DEMAND PATTERNS ACROSS THE DEVELOPMENT SPECTRUM:

ESTIMATES OF THE AIDADS SYSTEM

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ABSTRACT

This is a companion paper to Impact Preliminary Working Paper No OP-73 in which Rimmer and Powell report on a new implicitly directly additive demand system (AIDADS) which (in Cooper and McLaren's 1992b terminology) is effectively globally regular. In OP-73 AIDADS is fitted to a six-commodity disaggregation of a 35-year Australian time series of consumption. Unlike the linear expenditure system and the Rotterdam model, the new system allows marginal budget shares to vary as a function of income.

In the current paper we also work at a six-commodity level, fitting AIDADS to an international cross section of 30 countries in 1975. The data are from the International Comparisons Project of Kravis, Heston and Summers (1982) and previously were analyzed by Theil and Clements (1987) using a combination of additive preferences and Working's (1943) model in differential form. The present results overcome two potential shortcomings of the earlier work by replacing Working's model with a more regular specification of Engel effects and by providing and estimating an explicit functional form in the levels of the variables.

A rough comparison can be made between the time-series estimates of OP-73 and the cross-sectional ones reported here. We found the two sets of results broadly consistent (although the rate of decline in Food's marginal budget share was less in the Australian time series than in the international cross section). Overall, the new system performed well empirically. It seems suitable for modelling demand for broad consumption aggregates (say up to about a dozen commodities) in situations in which there may be very large variations in income per head.
DEMAND PATTERNS ACROSS THE DEVELOPMENT SPECTRUM:
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1. Introduction

Theil and Clements (1987) employed a novel approach to estimate an additive preferences demand system from the international comparisons data constructed by Kravis, Heston and Summers (1982). The motivation for this work sprang partly from the observation that the marginal budget share of food seemed to decline with increasing per capita real consumption. Since marginal budget shares are constant both in the Rotterdam model (see e.g., Theil 1965) and in Stone's (1954) Linear Expenditure System (LES), neither was suitable for Theil and Clements' estimation.

A better Engel specification, and the one used by Theil and Clements, was Working's (1943) formulation in which the budget share of food was written as a linear function in real per capita total expenditure. This is the Engel specification employed in Deaton and Muellbauer's Almost Ideal Demand System (AIDS) (1980).

Theil and Clements did not attempt to make their demand system explicit in the levels of the variables, preferring to work in terms of differentials. Thus the differences in per capita demands for broad commodity aggregates between the thirty or so countries of the international comparisons data were explained by the differences in per capita real total consumption and in relative prices across the cross section. The relatively high level of aggregation (10 commodities) and the need to preserve degrees of freedom led them to impose directly additive preferences. Under the latter specification, it is known (Houthakker 1960) that in any given price/total expenditure locality, the Allen-Uzawa cross partial substitution elasticities are proportional to the product of the Engel elasticities; that is,

\[ \sigma_{ij} = -\phi E_i E_j \quad (\phi < 0; i \neq j; i, j = 1, ..., n) \]

where the constant of proportionality \( \phi \) is independent of the particular pair \( (i, j) \) of commodities. Whilst \( \phi \) (closely related to the so-called Frisch (1959) 'parameter') in principle is a function of all prices and total expenditure, Theil and Clements concluded that the dependency was weak in their sample, and treated \( \phi \) as an absolute constant (which turned out empirically to be about \(-0.5\)).

Working's formulation and AIDS share the problem that, under large changes in real incomes, budget shares can stray outside the \([0,1]\) interval. It was such irregular behaviour that led Cooper and McLaren (1987, 1988, 1991, 1992a) to modify the AIDS system to become MAIDS, a system with regular properties over a much wider subset of the price-expenditure space and to propose (1992b) a new class of effectively globally regular (EGR) demand systems. In our current contribution we fit an EGR demand system — one which has proved promising with

* The authors would like to thank Russel Cooper and Keith McLaren for helpful suggestions.
relatively long (35 years) Australian time series data (Rimmer and Powell, 1992) —
to the international comparisons data. Our system goes by the acronym AIDADS (an
implicitly directly additive demand system)

By effectively global regularity in the current context we mean that AIDADS is
regular throughout that part of the price-expenditure space in which the consumer
is at least affluent enough to meet subsistence requirements.

The remainder of this paper is structured as follows. In Section 2 the
essentials of AIDADS are set out. Results are given in Section 3, which includes a
brief description of the data and a comparison with recent time-series work using
AIDADS (Rimmer and Powell, 1992).

2. Brief Description of AIDADS

2.1 The new expenditure system

Hanoch (1975) defines implicit direct additivity by the utility function:

\[ \sum_{i=1}^{n} U_i(x_i, u) = 1, \]

where \( (x_1, x_2, \ldots, x_n) \) is the consumption bundle, \( u \) is the level of utility, and the \( U_i \)
are twice-differentiable monotonic functions satisfying appropriate concavity
conditions. Using some intuition stemming from Cooper and McLaren's (1992a) and
from the LES, we choose the \( U_i \) as follows:

\[ U_i = \left[ 1 + \frac{1}{A} \right] \ln \frac{x_i}{\phi_i} - \gamma_i, \quad (i = 1, 2, \ldots, n) \]

where \( G(u) \) is a positive, monotonic, twice-differentiable function, and the lower-
case Greek letters and \( A \) are parameters, with

\[ 0 \leq \alpha_i, \beta_i \leq 1; \quad \sum_{i=1}^{n} \alpha_i = \sum_{i=1}^{n} \beta_i = 1. \]

The first-order conditions for minimizing the cost \( M \) of obtaining a given level of
utility \( u \) subject to given prices \( \{p_1, p_2, \ldots, p_n\} \) are (2.1.1) and:

\[ \frac{\lambda [\alpha_i + \beta_i G(u)]}{x_i - \gamma_i} \left( 1 + \frac{1}{G(u)} \right) = p_i, \quad (i = 1, 2, \ldots, n) \]

Hence

\[ \lambda^{-1} p_i (x_i - \gamma_i) = \frac{[\alpha_i + \beta_i G(u)]}{[1 + G(u)]} \cdot \]

Using the budget identity

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1 For a fuller account, see Rimmer and Powell (1992).
An Implicitly Additive Demand System

(2.1.6) \[ \sum_{i=1}^{n} p_i x_i = M, \]

by adding (2.1.5) across i, and using (2.1.3) to solve for \( \lambda \) we obtain:

(2.1.7) \[ \lambda = (M - p' \gamma), \]

where \( p' \gamma \) is shorthand for \( \sum_{i=1}^{n} p_i \gamma_i \). Back-substituting from (2.1.7) into (2.1.5), after rearrangement we obtain

(2.1.8) \[ p_i (x_i - \gamma_i) = \phi_i (M - p' \gamma) \quad (i = 1, 2, ..., n) \]

where from (2.1.2):

(2.1.9) \[ [a_i + [3_i G(u)] [1 + G(u)] = \frac{[\alpha_i + \beta_j G(u)]}{[1 + G(u)]} \quad (i = 1, 2, ..., n) \]

2.2 Substitution properties

The substitution elasticities associated with implicit direct additivity are:

(2.2.1) \[ \sigma_{ij} = \frac{(x_i - \gamma_i)(x_j - \gamma_j)}{x_i x_j} / \frac{[M - p' \gamma]}{M} \quad (i \neq j, i, j = 1, 2, ..., n) \]

where

(2.2.2) \[ \phi^* = -\frac{[M - p' \gamma]}{M} \]

and

(2.2.3) \[ E^*_i = \frac{\phi_i}{W_i} \quad (i = 1, 2, ..., n) \]

in which \( W_i \) is the budget share of i.\(^2\)

2.3 Engel properties

Not much further progress can be made without specifying a functional form for \( G \). Here we keep the LES interpretation of \( \gamma \) as the subsistence bundle, and require as well that

(2.3.1a) \[ \lim_{x \to \infty} u(x) = \infty; \]

(2.3.1b) \[ \lim_{x \to \gamma^+} u(x) = -\infty; \]

\[ ^2 \text{Note that } \phi^* \text{ in (2.2.3)} \text{ has the same form and interpretation as the } \phi \text{ of (2.1.1) in the LES; the } E^*_i \text{, however, are the ratios of two average budget shares: namely, of } \phi_i = [\text{supernumerary expenditure } p_i(x_i - \gamma_i) \text{ on } i]/[\text{total supernumerary expenditure } (M - p' \gamma)] \text{ to the ordinary average budget share } W_i = p_i x_i / M. \]
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(2.3.1c) \[ \lim_{u \to \infty} G(u) = \infty; \]
and

(2.3.1d) \[ \lim_{u \to -\infty} G(u) = 0. \]

\( x \) is the bundle \( \{x_1, x_2, \ldots, x_n\} \), and the notation \( x \to \infty \) implies that every \( x_i \) grows without limit, while \( x \to \gamma^+ \) implies that each \( x_i \) converges to its corresponding \( \gamma_i \) from above. G's monotonicity together with the bounds imposed on it above ensure that \( \phi_i \) behaves logarithmically, remaining always in the \([\alpha_i, \beta_i]\) interval. It can be shown that if \( \alpha_i < \beta_i \), the logistic behaviour of \( \phi_i \) implies that the lowest value of \( i \)'s marginal budget share is \( \alpha_i \), occurring when total expenditure is just enough to cover purchase of the subsistence bundle \( \gamma \); the upper asymptote of MBS\(_i\) as expenditure grows without limit is \( \beta_i \). If, on the other hand, \( \alpha_i > \beta_i \), the largest value of \( i \)'s marginal budget share is \( \alpha_i \), occurring at the subsistence expenditure level; its asymptote as expenditure grows indefinitely large and lowest value is \( \beta_i \).

The Engel elasticities in AIDADS are:

(2.3.2) \[ \varepsilon_i = \frac{\phi_i M}{p_i \gamma_i + \phi_i (M - p' \gamma)} + \frac{[\partial \phi_i / \partial M] M (M - p' \gamma)}{p_i \gamma_i + \phi_i (M - p' \gamma)} \]
\[ = \frac{\phi_i M}{p_i \gamma_i + \phi_i (M - p' \gamma)} + \frac{[\partial \phi_i / \partial u] [\partial u / \partial M] M (M - p' \gamma)}{p_i \gamma_i + \phi_i (M - p' \gamma)}. \]

\((i = 1, 2, \ldots, n)\)

Further progress cannot be made without specifying a functional form for \( G \). The simplest \( G(*) \) satisfying (2.3.1c&d) is:

(2.3.3) \[ G(u) = e^u. \]

In this case

(2.3.4) \[ \partial \phi_i / \partial u = (\beta_i - \phi_i) e^u / (1 + e^u). \]

\((i = 1, 2, \ldots, n)\)

An expression for \( \partial u / \partial M \) is given in Rimmer and Powell (1992).

2.4 Estimation of AIDADS

Because the expenditure system (2.1.9) depends via \( \phi_i \) on the implicitly defined and unobservable variable \( u \), estimation of AIDADS requires an initial level \( u_1 \) of \( u \) to be treated as a parameter. When moving from an initial data point (here, read 'country') with exogenous variable coordinates \( \{M; p_1, p_2, \ldots, p_n\}_1 \) to the next data point with exogenous variable coordinates \( \{M; p_1, p_2, \ldots, p_n\}_2 \), we need to be able to express the change in utility level \( \Delta u_1 \) as a function of the parameters (which are common across countries) of the utility function and of the differences between total per capita expenditure and prices in the two. In Rimmer and Powell (1992) it is shown that this can be achieved using the scheme shown in Figure 2.4.1. Note that in this figure \( \hat{x}_{it} \) is short-hand for the solution of (2.1.8) in terms of \( x_{it} \), while the \( A_{jt} \) are:

(3.3.7) \[ A_{jt} = (M_t - p'_t \gamma) / p_{jt}. \]

\((l=1, 2, \ldots, n)\)

\((j=1, 2, \ldots, n)\)
An Implicitly Additive Demand System

Start: $t = 1$

Set values of parameters $\alpha, \beta, \gamma, \rho$

Set $u_1$

Compute $\phi_{it} = \frac{[\alpha_i + \beta_i G(u_{it})]}{[1 + G(u_{it})]}$

Compute $\gamma_j + \frac{\phi_{it}}{p_{jt}} (M_t - p'_t \gamma)$

Compute $x_{jt} = \gamma_j + \frac{\phi_{it}}{p_{jt}} (M_t - p'_t \gamma)$

Compute $c_{jt} = -\frac{p_{jt}}{(M_t - p'_t \gamma)} \left[ \frac{e^{u_{it}}}{1 + e^{u_{it}}} \sum_{i=1}^{n} \phi_i \ln (x_{it} - \gamma_i) - 1 \right]^{-1}$

Compute $\Delta u_t = \sum_{j=1}^{n} c_{jt}(u_t) (A_{jt+1} - A_{jt}) \phi_{jt}(u_t)$

Compute $u_{t+1} = (u_t + \Delta u_t)$

Exit when $t = T - 1$

[after Rimmer and Powell (1992)]

Figure 2.4.1: Flow chart for data/parameter transformations in computation of the ML estimates
To keep the differences between sample points small (and therefore amenable to the use of differential analysis as in Figure 2.4.1), we followed Theil and Clements (1987) and arranged the cross-section of countries in a series according to increasing order of real per capita total expenditure. The plot of rank against real (international dollar) expenditure per head is shown in Figure 2.4.2 (the actual data are shown below in Table 3.1.1).

natural log of real per capita consumption

Figure 2.4.2 Sample variation in real per capita consumption in the international comparisons data

3. Empirical Estimates

3.1 The Data

The estimation of AIDADS requires data on nominal total expenditure as well as commodity prices. For each of thirty countries from the International Comparison Project listed in Table 3.1.1, Theil and Clements extracted data on real total consumption expenditure and real expenditure by commodity in 1975 expressed in "international dollars", $Q_t$ being real total consumption expenditure in country $t$ and $x_{it}$ being real consumption expenditure on commodity $i$ in country $t$. In addition they used data on the budget shares $w_{it}$ of commodity $i$ in country $t$. The commodity data were at the ten commodity level of disaggregation as listed in Table 3.1.2.

3 There were thirty-four countries in the Kravis, Heston and Summers study. Of these four countries were rejected by Theil and Clements. Kenya, Zambia and Malawi were omitted as outliers and because Kravis et al. regarded the data for these countries as beset with numerous problems. The fourth country, Jamaica, was omitted because it had an exceptionally large budget share devoted to “other” expenditure.

4 International prices are used to value the category quantities in each country under study in international dollars. The international price for a category is the quantity weighted average of detailed category Purchasing Power Parities (PPP's) after standardization.
Table 3.1.1

Consumption Per Capita For Thirty Countries in 1975
(from the International Comparisons Project)

<table>
<thead>
<tr>
<th>Country</th>
<th>Gross consumption per capita (in 1975 International dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. India</td>
<td>338.0</td>
</tr>
<tr>
<td>2. Pakistan</td>
<td>441.8</td>
</tr>
<tr>
<td>3. Sri Lanka</td>
<td>510.1</td>
</tr>
<tr>
<td>4. Philippines</td>
<td>693.6</td>
</tr>
<tr>
<td>5. Thailand</td>
<td>699.3</td>
</tr>
<tr>
<td>6. Malaysia</td>
<td>939.5</td>
</tr>
<tr>
<td>7. Korea</td>
<td>1019.4</td>
</tr>
<tr>
<td>8. Brazil</td>
<td>1219.0</td>
</tr>
<tr>
<td>9. Colombia</td>
<td>1265.3</td>
</tr>
<tr>
<td>10. Syria</td>
<td>1295.4</td>
</tr>
<tr>
<td>11. Iran</td>
<td>1345.3</td>
</tr>
<tr>
<td>12. Romania</td>
<td>1435.6</td>
</tr>
<tr>
<td>13. Yugoslavia</td>
<td>1712.6</td>
</tr>
<tr>
<td>14. Mexico</td>
<td>1839.4</td>
</tr>
<tr>
<td>15. Poland</td>
<td>2154.8</td>
</tr>
<tr>
<td>16. Uruguay</td>
<td>2234.4</td>
</tr>
<tr>
<td>17. Ireland</td>
<td>2299.3</td>
</tr>
<tr>
<td>18. Hungary</td>
<td>2313.2</td>
</tr>
<tr>
<td>19. Italy</td>
<td>2636.4</td>
</tr>
<tr>
<td>20. Japan</td>
<td>2912.7</td>
</tr>
<tr>
<td>21. Spain</td>
<td>3001.0</td>
</tr>
<tr>
<td>22. United Kingdom</td>
<td>3173.9</td>
</tr>
<tr>
<td>23. Netherlands</td>
<td>3398.1</td>
</tr>
<tr>
<td>24. Belgium</td>
<td>3715.1</td>
</tr>
<tr>
<td>25. Austria</td>
<td>3721.4</td>
</tr>
<tr>
<td>26. Germany</td>
<td>3743.4</td>
</tr>
<tr>
<td>27. France</td>
<td>3745.6</td>
</tr>
<tr>
<td>28. Denmark</td>
<td>3887.2</td>
</tr>
<tr>
<td>29. Luxembourg</td>
<td>3934.6</td>
</tr>
<tr>
<td>30. United States</td>
<td>4954.5</td>
</tr>
</tbody>
</table>

For comparison purposes with the AIDADS estimates obtained by Rimmer and Powell (1992) from Australian time series, this international data set of Theil and Clements was aggregated to six commodity groups as shown in Table 3.1.3. They are very roughly comparable to the commodity categories in the Australian study.

Relative price data $\pi_{it}$ (with the price of food the numeraire) were obtained for each of the six commodity groups in Table 3.1.3 in each country as the ratios $\frac{w_{it}x_{1t}}{w_{1t}x_{it}}$ of each budget share other than for Food to that of Food multiplied by the ratio of the per capita real consumption of food to that of each other commodity.
Table 3.1.2
The Theil and Clements Ten Commodity Level of Disaggregation of Final Consumption Expenditure

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>Food</td>
</tr>
<tr>
<td>(2)</td>
<td>Beverages and tobacco</td>
</tr>
<tr>
<td>(3)</td>
<td>Clothing and footwear</td>
</tr>
<tr>
<td>(4)</td>
<td>House furnishings and operation</td>
</tr>
<tr>
<td>(5)</td>
<td>Medical care</td>
</tr>
<tr>
<td>(6)</td>
<td>Transport and communication</td>
</tr>
<tr>
<td>(7)</td>
<td>Recreation</td>
</tr>
<tr>
<td>(8)</td>
<td>Education</td>
</tr>
<tr>
<td>(9)</td>
<td>Gross rent and fuel</td>
</tr>
<tr>
<td>(10)</td>
<td>All other expenditure</td>
</tr>
</tbody>
</table>

Table 3.1.3
The Six-Commodity Disaggregations used for the International Comparisons Data and the Australian Time-series Data

<table>
<thead>
<tr>
<th>International cross section</th>
<th>Australian time series</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Food</td>
<td>Food</td>
</tr>
<tr>
<td>(2) Beverages and tobacco</td>
<td>Alcohol &amp; tobacco</td>
</tr>
<tr>
<td>(3) Clothing and footwear</td>
<td>Clothing and footwear</td>
</tr>
<tr>
<td>(4) House furnishings &amp; operation</td>
<td>Durables</td>
</tr>
<tr>
<td>(5) Gross rent and fuel</td>
<td>Rent</td>
</tr>
<tr>
<td>(6) All other expenditure</td>
<td>All other expenditure</td>
</tr>
</tbody>
</table>

3.2 Estimating Equation

The expenditure system (2.1.8) was estimated in the form:

\[ W_{it} = \phi_{it} + \left( \frac{P_{it} Y_t - \phi^t P_t' Y}{M_t} \right) + v_{it}, \quad (1 = 1, 2, ..., n) \]

where \( W_{it} \) is the budget share of the \( t \)th commodity in the \( t \)th country, and \( v_{it} \) is a zero-mean random error, assumed normally distributed and independent of \( v_{it+t} \) (for all integral \( t \neq 0 \)). Full information maximum likelihood estimates were computed with the variance-covariance matrix of the \( v_{it} \)'s constrained à la Selvanathan (1991).\(^5\) Because the variables \( \{\phi_{1t}, \phi_{2t}, ..., \phi_{nt}\} \) are functions of \( u_t \), and...
because at any given parameter setting the values of \( u_1, u_2, \ldots, u_{t-1} \) must be computed before \( u_t \) can be evaluated (see Figure 2.4.1). Flexible software is needed. We found GAUSS 386 on a 486 personal computer up to the task.

### 3.3 The ML Estimates

We fitted AIDADS in the levels using the Theil and Clements cross section data described above. The results of that estimation appear in Table 3.3.1. The fitted and actual budget shares are plotted in Figure 3.3.1.

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#### Table 3.3.1

Maximum Likelihood Estimates of AIDADS fitted in the Levels, 30 Country Cross Section Data for 1975

<table>
<thead>
<tr>
<th>Item (^{(a)}) (^{(b)})</th>
<th>Food</th>
<th>Beverage &amp; Tobacco</th>
<th>Clothing &amp; Footwear</th>
<th>Household Furnishings &amp; Operation</th>
<th>Gross Rent &amp; Fuel</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>.693</td>
<td>.125</td>
<td>.182</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
</tr>
<tr>
<td>( t ) ratio</td>
<td>18.15</td>
<td>4.47</td>
<td>6.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>.000</td>
<td>.023</td>
<td>.049</td>
<td>.125</td>
<td>.183</td>
<td>.620</td>
</tr>
<tr>
<td>( t ) ratio</td>
<td>0.00</td>
<td>0.29</td>
<td>1.57</td>
<td>11\times10^3</td>
<td>64\times10^3</td>
<td>24\times10^3</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>112.33</td>
<td>0.91</td>
<td>0.07</td>
<td>0.02</td>
<td>13.13</td>
<td>0.08</td>
</tr>
<tr>
<td>( t ) ratio</td>
<td>7.59</td>
<td>1.82</td>
<td>0.71</td>
<td>0.31</td>
<td>2.05</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Marginal budget shares \( \psi_{11} \):

- poorest country: .33 .07 .11 .07 .10 .33
- richest country: .02 .03 .05 .12 .18 .60

Durbin-Watson statistic: 1.96 1.97 0.69 1.79 1.55 1.95

utility level for the poorest country, \( u_1 = -0.869 \): utility level in for the richest country, \( u_T = +1.290 \).

\( t \) value for \( u_T = 16.44 \).

While naturally there is much more variability in the cross section budget share data (see Figure 3.3.1) than in the Australian time series studied by Rimmer and Powell (1992), the same trends of decreasing shares for Food, Beverages and Tobacco, and Clothing and Footwear, and increasing shares for House furnishings an operation, Gross rent and fuel, and All other expenditure, are observed. Little overall change in budget share can be seen in Beverages and tobacco, and Clothing and footwear although, in keeping with the Australian study, for the most affluent
Figure 3.3.1: Actual and fitted budget shares for international comparisons data
Figure 3.3.1 (continued)
half of the countries, a downward trend in budget share with increasing affluence can be detected.

Corner solutions were obtained for the \( \alpha \) value for House furnishings and Operation, Gross rent and Fuel, and All other expenditure, while effectively zero subsistence quantities were found for all commodities except Food and All other expenditure. In keeping with the findings of Theil and Clements (1987) and the Adams, Chung and Powell (1988), the estimates show a strongly decreasing marginal budget share (MBS) for Food, from 0.33 to 0.02, whereas the AIDADS estimates for time series Australian data show an almost constant marginal budget shares for Food of around 0.07. The difference between the two AIDADS results can be explained substantially in terms of the span of real income across the samples. Figure 3.3.2 shows the MBS's for the countries in the cross section study. From Figure 3.3.2 it can be seen that for the majority of the countries very little change in the MBS for Food was observed and for countries within the time-series income range of Australia\(^6\), values ranged between 0.08 and 0.03.

Own and cross price elasticities of demand and Engel elasticities are given in Tables 3.3.2 and 3.3.3 respectively for the least affluent and most affluent countries in the sample. At constant prices it can be shown that the Engel elasticities for AIDADS tend to unity as expenditure grows without limit. However this movement towards unity need not be monotonic. With substantial price variation across countries, as is occurring in this cross section study, this pattern of movement to unity in Engel elasticities can be further obscured. The Engel elasticities are graphed in Figure 3.3.3. Movement of the Engel elasticity away from unity with increasing affluence is particularly evident for the necessity Food. As for the AIDADS estimation on Australian time series data, several of the cross price elasticities are positive indicating gross substitutability rather than gross complementarity. The estimated elasticities of substitution are shown for the richest and poorest countries in Table 3.3.4.

<table>
<thead>
<tr>
<th>Commodity</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Engel Elasticity ( \varepsilon_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Food</td>
<td>Alcohol</td>
<td>Clothing</td>
<td>Durables</td>
<td>Rent</td>
<td>Other</td>
<td></td>
</tr>
<tr>
<td>Food</td>
<td>-0.530</td>
<td>0.010</td>
<td>0.1853</td>
<td>0.005</td>
<td>-0.008</td>
<td>0.024</td>
<td>.485</td>
</tr>
<tr>
<td>2</td>
<td>-0.400</td>
<td>-0.902</td>
<td>0.032</td>
<td>0.008</td>
<td>-0.026</td>
<td>0.041</td>
<td>1.246</td>
</tr>
<tr>
<td>3</td>
<td>-0.473</td>
<td>0.013</td>
<td>-0.966</td>
<td>0.008</td>
<td>-0.032</td>
<td>0.039</td>
<td>1.411</td>
</tr>
<tr>
<td>4</td>
<td>-1.678</td>
<td>-0.088</td>
<td>-0.111</td>
<td>-1.0265</td>
<td>-0.139</td>
<td>-0.142</td>
<td>3.184</td>
</tr>
<tr>
<td>5</td>
<td>-0.833</td>
<td>-0.044</td>
<td>-0.055</td>
<td>-0.014</td>
<td>-0.564</td>
<td>-0.010</td>
<td>1.581</td>
</tr>
<tr>
<td>6</td>
<td>-1.681</td>
<td>-0.089</td>
<td>-0.111</td>
<td>-0.029</td>
<td>-0.139</td>
<td>-1.141</td>
<td>3.190</td>
</tr>
</tbody>
</table>

\(^6\) Australia's per capita income range across the sample period 1954-55 through 1988-89 is roughly spanned by the 1975 levels for Mexico and Luxembourg.

* Based on parameter estimates shown in Table 3.3.1.
Table 3.3.3
Estimated Engel and Own and Cross-Price Elasticities
for the Highest Income Country

<table>
<thead>
<tr>
<th>Commodity</th>
<th>1 Food</th>
<th>2 Alcohol &amp; Tobacco</th>
<th>3 Clothing &amp; Footwear</th>
<th>4 Durables</th>
<th>5 Rent</th>
<th>6 Other</th>
<th>Engel Elasticity $\varepsilon_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.773</td>
<td>0.034</td>
<td>0.059</td>
<td>0.075</td>
<td>0.1096</td>
<td>0.371</td>
<td>0.124</td>
</tr>
<tr>
<td>2</td>
<td>0.051</td>
<td>-0.078</td>
<td>0.032</td>
<td>0.041</td>
<td>0.058</td>
<td>0.202</td>
<td>0.596</td>
</tr>
<tr>
<td>3</td>
<td>0.034</td>
<td>0.014</td>
<td>-0.975</td>
<td>0.031</td>
<td>0.043</td>
<td>0.154</td>
<td>0.698</td>
</tr>
<tr>
<td>4</td>
<td>-0.054</td>
<td>-0.011</td>
<td>-0.018</td>
<td>-1.023</td>
<td>-0.037</td>
<td>-0.115</td>
<td>1.264</td>
</tr>
<tr>
<td>5</td>
<td>-0.058</td>
<td>-0.011</td>
<td>-0.018</td>
<td>-0.023</td>
<td>-1.017</td>
<td>-0.113</td>
<td>1.240</td>
</tr>
<tr>
<td>6</td>
<td>-0.059</td>
<td>-0.011</td>
<td>-0.018</td>
<td>-0.023</td>
<td>-0.037</td>
<td>-1.115</td>
<td>1.264</td>
</tr>
</tbody>
</table>

* Based on parameter estimates shown in Table 3.3.1.

Table 3.3.4
Estimated Substitution Elasticities from AIDADS
for Richest and Poorest Countries*

<table>
<thead>
<tr>
<th>i=1</th>
<th>1 Food</th>
<th>2 Beverage &amp; Tobacco</th>
<th>3 Clothing &amp; Footwear</th>
<th>4 Household Furnishings &amp; Operations</th>
<th>5 Cross Rent &amp; Fuel</th>
<th>6 Other</th>
<th>$\sigma_{ii}$ poorest</th>
<th>$\sigma_{ii}$ richest</th>
</tr>
</thead>
<tbody>
<tr>
<td>j</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>see last 2 cols 0.901</td>
<td>0.904</td>
<td>0.904</td>
<td>0.886</td>
<td>0.904</td>
<td>-0.300</td>
<td>-4.58</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.659</td>
<td>see last 2 cols</td>
<td>1.019</td>
<td>1.019</td>
<td>0.999</td>
<td>1.019</td>
<td>-14.43</td>
<td>-21.45</td>
</tr>
<tr>
<td>3</td>
<td>0.716</td>
<td>1.644</td>
<td>see last 2 cols</td>
<td>1.022</td>
<td>1.002</td>
<td>1.022</td>
<td>-10.75</td>
<td>-12.16</td>
</tr>
<tr>
<td>4</td>
<td>0.716</td>
<td>1.646</td>
<td>1.787</td>
<td>see last 2 cols</td>
<td>1.002</td>
<td>1.023</td>
<td>-46.68</td>
<td>-9.41</td>
</tr>
<tr>
<td>5</td>
<td>0.356</td>
<td>0.817</td>
<td>0.889</td>
<td>see last 2 cols</td>
<td>0.888</td>
<td>1.002</td>
<td>-7.73</td>
<td>-5.87</td>
</tr>
<tr>
<td>6</td>
<td>0.718</td>
<td>1.648</td>
<td>1.990</td>
<td>see last 2 cols</td>
<td>1.792</td>
<td>0.890</td>
<td>-8.00</td>
<td>-1.08</td>
</tr>
</tbody>
</table>

*The lower triangle shows values estimated for the poorest country; the upper triangle values estimated for the richest country.
Figure 3.3.2: Estimated marginal budget shares as a function of affluence
Figure 3.3.2 (continued)
Figure 3.3.3: Estimated total expenditure elasticities as a function of affluence (not standardized for relative price variation)
Figure 3.3.3 (continued): Estimated total expenditure elasticities as a function of affluence (not standardized for relative price variation)
3.4 **Comparison of the international cross section and the Australian time series**

Strict comparison between the international cross section results and the time-series results of Rimmer and Powell (1992) is not possible because of different definitions of the broad commodity groups (see Table 3.1.3). However, in order to make a rough comparison, price variations across countries and over time must be removed and a common measure of total expenditure adopted. Two separate computations were carried out in order to produce synthetic 'samples' in which the relative price variation had been removed: one for the cross section study, and one for the time series study.

For the cross section, prices for the six commodities under study were fixed at their geometric mean (across countries) and for each country nominal total expenditure was adjusted so that the country was 'just as well off' as in the original sample used in the AIDADS estimation above. In the case of the Australian time series, the relative prices of the commodities were fixed at the same values as in the synthetic cross section and total nominal consumption expenditure was adjusted in each sample year to leave each time-series value of per capita utility constant after the price change.

From the two synthetic samples, budget shares of commodities were obtained as a function of real total consumption expenditure — valued in 1975 international dollars in the case of the cross section study, and in 1984-85 Australian dollars for the time series. Real total consumption expenditure for Australia in 1975 in international dollars was available in Summers and Heston (1984). This figure was used to scale the other years of the Australian study to the 1975 international dollar base.

The budget shares from the cross-section and the time-series synthetic samples are plotted in Figure 3.4.1. In the case of Food, the Australian sample has been extrapolated backward to examine whether at very low income levels the time-series estimate of Food's budget share would be substantially different from the value implied by the cross sectional fit. Although the time-series and the cross section show substantial differences in curvature, there is very little difference in Food's budget share at very low and at very high incomes.

However, the difference in shape significantly colours perceptions of the rate of decline of Food's marginal budget share as a function of real per capita expenditure. Over the thirty or so years of the Australian sample, there was a decline in this MBS from 0.080 to 0.076 (Rimmer and Powell, Table 5.2), whereas the poorest and the richest country's MBSs for Food are estimated at 0.33 and 0.02 respectively. The gap between contemporary Australian behaviour (MBS_{Food} = 0.076) and American (MBS_{Food} = 0.02) is probably a consequence of the inclusion of the service component of restaurant meals in the Australian (but not the American) data.8

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7 In the AIDADS estimation each country's utility is estimated: the first country's utility level is an estimated parameter of the system and the utility levels for the remaining countries are obtained using the scheme set out in Figure 2.4.1.

8 It may also be a reflection of the relative prices at which the MBS were evaluated — the Australian figures quoted above were evaluated at actual Australian prices (the price of food is relatively low in Australia).
Figure 3.4.1: Comparison of behaviour of estimated budget shares in the international cross section and in the Australian time series data (relative price variation removed). Note that the Australian estimates refer to commodity groups which are not strictly comparable with those for the cross section (see Table 3.1.3).
Figure 3.4.1 (continued): Comparison of behaviour of estimated budget shares in the international cross section and in the Australian time series data (relative price variation removed). Note that the Australian estimates refer to commodity groups which are not strictly comparable with those for the cross section (see Table 3.1.3).
References


Cooper, R.J. and K.R. McLaren (1987) "Regular Alternatives to the Almost Ideal Demand System", Monash University, Department of Econometrics and Operations Research, second draft, mimeo (December).


Hanoch, Giora (1975) "Production or Demand Models with Direct or Indirect Implicit Additivity", Econometrica, Vol. 43, No. 3 (May), pp. 395-419.


