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From Dornbusch to Murphy: STYLIZED MONETARY DYNAMICS OF A CONTEMPORARY MACROECONOMETRIC MODEL

by

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ABSTRACT

Dornbusch's 1976 overshooting exchange rate model (hereafter, DBM) has long been known to underpin several large macro models, including the Murphy Model (MM). But the dynamic adjustment paths of variables in MM differ markedly from those in DBM, even qualitatively. A leading case in point is the exchange rate which in MM *under*shoots its new long run-equilibrium value after the injection of a monetary shock, and then actually moves *away* from this equi–librium for a time before approaching it via a damped cyclical adjustment path (whereas the corresponding path in DBM is monotonic).

This paper gives a simplified account of how this comes about. The emphasis is not so much on theoretical rigour but on providing a convincing practical demonstration. Using the simplest form of DBM as a starting point, it is shown how one can develop a miniature model exhibiting an MM-like response to a monetary shock. The key idea is that aggregate demand does not respond *instantaneously* (as in DBM) to shocks in the macroeconomic environment, but shows some degree of inertia. Nothing more is required to reconcile the qualitative dynamics of MM with DBM.

J.E.L classification numbers: C5, E4

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STYLIZED MONETARY DYNAMICS OF A CONTEMPORARY MACROECONOMETRIC MODEL

by

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1. Introduction

Dornbusch's 1976 overshooting exchange rate model (hereafter, DBM) has long been known to underpin the Murphy Model (MM[1]) — see, e.g., Murphy (1988a&b). But the dynamic adjustment paths of variables in MM differ markedly from those in DBM. A leading case in point is the exchange rate which in MM actually *under*-shoots its new equilibrium value after the injection of a monetary shock.

Figure 1.1 shows the trajectories in DBM and in MM of the nominal exchange rate (foreign dollars per local dollar) following an unanticipated, permanent, 1 per cent shock to the money supply. Apart from undershooting, the MM trajectory shows a damped cyclical response rather than monotonic convergence to the new equilibrium exchange rate (which is observed in the case of DBM). Moreover, after its initial jump, the exchange rate in MM actually moves *away* from the new equilibrium value.

My aim in this paper is to give a simplified account of how this comes about. In doing so, I draw liberally on my book with Chris Murphy (Powell and Murphy, 1995). The emphasis is not so much on theoretical rigour but on providing a convincing practical demonstration. Using the simplest form of DBM as a starting point, I show how one can progressively develop a miniature model exhibiting an MM-like response to a monetary shock. The key idea in this development is that aggregate demand does not respond instantaneously to shocks in the macro-economic environment. The vehicle used to implement the numerical miniature model is a computer spreadsheet.

The remainder of this paper is organized as follows. In Section 2, the details of DBM are briefly sketched. Section 3 contains an initial extension of DBM which adds only very marginally to the dimensions of the model, but which is capable of producing Murphy-like dynamics. In section 4, further refinements are made which result in a miniature model (*D4M*) that can be calibrated to track most major aggregates in MM under a monetary shock. The paper is summarized in Section 5.

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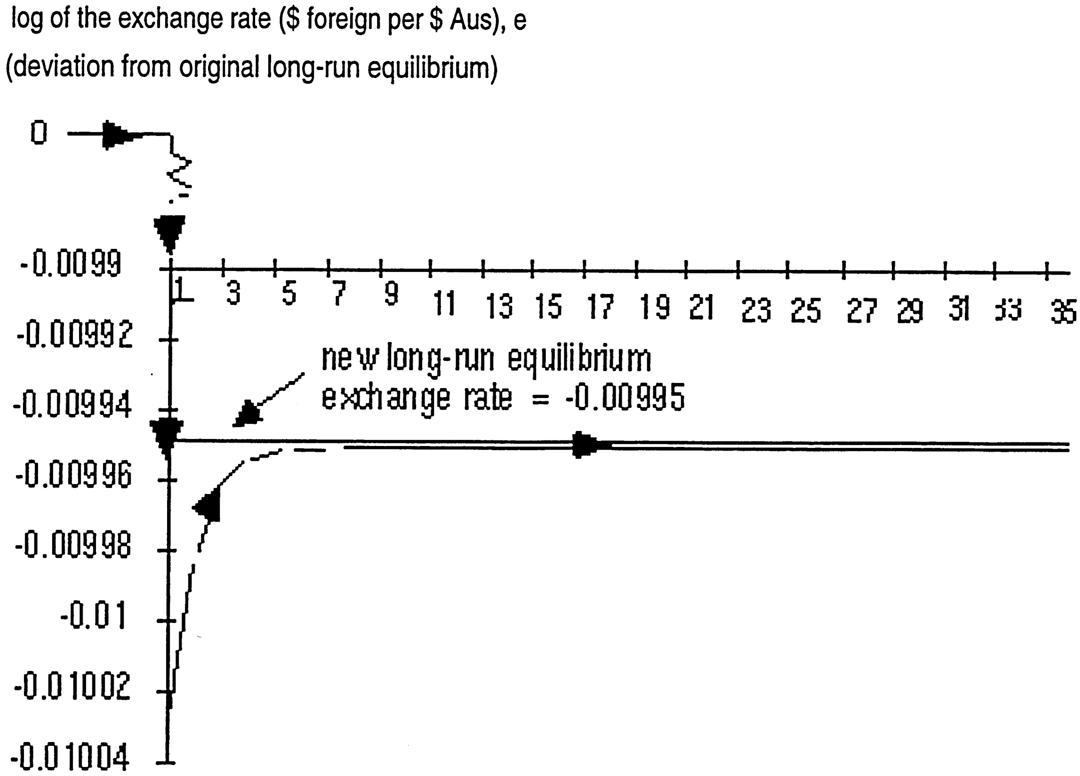
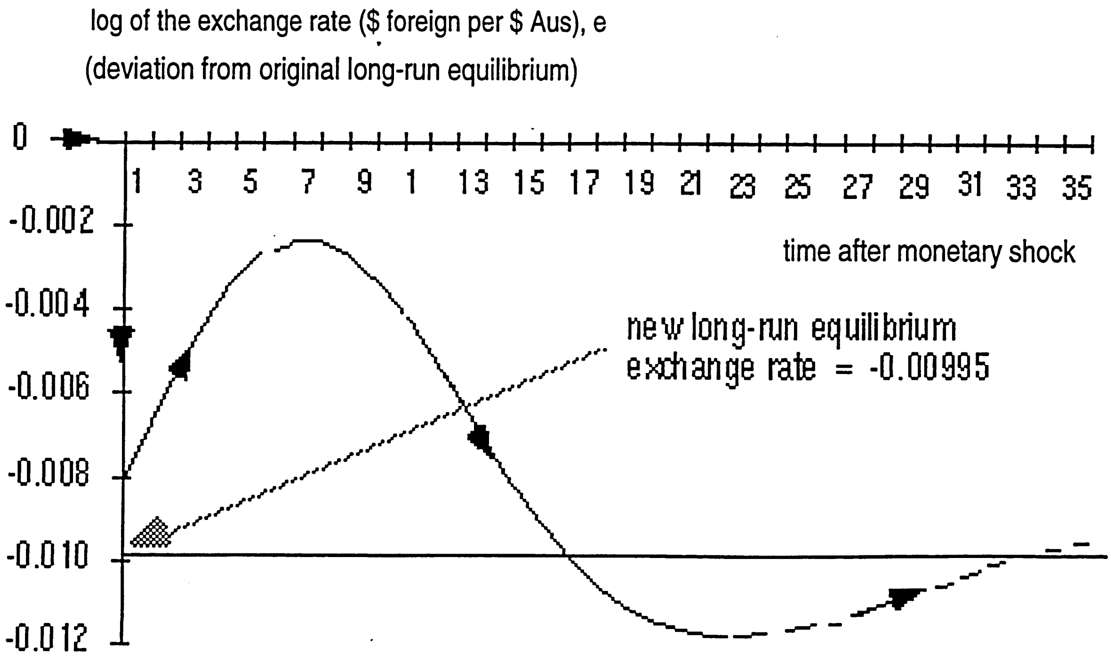
* This paper is built from edited extracts from my book with Chris Murphy [Powell and Murphy (1995,1997). I am grateful to Chris Murphy for helpful comments.

1 Throughout this paper, the acronym MM means the 100-equation macro model documented in Murphy (1992a).



(a) Typical trajectory of the nominal exchange rate in the Dornbusch model under an unanticipated, permanent 1 per cent expansion in the money supply.



(b) Trajectory of the nominal exchange rate in the Murphy model under an unanticipated, permanent 1 per cent expansion in the money supply

Figure 1.1: Comparison of exchange rate responses to a monetary expansion in the Dornbusch and Murphy models

2. Dornbusch's 1976 Overshooting Exchange Rate Model (DBM)

The equations and variables of DBM are shown in Tables 2.1 and 2.2 respectively. The dynamic equations of the model consist of *IS*, *LM* and Phillips curves, plus uncovered interest parity (UIP) and an expectations equation for the exchange rate.

Two versions of the last-mentioned equation are given: the first, (2.4a), is an error-correction mechanism in which the domestic currency is expected to appreciate if the current exchange rate values it below its long-run value, and to depreciate in the contrary situation. The second, the rational-expectations relationship (2.4b), puts the expected rate of depreciation of the local dollar, x, to its actually realized value, $-\dot{e}$. In this paper, all model solutions discussed assume that rational expectations prevail in financial markets. It can be shown that equations (2.4a) and (2.4b) are equivalent when DBM is solved under rational expectations provided that the value of the coefficient θ is chosen appropriately.[2]

Table 2.1

*Equations of the Dornbusch Model**

Eq'n no	Description	
	SHORT-RUN EQUATIONS OF THE DORNBUSCH MODEL PROPER	
(2.1)	*IS curve*	$a = \mu g + \delta(p^* - e - p) - \sigma r + \gamma y + \tau y^*$
(2.2)	*LM curve*	$m - p = \phi y - \lambda r$
(2.3)	*Uncovered interest parity*	$r = r^* + x$
(2.4a) or	*Expected rate of depreciation of the domestic currency*	$x = \theta(e - \bar{e})$ or
(2.4b)		$x = -\dot{e}$
(2.5)	*Phillips curve*	$\dot{p} = \pi(a - y)$
	LONG-RUN EQUATIONS FOR THE PRICE LEVEL AND NOMINAL EXCHANGE RATE	
(2.6)	*Long-run neutrality of money*	$\bar{p} = m - y$
(2.7)	*Purchasing Power Parity*	$\bar{p} = p^* - \bar{e}$

Source: Based on Powell and Murphy (1995), p. 27. The sign convention adopted in this table implies that the parameters (denoted by lower-case Greek letters) all are expected to be positive.

2 The appropriate choice is: $\theta = \pi \left(\frac{(\sigma/\lambda) + \delta}{2}\right) + \left\{ \pi^2 \left[\frac{(\sigma/\lambda) + \delta}{2}\right]^2 + \frac{\pi\delta}{\lambda} \right\}^{1/2}$ — see Powell and Murphy (1995), pp. 36-38.

Table 2.2

Notation for variables in the Dornbusch Model

Variable	Description[a]
a	real aggregate demand in the home country
g	real aggregate spending by government in the home country (exogenous)
e	current spot exchange rate, defined as the foreign currency price of a unit of domestic currency[b]
$\bar{e}$	long-run exchange rate
m	nominal quantity of money (exogenous)
p	domestic price level (initial value exogenous; endogenous thereafter)
$\bar{p}$	domestic price level in the long run
p^*	foreign price level (exogenous, and normalized so that $p^* = 0$)
$\dot{p}$	rate of domestic price inflation
r	domestic nominal interest rate
r^*	foreign nominal interest rate (exogenous)
x	expected rate of depreciation of the local currency
y	real aggregate supply at 'normal' rates of utilization of capital and labour in the local economy (exogenous) — also interpreted as permanent income
y^*	real income in the rest of the world (exogenous)

Source: Powell and Murphy (1995), p. 28.

[a] All variables, except the nominal interest rates r and r^* (which are measured as proportions per unit time), are expressed in logarithmic form. Leaving aside p (which has an effectively exogenous initial value, but is endogenous thereafter), the exogenous variables in the standard closure of DBM are the ones shown in shaded rows. Again with the same exceptional variable, the endogenous variables in the standard closure are those in *un*shaded rows.

[b] Dornbusch actually defines the exchange rate as the domestic currency price of a unit of foreign money, but this is inverted here for consistency with the MM.

The dynamics of DBM can be found analytically. The algebraic solution of DBM for a monetary expansion is given in Table 2.3, and illustrated for one set of parameters in Figure 2.1. Notice that all triples $\{\sigma, \pi, \delta\}$ that yield the same θ for a given λ are observationally equivalent.

3. An extended Dornbusch model (EDBM)

Perhaps the least empirically plausible feature of DBM is illustrated by the last panel of Figure 2.1 which shows that economic activity (*think:* GNE) jumps

Table 2.3

Analytical Solution of the Dornbusch Model under a Monetary Shock

Description	No.	Equation	
Sticky price of domestic good	(2.8)	$p(0\ [\text{post shock}]) = p(0\ [\text{pre shock}]) = p(0)$	
Size of the initial exchange rate jump	(2.9)	$\{\textit{jump in the log of the exchange rate}\} = -\left(1+\frac{1}{\lambda\theta}\right) \times \{\textit{jump in the log of the money supply}\}$	
Exchange rate immediately after the shock	(2.10)	$e(0\ [\text{post shock}]) = \bar{e}^{old} + \{\textit{jump in the log of the exchange rate}\}$	
New long-run equilibrium price of domestic good	(2.11)	$\bar{p} = \bar{p}^{old} + \{\textit{jump in the log of the money supply}\}$	
New long-run equilibrium exchange rate	(2.12)	$\bar{e} = p^* - \bar{p}$	
Path of nominal exchange rate	(2.13)	$e(t) = \bar{e} + (e(0[\text{post shock}]) - \bar{e})\ e^{-\theta t}$	$(0 \le t < \infty)$
Path of price of the domestic good	(2.14)	$p(t) = \bar{p} + (p(0) - \bar{p})\ e^{-\theta t}$	$(0 \le t < \infty)$
Path of the rate of price inflation of the domestic good	(2.15)	$\dot{p}(t) = -\theta\ (e(t) - \bar{e})$	$(0 \le t < \infty)$
Path of the expected rate of domestic currency depreciation	(2.16)	$x(t) = \theta\ (e(0[\text{post shock}]) - \bar{e})\ e^{-\theta t}$	$(0 \le t < \infty)$
Path of the domestic interest rate	(2.17)	$r(t) = r^* + \theta\ (e(0[\text{post shock}]) - \bar{e})\ e^{-\theta t}$	$(0 \le t < \infty)$
Path of activity level	(2.18)	$a(t) = y + \frac{\dot{p}(t)}{\pi}$	$(0 \le t < \infty)$

instantaneously in response to the monetary shock. The extension of DBM presented here dampens the initial jump in activity by introducing inertia into the response of aggregate activity a(t).

The new relationship determining aggregate demand is:

(3.1) $a(t) = \psi\,\alpha(t) + \varphi(t)\,[1-\psi(t)]\,a(t-1) + [1-\varphi(t)]\,[1-\psi(t)]\,a(t-2)$

in which

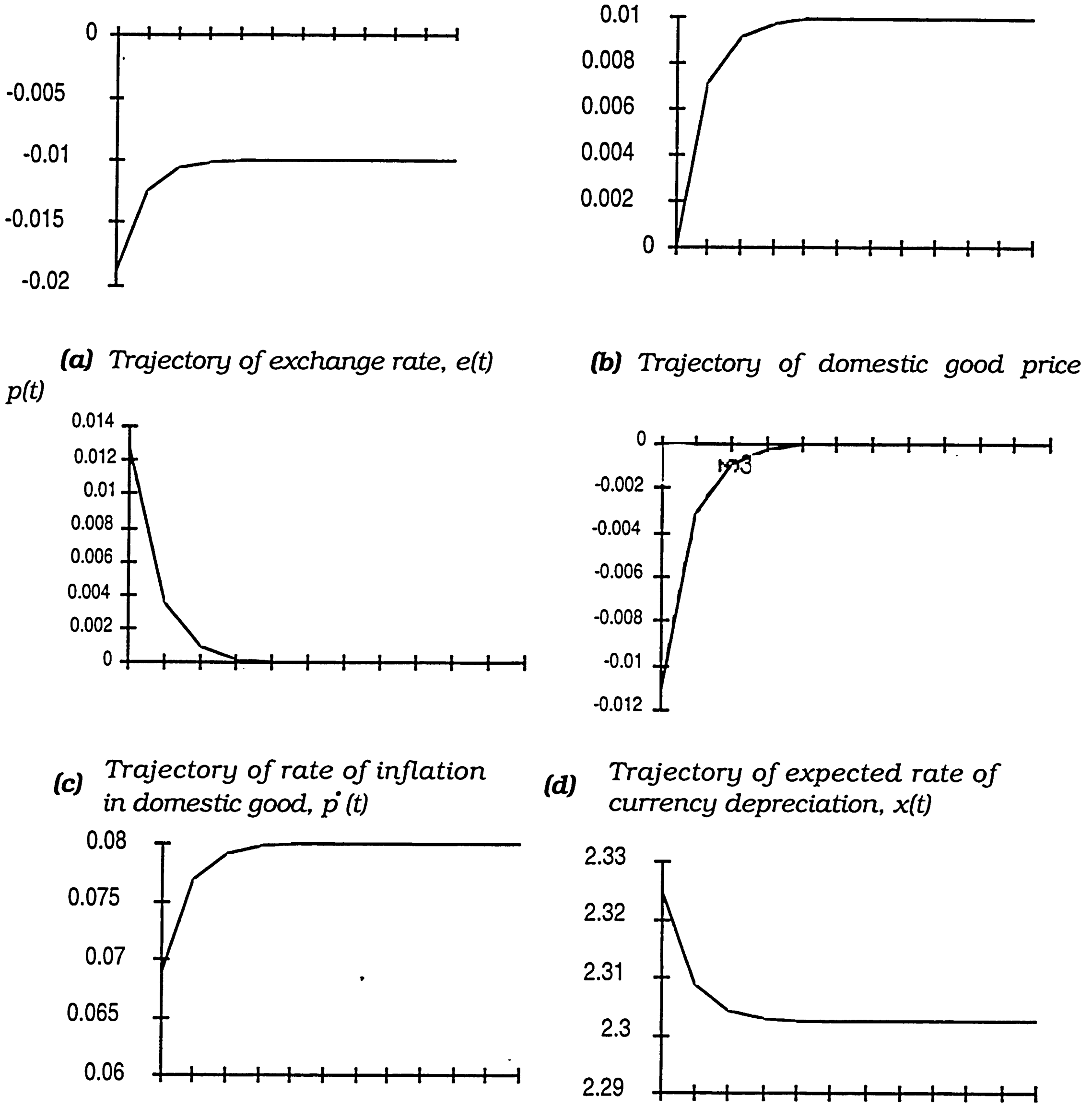
(3.2) $1 > \psi > 0;$

and

(3.3) $\alpha(t) = \mu\, g(t) + \delta\{p^*(t) - e(t) - p(t)\} - \sigma r(t) + \gamma y(t) + \tau y^*(t)$.

Notice that (3.3) is just DBM's IS curve (2.1); the variable $\alpha(t)$ accordingly is identified as what aggregate demand would have been in period t if

Dornbuschian short-run equilibrium had prevailed throughout the post-shock epoch.[3]



***(a)** Trajectory of exchange rate, e(t)*

***(b)** Trajectory of domestic good price p(t)*

***(c)** Trajectory of rate of inflation in domestic good, $\dot{p}$ (t)*

***(d)** Trajectory of expected rate of currency depreciation, x(t)*

***(e)** Trajectory of interest rate, r(t)*

***(f)** Trajectory of economic activity, a(t)*

Figure 2.1: Dynamics of main variables in DBM under a one per cent monetary shock

The parameter settings are:

λ	θ	π
0.9	1.260526	0.5714

Note that triples $\{\sigma, \pi, \delta\}$ that yield the same θ for a given λ are observationally equivalent.

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3. The inertia in the real side of the economy modelled via (3.3) effectively introduces a third length of run into the DBM: a period so short that the model's short-run equilibrium equations fail to be satisfied.

If $\psi = 1$, then EDBM collapses back to DBM as set out above in Table 2.1. As an empirical matter, suppose that $\psi \neq 1$. Then the DBM IS curve (2.1) is misspecified and the short-run equilibrium implied by it is not established until some time after the shock impinges (rather than instantaneously).

The full set of equations of EDBM, and a procedure for implementing them, are shown in Tables 3.1 and 3.2 respectively. The former differs from Table 2.1 in the following respects:

- Two additional variables appear; namely, $\alpha(t)$ and $\varphi(t)$. The former has been defined above; the latter is a lag weighting function which is explained below.
- Two additional equations are added; namely, (3.1) and (3.3).
- The explicit dynamic adjustment equation for expected exchange-rate changes (2.4a) is replaced in (3.7) by equality between the actual and the expected rates of depreciation of the value of the local currency — that is, rational expectations is enforced by inclusion of (3.7) [which is just (2.4b) again].
- Time is treated as finite, but the time grid is very fine, so that while lags are defined, the derivatives $\dot{e}$ and $\dot{p}$ are well approximated by finite difference quotients.

Table 3.1

*Equations of the extended Dornbusch model (EDBM)**

Eq'n no	Description	Equation/Inequality
	SHORT-RUN DISEQUILIBRIUM EQUATION	
(3.1)	*Actual aggregate demand*	$a(t) = \psi\,\alpha(t) + \varphi(t)\,[1-\psi]\,a(t-1) + [1-\varphi(t)]\,[1-\psi]\,a(t-2)$
(3.2)	*Convex weighting coefficient*	$1 \leq \psi \leq 0$
(3.3)	*Lag weighting function*	$\varphi(t) = R\,exp(-\rho t) \quad [\rho \geq 0]$
	SHORT-RUN EQUILIBRIUM EQUATIONS	
(3.4)	*Equilibrium IS curve*	$\alpha = \mu g + \delta(p^* - e - p) - \sigma r + \gamma y + \tau y^*$
(3.5)	*LM curve*	$m - p = \phi y - \lambda r$
(3.6)	*Uncovered interest parity*	$r = r^* + x$
(3.7)	*Expected rate of depreciation of the domestic currency*	$x = -\dot{e}$
(3.8)	*Phillips curve*	$\dot{p} = \pi(a - y)$
	LONG-RUN EQUILIBRIUM EQUATIONS FOR THE PRICE LEVEL AND EXCHANGE RATE	
(3.9)	*Long-run neutrality of money*	$\bar{p} = m - y$
(3.10)	*Purchasing Power Parity*	$\bar{p} = p^* - \bar{e}$

Source: Powell and Murphy (1995), p. 319 (modified).

* Time indexing of variables is suppressed in all equations other than (3.1&3), since in the other equations all variables are contemporaneous. Variables other than $\alpha(t)$ and $\varphi(t)$ — which respectively are the value that aggregate demand would take at t if the short-run equilibrium represented by the IS curve in DBM were reached instantaneously after a shock, and the value of a weighting function to be used in the lag distribution — are identified in Table 2.2 above. Note that the timing convention implicit in the notation for (3.1&3) assumes that the shock impinges at t = 0.

Table 3.2

Sequence of equations used to solve EDBM using a shooting algorithm

Eq'n no	Description	Equation
Variables at t = 0		
(3.11)	*Long-run neutrality of money*	$\bar{p} = dm$
(3.12)	*Purchasing power parity*	$\bar{e} = -dm$
Variables at t = 0		
(3.13)	*Time rate of appreciation of the local currency*	$\dot{e}\,(0_{\text{[post shock]}}) = dm/\lambda$
(3.14)	*Choose* $e(0\ \text{[post shock]})$ *to minimiz*	$\{\, e(100) - \bar{e}\ \}^2$
(3.15)	*Sticky initial price level*	$p(0) = \bar{p}^{\,old} = 0$
(3.16)	*Interest rate*	$r(0_{\text{[post shock]}}) = r^* - \dot{e}(0_{\text{[post shock]}})$
(3.17)	*Short-run equilibrium aggregate demand*	*Use equation (3.4) to find* $\alpha(0_{\text{[post shock]}})$
(3.18)	*Actual aggregate demand*	*Use equation (3.1) to find* $a(0_{\text{[post shock]}})$
(3.19)	*Initial rate of inflation*	$\dot{p}\,(0_{\text{[post shock]}}) = \pi(a(0_{\text{[post shock]}}) - y)$
Variables at t = 1		
(3.20)	*Update price level*	$p(1) = p(0) + \dot{p}(0_{\text{[post shock]}})$
(3.21)	*Update exchange rate*	$e(1) = e(0\ \text{[post shock]}) + \dot{e}\,(0)$
(3.22)	*Time rate of appreciation of the local currency*	$\dot{e}\,(1) = \{\bar{p} - p(1)\}/\lambda$
(3.23)	*Update exchange rate*	$e(2) = e(1) + \dot{e}\,(1)$
(3.24)	*Update interest rate*	$r(1) = r^* - \dot{e}(1)$
(3.25)	*Update short-run equilibrium aggregate demand*	*Use equation (3.4) to find* $\alpha(1)$
(3.26)	*Update actual aggregate demand*	*Use equation (3.1) to find* $a(1)$
(3.27)	*Update rate of inflation*	$\dot{p}\,(1) = \pi(a(1) - y)$
Variables at t = 2 and subsequently		
Replace $\dot{p}(0\ \text{[post shock]})$ and $e(0\ \text{[post shock]})$ in (3.20) and (3.21) by $\dot{p}(1)$ and $e(1)$ respectively, and advance all other time markers by one; continue recursively for t = 3, ..., etc.		

Source: Powell and Murphy (1995), p. 320 (modified).

Form of the lag distribution for aggregate demand in EDBM

The lag structure in (3.3) involves placing a weight of ψ (where ψ is non-negative and does not exceed unity) on the current short-run equilibrium value $\alpha(t)$ of aggregate demand and distributing the remaining weight $[1 - \psi]$ on the previous two values of *actual* aggregate demand, a(t-1) and a(t-2). Notice that in (3.3) the lag coefficients depend not only on the order of the lag, but also on the length of time t that has elapsed since the monetary shock first impinged on the economy.

The elapsed-time-after-the-shock-dependent weighting function $\varphi(t)$ in (3.3) is purposely chosen to allow the possibility that the weight on a(-1), $\{\varphi(t)\ [1 - \psi]\}$, exceeds 1 at t=0 and for some time afterwards. Note that in this case, the weight $\{\ [1 - \varphi(0)]\ [1 - \psi]\}$ on a(-2) in the formation of a(0) is negative. Also note that irrespective of the value of $\varphi(t)$, the weights on $\alpha(t)$, a(t-1) and a(t-2) by construction sum to unity. Finally, note that by setting $\rho = 0$, the three weights become just ψ, $\{[1 - \psi]\ R\}$, and $\{[1 - \psi]\ [1 - R]\}$. In this case a choice of R exceeding unity can lead to an explosive time path for e(t).

The above form of lag distribution was chosen with the following objectives in mind:

- The dimensionality (measured in terms of number of equations and number of parameters) must be kept low if EDBM is to be useful as an expository device. The very complex lag distributions operating in MM, while not dependent (as here) on elapsed time, involve many equations and parameters; moreover, the pervasive influence in MM of that model's production enterprise block implies that the reduced forms of the equations explaining many variables cannot be written as polynomials in the lag operator.

- The requirement that

  (3.28) *weight on* a(t-1) *in determination of* a(t) *at* $t = 0 \equiv [1 - \psi]\ \varphi(0) > 1$

  seems to be necessary for EDBM to be capable of <u>under</u>shooting.

- The chosen lag distribution must be capable of producing the damped oscillations evident in MM. The form of $\varphi(t)$ may also be regarded as an *ad hoc* (but simple) device designed to moderate the size of the initial jump in economic activity exp(a) *without* cutting off the possibility of further increases in exp(a) after the initial jump.[4]

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4 In a growing economy, adaptive lag distributions (in which the weights all lie between zero and one) cannot yield a current value for a variable that is above (below) the largest (smallest) lagged value entering with a finite weight into its determination. If the most recent value of a(t) is the largest such lagged value, as it is at t=1 in the case of a monetary shock impinging on a previously prevailing balanced growth path, then an adaptive lag structure necessarily would lead to a current value a(1) which is less than a(0).

Factors responsible for inertia in aggregate demand in MM

What are the frictions in MM that account for its failure to stay on the IS curve a la Dornbusch? Other than government consumption (which is set exogenously), each of the components of aggregate demand in MM exhibits inertia in responding endogenously to the changed economic environment. The details are given in Table 3.3.

Table 3.3

*Main lag mechanisms responsible for inertia in aggregate economic activity in MM**

Component of aggregate demand	Notes
consumption	Real consumption adjusts via a partial adjustment mechanism [see (7.3.2), p. 121] to equilibrium consumption. Interest rates appear explicitly with lags of 3, 4 and 5 quarters.
investment	The business investment to capital ratio displays inertia, being partially determined by its lagged value [see (11.4.2), p. 182]. Housing investment [(9.7.16), p.144] is similar, involving lags at one and at four quarters. Tobin's q enters both equations with a 2-quarter lag, and other right-hand variables are lagged up to 4 quarters.
exports	Exports X adjust partially to their medium-run equilibrium value X^{MR} [see (14.1.2), p. 197]. X^{MR} is determined in the enterprise production block [see (14.1.2), p. 197].
- imports	Imports M adjust partially to last quarter's medium-run equilibrium value M_{MR} [see (13.5.1), p. 192]. M_{MR} is determined in the enterprise production block [see (10.4.22), p.164].

Source: Powell and Murphy (1995), p. 320 (modified).
Page references are to Powell and Murphy (1995).

Solutions of EDBM using shooting

The method used to solve EDBM is a variant of the shooting algorithm[5]. Terminal values are known for several variables — for example, a one per cent monetary shock according to Table 2.3 leads via (2.11) to a one per cent rise in the domestic good's price $(\exp\{p(\infty)\})$ and via (2.12) to a 0.995033 per cent fall in the nominal exchange rate $(\exp\{e(\infty)\})$. Hence it is practicable to determine the size of the unknown initial jump in the exchange rate by minimizing the gap between p(T) and

[5] For a brief account of this and other methods for solving models under rational expectations, see Dixon, Parmenter, Powell and Wilcoxen (1992), Ch.5, pp. 334-345.

0.01 or between e(T) and 0.995033 for some arbitrarily high T; in the computations reported below, we use T = 100.

Such a solution in principle can be computed using any reasonably powerful non-linear solution package. The results presented below were obtained using the *Solver* facility in Microsoft Excel 4.0. Because the sequence of computation is important, and since spreadsheets (like Excel) cannot always resolve inherently circular references, there are a limited number of feasible ways of setting the problem up; Table 3.2 is the basis of one feasible method.

The start-up computations shown under ***t = 0*** in Table 3.2 are ordered as shown down the rows in a left-hand column of the spreadsheet; this is followed by the ***t = 1*** computations in the next column to the right; subsequent columns just extend the ***t = 1*** column recursively. At the commencement of the solution procedure, a guess is entered in the spreadsheet for the unknown initial post-shock value of the exchange rate. This is then made the choice variable in the minimization problem presented to *Solver*[6].

As noted above, quite a lot is known about the steady state of EDBM; hence there are many possible choices for the minimand. Some obvious choices are $\{ e(100) - \bar{e} \}^2$, $\{ p(100) - \bar{p} \}^2$, $\{ a(100) - \bar{a} \}^2$ and $\{ r(100) - r^* \}^2$, or some additive combination of them. Experience with EDBM suggests that the choice usually has negligible effects on the results (and certainly does not affect their qualitative properties).

Figures 3.1–3.3 show trajectories for the exchange rate, price level, interest rate, and aggregate economic activity (= aggregate demand) produced by EDBM under an unanticipated, permanent expansion of ***5 per cent*** in the money supply. When $\psi = 1$ these variables evolve as in the 'pure' DBM shown in Figure 2.1. Alternative calculations with the lag parameters set at

$$\boxed{\psi = 0.2, \quad R = 3 \quad \text{and} \quad \rho = 0.2}$$

exhibit qualitative features similar to MM — compare panel (b) of Figure 1.1 with the lower panel of Figure 3.1, noting in particular that the latter replicates the undershooting evident in the former. Not shown here are solutions with $R \leq 1$; the exchange rate <u>over</u>shoots in all the computations which we carried out with this setting.

In Figure 3.3, the lower initial jump in aggregate economic activity in EDBM with inertia seems more plausible as a likely description of how an actual economy would behave in the presence of a monetary stimulus than the pure DBM result shown by the broken curve.

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[6] Extracts from the Excel spreadsheet are given in Appendix 3.2 of Powell and Murphy (1995).

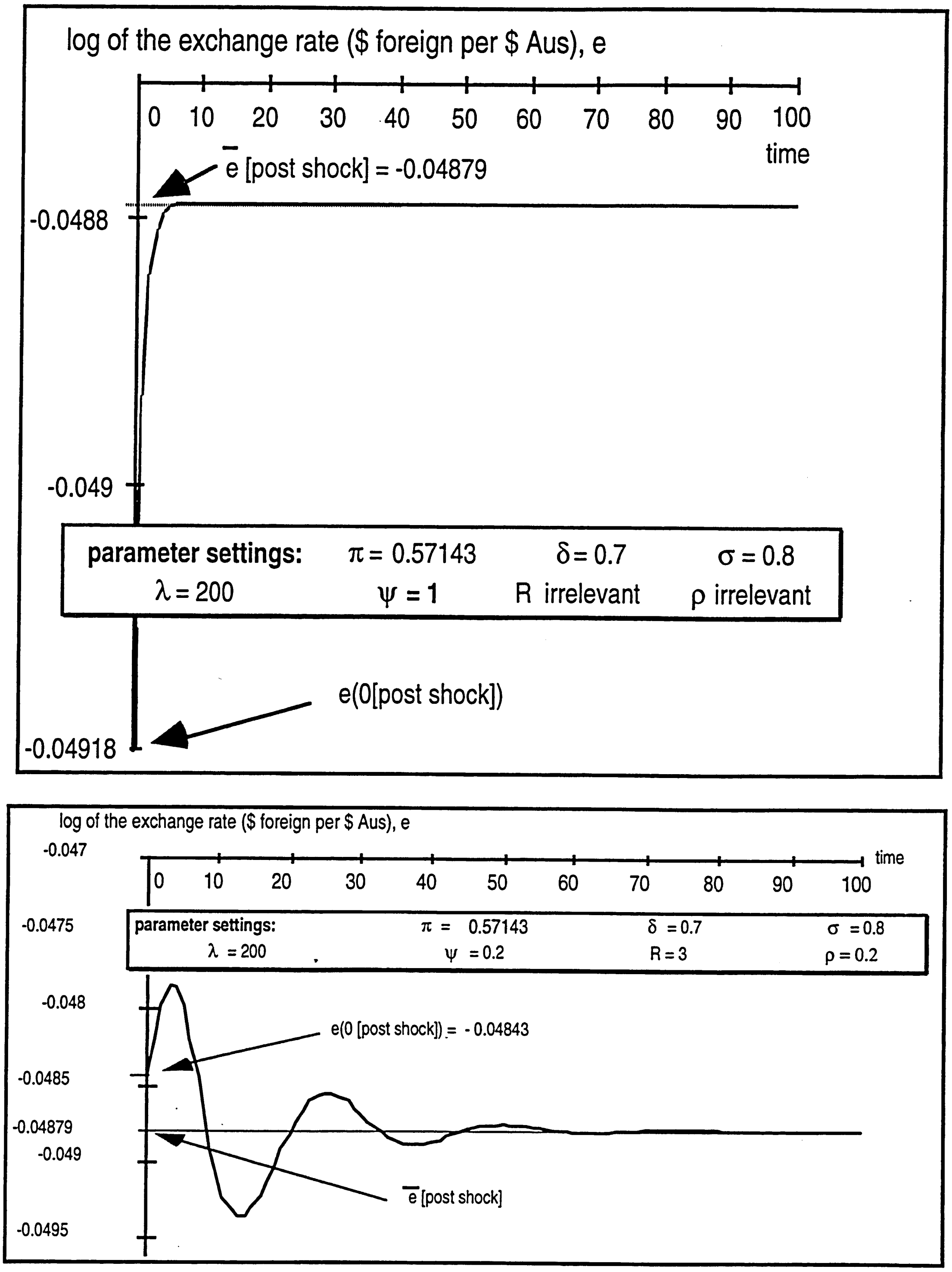

Figure 3.1 Exchange rate behaviour in the extended Dornbusch model — from Powell and Murphy (1995, p. 323). The upper panel shows EDBM with no inertia in demand (ψ = 1); this just replicates the behaviour of DBM shown in Figure 2.1. The lower panel shows MM-like behaviour: exchange rate undershooting coupled with damped oscillations towards the new long-run equilibrium after initially diverging away from that equilibrium. Qualitatively the EDBM result for a(t) is fairly similar to the MM result for real GDP and for output of the domestic good (for the latter, see Figure 3.4).

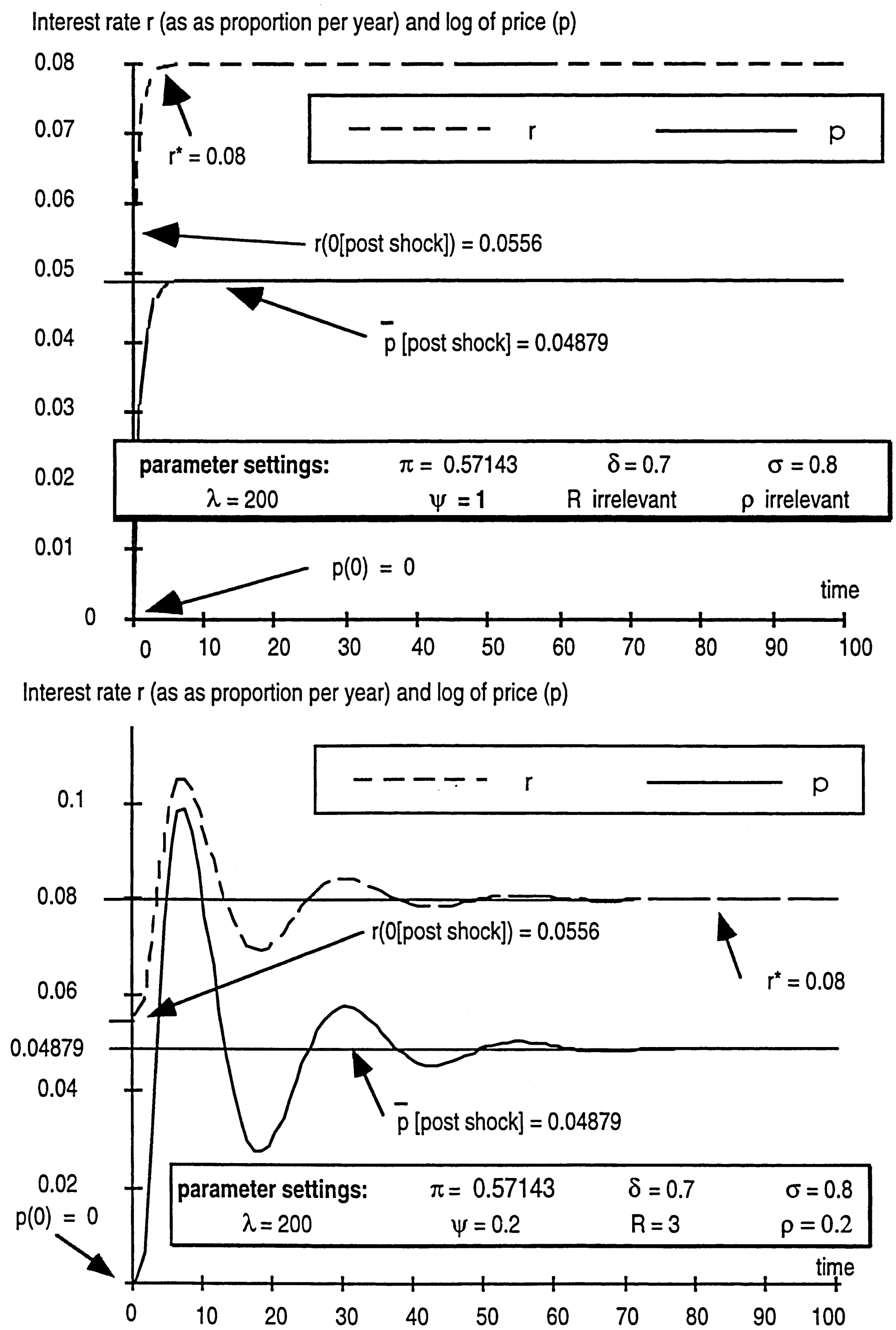

Figure 3.2 Interest rate and price behaviour in the extended Dornbusch model — from Powell and Murphy (1995, p. 324). The upper panel shows EDBM with no inertia in demand (ψ = 1); this just replicates the DBM — see Figure 2.1. The lower panel (ψ = 0.2) shows MM-like damped oscillations towards long-run equilibrium values. Note that the initial post-shock value of the interest rate, obtained from (3.13&16) as r(0 [post shock]) = r^* - λ dm, does not depend on whether or not inertia is present in aggregate demand. This is because the size of the jump in r depends only on the LM curve and on uncovered interest parity; hence inertia in the goods market has no effect on the size of r(0 [post shock]).

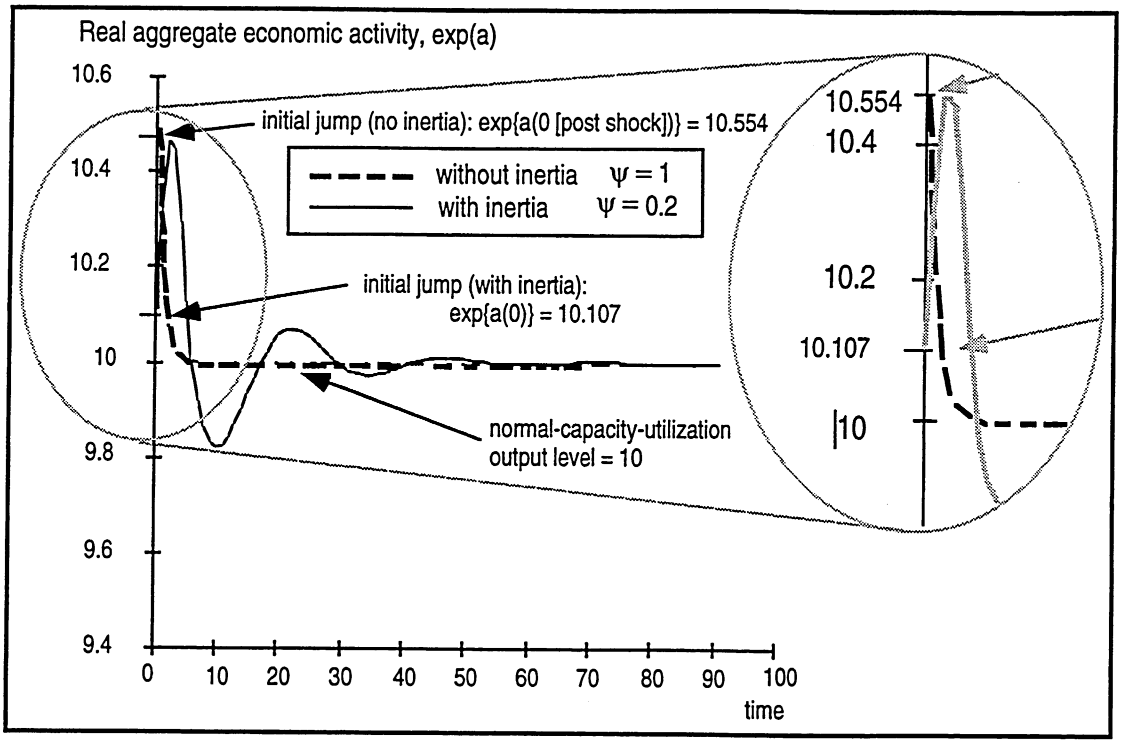

Figure 3.3 Aggregate economic activity in the extended Dornbusch model — from Powell and Murphy (1995, p. 325). Before the shock, exp(a) is in stationary equilibrium, and equals 10. The dashed line shows EDBM with no inertia in demand (ψ = 1); this just replicates DBM. The unbroken curve shows Murphy-Model-like behaviour: the initial jump in demand is followed by a further expansion, then a downturn after period 2, and thereafter an approach to equilibrium via damped oscillations. The parameters for the unbroken curve are the same as those in the lower panel of Figure 3.2.4.

4. D4M — a miniature MM for the analysis of monetary shocks

D4M is a special-purpose or shock-specific miniature expressly designed to simplify explanation of how a monetary shock works in MM. The success of *D4M* in building intuition about the principal mechanisms at work in MM depends on the plausibility of the stylized equations included in it as representations of a much larger number of more complex equations in MM. In part this plausibility might be judged *a priori* by the nature of the relationships and variables chosen for inclusion in any particular miniature, and *ex post* by the comparative tracking performance of the miniature in question *viz à viz* MM. Because quite a lot is known about the initial situation in which the shock impinges, and also about the terminal (steady-state) properties of MM, at least in the case of a monetary shock it is relatively easy to ensure that these facets are replicated in the construction of the miniature. Developing an intuitive understanding of the qualitative dynamics connecting the initial and terminal states of variables, however, is not so straightforward.

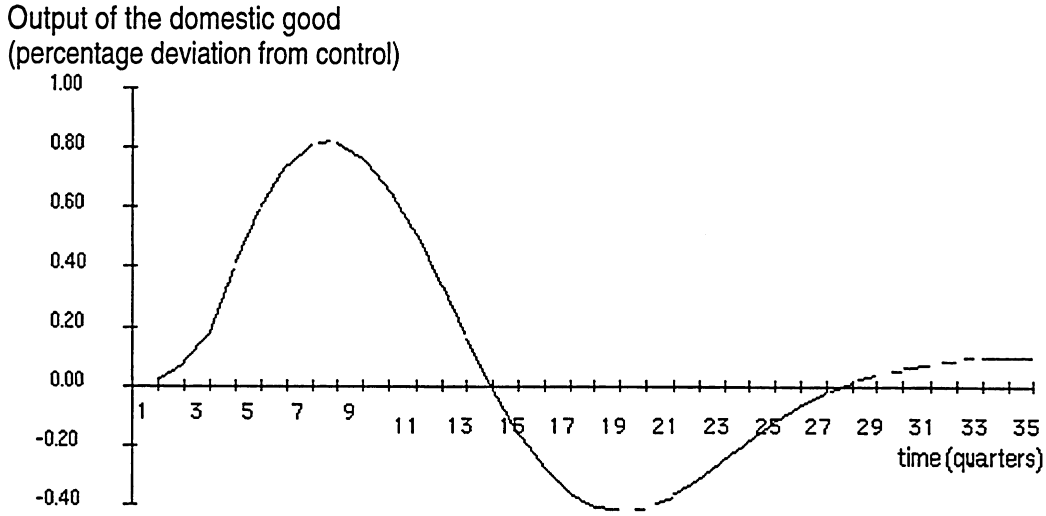

Figure 3.4 Response of output of the domestic good in MM to a monetary shock — from Powell and Murphy (1995, p. 326). Note the following similarities with EDBM with inertia: (a) the initial jump in activity produces a value for output which is much less than that ultimately reached; (b) output cycles below control at least once before converging back to the control solution.

The starting point for *D4M* is a simplified two-equation price system based on MM's specification of price dynamics. This allows the inflationary impact of the initial devaluation to be calculated and then used as the stimulus which leads, via the money demand function, to the post-devaluation rise in the interest rate.

Because the principal equations of MM's financial sector (specifically money demand and uncovered interest parity) take relatively simple forms, it is possible to incorporate them directly within *D4M*. Similarly MM's dynamic equation for the price of the domestic good, which is one of the two equations of the simplified price system mentioned above, is incorporated directly within *D4M*. The remaining equations of the miniature model give a very stylized account of MM's IS- and Phillips-curve mechanisms.

GNE is formulated as depending on two lags of itself plus a term driven entirely by the short-term interest rate at a lag of three quarters. The real trade balance is modelled as depending on just real GNE and the real exchange rate. Notwithstanding these drastic simplifications, *D4M* replicates the qualitative features of MM's dynamics. *D4M* even gives passable quantitative approximations to the MM results for most major macro variables over the first four years following the shock.

4.1 Simplified price system

MM is characterized by a set of underlying neoclassical equilibrium concepts which act as moving targets towards which the corresponding actual variables adjust over time. A case in point is p_Y^{MR}, which is the medium-run equilibrium level (not logarithm) of the price of the domestic good p_Y (loosely speaking, its marginal cost). In the transition from EDBM to *D4M* we need p_Y^{MR} as an additional variable of the model.

We will be working mostly in terms of deviations from control. The operational notation in Table 4.1 is used to save space. Note that for the small changes involved here, we use the approximations

$$(4.1) \qquad \frac{\%\Delta(\mathit{variable})}{100} \quad = \quad \Delta_C \log (\mathit{variable}) \quad = \quad \Delta_{propn} \; (\mathit{variable}) \quad .$$

The simplified price system consists of an equation for $\underline{\underline{\Delta}} \log p_Y$ and one for $\underline{\underline{\Delta}}$_log p_Y^{MR}. The first equation is just the actual MM price dynamics equation for the domestic good after the removal of the intercept[7] and the stochastic error term. Using MM parameter values, we obtain:

$$(4.2) \quad \underline{\underline{\Delta}} \log p_Y(t) \; = \; 0.3774 \, \underline{\underline{\Delta}} \log p_Y^{MR}(t\text{-}1) \; + \; 0.2508 \, \underline{\underline{\Delta}} \log p_Y^{MR}(t\text{-}2)$$
$$+ \; 0.3718 \, \underline{\underline{\Delta}} \log p_Y^{MR}(t\text{-}3) \; + \; 0.1259 \, \Delta_C \log \left(\frac{p_Y^{MR}(t\text{-}4)}{p_Y(t\text{-}4)} \right)$$

Table 4.1

Notational shorthand used for deviations from control

	Meaning
Δ...	change between last quarter and the present quarter in ...
Δ_C...	deviation from control in ...
% Δ_C...	percentage deviation from control in ...
%o ...	percentage-point deviation from control in ...
Δ_{propn}...	deviation from control in ... expressed as a proportion of the control value of this variable
$\underline{\underline{\Delta}}$...	deviation from control in the change between the current quarter and last quarter in ... (≡ the change between the current quarter and last quarter in the deviation from control in ...)

Source: Powell and Murphy (1995, p. 329).

The second equation has two components: (1) a feedback from the rate of growth of the price of the domestic good last quarter onto the rate of

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7 The intercept is the same in the base case (i.e., control solution) and in the shocked solution; when the latter is expressed as a deviation from the former, the intercept drops out.

growth this quarter of the corresponding medium-run equilibrium price; and (2) an error-correction term designed to ensure that p_Y^{MR} converges to the new long-run price $\bar{p}_Y(t)$ of the domestic good. Thus

$$(4.3) \qquad \underline{\underline{\Delta}} \log p_Y^{MR}(t) = \beta^* \underline{\underline{\Delta}} \log p_Y(t-1) + \gamma^* \left\{ \Delta_c \log \bar{p}_Y(t) - \Delta_c \log p_Y^{MR}(t) \right\}$$
$$(\beta^*, \gamma^* > 0)$$

where $\bar{p}_Y$ is the long-run equilibrium price of the domestic good, and β^* and γ^* are parameters (unknown at this stage). In the case of an unexpected, permanent 1 per cent monetary expansion we know that the new long-run-equilibrium domestic good price $\bar{p}_Y^{new}(t)$ after the shock is 1 per cent above its control value in every quarter; that is, $\Delta_c \log \bar{p}_Y(t) = 0.01$ for t =1, 2, 3, ...

Equation (4.3) is meant as a rough proxy for the much more complicated causal chain in MM in which price expectations feed into wage determination in and thence spread widely via MM's *enterprise production block* into all domestic prices (and in particular, into p_Y).

To make the system (4.2&3) operational, we need the following ingredients:

- an initial impulse in $\underline{\underline{\Delta}} \log p_Y^{MR}(1)$, and
- accumulation identities to update $\log p_Y$ and $\log p_Y^{MR}$ recursively over time.

The latter are straightforward:

$$(4.4) \qquad \Delta_c \log p_Y(t) = \Delta_c \log p_Y(t-1) + \underline{\underline{\Delta}} \log p_Y^{MR}(t-1) \ ;$$

$$(4.5) \qquad \Delta_c \log p_Y^{MR}(t) = \Delta_c \log p_Y^{MR}(t-1) + \underline{\underline{\Delta}} \log p_Y^{MR}(t-1) \ .$$

In the case of a monetary shock, the initial impulse in $\underline{\underline{\Delta}} \log p_Y^{MR}(1)$ comes from the instantaneous post-shock fall in the value of the local dollar. The medium-run equilibrium price p_Y^{MR} is a valid candidate for such an impulse since (unlike the actual price of the domestic good p_Y), p_Y^{MR} is a jumping variable in MM.

In response to a one per cent monetary shock, the nominal exchange rate falls at t = 1 in MM by 0.82 per cent. Hence the impact effect of the devaluation is for local-currency prices of traded goods to rise by 0.82 per cent.

The production technology in MM follows constant returns to scale. Labour, capital and imports combine to produce gross output, Y_B. It can be shown that the percentage change in the medium-run equilibrium price p_B^{MR} of gross output is a share-weighted sum of the percentage changes in the wage rate W_B, in the medium-run rental price of capital r_B^{MR}, and in the price of imports p_M.[8,9] Thus using the notation of Table 4.1, we find

$$\text{(4.6a)}\quad \%\Delta\, p_B^{MR} = \textit{labour share} \times (\%\Delta\, W_B)$$

$$+\ \textit{capital share} \times (\%_0\, \Delta\, r_B^{MR}) + \textit{import share} \times (\%\Delta\, p_M)\ ,$$

where the shares are taken from MM's control solution (along which they are constant). Because of the lags built into MM's Phillips curve, the nominal wage rate W_B does not respond at all in the initial quarter after the shock, and so $(\%\Delta\, W_B) = 0$. On the control path, imports' share in the sum of wages in the enterprise sector plus rentals on business capital plus imports is 0.2300; the shares of labour and capital respectively in this total are 0.5491 and 0.2209. The percentage deviation from control in the local-currency price of imports is $(\%\Delta\, p_M) = 0.8218$, being equal but opposite in sign to the jump in the exchange rate. Hence we reckon the initial impulse in the price p_B^{MR} of gross output to be:

$$\text{(4.6b)}\quad \%\Delta p_B^{MR} = (0.2209\,(\%_0\Delta r_B^{M}) + 0.2300 \times 0.8218\,)$$

$$= \quad (0.2209(\%_0\Delta r_B^{MR}) + 0.1890)\,.$$

The variable $\%_0\Delta r_B^{MR}$, unfortunately, cannot be eliminated from the above equation by a simple algebraic substitution from the equations of MM's enterprise production block; if it could, we would be able to convert the impulse in the price p_B^{MR} of gross output into an impulse in p_Y^{MR} as follows. Having noted that in MM gross output is a constant-returns-to-scale CET aggregate of domestic good output and exports, we would use

$$\text{(4.7)}\quad \%\Delta\, p_B^{MR} = \left\{\begin{matrix}\textit{domestic good's share}\\ \textit{in value of gross output}\end{matrix}\right\} \times (\%\Delta\, p_Y^{MR}) + \left\{\begin{matrix}\textit{exports' share in}\\ \textit{value of gross output}\end{matrix}\right\} \times (\%\Delta\, p_X)\ .$$

---

8 "Medium-run equilibrium" is used here in the sense defined by the medium-run closure of MM's enterprise production block — see Chapter 10 of Powell and Murphy (1995); the variables whose current values drive these equilibria are those shown in Table 10.4.2, p. 159 of Powell and Murphy (1995).

9 For any constant-returns-to scale production function $Y \equiv Y(X_1, X_2, \ldots, X_n)$ in a world where factors receive their marginal products and zero pure profits prevail,

$$\sum_{i=1}^{n} \frac{P_i X_i}{P_Y Y} (dP_i/P_i) = \frac{dP_Y}{P_Y}\,.$$

In MM's control-solution data base, the two shares on the right have the values 0.7528 and 0.2472. Putting ($\%\Delta\ p_X$) = 0.8218 in (4.7) and rearranging, we would obtain:—

$$\text{(4.8)} \qquad \%\Delta\ p_Y^{MR} = \frac{\{\%\Delta\ p_B^{MR}\} - 0.2472 \times 0.8218}{0.7528}$$

[from (4.6b)]

$$= \frac{\{0.2209\ \%\text{o}\Delta r_B^{MR} + 0.1890\} - 0.2472 \times 0.8218}{0.7528} \quad .$$

This is as far as elementary methods will take us. Because the eight endogenous variables in the medium-run closure of MM's enterprise block must be solved simultaneously as a system, we either have to undertake this chore to obtain a value for $\%\text{o}\Delta\ r_B^{MR}$, or equivalently, we can examine the MM solution at t = 1. It is convenient here to do the latter, although, in the light of the fact that r_B^{MR} is not a formally recognized (i.e., computed) variable in MM, even this approach involves one more step .

The additional step makes use of the percentage-change version of a first-order condition from MM's underlying neoclassical production structure. This takes the form:

$$\text{(4.9)} \qquad \%\text{o}\Delta\ r_B^{MR} = \%\Delta\ W_B + \frac{1}{\sigma_1}(\%\Delta E_B^{MR} - \%\Delta K_B)\ ,$$

where E_B^{MR} is medium-run equilibrium business-sector employment, K_B is the size of the business-sector capital stock, and σ_1 is the elasticity of substitution between labour and capital (= 0.75 in MM's parameter file). Since the capital stock is not a jumping variable, the last term on the right of (4.9) is zero at t = 1; as well, we have noted above that $\%\Delta\ W_B = 0$ at t = 1. The value of ($\%\Delta E_B^{MR}$) projected for t = 1 by MM is 0.3694 per cent. Hence we find

$$\text{(4.10)} \qquad \%\text{o}\Delta\ r_B^{MR} = 0.4925 \text{ percentage points per year.}$$

Substituting this value into (4.7) ; we find:

$$\text{(4.11)} \qquad \%\Delta\ p_Y^{MR} = 0.1266 \text{ per cent.}$$

When we check this against the MM result for t = 1 we find that $\%\Delta\ p_Y^{MR} = 0.1344$. The misclosure is acceptable, given the rounding involved in our arithmetic and the fact that the percentage-change linearizations used above are approximations. For the time-being we will use the value 0.1344 per cent for the initial impulse shock in p_Y^{MR}. Thus, starting at t=1, we put the simplified price dynamics into motion by setting $\Delta \log p_Y^{MR}(0) = 0.001344$ in (4.2) (the other lagged values of this variable being zero).

4.2 Calibrating the simplified price system

The parameters β^* and γ^* of (4.3) must be selected before we can use the simplified 2-equation price system. MM's standard software computes results for the first 36 quarters after a shock, as well as giving results in a notional long-run quarter (arbitrarily located 44 quarters out). The calibration method used here is to fit the simplified system, launched with an initial impulse shock in p_Y^{MR} of 0.1344 per cent above control, to the actual MM simulation results for the logarithms of p_Y and of p_Y^{MR}. The chosen criterion of fit is the sum of the squared differences over 36 quarters between $\%\Delta p_Y$ as projected by MM and by the simplified system (4.2&3). As in EDBM, we use the *Solver* routine (of Microsoft Excel 4.0 or later) for this purpose. The results are summarized in Figure 4.1 and the caption thereto.

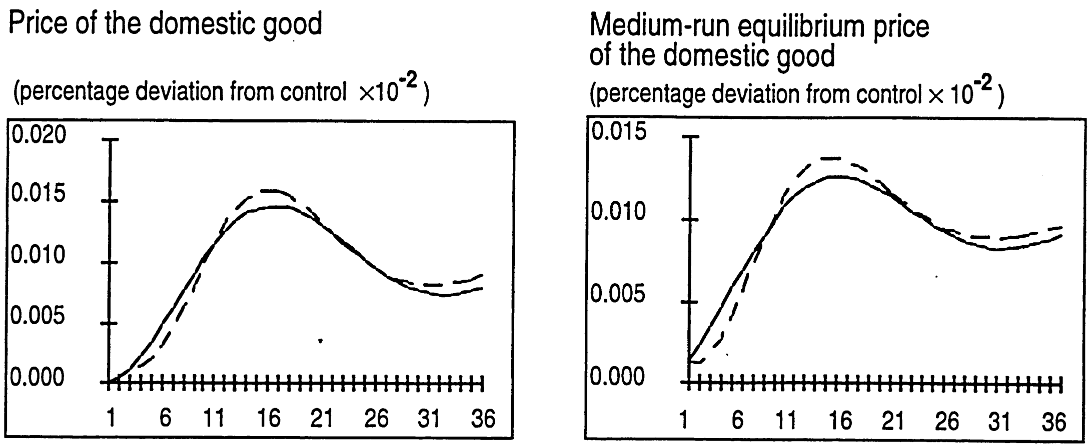

Figure 4.1 Fit of the simplified 2-equation price system (4.2)-(4.3) to the MM simulations of a one per cent monetary expansion — from Powell and Murphy (1995, p. 332). The panel on the left shows the MM trajectory of the price p_Y of the domestic good (dashed line) and the trajectory for the same variable from the simplified price system. The right-hand panel shows the trajectories for the medium-run equilibrium price — again, the MM projection is the dashed line. The values of the calibration parameters are $\beta^* = 0.5755$ and $\gamma^* = 0.1116$.

4.3 Explaining short-term interest rate movements

The interest-rate trajectory generated by substituting

- the $\%\Delta p_M$ values generated by the simplified price system,
- $\%\Delta M = 1$, and
- $\%\Delta$ *real GNE* $= 0$

into MM's money demand equation, is shown in Figure 4.2. The actual MM 3-month interest rate projections are shown in the same figure.

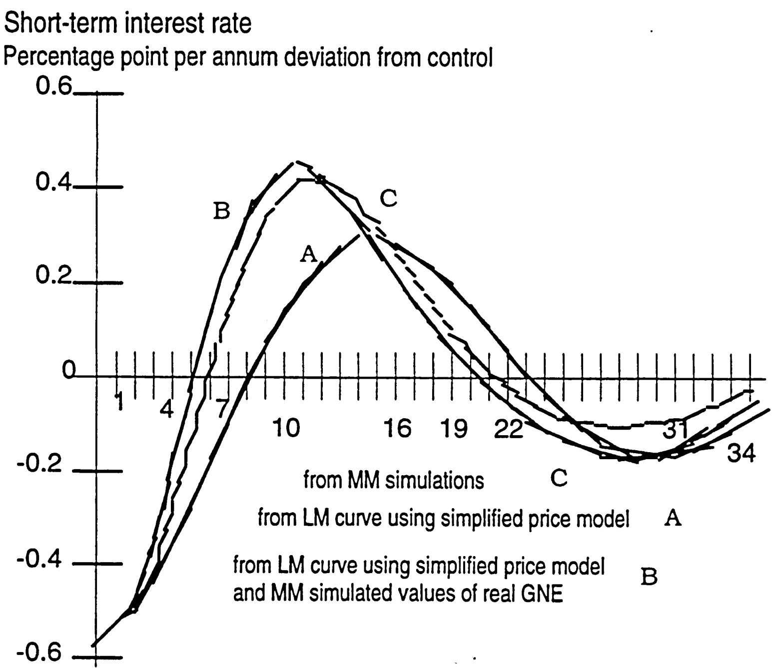

Figure 4.2 Interest rate response to a monetary shock in MM and its rationalization in terms of the operation of a simplified version of the MM subsystem determining prices from Powell and Murphy (1995, p. 333).

4.4 GNE, the price of the domestic good, and the interest and exchange rates

The simplified price system (curve "A" in Figure 4.2) produces only a poor approximation to the full MM model (curve "B"). Most of the misclosure, however, is due to the fact that curve "A" is computed with real GNE held constant. Using the actual MM results for real GNE and the simplified price model's results for p_Y in MM's money demand equation produces curve "C". Clearly, to obtain a tight story for the interest rate, we need to endogenize GNE.

What of the steady-state requirement that p_Y converge to 1 per cent above control in the long run? Taking t=100 (25 years) as being sufficiently far into the future for the steady state to have set in, we find $\%\Delta\ p_Y(100) = 0.982$ (per cent above control). Thus effectively the steady-state requirement is met. This should come as no surprise; the error-correction term on the right of (4.3) is designed to ensure just this outcome.

4.5 Equations of D4M

D4M is synthesized from two ingredients introduced above:

(i) the lag structures in aggregate demand introduced in EDBM (see especially Table 3.1), and

(ii) the simplification (4.2&3) of MM's price system.

Table 4.2 contains most of the equations of *D4M* that are involved in determining real spending, the price of the domestic good, and the interest and exchange rates. Relative to EDBM (see Table 3.1), *D4M* exhibits the following differences:

4.6 Calibrating D4M to MM

The simplified price system has already been calibrated above in subsection 26.5(c), yielding coefficients $\beta^* = 0.5755$ and $\gamma^* = 0.1116$. However, when the simplified price system is incorporated into *D4M*, there are potentially eight criteria to be satisfied; namely, the goodness of fit to the MM trajectories for each of the miniature model's four endogenous variables, and the steady-state conditions for each of them.

The objective function chosen for calibration is the sum over *gne*, p, r and e of the squared discrepancies between these variables and the corresponding values on the respective MM trajectories. The following side constraints are imposed on the terminal behaviour of the exchange rate and the price of the domestic commodity:

$$0.95 \leq p(100) \leq 1.05 \;; \tag{4.23}$$

and

$$-1.05 \leq e(100) \leq -0.95 \,. \tag{4.24}$$

With the initial impulse shock in the inflation rate in the price of the domestic good set to -0.1344, the combined fit (as defined in the last paragraph) of *gne*, p, r and e to MM was optimized over the first 36 quarters. The resulting calibrated parameter values are:

$$\boxed{\begin{array}{c}\beta^* = 0.5568\,, \quad \gamma^* = 0.1133, \quad \sigma = 4.0391, \quad \psi = 0.0449, \\ R = 1.9661 \quad \text{and} \quad \rho = 0.0486\end{array}}$$

Note that the responsiveness of *GNE* to the interest rate depends on *all four* of the

coefficients σ, ψ, R and ρ, not just on σ; moreover the individual parameters are not sharply identified.

Table 4.2

*Core equations of D4M**

Eq'n no	Description	
SHORT-RUN DISEQUILIBRIUM EQUATIONS FOR AGGREGATE ACTIVITY		
(4.12)	*Real GNE*	$gne\,(t) = \psi\,\alpha_1(t) + \varphi(t)\,[1-\psi]\,gne\,(t-1) + [1-\varphi(t)]\,[1-\psi]\,gne\,(t-2) \quad [0\ ^2\ \psi\ ^2 1]]$
(4.13)	*Lag weighting function*	$\varphi(t) = R\ exp\,(-\rho\,[t-1]) \quad [\rho \geq 0]$
(4.14)	*Component of GNE responsive to lagged interest rate*	$\alpha_1(t) = -\,\sigma r(t-3)$
(4.15)	*Balance of trade: responsive to lagged real exchange rate and lagged gne*	$\alpha_2(t) = H\,(\,[e(t-1)] + p(t-1)],\ gne(t-1)\,)$
FINANCIAL-MARKET EQUILIBRIUM EQUATIONS		
(4.16)	*LM curve*	$m - p = gne - \lambda r$
(4.17)	*Uncovered interest parity*	$r = r^* + x$
(4.18)	*Expected rate of depreciation of the domestic currency*	$x = -\,\dot{e}$
SIMPLIFIED PRICE SYSTEM		
(4.19)	*Influence of medium-run equilibrium price of the domestic good on its actual price*	$\dot{p}(t) = 0.3774\ \dot{p}^{MR}(t-1) + 0.2508\ \dot{p}^{MR}(t-2) + 0.3718\ \dot{p}^{MR}(t-3) + 0.1259\{p^{MR}(t) - p(t)\}$
(4.20)	*Feedback from p onto p^{MR}*	$\dot{p}^{MR}(t) = \beta^*\,\dot{p}(t-1) + \gamma^*\{\,\bar{p}\,(t) - p^{MR}(t)\}$
LONG-RUN EQUILIBRIUM EQUATIONS FOR THE PRICE LEVEL AND EXCHANGE RATE		
(4.21)	*Long-run neutrality of money*	$\bar{p} = m - y$
(4.22)	*Purchasing Power Parity*	$\bar{p} = p^* - \bar{e}$

Source:Powell and Murphy (1995, p. 325).

* The updating equations for the price of the domestic good and for the exchange rate, not shown in the body of the Table, take the forms $p(t) = p(t-1) + \dot{p}(t)$ and $e(t) = e(t-1) + \dot{e}(t)$ respectively. Equation (4.26) below, which endogenizes the initial impulse shock in the inflation rate for arbitrary m, also is a part of *D4M* proper.

Except for r, r* and α_2, all variables in this table are to be interpreted as percentage deviations from control. Deviations in the nominal interest rates at home and abroad, denoted r and r* respectively, are measured in percentage points per annum (for α_2, see point (f) in text opposite). Time indexing of variables is suppressed in all equations other than (4.12-15) and (4.19&20), since in the other equations all variables are contemporaneous. Note that the timing convention implicit in the notation assumes that the shock impinges at t = 1. The other variables are: *gne* (real gross national expenditure); α_1 (component of GDP which is responsive to the interest rate at a lag of three quarters); $\alpha_2(t)$ (the real trade balance = component of GDP which is responsive to the real exchange rate and to GNE, both lagged one quarter); e (the nominal exchange rate, foreign dollars per local dollar); $\dot{e}$ (the time-rate of change of e) ; $\bar{e}$ (the long-run equilibrium value of e); (φ, a lag weighting function); m (the money supply); p (the local-currency price of the domestic good — sometimes denoted elsewhere in this ppaper by p_Y); p^{MR} (the medium-run equilibrium value of p);

Notes to Table 4.2 (continued)

$\bar{p}$ (the long-run equilibrium value of p); $\dot{p}$ (the time-rate of change of p); $\dot{p}^{MR}$ (the time-rate of change of p^{MR}); and p* (the foreign-currency price of internationally traded commodities). Note that here the notation $\dot{e}$, $\dot{p}$, etc., indicates deviations from control in finite changes between successive quarters in the logarithm of the variable concerned, and so corresponds to the $\underline{\underline{\Delta}}$ operator of Table 4.1. H( ) is a function developed below in equation (4.27).

(a) the activity variable a (which in DBM and its extension may be identified with GDP) is disaggregated into *gross national expenditure [gne]* and the *balance of trade surplus*. The former is made the subject of the main dynamic equation for economic activity, (4.12). The determination of GDP from these components is left until later as an add-on equation.

(b) The variable y [= domestic supply] in the LM curve (3.5) is replaced in (4.16) by *gne*, making it conform to MM.

(c) The parameters ϕ and λ of (3.5) are put equal to their MM values in (4.16); namely, 1 and 0.5699 (see Appendix 3.1);

(d) Since they are not being shocked in this simulation, all of the exogenous variables on the right of (3.4) are omitted.

(e) As we have seen above in subsections 26.5(f)&(g), the interest rate enters the various components of gross national expenditure with lags of 2, 3, 4 and 5 quarters. In (4.14) we enter r with just *one* lag, namely a lag of 3 quarters, in the component α_1 of *gne*.

(f) The trade account is captured via the variable α_2 in (4.15), which is the change in the balance of trade surplus (expressed as a percentage of GDP). A drop in international competitiveness — i.e., a rise in the real exchange rate (e + p) — causes α_2 to fall. α_2 and *gne* are used to determine *gdp* in sub-section 26.6 (d) below.

(g) The Phillips curve (3.8) is replaced by the simplified MM price system (4.19&20).

(h) Finally, as in DBM and *D4M*, (4.12) implies that the long-run change in real GNE is zero under a monetary shock[10].

Neither of the terminal conditions (4.23&13) were binding in the solution. The values of the endogenous variables at t = 100 were:

Variable	*D4M-value at t = 100*	*Theoretical steady-state value*
p *(per cent)*	0.990	1
gne (per cent)	0.002	0
e *(per cent)*	-0.974	-1
r *(percentage points per year)*	-0.003	0 .

The resultant fits of the *D4M* trajectories to those from MM are shown in Figure 4.3.

One final step remains; namely, to check that *D4M* is capable of producing the trajectories shown in Figure 4.3 if a one per cent monetary shock is imposed on the model. For this we need to formalize

10 Note that the steady-state of (4.12) is { $gne(\infty) = -\sigma\, r(\infty)$ }. If the world interest rate is not shocked, then $r(\infty) = 0$ in MM's standard closure; in the case of a monetary shock, the steady state of (4.12) thus accords with MM's long run in which $gne(\infty) = 0$.

the link between the monetary shock and the initial impulse in inflationary pressure.

Above we found that $\underline{\underline{\Delta}}$ log $p_Y^{MR}(0) = 0.001344$; in current notation, this is $\dot{p}^{MR}(0) = 0.001344$. The calculation that produced this result can be summarized by writing out the components of (4.19) in full:

$$(4.25) \quad \dot{p}^{MR}(0) = \frac{\{0.2209\ [\%\Delta E_B^{MR}(1)/\sigma_1 - (0.23 - 0.2472)\%\textit{jump in } E\ \}}{0.7528},$$

where E is the exchange rate (foreign \$ per local \$). The results for $\%\Delta E_B^{MR}(1)$ for a 1 and a 2 per cent monetary expansion respectively are 0.3694 and 0.7384; thus for small changes $\%\Delta E_B{}^{MR}(1)$ is close to proportional to the percentage monetary shock. Hence, using this property and letting *jump in E(m)* denote the exchange rate jump associated with an m per cent monetary expansion, we can write the initial inflationary impulse shock as a function of m as follows:

$$(4.26)\ \dot{p}^{MR}(0) = \frac{\{0.2209\ [0.3694\,m/0.75] - (0.23 - 0.2472)\%\textit{jump in } E(m)\ \}}{0.7528},$$

where we have substituted the MM value of σ_1 (namely, 0.75).

We now add equation (4.26) to the equations of *D4M* and solve for the initial jump in the exchange rate (rather than taking it as given from MM, as previously). Again using *Solver* in Microsoft Excel to minimize the squared difference between e(100) and (-m), we obtain the following result:

jump in exchange rate = -0.817 per cent
e(100) = -0.995 per cent.

At the level of resolution of Figure 4.3, the graphed trajectories of *gne*, p, r and e do not change when the model is solved in this way. We conclude that with (4.26) added to Table 4.2, *D4M* captures, to a good first approximation, the dynamics of MM *for a one per cent* monetary shock (m = 1). Experiments with m = 2 and m = -2 yield comparably satisfactory results.

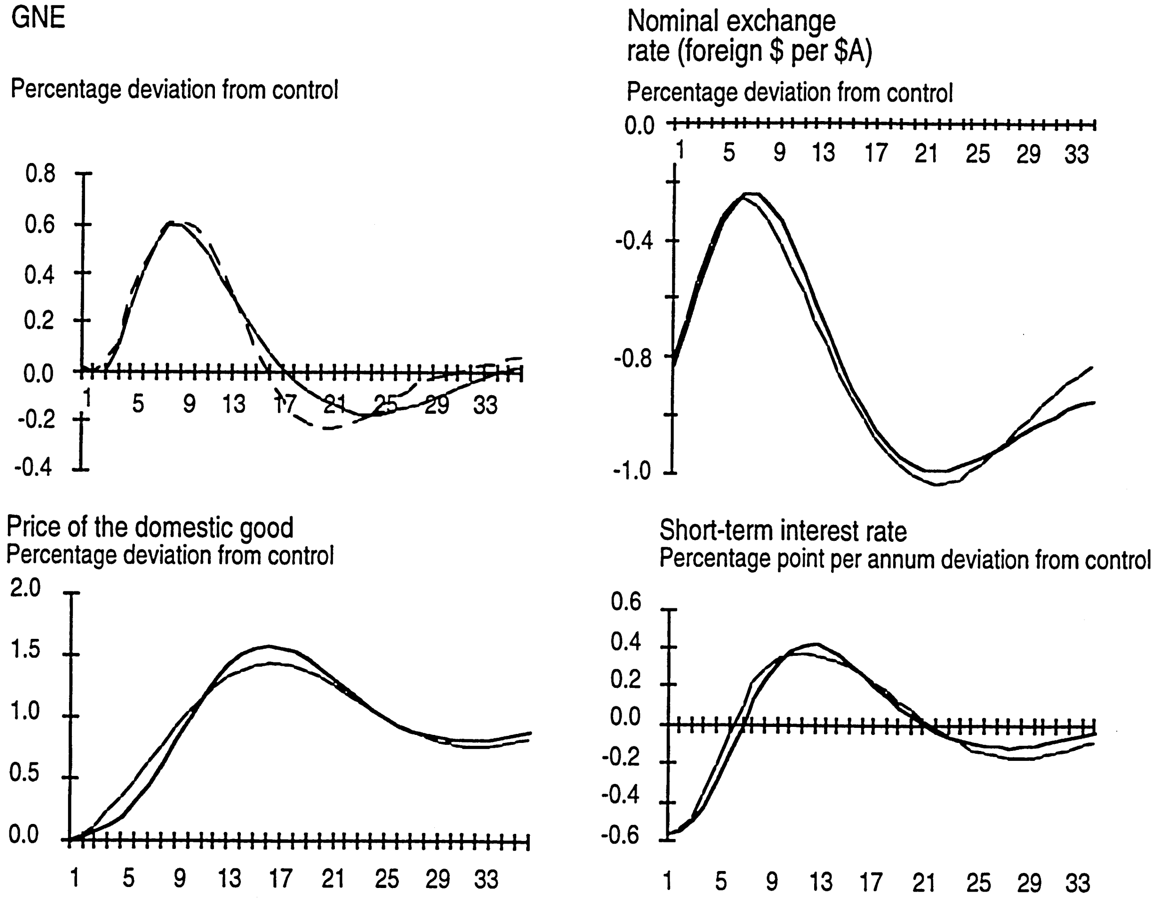

Figure 4.3 Fit of *D4M* to GNE, the exchange rate, the price of the domestic good and the short-term interest rate in MM. The solid lines are from the calibrated *D4M* model; the broken or greyed lines are the MM projections.

4.7 Completing the miniature model: the trade account and GDP

The MM trajectories for real imports, exports, and the trade balance are shown in Figure 4.4.[11]

GDP remains to be endogenized. It is the sum of GNE and the balance of trade surplus. The latter is denoted by α_2 in Table 4.2. We complete *D4M* by specifying the form of the function H( ) in (4.15).

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[11] In GDP accounting, real imports and exports can simply be defined as commodity indexes; alternatively, valuation effects can be imputed that take account of changes in the rate at which exports exchange for imports. In this paper the former approach is followed. Thus the real trade balance is here defined as the difference (in millions of 1984-85 $A) between what the current physical volume of exports would have yielded at 1984-85 prices and what the current physical volume of imports would have cost at prices of the same year.

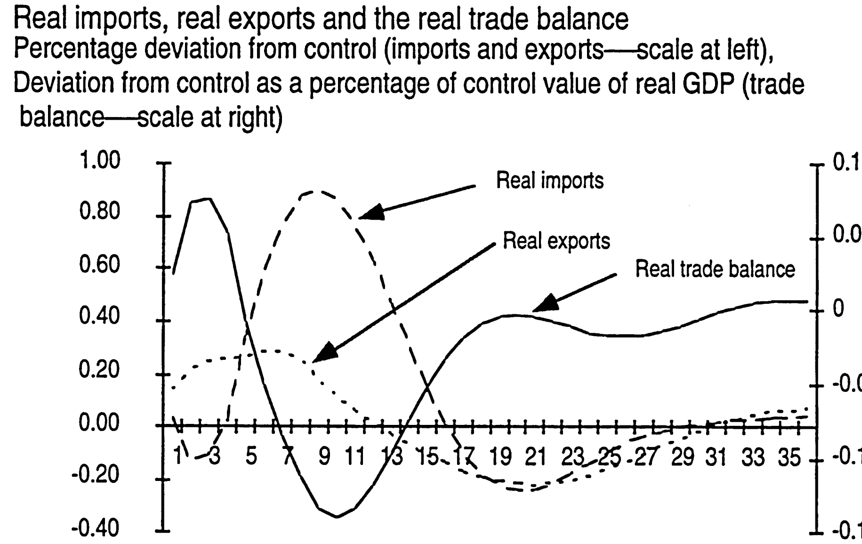

Figure 4.4 MM trajectories for real imports, exports, and the trade balance under the monetary shock

Imports and exports in MM respond to the real exchange rate. In terms of the notation of EDBM, the local economy's international competitiveness (i.e., the reciprocal of the real exchange rate) is measured by (p* - e - p). At any given level of local aggregate spending, imports and exports respectively are a decreasing and an increasing function of (p* - e - p). Because demand tends to spill over into the trade account, at any given level of (p* - e - p), increases in real demand lead to higher imports and lower exports. In MM such influences are captured by including a capacity utilization variable on the right of the import demand and of aggregate export supply equations.

The above implies that the real trade balance should be an increasing function of competitiveness and a diminishing function of *gne*. We allow a lag of one quarter in (4.15). In terms of percentage deviations from control, by including quadratic terms we also allow for the idea that the trade balance may respond non-linearly to these variables. Because the foreign-currency price of foreign goods does not change in these simulations, we delete p* from our expressions for competitiveness (and for the real exchange rate). The resultant detailed specification of (4.15) is:

$$(4.27) \qquad \alpha_2(t) = \omega_1 \times \{-e(t-1) - p(t-1)\} + \omega_2 \times \{-e(t-1) - p(t-1)\}^2 + \omega_3\, gne + \omega_4\, gne^2 ;$$

$$[\omega_1 - 2\omega_2 \{ e(t-1)+p(t-1)\} > 0;\ \omega_3 + 2\ \omega_4\, gne < 0] .$$

Calibration was achieved using the values of e and *gne* from *D4M* by minimizing the squared sum of discrepancies between the values of the real trade balance generated by (4.27) and the corresponding values from MM over the first 36 quarters. The resulting values for the ω coefficients are: $\omega_1 = 0.05176$; $\omega_2 = 0.03073$; $\omega_3 = -0.0265$; and $\omega_4 = -0.2923$. At t = 100, the projected real balance of trade surplus is 0.0003 per cent of GDP (versus a theoretical steady-state value of 0). The fit is shown in left-hand panel of Figure 4.5. The resultant fit of real GDP in *D4M* to the trajectory of the same variable in MM is shown in the right-hand panel of the same figure. At t = 100, the *D4M* projection for GDP is 0.003 per cent above control.

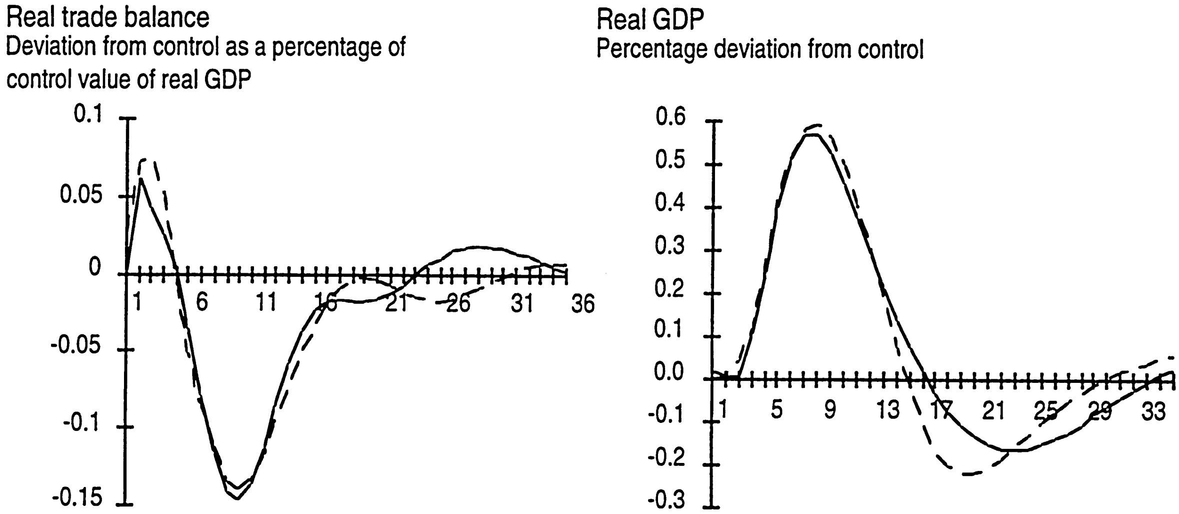

Figure 4.5 *D4M* fit of real trade balance and real GDP to MM. The unbroken lines show the *D4M* trajectories.

4.8 Extending the coverage of D4M: investment, imports and employment

In the interests of space, the account given in this section is synoptic. No macro model (not even a miniature) would be complete without an attempt to endogenize employment. The approach here is via the production function for MM's enterprise sector (schematically set out in Figure 10.2.1 [p. 151]). Given values of business capital K_B, gross output Y_B and imports M , the production function can be inverted to solve for business sector employment, E_B. But first we must endogenize K_B, Y_B and M .

We start with K_B — this first requires us to endogenize business fixed investment (I_B) so that the capital stock can be found from the accumulation identity. I_B is endogenized by equation (4.28). The lagged percentage deviation from control of the capital stock ($\%\Delta K_B$) enters on the right of (4.28) in order to capture the negative feedback from the size of the capital stock onto the rate of return (and thence, investment) at any given level of GNE. To allow for a non-linear response, the variable $\%\Delta K_B(t-1)$ also appears in squared form. We expect the slope $\{\omega_6 + 2\,\omega_7\, \%\Delta K_B(t-1)\}$ to be negative.

$$(4.28) \qquad \%\Delta I_B(t) = \omega_5\, gne\,(t) + \omega_6\, \%\Delta K_B(t-1) + \omega_7\, (\%\Delta K_B(t-1))^2 \quad ;$$

$$\omega_5 = 4.1526; \quad \omega_6 = -3.6936; \quad \omega_7 = 4.5258.$$

The above equation must be solved simultaneously with the accumulation identity:

$$(4.29) \qquad (1+\gamma)\, \%\Delta K_B(t) = (1-\delta_B)\, \%\Delta K_B(t-1) + \{\frac{I_B}{K_B}\}_{t-1}\, \%\Delta I_B(t-1) \ ,$$

where γ is the rate of growth of the capital stock between (t-1) and t (here set equal to the natural growth rate on the control solution, 0.396 per cent per quarter). (Note that this rate is higher than used for the estimation in Part 2 — see p. 279.)

Real imports are modelled along the lines used above for the real trade balance:

$$(4.30)\ M = \omega_8 \times (-e(t-1) - p(t-1)) + \omega_9 \times \{-e(t-1) - p(t-1)\}^2 + \omega_{10}\, gne + \omega_{11} gne^2;$$

$$\omega_8 = -0.02 (imposed); \quad \omega_9 = -0.3118 \quad \omega_{10} = 0.9612 \quad \omega_{11} = 1.3099.$$

In spite of having to impose an arbitrary value on ω_8 to ensure negativity, this had only a small impact on the goodness of fit.

The business-sector gross output variable Y_B of MM (see Figure 10.2.1, p. 151) is taken here as proportional to GDP; on MM's control solution the constant of proportionality is 0.9338. Using the constant-returns-to-scale assumption underpinning production in MM, we may write (see Exercise 5.4, p. 101):

$$(4.31)\quad \%\Delta Y_B(t) = S_K\%\Delta K_B(t) + S_L\%\Delta E_B(t) + S_M\%\Delta M(t);$$

$$S_K = 0.2209; \ S_L = 0.5491; \ S_M = 0.2300 .$$

Above, the shares S_K, S_L and S_M of capital, labour and imports respectively in the cost structure for gross enterprise output are taken from the control solution of MM. Solving for E_B, and making the substitution $\%\Delta Y_B = 0.9338 gdp$ ($gdp \equiv \%\Delta GDP$), we find:

$$(4.32)\qquad \%\Delta E_B(t) = \{\ 0.9388 gdp - S_K\%\Delta K_B(t) - S_M\%\Delta M(t)\ \}/S_L .$$

As Figure 4.6 shows, the employment fit obtained using (4.32) is poor over the first two years. Given that short-run employment in MM is not constrained to lie on the production function, this is not unexpected. Business employment in MM adjusts partially towards its medium-run equilibrium value, E_B^{MR}, which also is plotted along with the MM projections for E_B in Figure 4.6.6. MM also includes the time-rates of change of capacity utilization as an explanator. There is no equivalent variable in *D4M*.

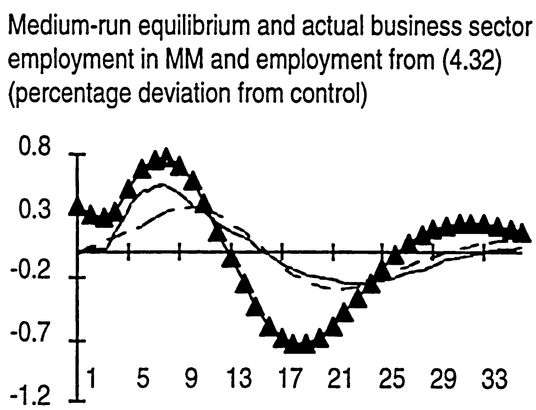

Figure 4.6 Medium-run equilibrium and actual business-sector employment projections from MM compared with employment projections from (4.32). The shaded triangles give the path of E_B^{MR}; the unbroken line gives the E_B path from*D4M*.

However, the fit over the first two years can be improved by including the time-rate of change of real GDP.

The rationale for including the time rate of change of output in the employment equation is as follows. In MM's shortest run (1 quarter), output of the domestic good is demand-determined a la Keynes. Inverting the production function as in (4.32) then produces input levels consistent with output being on the production function. However, no such constraint operates in MM's shorter lengths of run. With capital responding slowly due to a gestation lag for investment, plus distributed adjustment lags in investment itself, inverting the production function implies movements in imports and/or employment that validate the change in output. Imports, however, are also subject to lags in adjustment; consequently most of the increase in inputs that would be required to make the early MM values of GDP consistent with its production function show up in (4.32) as extra employment.

MM includes the contemporaneous value of the time-rate of change of capacity utilization via a variable in its business-sector employment equation and the same variable lagged one quarter. In *D4M* we include just one lag of a linear and a quadratic term in the time-rate of change of the percentage deviation from control in real GDP. The resultant equation is:

$$(4.33) \qquad \%\Delta E_B(t) = \{\ 0.9388\,gdp\ -\ S_K\%\Delta K_B(t)\ -\ S_M\%\Delta M(t)\ \}/S_L + \omega_{12}\,\Delta\{\%\Delta Y_B(t-1)\} + \omega_{13}\,[\Delta\{\%\Delta Y_B(t-1)\}]^2\ .$$

The calibrated values of ω_{12} and ω_{13}, namely - 0.880 and - 4.305, were found by optimizing the fit to the MM values over t = 1, 2, ... , 36.

The fitted plots for I_B, K_B, M , and E_B are shown in Figure 4.7.

5. Summary

MM’s dynamic response to an unanticipated, permanent 1 per cent increase in the money supply has been explained in two parts. In the first, the Dornbusch model (DBM) was extended by including frictions in the response of aggregate demand to the monetary stimulus. In the second part of the explanation, a miniature model (*D4M*) was developed by further extension of DBM and calibration to simulation results from MM.

The three major qualitative differences of MM’s response from DBM’s were resolved as follows:

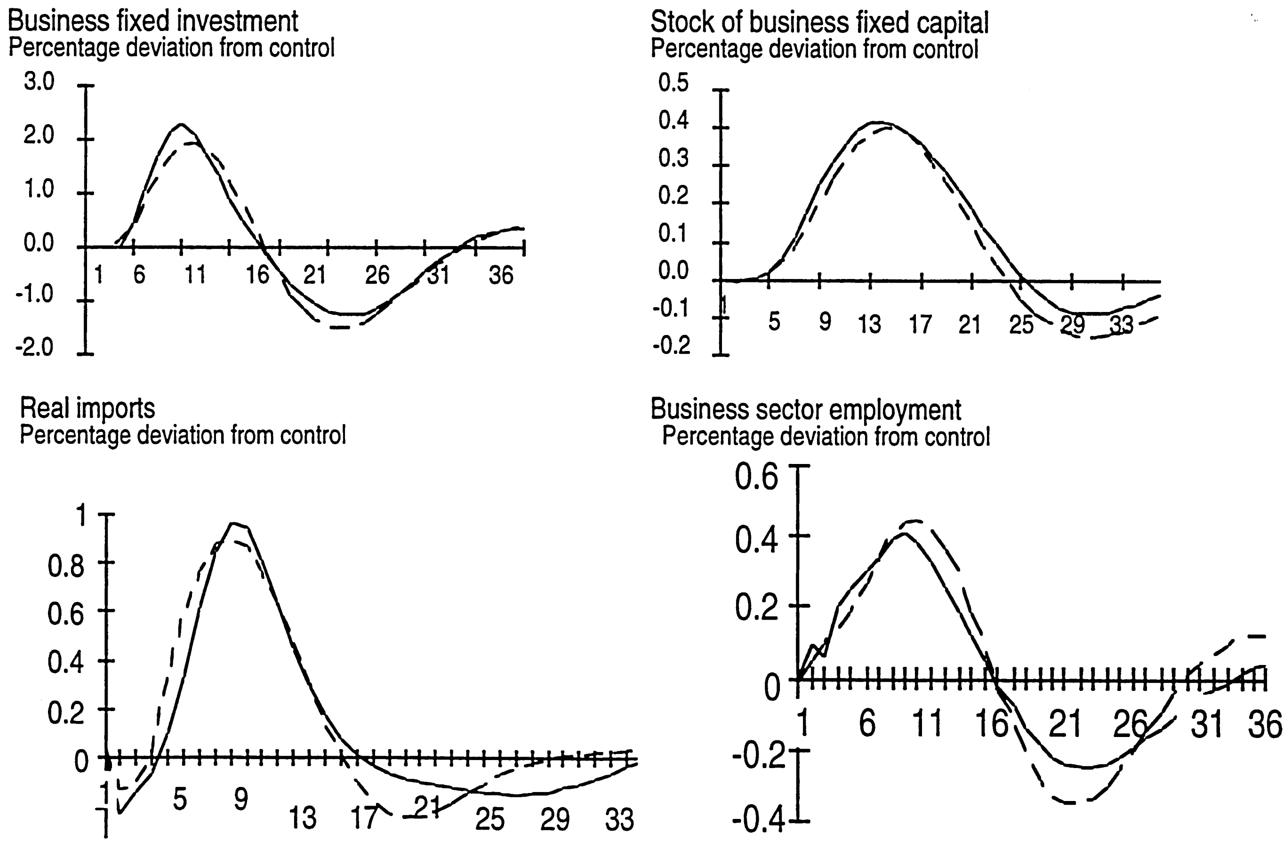

Figure 4.7 *D4M* solutions for investment, the business fixed capital stock, imports and business-sector employment. The solid lines show *D4M* trajectories; the broken lines are the corresponding time paths in MM.

(1) MM's cyclical (rather than monotonic) adjustment paths arose naturally from the lag structures in the model's structural form. Moreover, at least in the case of the real exchange rate, a cyclical path is inevitable since a continuously appreciating currency in the post-shock regime would lead to the economy entering the steady state with less foreign debt than on the control path. This would result in a higher steady-state ratio of consumption to GDP, violating the terminal conditions of MM's standard closure.

(2) MM's initially undershooting exchange rate was traced to the dynamics of the adjustment of aggregate demand. Endowing DBM with a gradual (rather than an instantaneous) adjustment of demand leads to a damped cyclical (rather than a monotonic) adjustment path for the nominal exchange rate. Both undershooting and overshooting are possible in this extended DBM, depending on how the lag structure in aggregate demand is specified.

(3) MM's seeming errancy (relative to DBM) in its movement of the nominal exchange rate *away* from the new equilibrium in the immediate post-shock era again can be traced to the sluggish response of aggregate demand. Real GNE in MM is a jumping variable, but the jump is small. Given MM's money demand curve, the subsequent gradual increase for about two years in real GNE means that the interest rate rises over this period. Uncovered interest parity then implies that the nominal exchange rate must rise during these two years.

The mechanism driving the (on the whole, successful) emulation of MM's response by the miniature model *D4M* was the elaboration of the initial inflationary impact of the devaluation at t=1. The equations of MM's production enterprise block allow the initial jump in the local prices of traded goods to be translated into a corresponding jump in the medium-run equilibrium price of the domestic good. Via a simplified 2-equation price system, this generates a trajectory for the price of the domestic good conditional on the size of the initial devaluation.

The variables appearing in MM's LM curve are nominal money, the price of the domestic good, real GNE and the (short-term) interest rate. Real GNE is formulated in *D4M* as a function of itself lagged and of lagged interest rates. The initial devaluation causes an instantaneous fall in the interest rate. With a lag, this leads to a response in GNE[12]. Given an exogenous trajectory for money and a price trajectory predetermined by the simplified price system, plus a response in GNE, the response of the interest rate is determined. This system is recursively solved forward for GNE and the interest rate. Note that this solution is everywhere conditioned on the size of the initial devaluation.

The core of *D4M* (as described in the preceding paragraph) is solved for the size of the initial devaluation by minimizing the discrepancy between the model's value of the exchange rate at t=100 and its (known) steady-state value (namely, one per cent below control). Additional equations are then added for the trade account, GDP and employment. These can be solved from the values of variables in the core of *D4M*. The miniature is able to reproduce a fair approximation to the MM solution for most major aggregates, the exceptional case being employment.

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[12] Or lack of one (i.e., a zero response) for the first few quarters.