 (3.8.2)	$dompq320_q = imp_q + natphi$	q∈RDES	8	International imports

Identifier	Equation .	Subscript Range	Number	Description
E_dompq300 (3.8.2)	C_DOMPQ300qdompq300q = C_DOMPQ310qdompq310q - C_DOMPQ320qdompq3	320 _q q∈RDES	8	International trade balance
E_dompq000 (3.8.2)	$\begin{split} \text{C}_{\text{DOMPQ000}_{q}\text{dompq000}_{q}} = \text{C}_{\text{DOMPQ100}_{q}\text{dompq100}_{q}} + \text{C}_{\text{DOMPQ200}_{q}\text{dompq}} \\ + \text{C}_{\text{DOMPQ300}_{q}\text{dompq300}_{q}} \end{split}$	l200 _q q∈ RDES	8	GDP at market prices (expenditure side)
2.3.3. Miscella	aneous equations			•
E_tir (3.8.3)	$i_r_q = dompy320_q$	q∈RDES	8	Commodity taxes less subsidies (excl. tariffs)
E_ti (3.8.3)	C_TI ti = $\sum_{q \in RDES} C_DOMPY320_q dompy320_q$		1	Commodity taxes less subsidies (excl. tariffs)
E_yn_r (3.8.3)	$yn_r_q = dompq000_q$	q∈RDES	8	Nominal regional domestic product
E_yn (3.8.3)	$C_YN yn = \sum_{q \in RDES} \sum_{q \in$		1	Nominal GDP
E_xiy_r (3.8.3)	$\begin{split} \textbf{C_DOMPQ000}_q \textbf{xiy}_{r_q} &= \textbf{C_DOMPQ110}_q \textbf{xi3}_q + \textbf{C_DOMPQ120}_q \textbf{xi2}_q + \textbf{C_DOMPQ130}_q \textbf{xi5}_q \\ &+ \textbf{C_DOMPQ140}_q \textbf{xi6}_q + \textbf{C_DOMPQ150}_q \textbf{xi2}_q \\ &+ \textbf{C_DOMPQ210}_q \textbf{psexp}_q - \textbf{C_DOMPQ220}_q \textbf{psimp}_q \\ &+ \textbf{C_DOMPQ310}_q \textbf{xi4}_q - \textbf{C_DOMPQ320}_q \textbf{xim}_q \end{split}$	q∈ RDES	8	GDP deflator
E_xiy (3.8.3)	$C_{YN xiy} = \sum C_{DOMPQ000_q xiy_r_q}$ $q \in RDES$		I	GDP deflator
E_yr_r (3.8.3)	$yr_r_q = yn_r_q - xiy_r_q$	q∈RDES	8	Real regional domestic product
E_yr (3.8.3)	yr = yn - xiy		1	Real GDP
	yr = yn - xiy		i	Real C

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Identifier	Equation	Subscript Range	Number	Description
E_yf (3.8.3)	$C_YFyf = \sum_{q \in RDES} C_DOMPY100_q dompy100_q) + \sum_{q \in RDES} C_DOMPY200_q dompy200_q$		I	GDP at factor cost
E_bstar (3.8.3)	$C_{YN bstar} = \sum_{q \in RDES} \sum$		1	Balance of trade surplus to GDP: percentage - point change
E_ty (3.8.3)	$C_TY iY = \sum C_DOMPY120_q dompy120_q) + \sum C_DOMPY220_q dompy220_q$ $q \in RDES \qquad q \in RDES$		1	Income taxes
E_yl (3.8.3)	$C_YL yl = \sum_{q \in RDES} \sum_{q \in$		1	Pre - tax wage income
E_wn (3.8.3)	wn = yl - nati		1	Nominal pre - tax wage rate
E_ylstar (3.8.3)	$C_YLSTAR y lstar = \sum_{q \in RDES} \sum_{q \in RDE$		1	Post - tax wage income
E_wnstar (3.8.3)	wnstar = ylstar - natl		1	Nominal post - tax wage rate
E_wrstar (3.8.3)	wrstar = wnstar - natxi3		t	Real post - tax wage rate
E_g_rA (3.8.6)	$g_r_q = othnom S_q$	q∈RDES	8	Nominal government consumption - regions
E_g_rB (3.8.6)	$g_r_q = natothnom6$	q=federal	1	Nominal government consumption - federal
E_ip (3.8.6)	$C_{IP ip} = \sum_{q \in RDES} C_{IP}R_{q}in_{q}$		1	Aggregate nominal private investment

Identifier	Equation	Subscript Range	Number	Description
E_ig_r_reg (3.8.6)	$ig_r_q = in_q$	q∈RDES	8	Nominal government investment - regions
E_ig (3.8.6)	NATAGGINVnatin = C_IPip + C_IGig			Aggregate nominal government investment (residual)
E_ig_r_fed (3.8.6)	$C_{IG ig} = \sum_{q \in DDES} C_{IG_{q}ig_{r_{q}}}$		I	Nominal government investment - federal (residual)
E_c_b (3.8.6)	$c_q = yd_r_q + miscf001_q$	q∈ RDES	8	Consumption function
E_rl (3.8.6)	rl = rk + miscf002		1	Relative income tax rates
2.3.4. Summar	y Of Financial Transactions: the SOFT accounts		•	
E_softy111 (3.8.4)	softy 111 _q = ty	q=federal	1	Income taxes
E_softy112A (3.8.4)	$softy 112_q = yn_r_q + softf001_q$	q∈RDES	8	Other direct taxes
E_softy112B (3.8.4)	$softy 112_q = yn + softf011$	q=federal	1	Other direct taxes federal
E_softy110 (3.8.4)	$C_SOFTY110_q softy110_q = C_SOFTY111_q softy111_q + C_SOFTY112_q softy112_q$	q∈DDES	9	Direct taxes
E_softy121 (3.8.4)	$softy 121_q = nattaxrevm$	q=federal	1	Tariff revenue
E_softy122A (3.8.4)	softy $122_q = ti_r_q$	q∈RDES	8	Other commodity taxes - regions
E_softy122B (3.8.4)	C_TI ti + C_SUBSIDIES tI = $\sum_{q \in DDES} C_SOFTY122_q softy122_q$		I	Other commodity taxes - federal (residual)

Identifier	Equation	Subscript Range	Number	Description
E_softy123a (3.8.4)	$softy 123_q = z03_r_q$	q∈ RDES	8	Payroll taxes - regions
E_softy123b (3.8.4)	softy $123_q = 209$	q=federal	I	Fringe benefits taxes - federal
E_softy124 (3.8.4)	$softy 124_q = z05_r_q$	q∈RDES	8	Property taxes - regions
E_softy125 (3.8.4)	$softy 125_q = z07_r_q$	q∈RDES	8	Land taxes - regions
E_softy126A (3.8.4)	$softy 126_q = 209_r_q$	q∈RDES	8	Other indirect taxes - regions
E_softy126B (3.8.4)	$C_Z09 \ z09 = \sum C_SOFTY126_q \text{softy126}_q$ $q \in DDES$		1	Other indirect taxes - federal (residual)
E_softy120 (3.8.4)	$\begin{split} \textbf{C}_{SOFTY120_q} &\text{softy120}_q = \textbf{C}_{SOFTY121_q} \text{softy121}_q + \textbf{C}_{SOFTY122_q} \text{softy122}_q \\ &+ \textbf{C}_{SOFTY123_q} \text{softy123}_q + \textbf{C}_{SOFTY124_q} \text{softy124}_q \\ &+ \textbf{C}_{SOFTY125_q} \text{softy125}_q + \textbf{C}_{SOFTY126_q} \text{softy126}_q \end{split}$	q∈ DDES	9	Indirect taxes
E_softy130 (3.8.4)	softy $130_q = softq 400_q$	q∈ DDES	9	Interest received
E_softy141 (3.8.4)	$\operatorname{softy} 141_q = \operatorname{yn}_r q + \operatorname{soft} 002_q$	q∈ RDES	8	Commonwealth grants to regions - current
E_softy142 (3.8.4)	$softy 142_q = yn_r_q + soft f003_q$	q∈ RDES	8	Commonwealth grants to regions - capital
E_softy140 (3.8.4)	$C_{SOFTY140_q} softy140_q = C_{SOFTY141_q} softy141_q + C_{SOFTY142_q} softy142_q$	q∈ RDES	8	Commonwealth grants to regions
E_softy150A (3.8.4)	$softy 150_q = yn_r_q + softf004_q$	q∈ RDES	8	Other revenue - regions

Identifier	Equation	Subscript Range	Number	Description
E_softy150B (3.8.4)	$softy150_q = yn + softf004_q$	q=federal	1	Other revenue - federal
E_softy100 (3.8.4)	$\begin{split} C_SOFTY100_{q}softy100_{q} &= C_SOFTY110_{q}softy110_{q} + C_SOFTY120_{q}softy120_{q} \\ &+ C_SOFTY130_{q}softy130_{q} + C_SOFTY140_{q}softy140_{q} \\ &+ C_SOFTY150_{a}softy150_{a} \end{split}$	q∈ DDES	9	Government revenue
E_softy200 (3.8.4)	$softy200_q = g_r_q$	q∈DDES	9	Consumption of fixed capital - general government
E_softy300 (3.8.4)	$C_SOFTY_{300}_{q} \circ ofiy_{300}_{q} = C_SOFTQ_{000}_{q} \circ ofiq_{000}_{q} - C_SOFTY_{100}_{q} \circ ofiy_{100}_{q}$ $- C_SOFTY_{200}_{q} \circ ofiy_{200}_{q}$	q∈ DDES	9	Financing transactions
E_softy320 (3.8.4)	$softy320_q = softq100_q + softf005_q$	q∈DDES	9	Increase in provisions
E_softy330 (3.8.4)	$softy330_q = softy300_q + f_oft_q$	q∈DDES	9	Other financing transactions
E_softy310 (3.8.4)	$C_{SOFTY300_{q}sofiy300_{q}} = C_{SOFTY310_{q}sofiy310_{q}} + C_{SOFTY320_{q}sofiy320_{q}} + C_{SOFTY330_{q}sofiy330_{q}}$	q∈DDES	9	Net borrowing (residual)
E_softy000 (3.8.4)	$C_{SOFTY000_{q}} \text{sofiy000}_{q} = C_{SOFTY100_{q}}^{q} \text{sofiy100}_{q} + C_{SOFTY200_{q}}^{q} \text{sofiy200}_{q}$ $+ C_{SOFTY300_{q}}^{q} \text{sofiy300}_{q}$	q∈ DDES	9	Summary of financial transactions : income - side total
E_softq110 (3.8.4)	$softq110_q = g_r_q$	q∈DDES	9	Government consumption
E_softq120 (3.8.4)	$softq120_q = ig_r_q$	q∈ DDES	9	Government investment
E_softq100 (3.8.4)	$C_SOFTQ100_q softq100_q = C_SOFTQ110_q softq110_q + C_SOFTQ120_q softq120_q$	q∈ DDES	9	Expenditure on goods and services
E_softq210 (3.8.4)	$softq210_q = upb$	q=federal	1	Unemployment benefits

Identifier	Equation	Subscript Range	Number	Description
E_softq200 (3.8.4)	$softq200_q = pbp_r_q$	q∈DDES	9	Personal benefit payments
E_softq220 (3.8.4)	$C_{SOFTQ200_{q}}softq200_{q} = C_{SOFTQ210_{q}}softq210_{q} + C_{SOFTQ220_{q}}softq220_{q}$	q∈DDES	9	Other personal benefits (residual)
E_softq300A (3.8.4)	$softq300_q = ti_r_q$	q∈ RDES	8	Subsidies - regions
E_softq300B (3.8.4)	C_SUBSIDIES t1 = $\sum C_SOFTQ300_q \operatorname{softq300}_q$ q DDES	•	1	Subsidies - federal (residual)
E_softq400A (3.8.4)	$softq400_q = yn_r_q + softf007_q$	q∈RDES	8	Interest payments - regions
E_softq400B (3.8.4)	$softq400_k = yn + softf007_k$	k=federal	1	Interest payments - Federal
E_softq510 (3.8.4)	$C_SOFTQ510_k \text{ softq510}_k = \sum_{q \in RDES} C_SOFTY141_q \text{ softy141}_q$	k=federal	1	Commonwealth grants to regions - current
E_softq520 (3.8.4)	$C_{SOFTQ520_{k}} \operatorname{softq520_{k}} = \sum_{q \in RDES} \sum_{q \in $	k=federal	I	Commonwealth grants to regions - capital
E_softq500 (3.8.4)	$C_{SOFTQ500_q} \operatorname{softq500_q} = C_{SOFTQ510_q} \operatorname{softq510_q} + C_{SOFTQ520_q} \operatorname{softq520_q}$	q=federal	1	Commonwealth grants to regions
E_softq600 (3.8.4)	$softq600_q = softq000_q + softf006_q$	q∈DDES	9	Other outlays
E_softq000 (3.8.4)	$\begin{split} \text{C}_{\text{SOFTQ000}_{q}\text{softq000}_{q}} &= \text{C}_{\text{SOFTQ100}_{q}\text{softq100}_{q}\text{softq200}_{q}\text{softq200}_{q}} \\ &+ \text{C}_{\text{SOFTQ300}_{q}\text{softq300}_{q}\text{softq200}_{q}\text{softq400}_{q}} \\ &+ \text{C}_{\text{SOFTQ500}_{q}\text{softq500}_{q}\text{softq200}_{q}\text{softq600}_{q}} \end{split}$	q∈ DDES	9	Summary of financial transactions : expenditure - side total

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Identifier	Equation	Subscript Range	Number	Description
E_realdefr (3.8.4)	realdef _q = $softy300_q - xi3_q$	q∈RDES	8	Real budget deficit for region
E_realdeff (3.8.4)	$realdef_q = softy300_q - natxi3$	q=federal	1	Real budget deficit for Fed.
E_dGstar (3.8.4)	$C_SOFTQ000_q dgstar_q = C_SOFTY310_q(softy310_q - softq000_q)$	q∈ DDES	9	Net borrowing to total outlays: percent point change
E_tod_r (3.8.4)	$tod_r_q = softy112_q$	q∈RDES	8	Other direct taxes
2.3.5. Househ	old disposable income			
E_hhldy110 (3.8.5)	hhldyl10 _q = $z01_r_q$	q∈ RDES	8	Wages, salaries and supplements
E_hhldy120 (3.8.5)	hhldy $120_q = zg_r_q$	q∈ RDES	8	Non - wage primary factor income
E_hhldy100 (3.8.5)	$C_{HHLDY100_{q}hhldy100_{q}} = C_{HHLDY110_{q}hhldy110_{q}} + C_{HHLDY120_{q}hhldy120_{q}}$	q∈ RDES	8	Primary factor income
E_hhldy210 (3.8.5)	$hhldy210_q = natxi3 + C_HHLDD001_qlabsup_q - C_HHLDD002_ql_q + hhldf001_q$	q∈ RDES	8	Unemployment benefit receipts
E_hhldy220 (3.8.5)	$hhldy220_q = natxi3 + pop_q + hhldf002_q$	q∈ RDES	8	Other personal benefit receipts
E_hhldy200 (3.8.5)	$\label{eq:c_HHLDY200} C_HHLDY200_q hhldy200_q = C_HHLDY210_q hhldy210_q + C_HHLDY220_q hhldy220_q$	q∈ RDES	8	Personal benefit receipts
E_hhldy300 (3.8.5)	$hhldy300_q = yn_r_q + hhldf003_q$	q∈RDES	8	Other Income (net)
E_hhldy410 (3.8.5)	$hhldy410_q = hhldy110_q + rl$	q∈ RDES	8.	PAYE taxes

Identifier	Equation	Subscript Range	Number	Description
E_hhldy420 (3.8.5)	hhldy $420_q = hhldy 120_q + rk$	q∈RDES	8	Taxes on non - wage primary factor income
E_hhldy430 (3.8.5)	hhldy $430_q = tod_r_q$	q∈ RDES	8	Other direct taxes
E_hhldy400 (3.8.5)	$C_{HHLDY400_{q}hhldy400_{q}} = C_{HHLDY410_{q}hhldy410_{q}} + C_{HHLDY420_{q}hhldy420_{q}} + C_{HHLDY430_{q}hhldy430_{q}}$	q∈ RDES	8	Direct taxes
E_hhldy000 (3.8.5)	$C_{HHLDY000_{q}hhldy000_{q}} = C_{HHLDY100_{q}hhldy100_{q}} + C_{HHLDY200_{q}hhldy200_{q}} + C_{HHLDY300_{q}hhldy300_{q}} - C_{HHLDY400_{q}hhldy400_{q}}$	q∈RDES	8	Disposable income
E_ydr (3.8.5)	$yd_r_q = hhldy000_q$	q∈RDES	8	Disposable income
E_upb (3.8.5)	C_UPB upb = $\sum C_HHLDY210_q hhldy210_q$ q RDES			Aggregate unemployment benefit payments
E_pbp_r (3.8.5)	$pbp_r_q = hhldy200_q$	q∈RDES	8	Personal benefit payments - regions
E_pbpA (3.8.5)	$C_{PBP \ pbp} = \sum_{q \in RDES} $			Aggregate personal benefit payments
E_pbpB (3.8.5)	$C_{PBP pbp} = \sum_{q \in DDES} C_{PBP_{q}pbp_{r}q}$		I	Personal benefit payments - federal (residual)
2.4. Dynamic	s for Forecasting			
2.4.1. Industr	y capital and investment			
E_yTA (4.4)	$\begin{aligned} \text{VALK}_{T1_{j,q}} \text{curcap}_{t1_{j,q}} &= \text{VALKT}_{j,q} \text{DEP}_{j} \text{curcap}_{j,q} + \text{INVEST}_{j,q} \text{v}_{j,q} \\ & - 100(\text{VALK}_{0_{j,q}}(1 - \text{DEP}_{j})) - \text{INVEST}_{0_{j,q}}) \text{delkfudge} \\ & + 100 \text{delf}_{rate}_{j,q} \end{aligned}$	j∈ IND q∈ RDES	13×8	Investment in period T: forecasting

Identifier	Equation	Subscript Range	Number	Description
E_curcapT1A (4.4)	$curcap_t l_{j,q} = K_TERMcurcap_{j,q}$	j∈ IND q∈ RDES	13×8	Capital stock in period T + 1: forecasting
E_yTB (4.4)	$curcap_{j,q} = y_{j,q} + 100delf_rate_{j,q}$	j∈ IND q∈ RDES	13×8	Investment in period T: comparative statics
E_curcapT1B (4.4)	$\operatorname{curcap}_{1j,q} = \operatorname{curcap}_{j,q}$	j∈ IND q∈ RDES	13×8	Capital stock in period T + 1: comparative statics
E_r0 (4.4)	$r_{j,q} = QCOEF_{j,q}(p1cap_{j,q} - pi_{j,q})$	j∈ IND q∈ RDES	13×8	Definition of rates of return to capital
E_f_rate_xx (4.4)	$r0_{j,q}$ - natr_tot = BETA_R _{j,q} [curcap _{j,q} - kt _q] + f_rate_xx _{j,q}	j∈ IND q∈ RDES	13×8	Capital growth rates related to rates of return
E_naty (4.4)	NATINVEST _j naty _j = \sum INVEST _{j,q} y _{j,q} q e RDES	j∈IND	13	Total real investment
2.4.2. Accumm	ulation of national foreign debt			
E_deldebt (5.5)	deldebt = {DEBT0(R_WORLD^PRIOD - 1) + B0N_DEBT}deldfudge + M_DEBTdelbt		1	Ordinary change in foreign debt
E_delbt (5.5)	100P_GLOBALdelbt = NATAGGIMP(natimpvol) - NATAGGEXP(natexpvol + natxi4 - natxim)		1	Ordinary change in Real trade deficit
E_deldebt_ratio (5.5)	deldebt_ratio = (DEBT_RATIO/DEBT)deldebt - (DEBT_RATIO/100)(natgdpexp - natxin	n)	1	Change in Debt/GDP ratio
2.5. Regional	Population and Labour Market Settings			
E_del_rm (6.5)	$C_{POP_q} pop_q = C_{PR1_q} delrpfudge_q + C_{PA2} (del_{rm_q} + del_{fm_q} + del_{g_q}) + f_{pop_q}$	q∈RDES	8	Accumulation of regional population
E_del_labsup (6.5)	$C_{labsup_q}del_{unr_q} = C_{EMPLOY_q}(labsup_q - cmploy_q)$	q∈RDES	8	Percentage-point changes in regional unemployment rates

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Identifier	Equation	Subscript Range	Number	Description
E_wpop (6.5)	$labsup_q = pr_q + wpop_q$	q∈RDES	8	Regional labour supply
E_pop (6.5)	$wpop_q = pop_q + f_wpop_q$	q∈ RDES	8	Regional working age population
E_rm_0 (6.5)	$del_rm_q = del_rm_0_q + delf_rm_0$	q∈RDES	8	ABS population forecasts can drive interregional migration
E_rempl_inter (6.5)	$fl_q = cmploy_q + f_l_q$	q∈ RDES	8	Interface employment in wage-bill weights and person weights.
E_pop_interf (6.5)	$qhous_q = pop_q + f_qhous_q$	q∈RDES	8	Interface population and householf formation.
E_rm_addup (6.5)	$delf_rm = \sum_{q \in RDES} \int del_rm_q$		1	Adding - up condition on interregional migration.
E_delnatfm (6.5)	$del_natfm = \sum del_fm_q$ q \in RDES		I	National forcign migration.
E_delnatg (6.5)	$del_natg = \sum_{q \in RDES} del_{g_q}$		1	National natural population change.
E_natlabsup (6.5)	$C_NATLABSUP \text{ natlabsup} = \sum C_LABSUP_q \text{ labsup}_q$ $q \in RDES$		t	National labour supply
E_natemploy (6.5)	$C_NATEMPLOY \text{ natemploy} = \sum_{q \in RDES} C_EMPLOY_q employ_q$		I	National employment
E_natunr (6.5)	C_NATLABSUP del_naturr = C_NATEMPLOY(natlabsup - natemploy)		I	Percentage-point change in national unemployment rate

Variable	Subscript Range	Number	Description
The CGE Co	ore		
Scalar Varia	bles		
deltax l all		1	Overall percent-point change in indirect tax rates, user 1
deltax2all		1	Overall percent-point change in indirect tax rates, user 2
deltax3all		1	Overall percent-point change in indirect tax rates, user 3
deltax4all		1	Overall percent-point change in indirect tax rates, user 4
deltax5all		1	Overall percent-point change in indirect tax rates, user 5
deltax6all		1	Overall percent-point change in indirect tax rates, user 6
natc		1	Nominal total household consumption
natcaprev		1	Aggregate payments to capital
nater		1	Real household consumption
natdelb		1	Ordinary change in balance of trade
natexport		1	Foreign-currency value of exports
natexpvol		1	Export volumes
natfep		1	Economy-wide shifter of export demand curves
natf5gen		1	Overall shift term for regional "Other" demands
natf6gen		1	Overall shift term for Federal "Other" demands
natfwage		1	Overall wage shifter
natgdpexp		1	Nominal GDP from expenditure side
natgdpinc		1	Nominal GDP from income side
natgdpreal		1	Real GDP from expenditure side
natimp		1	Foreign currency value of imports
natimpvol		1	Import volumes
natin		1	Aggregate nominal investment
natir		1	Aggregate real investment expenditure

Table 2.2. The MMRF Variables

MONASH-MRF: A multiregional model of Australia

Variable	Subscript Range	Number	Description
natkt		1	Aggregate capital stock, rental weights
natl		1	Aggregate employment, wage bill weights
natlabrev		1	Aggregate payments to labour
natIndrev		1	Aggregate payments to land
natoctrev		1	Aggregate other cost ticket payments
natothnom5		1	Aggregate nominal value of regional "Other" demands
natothnom6		1	Aggregate nominal value of Federal "Other" demands
natothreal5		1	Aggregate real regional "Other" demands
natothreal6		1	Aggregate real Federal "Other" demands
natplcap		1	Aggregate nominal capital rentals
natphi		1	Exchange rate
natpwage		1	Aggregate nominal wage paid to workers
natpwage_p		1	Aggregate nominal wage paid by producers
natrealwage		1	National consumer real wage
natrwage_p		I	National real wages for producers: deflated by GDP deflator
natrwage_w		1	National real wages for workers: deflated by CPI
nattaxind		1	Aggregate revenue from all indirect taxes
nattaxrev1		1	Aggregate revenue from indirect taxes on intermediate
nattaxrev2		1	Aggregate revenue from indirect taxes on investment
nattaxrev3		t	Aggregate revenue from indirect taxes on households
nattaxrev4		1	Aggregate revenue from indirect taxes on exports
nattaxrev5		I	Aggregate revenue from indirect taxes on regional "Other"
nattaxrev6		1	Aggregate revenue from indirect taxes on Federal "Other"
nattaxrevm		1	Aggregate tariff revenue
nattot		I	Economy-wide terms of trade
natxi2		1	Investment price index
natxi3		1	Consumer price index

Theoretical structure of MONASH-MRF

Variable	Subscript Range	Number	Description
natxi4		1	Exports price index
natxi5		1	Regional "Other" demands price index
natxi6		1	Federal "Other" demands price index
natxigdp		1	GDP price index, expenditure side
natxim		1	Imports price index
natximp0		1	Duty-paid imports price index
natxiplpk		1	Relative prices of labour and capital
natz_tot		1	Aggregate Output: Value-Added Weights
Vector Varial	bles		
aggnt_fep _s	s∈ RSOU	8	Price shifter on non-traditional exports
aggnt_p4r _s	s∈ RSOU	8	Aggregate price for non-traditional exports
aggnt_x4r _s	s∈RSOU	8	Demand for aggregate non-traditional exports
caprev _q	q∈ RDES	8	Aggregate payments to capital
cq	q∈RDES	8	Nominal total household consumption
cr_shr _q	q∈RDES	8	Regional/national consumption ratio
crq	q∈RDES	8	Real household consumption
delb_dom _q	q∈RDES	8	Change in interregional trade balance
delb_for_audq	q∈RDES	8	Change in AUD value of foreign trade balance
delb_tot	q∈RDES	8	Sum change in of domestic and foreign trade balance
deltaxdest _q	q∈ ADES	10	Tax shifter (percentage-point change) to all destinations including foreign and Federal
deltaxsource _s	s∈ ASOU	9	Tax shifter (percentage-point change) by all sources (domestic and foreign)
exp_for_aud _q	q∈RDES	8	AUD value of foriegn exports
export	q∈RDES	8	Foreign currency value of exports
expvolq	q∈ RDES	8	Export volumes
f5gen _q	q∈ RDES	8	Overall shift term for regional "Other" demands
f6gen _q	q∈RDES	8	Shifter, Federal "Other" demand
faggnt_i	i∈ NTEXP	11	Shifter by commodity for aggregate non-traditional exports
faggnt_p4rs	s∈ RSOU	8	Shifter on the price of aggregate non-traditional exports

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Variable	Subscript Range	Number	Description
faggnt_s	s∈RSOU	8	Shifter by region for aggregate non-traditional exports
fep _i	i∈COM	13	Price (upward) shift in export demands
feq _i	i∈COM	13	Quantity (right) shift in export demands
fwage _q	q∈ RDES	8	Overall real wage shifter
imp_for_aud _q	q∈RDES	8	AUD value of imports
imp _q	q∈RDES	8	Foreign currency value of imports
impvol _q	q∈RDES	8	Import volume index
in _q	q∈RDES	8	Aggregate nominal investment
ir _q	q∈RDES	8	Aggregate real investment expenditure
kt _q	q∈RDES	8	Aggregate capital stock, rental weights
labrev	q∈RDES	8	Aggregate payments to labour
Indrev _q	q∈RDES	8	Aggregate payments to land
l	q∈RDES	8	Aggregate employment- wage bill weights
luxexp _q	q∈RDES	8	Total supernumerary household expenditure
natlabind _j	j∈IND	13	Employment by Industry
natlambda _m	m∈OCC	8	Employment in occupation M
natx0imp _i	i∈COM	13	Import volumes
naty _j	j∈IND	13	Capital creation by using industry
natz _j	j∈ IND	13	Activity level or value-added
octrev _q	q∈RDES	8	Aggregate other cost ticket payments
othnom5 _q	q∈RDES	8	Aggregate nominal regional "Other" demands
othnom6 _q	q∈RDES	8	Aggregate nominal Federal "Other" demand
othreal5 _q	q∈ RDES	8	Aggregate real regional "Other" demands
othreal6 _q	q∈RDES	8	Aggregate real Federal "Other" demand
pmi	i∈COM	13	C.I.F. foreign currency import prices
powtaxm _i	i∈COM	13	Power of tariffs
psexp _s	s∈RSOU	8	Price indices for interregional exports
psimp _q	q∈RDES	8	Price indices for interregional imports
pwage _q	q∈RDES	8	Region-wide nominal wage received by workers
pwage_p _q	q∈ RDES	8	Region-wide nominal wage paid by producers
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....Table 2.2. continued

Theoretical structure of MONASH-MRF

Variable	Subscript Range	Number	Description
qhousg	q∈RDES	8	Number of households
realwage_w _q	q∈RDES	8	Real wages for workers: deflated by CPI
realwage_p _q	q∈RDES	8	Real wages for producers: deflated by the GDP deflator
reg_plcap _q	q∈RDES	8	Regional rental price of capital
totdomq	q∈RDES	8	Domestic terms of trade
totfor _q	q∈RDES	8	Foreign terms of trade
r0_tot _q	q∈RDES	8	Regional aggregate rate of return
taxind _q	q∈RDES	8	Aggregate revenue from all indirect taxes
taxrev l _q	q∈RDES	8	Aggregate revenue, indirect taxes on intermediate
taxrev2 _q	q∈RDES	8	Aggregate revenue, indirect taxes on investment
taxrev3 _q	q∈RDES	8	Aggregate revenue, indirect taxes on households
taxrev4 _s	s∈ RSOU	8	Aggregate revenue, indirect taxes on foreign exports
taxrev5 _q	q∈RDES	8	Aggregate revenue, indirect taxes on regional "Other"
taxrev6q	q∈RDES	8	Aggregate revenue, indirect tax on Federal "Other"
taxrevmq	q∈RDES	8	Aggregate tariff revenue
utilityq	q∈RDES	8	Utility per household
wage_diff _q	q∈RDES	8	Regional real wage differential
ximq	q∈RDES	8	Imports price index
ximp0 _q	q∈RDES	8	Duty-paid imports price index
xiplpk _q	q∈RDES	8	Index of relative price of labour and capital
xiy_r _q	q∈RDES	8	Regional GDP deflator
xi2 _q	q∈RDES	8	Investment price index
xi3 _q	q∈RDES	8	Consumer price index
xi4 _q	q∈RDES	8	Exports Price Index
xi5 _q	q∈RDES	8	Regional "Other" demands price index
xi6 _q	q∈RDES	8	Federal "Other" demands price index
xsexp	s∈RSOU	8	Exports volume in interregional trade
xsimp	q∈ RDES	8	Imports volume in interregional trade
z_tot _q	q∈RDES	8	Aggregate output: Value-added weights

....Table 2.2. continued

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Variable	Subscript Range	Number	Description
Matrix Vari	ables		
alcap _{j.q}	j∈ IND q∈ RDES	13×8	Capital augmenting technical change
al _{j.q}	j∈ IND q∈ RDES	13×8	All input augmenting technical change
allab _{j.q}	j∈ IND q∈ RDES	13×8	Labor augmenting technical change
alland _{j,q}	j∈ IND q∈ RDES	13×8	Land augmenting technical change
aloct _{j.q}	j∈ IND q∈ RDES	13×8	Other cost ticket technial change
alprim _{j.q}	j∈ IND q∈ RDES	13×8	All primary factor technical change
a2ind _{j.q}	j∈ IND q∈ RDES	13×8	Technical change in capital creation
a3com _{i.q}	i∈ COM q∈ RDES	13×8	Change in household tastes
a3lux _{i.q}	i∈ COM q∈ RDES	13×8	Change in household tastes, luxury
a3sub _{i.q}	i∈ COM q∈ RDES	13×8	Change in household taste, subsistence
a _{j.q}	j∈ IND q∈ RDES	13×8	Average of technical change terms in production
arpri _{j.q}	j∈ IND q€ RDES	13×8	Payroll tax adjustment factor
curcap _{j.q}	j∈ IND q∈ RDES	13×8	Current capital stock
deltax l _{i.s.j.q}	i∈ COM s∈ ASOU j∈ IND q∈ RDES	13×10×13×8	Percentage-point change in tax rate on sales of intermediate inputs
deltax2 _{i.s.j.q}	i∈COM s∈ASOU j∈IND q∈RDES	13×10×13×8	Percentage-point change in tax rate on sales for capital creation

Variable	Subscript Range	Number	Description
deltax3 _{is.q}	i∈ COM s∈ ASOU q∈ RDES	13×10×8	Percentage-point change in tax rate on sales to households
deltax4 _{i,s}	i∈COM s∈RSOU	13×8	Percentage-point change in export tax rates
deltax5 _{i.s.q}	i∈ COM s∈ ASOU q∈ RDES	13×10×8	Percentage-point change in tax rate on sales to regional "Other" demand
deltax6 _{i.s.q}	i∈ COM s∈ ASOU q∈ RDES	13×9×8	Percent-point change in sales tax rate, Federal government demands
deltax _i	i∈COM	13	Percentage-point change in the general sales tax rate
efflab _{j.q}	j∈ IND q∈ RDES	13×8	Effective labour input
floct _{j,q}	j∈ IND q∈ RDES	13×8	Shifters, other cost tickets
f5a _{i.s.q}	i∈ COM s∈ ASOU q∈ RDES	13×10×8	Shift in regional "Other" demands
f6a _{i.s.q}	i∈ COM s∈ ASOU q∈ RDES	13×9×8	Shift, Federal "Other" demand
faggnt_is _{i.s}	i∈ NTEXP s∈ RSOU	11×8	Commodity and source shifter for non-traditional exports
frPRi _{j.q}	j∈ IND q∈ RDES	13×8	Payroll tax rate shifter
fwagei _{j.q}	j∈ IND q∈ RDES	13×8	Industry-specific wage shifter
labind _{j.q}	j∈ IND q∈ RDES	13×8	Employment by industry
lambda _{mq}	m∈ OCC q∈ RDES	8×8	Employment by occupation
n _{j.q}	j∈ IND q∈ RDES	13×8	Use of land
p0a _{i,s}	i∈ COM s∈ ASOU	13×10	Basic price of good i, source s

Variable	Subscript Range	Number	Description
pla _{i,s.j.q}	i∈ COM s∈ ASOU j∈ IND q∈ RDES	13×10×13×8	Prices of inputs for current production
plcap _{j.q}	j∈ IND q∈ RDES	13×8	Rental price of capital
plc _{i,j.q}	i∈ COM j∈ IND q∈ RDES	13×13×8	Prices of domestic composite inputs for current production
pllab _{j.q}	j∈ IND q∈ RDES	13×8	Price of labour
p1laboi _{j.q.m}	j∈ IND q∈ RDES m∈ OCC	13×8×8	Wage of occupation type m in industry j
plland _{j,q}	j∈ IND q∈ RDES	13×8	Rental price of land
ploct _{j.q}	j∈ IND q∈ RDES	13×8	Price of other cost tickets
plo _{i.j.q}	i∈ COM j∈ IND q∈ RDES	13×13×8	Price, domestic/foreign composite inputs for current production
p2a _{i.s.j.q}	i∈ COM s∈ ASOU j∈ IND q∈ RDES	13×10×13×8	Prices of inputs for capital creation
p2c _{i.j.q}	i∈ COM j∈ IND q∈ RDES	13×13×8	Prices of domestic composite inputs for capital creation
p2o _{i.j.q}	i∈COM j∈IND q∈RDES	13×13×8	Price, domestic/foreign composite inputs for capital creation
рЗа _{і.s.q}	i∈ COM s∈ ASOU q∈ RDES	13×10×8	Purchasers prices by commodities and source for households
p3c _{i.q}	i∈COM q∈RDES	13×8	Prices of domestic composite inputs for households

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Variable	Subscript Range	Number	Description
p3o _{i.q}	i∈ COM q∈ RDES	13×8	Price, domestic/foreign composite inputs for households
p4r _{i,s}	i∈ COM s∈ RSOU	13×8	F.O.B. foreign currency export prices
p5a _{i.s.q}	i∈ COM s∈ ASOU q∈ RDES	13×10×8	Purchasers' prices for commodities (by source) by regional "Other"
рба _{і.s.q}	i∈ COM s∈ ASOU q∈ RDES	13×10×8	Purchasers' prices paid for commodities by Federal "Other"
pi _{j.q}	j∈ IND q∈ RDES	13×8	Costs of units of capital
psflo _{s.q}	s∈ RSOU q∈ RDES	8×8	Price indices in interregional trade flows
pwagei _{j.q}	j∈ IND q∈ RDES	13×8	Nominal wage rates
r0 _{j.q}	j∈ IND q∈ RDES	13×8	Current rates of return on capital
rpri _{j.q}	j∈ IND q∈ RDES	13×8	Payroll tax rate (in per cent)
x0imp _{i.q}	i∈ COM q∈ RDES	13×8	Import volumes
xla _{i.s.j.q}	i∈ COM s∈ ASOU j∈ IND q∈ RDES	13×10×13×8	Demands for inputs for current production
xlc _{i,j.q}	i∈ COM j∈ IND q∈ RDES	13×13×8	Demands for domestic composite inputs for current production
x11aboi _{j.q.m}	j∈ IND q∈ RDES m∈ OCC	13×8×8	Employment of occupation type m in industry j
x l marg _{i.s.j.q.} r	i∈ COM s∈ ASOU j∈ IND q∈ RDES r∈ MARG	13×10×13×8 ×2	Margins - current production

Variable	Subscript Range	Number	Description
xloct _{j,q}	j∈ IND q∈ RDES	13×8	Demand for other cost tickets
x1o _{i.j.q}	i∈ COM j∈ IND q∈ RDES	13×13×8	Demands for domestic/foriegn composite inputs for current production
xlprim _{j.q}	j∈ IND q∈ RDES	13×8	Demand for primary factor composite
x2a _{i,s,j,q}	i∈COM s∈ASOU j∈IND q∈RDES	13×10×13×8	Demands for inputs for capital creation
x2c _{i.j.q}	i∈COM j∈IND q∈RDES	13×13×8	Demands for domestic composite inputs for capital creation
x2marg _{i.s.j.q.r}	i∈ COM s∈ ASOU j∈ IND q∈ RDES r∈ MARG	13×10×13×8 ×2	Margins - capital creation
x2o _{i.j.q}	i∈ COM j∈ IND q∈ RDES	13×13×8	Demands for domestic/foriegn composite inputs for capital creation
x3a _{i.s.q}	i∈ COM s∈ ASOU q∈ RDES	13×10×8	Household demand for goods
x3c _{i.q}	i∈ COM q∈ RDES	13×8	Demands for domestic composite inputs for households
x3marg _{i.s.q.r}	i∈ COM s∈ ASOU q∈ RDES r∈ MARG	13×10×8×2	Margins - on sales to households
x30 _{i.q}	i∈ COM q∈ RDES	13×8	Demands for domestic/foriegn composite inputs for households
x4marg _{i.s.s}	i∈ COM s∈ RSOU r∈ MARG	13×8×2	Margins - on exports

Variable	Subscript Range	Number	Description
x4r _{i,s}	i∈ COM s∈ RSOU	13×8	Export volumes
x5a _{i.s.q}	i∈ COM s∈ ASOU q∈ RDES	13×10×8	Regional "Other" demands
x5marg _{i,s,q,r}	i∈ COM s∈ ASOU q∈ RDES r∈ MARG	13×10×8×2	Margins - regional "Other"
хба _{і.s.q}	i∈ COM s∈ ASOU q∈ RDES	13×10×8	Federal "Other" demands in each region
x6marg _{i,s.q.r}	i∈ COM s∈ ASOU q∈ RDES r∈ MARG	13×10×8×2	Margins - on sales to Federal " Other" demand
xi_fac _{j.q}	j∈ IND q∈ RDES	13×8	Index of factor costs
xiplpk_ind _{j.q}	j∈ IND q∈ RDES	13×8	Index of ratio of price of labour to price of capital
xsflo _{s.q}	s∈ RSOU q∈ RDES	8×8	Volume of inter and intraregional trade flows
У _{ј.q}	j∈ IND q∈ RDES	13×8	Capital creation by using industry
z _{j.q}	j∈ IND q∈ RDES	13×8	Activity level or value-added
Government	Finances		
Scalar Varia	bles		
bstar		1	Balance of trade surplus as percentage of GDP
ig		1	Nominal government investment
ip		1	Nominal private investment
miscf002		1	Shift variable : relative income tax rates
pbp		1	Personal benefit payments
rl		1	Tax rate - wages, salaries and supplements
rk		2 1	Tax rate - non-wage primary factor income

Variable	Subscript Range	Number	Description
softf011		1	Shifter for Federal collection of other indirect taxes
ti		1	Commodity taxes less subsidies (excl tariffs)
ty		1	Income taxes
upb		1	Unemployment benefits
wn		1	Nominal pre-tax wage rate
wnstar		1	Nominal post-tax wage rate
wrstar		1	Real post-tax wage rate
xiy		1	GDP deflator
yf		1	GDP at factor cost
yl		1	Pre-tax wage income
ylstar		1	Post-tax wage income
yn		1	Nominal GDP
yr		1	Real GDP
z03		1	Payroll taxes
z05		1	Property taxes
z07		1	Land taxes
z09		1	Other indirect taxes
z10		1	Sales by final buyers
Vector Varia	bles		
dgstar _q	q∈ DDES	9	Government net borrowing/total outlays
debtg _q	q∈ DDES	9	Government debt
g_r _q	q∈ DDES	9	Government consumption
ig_r _q	q∈ DDES	9	Government investment
labsup _q	q€ RDES	8	Labour supply
pbp_r _q	q∈ DDES	9	Personal benefit payments
rpr _q	q∈ RDES	8	payroll tax rate
ti_r _q	q∈ RDES	8	Commodity taxes less subsidies (excl tariffs)
tod_r _q	q∈ RDES	8	Other direct taxes
xisfb _q	q∈ RDES	8	price index : sales by final buyers
yn_r _q	q∈ RDES	8	Nominal GSP - regions
yr_r _q	q∈ RDES	8	Real GSP - regions

Variable	Subscript Range	Number	Description
yd_r _q	q∈RDES	8	Household disposable income
z01_r _q	q∈RDES	8	Wages, salaries and supplements
z02_r _q	q∈RDES	8	Imputed wages
z03_r _q	q∈RDES	8	Payroll taxes
z04_r _q	q∈RDES	8	Returns to fixed capital
z05_r _q	q∈ RDES	8	Property taxes
z06_r _q	q∈RDES	8	Returns to agricultural land
z07_r _q	q∈RDES	8	Land taxes
z08_r _q	q∈ RDES	8	Returns to working capital
z09_r _q	q∈RDES	8	Other indirect taxes
z10_r _q	q∈RDES	8	Sales by final buyers
zg_r _q	q∈RDES	8	Gross operating surplus
zt_r _q	q∈RDES	8	Production taxes
dompy000 _q	q∈RDES	8	GSP at market prices (income side)
dompy100 _q	q∈RDES	8	Wages, salaries and supplements
dompy110 _q	q∈RDES	8	Disposable wage income
dompy120 _q	q∈ RDES	8	PAYE taxes
dompy200 _q	q∈RDES	8	GOS : non-wage primary factor income
dompy210 _q	q∈RDES	8	Disposable non-wage primary factor income
dompy220 _q	q∈RDES	8	Taxes on non-wage primary factor income
dompy300 _q	q∈ RDES	8	Indirect taxes less subsidies
dompy310 _q	q∈ RDES	8	Tariff revenue
dompy320 _q	q∈RDES	8	Other commodity taxes less subsidies
dompy330 _q	q∈RDES	8	Production taxes
dompq000 _q	q∈RDES	8	GSP at market prices (expenditure side)
dompq100 _q	q∈RDES	8	Domestic absorption
dompq110 _q	q∈RDES	8	Private consumption
dompq120 _q	q∈RDES	8	Private investment
dompq130 _q	q∈RDES	8	Government consumption - regions
dompq140 _q	q∈RDES	8	Government consumption - federal
dompq150 _q	q∈RDES	8	Government investment

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Variable	Subscript Range	Number	Description
dompq200 _q	q∈RDES	8	Interregional trade balance
dompq210 _q	q∈RDES	8	Interregional exports
dompq220 _q	q∈RDES	8	Interregional imports
dompq300 _q	q∈ RDES	8	International trade balance
dompq310 _q	q∈RDES	8	International exports
dompq320 _q	q∈RDES	8	International imports
softy000 _q	q∈DDES	9	SOFT : income side total
softy 100 _q	q∈ DDES	9	Government revenue
softy110 _q	q∈DDES	9	Direct taxes
softy111 _q	q∈ DDES	9	Income taxes
softy112 _q	q∈ DDES	9	Other direct taxes
softy120g	q∈DDES	9	Indirect taxes
softy 121	q∈ DDES	9	Tariff revenue
softy 122	q∈DDES	9	Other commodity taxes
softy123	q∈DDES	9	Payroll taxes
softy124 _g	q∈DDES	9	Property taxes
softy125	q∈DDES	9	Land taxes
softy 126	q∈DDES	9	Other indirect taxes
softy130g	q∈ DDES	9	Interest received
softy 140 _q	q∈DDES	9	Commonwealth grants to regions
softy141 _q	q∈DDES	9	Current grants
softy 142 _g	q∈ DDES	9	Capital grants
softy150	q∈ DDES	9	Other revenue
softy200g	q∈ DDES	9	Consumption of fixed capital - general government
softy300g	q∈ DDES	9	Financing transactions
softy310g	q∈DDES	9	Net borrowing
softy320 _q	q∈DDES	9	Increase in provisions
softy330 _q	q∈ DDES	9	Other financing transactions
f_oft _q	q∈DDES	9	Other financing transactions shifter
realdef	q∈ DDES	9	Real government budget deficit
softq000 _q	q∈DDES	9	SOFT: expenditure side total

Variable	Subscript Range	Number	Description
softq100 _q	q∈DDES	9	Expenditure on goods and services
softq110 _q	q∈ DDES	9	Government consumption
softq120	q∈DDES	9	Government investment
softq200 _q	q∈DDES	9	Personal benefit payments
softq210 _q	q∈ DDES	9	Unemployment benefits
softq220 _q	q∈DDES	9	Other personal benefits
softq300 _q	q∈ DDES	9	Subsidies
softq400 _q	q∈DDES	9	Interest payments
softq500 _q	q∈ DDES	9	Commonwealth grants to regions
softq510 _q	q∈DDES	9	Current grants
softq520 _q	q∈ DDES	9	Capital grants
softq600 _q	q∈DDES	9	Other outlays
softf001 _q	q∈ RDES	8	Shift variable : other direct taxes
softf002 _q	q∈RDES	8	Shift variable : current Commonwealth grants
softf003 _q	q∈RDES	8	Shift variable : capital Commonwealth grants
softf004 _q	q∈ DDES	9	Shift variable : other revenue
softf005 _q	q∈ DDES	9	Shift variable : increase in provisions
softf006 _q	q∈DDES	9	Shift variable : other outlays
softf007 _q	q∈ DDES	9	Shift variable : government debt
hhldy000 _q	q∈ RDES	8	Disposable income
hhldy100 _q	q∈ RDES	8	Primary factor income
hhldy110 _q	q∈ RDES	8	Wages, salaries and supplements
hhldy120 _q	q∈ RDES	8	Non-wage primary factor income
hhldy200 _q	q∈ RDES	8	Personal benefit receipts
hhldy210 _q	q∈ RDES	8	Unemployment benefits
hhldy220 _q	q∈ RDES	8	Other personal benefits
hhldy300g	q∈RDES	8	Other income (net)
hhldy400 _q	q∈ RDES	8	Direct taxes
hhldy410 _q	q∈RDES	8	PAYE taxes
hhldy420 _q	q∈ RDES	8	Taxes on non-wage primary factor income
hhldy430 _q	q∈RDES	8	Other direct taxes

Variable	Subscript Range	Number	Description
hhldf001	q∈ RDES	8	Shift variable : unemployment benefits
hhldf002	q∈ RDES	8	Shift variable : other personal benefits
hhldf003	q∈ RDES	8	Shift variable : other income (net) - hholds
miscf001	q∈ RDES	8	Shift variable : consumption function
	Capital Accum	ulation	
Scalar Varia	bles		
delkfudge		1	Dummy variable to switch on capital accumulation equation
natr_tot		1	Average rate of return
Matrix Varia	ıbles		
delf_rate _{j.q}	j∈ IND q∈ RDES	13×8	A change shifter in capital accumulation equation
f_rate_xx _{j.q}	j∈ IND q€ RDES	13×8	Shifter, rate of return equation
curcap_t1 _{j.q}	j∈ IND q€ RDES	13×8	Capital stock in period T+1
National For	eign Debt		· · · ·
deldfudge		1	Dummy variable in equation E_delDebt
delbt		1	Ordinary change in Real trade deficit
deldebt_ratio		1	Change in Debt/GDP ratio
deldebt		1	Ordinary change in foreign debt
Regional Pop	oulation and L	abour Mark	ets
Scalar Varia	bles		
del_natfm		1	Ordinary change in national net foreign migration.
del_natg		1	Ordinary change in national net natural population.
del_natunr		1	Percentage-point change in the national unemployment rate
delf_rm		1	Shift variable in equation E_rm_addup
delf_rm_0		1	Shift variable in equation E_rm_0
natlabsup		1	National labour supply.
natemploy			National employment.

Variable	Subscript Range	Number	Description
Vector Varia	bles		
del_fm _q	q∈ RDES	8	Ordinary change in regional net foreign migration.
del_g	q∈ RDES	8	Ordinary change in regional net natural population.
del_rm _q	q∈ RDES	8	Ordinary change in net interregional migration.
del_rm_0 _q	q∈RDES	8	Exogenous forecast of interregional migration
delrpfudge _q	q∈RDES	8	Dummy variable in equation E_del_rm.
del_unr _q	q∈RDES	8	Percentage-point change in regional unemploy rate
employ _q	q∈RDES	8	Regional employment: persons.
f_la	q∈ RDES	8	Shift variable in equation E_remploy_interf.
f_pop _g	q∈RDES	8	Shift variable in equation E_del_rm.
f_wpop _g	q∈RDES	8	Shift variable in equation E_wpop
f_qhous _q	q∈ RDES	8	Shift variable in equation E_pop_interf
pop _q	q∈RDES	8	Regional population
pr _q	q∈RDES	8	Regional workforce participation rate
wpop _q	q∈ RDES	8	Regional population of working age

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Coefficient	Description	Value
σ1C _i	Elasticity of substitution between regional sources of good i for use as	Arbitrarily set at five times the value of the elasticity of substitution between domestic
SIA _{i.s.j.q}	an input in production. Share of purchasers-price value of good i from regional source s in industry j in region q's total purchases of good i from domestic sources for use in production	goods and foreign imported goods: The numerator of this share, for the isjq th element (s \neq 9), is the sum of the corresponding elements of the BAS1, MAR1 and TAX1 matrices. The denominator is the sum, over the 8 s regional sources (i.e., for all s; s \neq 9), of the sum of the isjq th elements of BAS1, MAR1 and TAX1.
σ10 _i	Elasticity of substitution between domestic composite and foreign import of good i for use as an input in production.	Econometric estimate from MONASH model.
PVAL10 _{i.j.q}	Purchaser's value of domestic composite and foreign import of good i as an input the jq th industry's production.	The ijq^{th} value is the sum, over the 9 (all domestic plus foreign) sources, of the sum of the $isjq^{th}$ elements of BAS1, MAR1 and TAX1.
PVALIA _{i.s.j.q}	Purchaser's value of good i from the s^{th} source (s = 1,,9) as an input in the jq th industry's production.	The isjq th value (s = 1,,9) is the sum of the corresponding elements of the BAS1, MAR1 and TAX1 matrices. Dividing by PVAL10. forms the share of the is th good in the firm's total expenditure on the ith good from all s sources (s = 1,,9) for use in production.
σ1LAB _{i.o}	Elasticity of substitution for labour of m occupational types used as an inputs to production by industry j in region q	Econometric estimate from MONASH model.
LABOUR _{j.q}	Total wage bill of the jq th industry.	Sum of the m elements of LABR for the jq th industry. Dividing by TOTFACIND gives the share of the wage bill in the value-added of the jq th industry.
LAB_OCC_IND _{m,jq}	Wage bill of the m th occupation used by the jq th industry.	The jqm th element of LABR. Dividing by LABOUR gives the share of the wage bill of the m th occupational group in the total wage bill of the jq th industry.

Table 2.3. The MMRF Coefficients and Parameters

Coefficient	Description	Value
σ1FAC _{i.α}	Elasticity of substitution for primary factors (agricultural land, labour and capital) used as inputs to production by industry j in region q.	Econometric estimate from MONASH model.
TOTFACIND _{j.q}	Total cost of primary factors used by industry j in region q.	Sum of jq th element of coefficient LABOUR and the corresponding jq th elements of the CPTL and LAND matrices.
CAPITAL _{j.q}	Value of rentals to capital for industry j in region q.	The jq th element of the CPTL matrix.
LAND _{j.q}	Value of rentals to agricultural land for industry j in region q.	The jq th element of the LAND matrix.
σ2C _i	Elasticity of substitution between regional sources of good i for use as an input in the creation of capital.	Arbitrarily set at five times the value of the elasticity of substitution between domestic goods and foreign imported goods:
σ20 _i	Elasticity of substitution between domestic composite and foreign import of good i for use as an input in creation of capital.	Econometric estimate from MONASH model.
PVAL2T _{i.ss.j.q}	Purchaser's value of domestic composite good i as an input the jq th industry's creation of capital.	The sum, over the 8 regional sources (ss = domestic; i.e., for all s; $s \neq 9$), of the sum of the isjq th elements of BAS2, MAR2 and TAX2.
PVAL2A _{i.s.j.q}	Purchaser's value of good i from the	The isjq th value (s = 1,,8) is the sum of the
(in E_p2c)	s th regional source (s = 1,,8) as an input in the jq th industry's creation of capital.	corresponding elements of the BAS2, MAR2 and TAX2 matrices. Dividing by PVAL2T. forms the share of the is th good in the firm's total expenditure on the ith good from all regional sources ($s = 1,,8$) for the purpose of capital creation.
PVAL2A _{i.s.j.q}	Purchaser's value of good i from the	The isjq th value (s = 1,,9) is the sum of the
(in E_p2o)	all sources (s = 1,,9) as an input in the jq th industry's creation of capital.	corresponding elements of the BAS2, MAR2 and TAX2 matrices. Dividing by PVAL20, forms the share of the is th good in the firm's total expenditure on the ith good from all sources (s = 1,,9) for the purpose of capital creation.
PVAL2O _{i.j.q}	Purchaser's value of domestic composite and foreign import of good i as an input the jq th industry's capital creation.	The ijq^{th} value is the sum over the 9 (all domestic plus foreign) sources of the sum of the $isjq^{th}$ elements of BAS2, MAR2 and TAX2.

....Table 2.3. continued

Coefficient	Description	Value
σ3C _i	Elasticity of substitution between regional sources of good i in regional household demand.	Arbitrarily set at five times the value of the elasticity of substitution between domestic goods and foreign imported goods:
σ20;	Elasticity of substitution between domestic composite and foreign import of good i in regional household demand.	Econometric estimate from MONASH model.
PVAL3T _{i.domestic.q}	Purchaser's value of domestic composite good i consumed by the household in region q.	The sum, over the 8 regional sources, of the sum of the isq th elements of BAS3, MAR3 and TAX3.
PVAL3A _{i.s.q}	Purchaser's value of good i from the	The isq th value ($s = 1,,8$) is the sum of the
(in E_p3c)	s th regional source (s = 1,8) consumed by the household in region q.	corresponding elements of the BAS3, MAR3 and TAX3 matrices. Dividing by PVAL3T, forms the share of the is th good in the household's total expenditure on the ith good from all regional sources (s = 1,,8).
PVAL3A _{i.s.q}	Purchaser's value of good i from the	The isq th value ($s = 1,,9$) is the sum of the
(in E_p30)	all sources (s = $1,,9$) consumed by the household in region q.	corresponding elements of the BAS3, MAR3 and TAX3 matrices. Dividing by PVAL30, forms the share of the is th good in the household's total expenditure on the ith good from all sources (s = 1,,9).
PVAL3O _{i.q}	Purchaser's value of domestic composite and foreign import of good i consumed by the household in region q.	The iq th value is the sum, over the 9 (all domestic plus foreign) sources of the sum of the isq th elements of BAS3, MAR3 and TAX3.
ALPHA_I _{i.q}	Supernumerary expenditure on good i as a share of supernumerary and subsistence expenditure on good i by the household in region q.	The formula for ALPHA_ $I_{i,n}$ is ALPHA_ $I_{i,n}$ =-(1/FRISCH _n)×(DELTA _{i,n} /S3COM _{i,n}) where FRISCH _n is an econometrically estimated parameter taken from the MONASH model database, and DELTA _{i,n} and S3COM _{i,n} are described below.
DELTA _{i.q}	Marginal household budget share of good i in total marginal expenditure by the household in region q.	Econometric estimate from MONASH model.
S3COM _{i.q}	Share of good i in total household expenditure by the household in region q.	The formula for S3COM _{in} is S3COM _{in} = PVAL3O _{in} /AGGCON _n where PVAL3O _{in} is described above and AGGCON _n is described below.

Coefficient	Description	Value
EXP_ELAST _i	Foreign export demand elasticity of good i.	Econometric estimate from MONASH model.
AGGEXPNT _s	Total regional non-traditional export earnings.	The sum, over all non-traditional export commodities $(1 = 3,,13)$ of PVAL4R _i , (which is described below).
PVAL4R _{is}	Purchaser's value of non-traditional	The is th value (i = 3,13, s = 1,,8) is the
(in E_aggnt_p4r)	export good i (i = 3,13) from the s th regional source (s = 1,,8) consumed foreigners.	sum of the corresponding elements of the BAS4, MAR4 and TAX4 matrices. Dividing by AGGEXPNT, forms the share of the i^{th} good in the foreigners expenditure on the all goods from region s (s = 1,,8).
COSTS _{j.q}	Total costs of production in industry j in region q	The jq th value is the sum over the 9 (all domestic plus foreign) sources of the sum of the isjq th elements of the BAS1, MAR1 and
		TAX1 matrices, plus the sum over m occupational types of the corresponding element of the LABR matrix, plus the corresponding elements of CPTL and OCTS matrices.
OTHCOST _{j.q}	Other cost tickets paid by industry j in region q.	The jq th value is the corresponding element in the OCTS matrix.
INVEST _{j.q}	Value of total capital created for industry j in region q.	The jq^{th} value is the sum over i (i = 1,,13) of PVAL2O.
BAS1 _{i.s.j.q}	Basic value of good i from source s (domestic and foreign) used in	The isjq th value is the corresponding element in the BAS1 matrix.
TAX1 _{i.s.j.q}	production by industry j in region q. Value of commodity taxes paid on good i from source s (domestic and foreign) used in production by	The isjq th value is the corresponding element in the TAX1 matrix.
MAR1 _{i.s.j.q.r}	industry j in region q. Value of expenditure on margin commodity r, to facilitate the transfer of good i from source s	The isjqr th value is the corresponding element in the MAR1 matrix.
BAS2 _{i.s.j.q}	(domestic and foreign) to industry j in region q to be used in production. Basic value of good i from source s (domestic and foreign) used in capital creation by industry j in region q.	The isjq th value is the corresponding element in the BAS2 matrix.

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Coefficient	Description	Value
TAX2 _{i.s.j.q}	Value of commodity taxes paid on	The isjqth value is the corresponding element
	good i from source s (domestic and	in the TAX2 matrix.
	foreign) used in capital creation by	
	industry j in region q.	
MAR2 _{i.s.j.q.r}	Value of expenditure on margin	The isjqr th value is the corresponding
	commodity r, to facilitate the	element in the MAR2 matrix.
	transfer of good i from source s	
	(domestic and foreign) to industry j	
	in region q to be used in capital creation.	
BAS3	Basic value of good i from source s	The isq th value is the corresponding element
BAS3 _{i,s.q}	(domestic and foreign) consumed by	in the BAS3 matrix.
	the household in region q.	in the DASS matrix.
TAX3 _{i.s.q}	Value of commodity taxes paid on	The isq th value is the corresponding element
1.3.9	good i from source s (domestic and	in the TAX3 matrix.
	foreign) consumed by the household	
	in region q.	
MAR3 _{i.s.q.r}	Value of expenditure on margin	The isqr th value is the corresponding element
	commodity r, to facilitate the	in the MAR3 matrix.
	transfer of good i from source s	4
	(domestic and foreign) to the	
	household in region q.	
PVAL4R _{is}	Purchaser's value of good i (i =	The is th value (i = 1,13, s = 1,,8) is the
(in E_p4r)	1,13) from the s th regional source	sum of the corresponding elements of the
	(s = 1,,8) consumed foreigners.	BAS4, MAR4 and TAX4 matrices.
BAS4 _{i,s}	Basic value of good i from source s	The is th value is the corresponding element
	(domestic and foreign) sold to the	in the BAS4 matrix.
m	foreigner.	more that the second second
TAX4 _{i.s}	Value of commodity taxes paid on	The is th value is the corresponding element
	good i from source s (domestic and	in the TAX4 matrix.
MADA	foreign) sold to the foreigner. Value of expenditure on margin	The isr th value is the corresponding element
MAR4 _{i,s,r}	commodity r, to facilitate the	in the MAR4 matrix.
	transfer of good i from source s	in the provide matrix.
	(domestic and foreign) to the	
	Australian port of departure for	
	shipment to the foreign consumer.	·

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Coefficient	Description	Value
PVAL5A _{i.s.q}	Purchaser's value of good i (i = 1,13) from the s th source (domestic and foreign) to be used by the regional government in region q.	The isq th value is the sum of the corresponding elements of the BAS5, MAR5 and TAX5 matrices.
BAS5 _{i.s.q}	Basic value of good i from source s (domestic and foreign) consumed by the regional government in region q.	The isq th value is the corresponding element in the BAS5 matrix.
TAX5 _{i.s.q}	Value of commodity taxes paid on good i from source s (domestic and foreign) consumed by the regional government in region q.	The isq th value is the corresponding element in the TAX5 matrix.
MAR5 _{i,s.q.r}	Value of expenditure on margin commodity r, to facilitate the transfer of good i from source s (domestic and foreign) to the regional government in region q.	The isqr th value is the corresponding element in the MAR5 matrix.
PVAL6A _{is.q}	Purchaser's value of good i (i = 1,13) from the s th source (domestic and foreign) to be used by the Federal government in region 9.	The isq th value is the sum of the corresponding elements of the BAS5, MAR5 and TAX6 matrices.
BAS6 _{i.s.q}	Basic value of good i from source s (domestic and foreign) consumed by the Federal government in region q.	The isq th value is the corresponding element in the BAS6 matrix.
TAX6 _{i.s.q}	Value of commodity taxes paid on good i from source s (domestic and foreign) consumed by the Federal government in region q.	The isq th value is the corresponding element in the TAX6 matrix.
MAR6 _{i.s.q.r}	Value of expenditure on margin commodity r, to facilitate the transfer of good i from source s (domestic and foreign) to the Federal government in region q.	The isqr th value is the corresponding element in the MAR6 matrix.

Coefficient	Description	Value
SALES	Total value of sales received by the	The rsth value is the sum of sales for direct
(in E_mkt_clear_	producer of the r th margin	usage and sales for use as a margin. The
margins)	commodity produced in the s th	direct sales are the sum of: the sum over j
-	region. Note that we distinguish two	industries and q regions of the corresponding
	purposes for which margin	isjq th elements of the BAS1 and BAS2
	commodities are purchased (i) for	matrices (for $i = r$); the sum over q regions of
	direct usage and (ii) to facilitate	the corresponding isq th elements of the
	trade. Our convention regarding the	BAS3, BAS5 and BAS6 matrices (for i = r);
	second form of usage is that the	and the corresponding element the BAS4
	margin commodity is produced by	matrix (for $r = i$). For the margin sales
	the corresponding margin industry	component, we are adding the purchases of
	in the purchasing region of the	the r th margin commodity facilitating the transfer of all i commodities from all ss
	traded commodity.	(regional and foreign) sources to be used by
		(i) the j industries in the q th region (MAR1
		and MAR2); (ii) the household in the q th
		region (MAR3); (iii) the governments in the
		q th region (MAR5 and MAR6); and the
		exporter in the q th where it is assumed that
		the q th region is the region producing the
		margin (i.e., for $q = s$).
SALES	Total value of sales received by the	The rs th value is the sum of: the sum over j
(in E_mkt_clear_	producer of the r th nonmargin	industries and q regions of the isjq th elements
nonmargins)	commodity produced in the s th	of the BAS1 and BAS2 matrices (for $i = r$);
U ,	region. Note that nonmargin	the sum over q regions of the isq th elements
	commodities are purchased only for	of the BAS3, BAS5 and BAS6 matrices (for
	direct usage.	i = r); and the corresponding element the
		BAS4 matrix (for $r = i$).
IMPORTS _{i.q}	Total basic-value imports of good i	The iq th value is the sum of: the sum over j
	into region q.	industries of the corresponding isjq th (s = $\frac{1}{2}$
		foreign) elements of the BAS1 and BAS2
		matrices; and (ii) the corresponding isq th
ACCTAVI	Salas tay on ourrant production	elements of BAS3, BAS5 and BAS6. The q^{th} value is the sum over i, s (s = 1,,9)
AGGTAXI _q	Sales tax on current production collected in region q.	and j of the corresponding is jq^{th} elements of
	concerca în region q.	the TAX1 matrix.
AGGTAX2 _q	Sales tax on capital creation	The q th value is the sum over i, s (s = 1,,9)
q	collected in region q.	and j of the corresponding isigth elements of
		the TAX2 matrix.
		Table 2.3 continued

Coefficient	Description	Value
AGGTAX3 _q	Sales tax on household consumption collected in region q.	The q^{th} value is the sum over i and s (s = 1,,9) of the corresponding isq th elements of the TAX3 matrix.
AGGTAX4 _q	Sales tax on exports collected in region q.	The q^{th} value is the sum over i, for $s = q$, of the corresponding iq^{th} elements of the TAX4 matrix.
AGGTAX5 _q	Sales tax on regional government consumption collected in region q.	The q^{th} value is the sum over i and s (s = 1,,9) of the corresponding isq th elements of the TAX5 matrix.
AGGTAX6 _q	Sales tax on Federal government consumption collected in region q.	The q^{th} value is the sum over i and s (s = 1,,9) of the corresponding isq th elements of the TAX6 matrix.
AGGCAPq	Total payments to capital in region q.	The q^{th} value is the sum over j of the corresponding j q^{th} elements of CAPITAL _{in} .
AGGLAB _q	Total payments to labour in region q.	The q th value is the sum over j of the corresponding jq th elements of LABOUR _{in} .
AGGLND _q	Total payments to agricultural land in region q.	The q th value is the sum over j of the corresponding jq th elements of LAND _{in} .
AGGOCT _q	Total payments to other cost tickets in region q.	The q th value is the sum over j of the corresponding jq th elements of OTHCOST _{in} .
AGGTAX _q	Aggregate indirect tax revenue collected by region q.	The q th value is the sum of the corresponding q th elements of AGGTAX1 _a , AGGTAX2 _a , AGGTAX3 _a and AGGTAX5 _a .
TARIFF _{i.q}	The value of tariffs collected on good i in region q.	The iq th value is the corresponding element in the MMRF database file TARF.
AGGTAXM _q	The total value of tariffs collected in region q.	The q^{th} value is the sum over i of the corresponding i q^{th} elements of TARIFF _{i.a} .
AGGCONq	Total value in purchaser's prices of household expenditure in region q.	The q^{th} value is the sum over i of the corresponding i q^{th} elements of PVAL3O _{i a} .
AGGINV _q	Total value in purchaser's prices of capital creation in region q.	The q^{th} value is the sum over i of the corresponding i q^{th} elements of INVEST _{i o} .
AGGOTH5 _q	Total value in purchaser's prices of regional government expenditure in region q.	The q^{th} value is the sum over i and s of the corresponding isq th elements of PVAL5A _{i s o} .
AGGOTH6 _q	Total value in purchaser's prices of Federal government expenditure in region q.	The q th value is the sum over i and s of the corresponding isq th elements of PVAL6A _{i s n} .

....Table 2.3. continued

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Coefficient	Description	Value
C_XSFLO _{s,q}	Basic value of inter and intra regional trade flows.	The sq th value is the sum of: the sum over i and j of the corresponding isjq th elements of BAS1 _{i s in} ; the sum over i and j of the corresponding isjq th elements of BAS2 _{i s in} ; and the sum over i of the corresponding isq th elements of BAS3 _{i s o} and BAS5 _{i s o} .
C_XSEXP _s	Total value of interregional exports from region s.	The s th value is the difference between the sum over q of the corresponding sq th elements of C_XSFLO and the corresponding ss th element of C_XSFLO.
C_XSIMP _q	Total value of interregional imports to region q.	The q^{th} value is the difference between the sum over s of the corresponding sq th elements of C_XSFLO and the corresponding qq^{th} element of C_XSFLO.
AGGEXP _q	Total value of export earnings in region q.	The q th value is the sum over i of corresponding ig th elements of PVAL4R.
IMPCOST _{i.q}	Total ex-duty value of imports of good i to region q.	The iq th value is the sum of: the sum over j of the corresponding isjq th elements (s = foreign) of the BAS1 and BAS2 matrices; and the corresponding isq th elements (s = foreign) of the BAS3, BAS5 and BAS6 matrices
AGGIMP _q	Total value of foriegn import expenditures in region q.	The q th value is the sum over i of corresponding iq th elements of IMPCOST.
TOTFACq	Total primary factor payments in region q.	The q th value is the sum of the corresponding elements of AGGLAB, AGGCAP and AGGLND.
LAB_OCC _{mq}	Total labour bill in occupation m in region q.	The mq th value is the sum over j of the corresponding mjq th elements of LAB_OCC_IND.
NAT coefficients	All coefficients with the NAT prefix are national add-ups formed by	Example:
	summing across the q elements of the corresponding regional variable.	$NATAGGTAX1 = \sum_{q=1}^{AGGTAX1} q$
NATGDPIN	National GDP from the income side.	The sum of NATTOTFAC, NATAGGOCT and NATAGGTAX.
NATGDPEX	National GDP from the expenditure side.	The sum of NATAGGCON, NATAGGINV, NATAGGOTH5, NATAGGOTH6, NATAGGEXP and NATAGGIMP.

Coefficient	Description	Value
C_Z01_I_R _{i.q}	Industry by region value of wages,	The jq th value is the corresponding element
54	salaries and supplements.	in the MMRF database file FZ01.
C_Z01_R _a	Regional value of wages, salaries	The q th value is the sum, over j, of the
4	and supplements.	corresponding jq th elements of C_Z01_I_R
C_Z02_I_R _{j.q}	Industry by region value of imputed	The jq th value is the corresponding element
	wages.	in the MMRF database file FZ02.
C_Z02_R _q	Regional value of imputed wages.	The q th value is the sum, over j, of the
-		corresponding jq th elements of C_Z02_I_R.
C_Z03_I_R _{j.q}	Industry by region value of payroll	The jq th value is the corresponding element
	taxes	in the MMRF database file FZ03.
C_Z03_R _q	Regional value of payroll taxes.	The q th value is the sum, over j, of the
		corresponding jq th elements of C_Z03_I_R.
C_Z03	National value of payroll taxes.	The sum of the elements of C_Z03_R.
C_Z04_I_R _{j,q}	Industry by region value of returns	The jq th value is the corresponding element
	to fixed capital.	in the MMRF database file FZ04.
C_Z04_R _q	Regional value of returns to fixed	The q th value is the sum, over j, of the
-	capital.	corresponding jq th elements of C_Z04_I_R.
C_Z05_I_R _{j.q}	Industry by region value of property	The jq th value is the corresponding element
	taxes	in the MMRF database file FZ05.
C_Z05_R _q	Regional value of property taxes.	The q th value is the sum, over j, of the
		corresponding jq th elements of C_Z05_I_R.
C_Z05	National value of property taxes.	The sum of the elements of C_Z05_R.
C_Z06_I_R _{j.q}	Industry by region value of returns	The jq th value is the corresponding element
	to agricultural land.	in the MMRF database file FZ06.
C_Z06_R _q	Regional value of returns to	The q th value is the sum, over j, of the
	agricultural land.	corresponding jq th elements of C_Z06_I_R.
C_Z07_I_R _{j.q}	Industry by region value of land	The jq th value is the corresponding element
	taxes.	in the MMRF database file FZ07.
C_Z07_R _q	Regional value of land taxes.	The q th value is the sum, over j, of the
		corresponding jq th elements of C_Z07_1_R.
C_Z07	National value of land taxes.	The sum of the elements of C_Z07_R.
C_Z08_I_R _{j,q}	Industry by region value of returns	The jq th value is the corresponding element
	to working capital.	in the MMRF database file FZ08.
C_Z08_R _q	Regional value of returns working	The q th value is the sum, over j, of the
	capital.	corresponding jq th elements of C_Z08_I_R.
C_Z09_I_R _{j.q}	Industry by region value of other	The jq th value is the corresponding element
	indirect taxes.	in the MMRF database file FZ09.
C_Z09_R _q	Regional value of other indirect	The q th value is the sum, over j, of the
	taxes.	corresponding jq th elements of C_Z09_I_R.

....Table 2.3. continued

Coefficient	Description	Value
C_Z09	National value of other indirect taxes.	The sum of the elements of C_Z09_R.
C_Z010_I_R _{j.q}	Industry by region value of sales by final buyers.	The jq th value is the corresponding elemen in the MMRF database file FZ010.
C_Z010_R _q	Regional value of sales by final buyers.	The q th value is the sum, over j, of the corresponding jq th elements of C_Z010_1_R.
C_Z010	National value of sales by final buyers.	The sum of the elements of C_Z010_R.
C_ZG_R _q	Regional value gross operating surplus.	The q th value is the sum of the corresponding elements of C_Z02_R. C_Z04_R. C_Z06_R and C_Z08_R.
C_ZT_R _q	Regional value production taxes.	The q th value is the sum of the corresponding elements of C_Z03_R, C_Z05_R, C_Z07_R and C_Z09_R.
C_DOMPY000 _q	Gross regional product: income side.	The q th value is the corresponding element in the MMRF database file PA01.
C_DOMPY100 _q	Regional wages, salaries and supplements.	The q th value is the corresponding element in the MMRF database file PA02.
C_DOMPY110 _q	Regional disposable wage income.	The q th value is the corresponding element in the MMRF database file PA03.
C_DOMPY120 _q	Regional PAYE taxes.	The q th value is the corresponding element in the MMRF database file PA04.
C_DOMPY200 _q	Regional GOS : non-wage primary factor income.	The q th value is the corresponding element in the MMRF database file PA05.
C_DOMPY210 _q	Regional disposable non-wage income.	The q th value is the corresponding element ir the MMRF database file PA06.
C_DOMPY220 _q	Regional tax collected on non-wage income.	The q th value is the corresponding element in the MMRF database file PA07.
C_DOMPY300 _q	Regional indirect taxes less subsidies.	The q th value is the corresponding element in the MMRF database file PA08.
C_DOMPY310 _q	Regional collection of tariff revenue.	The q th value is the corresponding element in the MMRF database file PA09.
C_DOMPY320 _q	Regional tax collected on other indirect taxes and subsidies.	The q th value is the corresponding element in the MMRF database file PA10.
C_DOMPY330 _q	Regional collection of production taxes.	The q th value is the corresponding element in the MMRF database file PA11.
C_DOMPQ000 _q	Gross regional product: expenditure	The q th value is the corresponding element in
C_DOMPQ100 _q	side. Regional domestic absorption.	the MMRF database file PA12. The q th value is the corresponding element in the MMRF database file PA13.

Coefficient	Description	Value
C_DOMPQ110	Regional private consumption.	The q th value is the corresponding element in
r		the MMRF database file PA14.
C_DOMPQ120	Regional private investment.	The q th value is the corresponding element in
л Т		the MMRF database file PA15.
C_DOMPQ130	Regional-government consumption.	The q th value is the corresponding element in
т. 		the MMRF database file PA16.
C_DOMPQ140	Federal-government consumption.	The q th value is the corresponding element in
7		the MMRF database file PA17.
C_DOMPQ150 _a	Government investment.	The q th value is the corresponding element in
ч		the MMRF database file PA18.
C_DOMPQ200	Interregional trade balance.	The q th value is the corresponding element in
ч		the MMRF database file PA19.
C_DOMPQ210	Interregional exports.	The q th value is the corresponding element in
· 4		the MMRF database file PA20.
C_DOMPQ220	Interregional imports.	The q th value is the corresponding element in
- 4		the MMRF database file PA21.
C_DOMPQ300	Regional international trade balance.	The q th value is the corresponding element in
ч	-	the MMRF database file PA22.
C_DOMPQ310	Regional international exports.	The q th value is the corresponding element in
4		the MMRF database file PA23.
C_DOMPQ320	Regional international imports.	The q th value is the corresponding element in
- 4		the MMRF database file PA24.
C_DOMPY330 _a	Regional collection of production	The q th value is the corresponding element in
- y	taxes.	the MMRF database file PA11.
C_TI	National collection of commodity	The sum of the elements of C_DOMPY320.
-	taxes less subsidies (excluding	•
	tariffs).	
C_YN	National nominal GDP.	The sum of the elements of C_DOMPY000.
C_YF	National nominal GDP at factor	The sum of the elements of C_DOMPY100
-	cost.	and C_DOMPY200.
NATBT	National nominal trade balance	The sum of the elements of C_DOMPQ300.
C_TY	National income tax collection.	The sum of the elements of C_DOMPY120
-		and C_DOMPY220.
C_YL	National pre-tax wage income.	The sum of the elements of C_DOMPY100.
C_YLSTAR	National post-tax wage income.	The sum of the elements of C_DOMPY110.
C_IP_R _a	Regional nominal private	The q th value is the corresponding element in
ч	investment.	the MMRF database file MI02.
C_IP	National nominal private	The sum of the elements of C_IP_R.
	investment.	

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Coefficient	Description	Value
C_IG_R _q	Regional nominal government	The q th value is the corresponding element in
1	investment (by regional and Federal	the MMRF database file MI03.
	governments).	
C_IG	National nominal government	The sum of the elements of C_IG_R.
	investment.	
C_SOFTY000 _q	SOFT: income-side total.	The q th value is the corresponding element in
		the MMRF database file GA01.
C_SOFTY100 _q	Government revenue.	The q th value is the corresponding element in
		the MMRF database file GA02.
C_SOFTY110 _q	Direct taxes.	The q th value is the corresponding element in
		the MMRF database file GA03.
C_SOFTY111 _q	Income taxes.	The q th value is the corresponding element in
		the MMRF database file GA04.
C_SOFTY112 _q	Other direct taxes.	The q th value is the corresponding element in
		the MMRF database file GA05.
C_SOFTY120 _q	Indirect taxes.	The q th value is the corresponding element in
		the MMRF database file GA06.
C_SOFTY121 _q	Tariff revenue.	The q th value is the corresponding element in
		the MMRF database file GA07.
C_SOFTY122 _q	Other commodity taxes.	The q th value is the corresponding element in
		the MMRF database file GA08.
C_SOFTY123 _q	Payroll taxes.	The q th value is the corresponding element in
		the MMRF database file GA09.
C_SOFTY124 _q	Property taxes.	The q th value is the corresponding element in
		the MMRF database file GA10.
C_SOFTY125 _q	Land taxes.	The q th value is the corresponding element in
		the MMRF database file GA11.
C_SOFTY126 _q	Other indirect taxes.	The q th value is the corresponding element in
		the MMRF database file GA12.
C_SOFTY130 _q	Interest received.	The q th value is the corresponding element in
		the MMRF database file GA13.
C_SOFTY140 _q	Commonwealth grants to the	The q th value is the corresponding element in
	regions.	the MMRF database file GA14.
C_SOFTY141 _q	Current grants.	The q th value is the corresponding element in
		the MMRF database file GA15.
C_SOFTY142 _q	Capital grants.	The q th value is the corresponding element in
		the MMRF database file GA16.
C_SOFTY150 _q	Other revenue.	The q th value is the corresponding element in
-		the MMRF database file GA17.

Coefficient	Description	Value
C_SOFTY200 _q	Depreciation, general government.	The q th value is the corresponding element in the MMRF database file GA18.
C_SOFTY300 _q	Financing transactions.	The q th value is the corresponding element in
ч		the MMRF database file GA19.
C_SOFTY310	Net borrowings.	The q th value is the corresponding element in
7		the MMRF database file GA20.
C_SOFTY320g	Increase in provisions.	The q th value is the corresponding element in
1		the MMRF database file GA21.
C_SOFTY330 _a	Other financing transactions.	The q th value is the corresponding element in
4		the MMRF database file GA22.
C_SOFTQ000a	SOFT: expenditure-side total.	The q th value is the corresponding element in
r		the MMRF database file GA23.
C_SOFTQ100	Expenditure on goods and services.	The q th value is the corresponding element in
4		the MMRF database file GA24.
C_SOFTQ110	Government consumption.	The q th value is the corresponding element in
4		the MMRF database file GA25.
C_SOFTQ120	Government investment.	The q th value is the corresponding element in
ч		the MMRF database file GA26.
C_SOFTQ200 _a	Personal benefit payments.	The q th value is the corresponding element in
		the MMRF database file GA27.
C_SOFTQ210	Unemployment benefits.	The q th value is the corresponding element in
		the MMRF database file GA28.
C_SOFTQ220	Other personal benefits.	The q th value is the corresponding element in
7		the MMRF database file GA29.
C_SOFTQ300	Subsidies.	The q th value is the corresponding element in
- 4		the MMRF database file GA30.
C_SOFTQ400	Interest payments.	The q th value is the corresponding element in
ч		the MMRF database file GA31.
C_SOFTQ500	Commonwealth grants to regions.	The q th value is the corresponding element in
4		the MMRF database file GA32.
C_SOFTQ510	Current grants.	The q th value is the corresponding element in
4		the MMRF database file GA33.
C_SOFTQ520	Capital grants.	The q th value is the corresponding element in
y		the MMRF database file GA34.
C_SOFTQ600	Other outlays.	The q th value is the corresponding element in
- 4	÷	the MMRF database file GA35.
C_SUBSIDIES	Subsidies.	The sum of the elements of C_SOFTQ300.
C_HHLDY000	Disposable income.	The q th value is the corresponding element in
q		the MMRF database file HA01.

Coefficient	Description	Value	
C_HHLDY100q	Primary factor income.	The q th value is the corresponding element in	
		the MMRF database file HA02.	
C_HHLDY110 _q	Wages, salaries and supplements.	The q th value is the corresponding element in	
		the MMRF database file HA03.	
C_HHLDY120 _q	Non-wage primary factor income.	The q th value is the corresponding element in	
-		the MMRF database file HA04.	
C_HHLDY200 _q	Personal benefit receipts.	The q th value is the corresponding element in	
-		the MMRF database file HA05.	
C_HHLDY210 _q	Unemployment benefits.	The q th value is the corresponding element in	
		the MMRF database file HA06.	
C_HHLDY220 _q	Other personal benefits.	The q th value is the corresponding element in	
		the MMRF database file HA07.	
C_HHLDY300 _q	Other income (net).	The q th value is the corresponding element in	
		the MMRF database file HA08.	
C_HHLDY400 _a	Direct taxes.	The q th value is the corresponding element in	
•		the MMRF database file HA09.	
C_HHLDY410	PAYE taxes.	The q th value is the corresponding element in	
• .		the MMRF database file HA10.	
C_HHLDY420	Taxes on non-wage primary factor	The q th value is the corresponding element in	
T	income.	the MMRF database file HAII.	
C_HHLDY430	Other direct taxes.	The q th value is the corresponding element in	
7		the MMRF database file HA12.	
C_HHLDY410	PAYE taxes.	The q th value is the corresponding element in	
л		the MMRF database file HA10.	
C_LABSUP	Labour supply.	The q th value is the corresponding element in	
ч		the MMRF database file RLBS.	
C_EMPLOY	Persons employed.	The q th value is the corresponding element in	
7		the MMRF database file REMP.	
C_HHLDD001	Reciprocal of the unemployment	The q th value is given by the formula	
4	rate.	C_LABSUP/(C_LABSUP-C_EMPLOY).	
C_HHLDD002	Reciprocal of the proportion of the	The q th value is given by the formula	
4	unemployed to the employed.	C_EMPLOY/(C_LABSUP-C_EMPLOY).	
C_UPB	National payments of	The sum of the elements of C_HHLDY210.	
	unemployment benefits.		
C_PBP_R _q	Personal benefit payments.	The q th value is the corresponding element in	
ч		the MMRF database file MI04.	
C_PBP	National payments of personal	The sum of the elements of C_PBP_R.	
	benefits.	-	
VALK_TI _{j.q}	Asset value of the capital stock in	The jq th value is the corresponding element	
- J.Q	•	in the MMRF database file VALK.	

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Coefficient	Description	Value
VALKT _{j.q}	Asset value of the capital stock in	The jq th value is the corresponding element
	period T.	in the MMRF database file VALK.
VALK_0	Asset value of the capital stock in	The jq th value is the corresponding element
	period 0.	in the MMRF database file VALK.
INVEST_0 _{j.q}	Asset value of investment in period	The jq th value is the corresponding element
	0.	of the coefficient INVEST.
DEPj	Depreciation factor, one minus the	The j th value is the corresponding element in
	rate of depreciation.	the MMRF database file DPRC.
K_TERM	A constant in equation	Its value is given by the formula $1+(1/T)$.
	E_curcapT1A.	
QCOEF	Ratio of gross to net rate of return.	The jq th value is the corresponding element
		in the MMRF database file P027.
BETA_R _{j.q}	Parameter to distribute investment.	The jqth value is the corresponding element
*	Can be thought of as risk premia.	in the MMRF database file BETR.
DEBT_RATIO	National foreign debt to GDP ratio.	Its value is stored in the MMRF database file DGDP.
R_WORLD	World real interest rate factor, one	Its value is stored in the MMRF database file
	plus the real world rate of interest.	RWLD.
P_GLOBAL	Converts \$A into 'real' terms.	It is set equal to NATXIM.
BO	National real trade deficit in year 0.	Its value is given by the formula (NATAGGIMP-NATAGGEXP+NATXI4- NATXIM)/P_GLOBAL.
M_DEBT &	Constants in the foreign debt	Functions of time; see section 2.4.2 for their
N_DEBT	accumulation equations.	formulas.
C_POP _q	Regional population.	The q th value is the corresponding element in the MMRF database file RPOP.
C_RM _q	Net interregional migration.	The q th value is the corresponding element in the MMRF database file RRGM.
C_FMq	Net regional foreign migration.	The q th value is the corresponding element in the MMRF database file RFRM.
C_Gq	Net regional natural population.	The q th value is the corresponding element in the MMRF database file RGRO.
C DD1	Constant in the normalizion	
C_PR1 _q	Constant in the population	The q th value is given by the formula
0.040	accumulation equation.	100T(C_RM+C_FM+C_RM).
C_PA2	Constant in the population	Its value is given by the formula 50(T+1).
C NATI ADDID	accumulation equation.	The sum of the elements of C 1 APSUD
C_NATLABSUP	National labour supply.	The sum of the elements of C_LABSUP.
C_NATEMPLOY	National employment: persons.	The sum of the elements of C_EMPLOY.

Set	TABLO Name	Description	Elements
СОМ	СОМ	Commodities	 13: Agriculture, Mining, Manufacturing, Public Utilities, Construction, Domestic Trade, Transport & Communication, Finance, Housing, Public Services, Community Services, Personal Services, Non-competing Imports
RSOU	REGSOURCE	Regional sources	8: NSW, Victoria, Queensland, South Australia, Western Australia, Tasmania, Northern Territory, ACT
IND	IND	Industries	13: Same as for COM
RDES	REGDEST	Regional destinations	8: Same as for RSOU
OCC	occ	Occupation types	8: ASCO categories
TEXP	TEXP	Traditional exports	2: Agriculture, Mining
NTEXP	NTEXP	Non-traditional exports	 11: Manufacturing, Public Utilities, Construction, Domestic Trade, Transport & Communication, Finance, Housing, Public Services, Community Services, Personal Services, Non-competing Imports
ASOU	ALLSOURCE	Origin of goods	9: Same as RSOU plus foreign
MARG	MARCOM	Margin commodities	2: Domestic Trade, Transport & Communication
NONMARG	NONMARCOM	Non-margin commodities	 11: Agriculture, Mining, Manufacturing, Public Utilities, Construction, Finance, Housing, Public Services, Community Services, Personal Services, Non-competing Imports
DDES	DOMDEST	Destination of goods	9: Same as for RSOU plus 'Federal'.

Table 2.4. Dimensions of MMRF

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