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Water Pricing and Investment in Melbourne: General Equilibrium Analysis with Uncertain Streamflow

by

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Preliminary Working Paper No. IP-63 December 1993

ISSN 0 062 10369 0

ISBN 0 642 10361 5

The Centre of Policy Studies (COPS) is a research centre at Monash University devoted to quantitative analysis of issues relevant to Australian economic policy. The Impact Project is a cooperative venture between the Australian Federal Government and Monash University, La Trobe University, and the Australian National University. During the three years January 1993 to December 1995 COPS and Impact will operate as a single unit at Monash University with the task of constructing a new economy-wide policy model to be known as *MONASH*. This initiative is supported by the Industry Commission on behalf of the Commonwealth Government, and by several other sponsors. The views expressed herein do not necessarily represent those of any sponsor or government.

ABSTRACT

We describe the theory, computation and results of a multiperiod general equilibrium model designed to assist an urban water authority in its pricing and investment decisions. The model includes gestation periods in the creation of dams, main sewers and treatment plants. It allows for lumpy capital items and recognizes cost differences in the provision of services in peak and non-peak times. Its general equilibrium framework is convenient for handling links between the water authority and the rest of the economy, especially the housing sector.

We have used two computational approaches. In the first, we reformulate the model as a single-entity optimization problem and then apply a linear programming package. We have found that a better approach is to apply Newton-Raphson methods to a formulation of the model as a set of equations depicting purely competitive behaviour in all productive activities.

A special feature of this paper is an integration of the model's results, obtained under the assumption of certainty, with data on weather-induced variations in streamflow and demand. Using Monte Carlo techniques we assess the risks of water shortages associated with the investment and pricing strategies that our model indicates.

Key Words and Phrases : water pricing and investment; uncertain streamflow; water policy in a general equilibrium model; water policy for Melbourne; linear programming; Newton-Raphson; Monte Carlo; peak and non-peak.

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WATER PRICING AND INVESTMENT IN MELBOURNE: GENERAL EQUILIBRIUM ANALYSIS WITH UNCERTAIN STREAMFLOW

by

Mark Horridge, Peter B. Dixon and Maureen Rimmer[1]

INTRODUCTION

Until recently, market forces have had little effect on the production and use of water in Melbourne. The Melbourne water authority derived most of its revenue from taxes levied on house and land values. Householders were entitled to a generous quota of "free" water and the charges for additional water were low. Water usage was little affected by price, making Melbourne water policy a clear example of what Hanke (1978) described as supply management. That is, extrapolating from past trends, the water authority predicted future needs and undertook whatever capital works were necessary to meet them.

Today that pattern is changing. Environmentalists decry the steady conversion of valleys to reservoirs, while a reduction in Federal capital grants to State Governments has curtailed all types of infrastructural investment. In this climate, water policy reform has accelerated. Volumetric charges are starting to replace taxes and large capital plans are being subjected to critical assessment. Catch-cries such as "user pays" and "corporatization" herald a shift in the water authority's role: from a tax-funded supplier of exogenous needs to a self-funding or even profit-making seller of water services.

This paper is part of a continuing project by Dixon *et al.* (1989, 1990, 1992) to set the determination of appropriate water policy for Melbourne on a more rigorous basis. In common with other contributors to the literature on optimal water pricing and investment [see, for example, Hirshleifer *et al.* (1969), Riordan (1971a & b), Gysi and Loucks (1971), Dandy *et al.* (1984) and Ng (1987)], we formulate a model in which maximization of community welfare requires marginal cost pricing. However, we try to extend the literature by calculating marginal costs in a model which includes a more detailed description of demand and technology than has previously appeared in applied studies dealing with the economics of urban water.

The previous literature relies on highly simplified partial equilibrium (one industry) models. Typically, it is assumed that the water authority sells a single commodity (or equivalently a group of commodities supplied in fixed proportions). This commodity ("water") is produced using materials, labour and capital. Material and labour inputs are usually assumed to be proportional to water output up to a point where output is constrained by capital capacity. Because of indivisibilities, the capital stock must expand in discrete jumps. This leads to the familiar saw-tooth path for the optimal price of water : the price falls after an expansion of capital capacity and rises until the next expansion is justified.

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1 The first two authors are members of the Centre of Policy Studies and the third of the IMPACT Project, both located at Monash University, Clayton, Victoria 3168, Australia.

Our model has six types of water commodities and three types of water authority capital. It includes creation processes for capital stocks and allows for gestation lags. Another distinguishing feature is our general equilibrium formulation. With this approach we recognize the roles of economy-wide resource constraints and of special links between the water industry and other parts of the urban economy, e.g., the housing industry. A final noteworthy feature of our model is its integration with data on streamflow. We have not presented this aspect of our work elsewhere and it is the main topic of the present paper. By using a long series of streamflow data we are able to include an important aspect of uncertainty in our analysis of Melbourne's water pricing and investment problem.

The remainder of the paper is laid out as follows. In the next section we provide an overview of our pricing and investment model and describe our solution method. We review some simulation results, and show how these are related to *ad hoc* safety margins which are designed to prevent unanticipated water shortages caused by drought. The effect on both water supply and demand of stochastic changes in weather are investigated in the third section. We estimate a regression model of these effects. The regression model is used in a series of Monte Carlo experiments which combine typical variations in weather with underlying water supply and demand patterns derived from our original pricing and investment model. We predict the frequency of water shortages for various safety margins. The final section summarizes our conclusions and mentions some areas for future research.

THE MODEL

Description

We base our description of the pricing and investment model on Figure 1. A complete listing of equations, parameter values and data sources for our Melbourne application is given in Dixon and Baker (1992).

At the top of the figure, the household sector chooses consumption levels for six products for each of 50 two-year periods. In making these choices, households maximize a multiperiod utility function subject to a budget constraint which limits the present value of expenditures to the present value of disposable income. The water authority influences the households' choices by setting prices for water products and by charging a tax on housing.

The six products entering the household utility function are housing services, inside water summer, inside water winter, outside water winter, outside water summer and all other goods. The distinction between inside water and outside water is that inside water must be taken away through the sewage system. Outside water is used, for example, on gardens and is not taken away.

In Melbourne, the water reticulation system is subject to peak load on summer evenings (November to March, 5 pm to 9 pm). To allow us to analyse peak load issues, we view the two products inside and outside water summer as being combinations of water consumed in peak and non-peak times. These combinations are specified by CES functions (Arrow *et al.*, 1961).

By appropriate choice of parameter values in the utility and CES functions we built into the Melbourne application the idea that households find the four products inside water summer, inside water winter, outside water summer and outside water winter to be poor substitutes for each other. On the other hand, they find outside water summer peak to be a good substitute for outside water summer non-peak. Consequently, we assume that a surcharge on outside

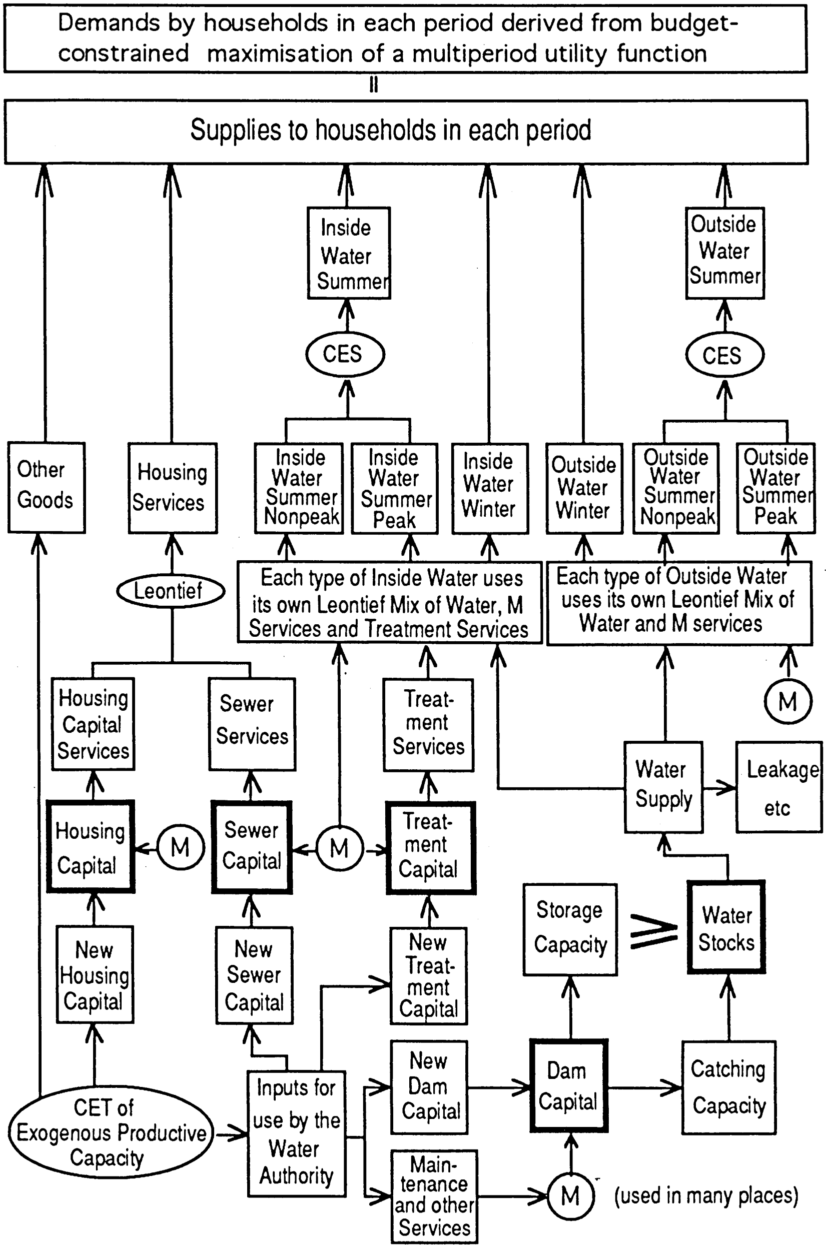

Figure 1: Overview of Water Model

summer peak usage would be an effective way of inducing households to switch some of their peak-period usage to non-peak times. Similarly, we specify good substitution possibilities between inside water summer peak and inside water summer non-peak.

The industrial sector in our model (bottom left corner of Figure 1) produces other goods, new houses and inputs for the water authority. Output of these three goods is limited by a CET production possibilities frontier (Powell and Gruen, 1968). This frontier moves out exogenously over the model's 100-year horizon reflecting labour force growth and technological progress. In each period, producers are assumed to choose the combination of other goods, new houses and water authority inputs to maximize the total value of output. Thus the relative supplies of these three goods are sensitive in our model to relative prices. For example, if the water authority wishes to expand its activities in any period, then it must offer a higher price for water authority inputs. This moves the urban economy's resources away from the production of other goods and new houses and into the production of water authority inputs. Although not shown in Figure 1, wages and profits generated in the industrial sector feed into the income side of the household sector's budget constraint.

Of the three goods emanating from the industrial sector, other goods undergo no transformation on their way to the household sector. Although this is not shown in the diagram, we normally allow other goods to be traded. In this case, household demand for other goods is equated to output from the industrial sector plus imports minus exports.

New houses produced in each period add to the housing stock. The housing stock includes the connecting pipes to water services. These connections are maintained by the water authority. This is indicated in Figure 1 by the M input to housing capital. Because of this maintenance role, the water authority is justified in levying a charge on house ownership.

Each period, the housing stock produces housing capital services. These must be combined with sewer services to produce housing services. The combination is specified by a Leontief function (i.e., a unit of housing service requires a given quantity of sewer service). Sewer services are produced each period by sewer capital. In effect, we assume that provided various capacity constraints are tight, then expansion in the supply of housing services requires expansions in both the housing stock and in the provision of main sewers. This link between the housing stock and main sewers provides a second justification in our model for non-volumetric changes to be levied on owners of houses.

The third product of the industrial sector, inputs to the water authority, is used by the authority to produce maintenance and operating services and three types of capital (main sewers, dams and sewage treatment plants). Gestation lags are introduced by specifying that water authority inputs are required in the creation of new units of capital in periods before these units are added to capital stocks. As indicated by the M inputs to the capital stock boxes in Figure 1, maintenance of water authority capital requires inputs of maintenance and operating services.

As we have seen already, sewer capital produces sewer services which are a necessary part of the production of housing services. Treatment capital produces treatment services. These, together with water and maintenance and operating services, are required to produce the three inside water products. For both inside and outside water products, differences in the use of maintenance and operating services per kilolitre of supply are used to introduce differences in the costs of meeting demands in peak and non-peak times. Notice that treatment services are not required for the outside water products.

Dam capital produces two services, storage capacity and catching capacity (i.e., access to streamflow). By linking dam capital with catching capacity we recognize that new dams are normally created in new catchment areas. Each period, catching capacity provides additions to water stocks while storage capacity provides an upper bound.

Water supply in each period is obtained by drawing on water stocks and is used in the production of water products. It is also dissipated in leakages.

The Water Authority Optimization Problem (WAOP)

To complete the model, we must specify the behaviour of the water authority. That is, we must specify how the authority plans its investments, sets the levy on house ownership and determines prices for different types of water products. In most applications of the model, we have assumed that the authority behaves as follows.

Problem A (the WAOP). The authority plans its investments and charges to maximize household utility subject to

- the behaviour of the industrial and household sectors,
- water technology constraints, and
- economy-wide resource and accounting constraints.

Sometimes we have added further constraints. For example, we have assumed that the authority is obliged to charge the same price per kilolitre for all types of water. A comparison of the solution of the one price problem with that of the multiprice problem gives an indication of the value of installing metering equipment to allow the multiprice solution to be implemented.

For computations, formulation A is inconvenient. Both the objective function and many of the behavioural constraints (coming from consumer demand and producer supply equations) are non-linear. However, the problem may be recast in a more tractable form. With the household sector being a price-taking, budget-constrained utility maximizer, with the industrial sector being a price-taking, production-possibilities-constrained revenue maximizer, and with the water authority setting policies which result in the highest possible level of household utility, the respective aims of all three entities are in complete harmony. The outcome is the same as if a benevolent dictator had arranged the activities of all so as to maximize consumer utility. Thus the following problem has just the same answer as the WAOP.

Problem B. A benevolent dictator chooses all production levels and consumption quantities, and plans investment in a way which maximizes consumer utility subject to:

- water technology constraints, and
- economy-wide resource constraints and accounting constraints.

That Problem A, which includes explicit behavioural assumptions for three economic entities, can be reduced to a single-entity problem (Problem B) is not surprising. The correspondence between multi-entity general equilibrium systems and single-entity optimization problems is well-known [Dixon (1975) and Ginsburgh and Waelbroeck (1976)]. The advantage of the reduction is that B is easier to solve than A. Apart from the CET transformation frontier, all B's constraints are linear. The few non-linear constraints may be approximated by a series of line segments, so that a standard linear programming package can be used. Most versions of the water model have been solved this way, with satisfactory results, using the MINOS package. However the LP problem is large (around 19,000 constraints). This has made computations slow, limiting our freedom to experiment with different variants of the model.

Another approach to solving the WAOP is to recast it as a general equilibrium system in which all activities of the water authority are conducted as though embedded in purely competitive markets. For example, as we will explain in the next subsection, using this approach we assume that there is a price-taking, cost minimizing, dam-operating entity which sells plain water at marginal cost. Similarly, we assume that there are price-taking cost-minimizing main sewer and treatment entities selling services at marginal cost. With the specification of the water authority broken up in this way, we can solve the WAOP via:

Problem C. Find prices and quantities to reconcile

- the demands of a utility-maximizing, budget-constrained, price-taking household sector with
- the outputs of resource-constrained, purely competitive water and industrial sectors.

That purely competitive behaviour in all markets will give the highest possible level of household utility is to be expected from the numerous theorems on the optimality of a purely competitive general equilibrium (Intriligator, 1971, chapter 10). In the present context, the equivalence of Problem C and the WAOP can be established by showing that the equation system to be solved in Problem C is the same as the set of Kuhn-Tucker or first-order conditions for a solution of Problem B.

Recently we have found that solutions of the WAOP can be computed much more quickly by applying Newton-Raphson techniques to solve Problem C than by using MINOS on Problem B. Compared with the approach using a standard LP package, successful application of Newton-Raphson techniques requires a more intimate knowledge of the equation system being solved and the underlying mathematics of the algorithm. Another disadvantage of Newton-Raphson is that convergence can occasionally be a problem. Nevertheless, for our model, these disadvantages are far outweighed by gains in computational efficiency.

The dam-operating entity

In this subsection we illustrate the Problem-C approach to the WAOP by describing the behaviour of the dam-operating entity. By choosing this entity, we are also able to set out some equations which are required for an understanding of our analysis in the next section concerning uncertain streamflow.

Part (a) of Table 1 shows a stripped-down version of the dam-operating entity's cost minimizing problem. For presentational convenience, the algebra is for one-year periods rather than the two-year periods used in our computations, and we omit various details such as terminal conditions and maintenance of dam capital. Although demands for water, X(s), are exogenous in the cost-minimizing problem, they are endogenous in the complete general equilibrium system. Our

Table 1

The Dam-operating Entity

(a) The cost minimizing problem

Choose non-negative values for

$I(s)$, increment to dam capital in period s, $s = 11, \ldots, 100$,
$K(s)$, dam capital in period s, $s = 11, \ldots, 100$,
$W(s)$, stock of water at the start of period s, $s = 2, \ldots, 100$,

to minimize

$$C = \sum_{s=1}^{100} r(s)\ P_{WAI}(s)\ \gamma \left[\sum_{i=1}^{10} I(s+i)\ V(s+i)/10 \right]$$

subject to

1	$\lambda_1(s)$:	$K(s-1) + I(s) - K(s) \geq 0$,	$s=11, \ldots, 100$,
2	$\lambda_2(s)$:	$W(s) - W(s+1) - X(s) + \beta K(s) \geq 0$,	$s=1, \ldots, 100$,
3	$\lambda_3(s)$:	$\alpha K(s) - W(s) \geq 0$,	$s=2, \ldots, 100$,
4	$\lambda_4(s)$:	$\varrho W(s) - X(s) \geq 0$,	$s=1, \ldots, 100$,

where

α is the storage capacity of unit of dam capacity,
β is the annual yield of water per unit of dam capital,
γ is the amount of water authority inputs needed to construct a unit of dam capital,
$X(s)$ is water supplied, period s,
$P_{WAI}(s)$ is the price of water authority inputs, period s,
ϱ is the drought safety factor, setting an upper bound on the fraction of the water stock which can be used,
$r(s)$ is the interest discount factor applying to period s,
the $\lambda_i(s)$'s are non-negative Lagrangean multipliers, and
the $V(s)$'s are parameters allowing for lumpiness.

We make $V(s)$ very large for most years and equal to one for the others. Increments will occur only in years where $V(s)$ is one. By placing these at, say, 10-yearly intervals, we produce results in which increments tend to be large and dam capacity increases in discrete jumps.

(b) The Lagrangean conditions for a solution of the cost minimizing problem

I	$\text{Min} \left\{ \sum_{i=1}^{10} r(s-i)\ P_{WAI}(s-i)\ \gamma V(s)/10 - \lambda_1(s)\ ;\ \ I(s) \right\} = 0$	$s=11, \ldots, 100$,
K	$\lambda_1(s) - \lambda_1(s+1) - \beta\lambda_2(s) - \alpha\lambda_3(s) = 0$,	$s=11, \ldots, 100$,
W	$-\lambda_2(s) + \lambda_2(s-1) + \lambda_3(s) - \varrho\lambda_4(s) = 0$,	$s=2, \ldots, 100$,
1	$K(s-1) + I(s) - K(s) = 0$,	$s=11, \ldots, 100$,
2	$\text{Min} \{W(s) - W(s+1) - X(s) + \beta K(s);\ \lambda_2(s)\} = 0$,	$s=1, \ldots, 100$,
3	$\text{Min} \{\alpha K(s) - W(s);\ \lambda_3(s)\} = 0$,	$s=2, \ldots, 100$,
4	$\text{Min} \{\varrho W(s) - X(s)\ ;\ \lambda_4(s)\} = 0$,	$s=1, \ldots, 100$.

(c) Marginal-cost pricing of the output of dam activities

$P_w(s) = (\lambda_2(s) + \lambda_4(s))/r(s)$, $s=1, \ldots, 100$,

where $P_w(s)$ is the price of plain water in period s.

assumption in Table 1 is that whatever the demands turn out to be, they will be satisfied at minimum cost.

The objective function is a discounted sum of costs of dam increments. The cost of the increment in any year is incurred over the previous 10 years. Consequently, in each year s, expenditure is associated with increments which come on line in years s+1 to s+10. Because we have adopted a 10 year gestation period for dams, we treat K(s) and I(s) for s=1,...,10 as pre-determined - they reflect decisions which have been made before year 1.

The first constraint allows for the accumulation of capital. The second constraint says that water accumulated from year to year cannot exceed the difference between water yield, $\beta K(s)$, and demand, X(s). If the dam becomes full, water overflows, and so the accumulation will be less - due to the third constraint.

The last constraint states that usage in any year must be less than some fraction Q of available stocks. In our model, the water authority plans on the basis of certainty, but retains sufficient water in reserve to meet the contingency of a drought.

Part (b) of Table 1 illustrates the equations that make up the general equilibrium system in Problem C. The equations shown in part (b) are the Kuhn-Tucker conditions for the cost-minimizing problem in part (a). The "Min" notation allows us to express each of the complementary slackness conditions in a single equation. Where we can be certain that a constraint will be binding or that a choice variable will be positive, we can avoid the "Min" formulation. We have done this in Table 1(b) for the Kuhn-Tucker conditions relating to K(s), W(s) and the capital accumulation constraint. In solving Problem C, there is a small computational advantage in avoiding unnecessary "Min" equations.

The "Min" equations are treated in the same way as other equations in implementing standard non-linear solution techniques. For example, in applying the Newton-Raphson method with analytic derivatives to a system containing the equation

$$\mathrm{Min}(a;b) = 0 \; , \tag{1}$$

we write

$$\mathrm{Min}_a(a;b).\Delta a + \mathrm{Min}_b(a;b).\Delta b = -\mathrm{Min}(a;b) \tag{2}$$

where

$$\mathrm{Min}_a(a;b) = 1 \text{ if } a < b, \text{ 0 otherwise, and } \mathrm{Min}_b(a;b) = 1-\mathrm{Min}_a(a;b) \; .$$

Part (c) of Table 1 shows another equation of the general equilibrium system in Problem C: the marginal-cost pricing condition for water. $[\lambda_2(s) + \lambda_4(s)]$ is the derivative of the Lagrangean with respect to X(s): this gives the effect on total discounted costs of a unit increase in X(s). Dividing by r(s) expresses this in terms of prices in period s.

Results from the model: sample simulations

This subsection contains a description of a set of solutions of the WAOP for Melbourne. The main assumptions are that

- dams can be brought on line only in the years 2002, 2012, 2022, ... By spacing the increments to dam capital (K) at 10 year intervals, we impose the idea of lumpiness with increments tending to be quite large.

- the water authority is cautious about running down its water stocks. We look at several cases. In the first we set Q at 0.37, i.e. in any year s, the water authority is assumed to be unwilling to allow its sales and leakages to be more than 37 per cent of the water stock on hand at the beginning of the year. This Q value was suggested by the authority. Then we make more adventurous settings, allowing Q to rise to 0.5.

The path of dam capital (K) with Q=0.37 is shown in Figure 2. The most striking feature is that no increment is made until 2022. Even then, the increment is negligible. The first large increment is in 2032 with significant increments at 10 year intervals in 2042, 2052, etc..

During the 1980s the Melbourne Water Authority completed the Thomson dam. This more than doubled the storage capacity of the Melbourne system taking it from about 800 gigalitres to 1773 gigalitres. (In terms of Table 1, $\alpha K(1992)$ is 1773.) The Thomson project also increased the annual yield of the Melbourne system from about 450 gigalitres to 650 gigalitres (i.e. $\beta K(1992) = 650$). Water sales and leakages in 1992, X(1992), were only 462 gigalitres. In view of these figures, it is not surprising that our model implies that there is no need for new dam capital for a very long time. This is despite expected annual growth in Melbourne's economy of 2.5 per cent (1.5 per cent population growth and 1 per cent productivity improvements).

Apart from the current water demand and supply situation, another factor contributing to the model's projection of a long pause in dam creation is pricing. In 1992, Melbourne residents paid the Water Authority about 20 cents for an extra kilolitre of water. This was for all types of water. Despite the abundance of water, our model indicates that this price was too low. Twenty cents is not sufficient to cover the delivery costs per kilolitre from the dams to households and is certainly insufficient to cover volume-related treatment costs of inside water. In other words, with its present pricing strategy, the water authority is not covering marginal costs. As can be seen from the heavy (Q=0.37) line in Figure 3, the model implies that the average price of water (averaged over the different types) should be raised immediately from $0.20 to about $0.45. This would reduce annual usage from 462 million kilolitres in 1992 to about 420 million in 1994.

Before significant dam expansion takes place in 2032, Figure 3 indicates that growth in water usage should be inhibited by a series of price rises. These reflect the onset of capacity shortages. For example, the rise in the average price of water between 2010 and 2014 reflects scarcity of inside water treatment capacity. Expansion of this capacity commences in 2014. By this time, population and income growth have increased the marginal value of inside water sufficiently to justify the capital expenditures needed to expand its supply.

It is not until 2032 that population and income growth have pushed the water-demand curve out far enough to justify a significant expansion of dam capital. Over the period 2018 to 2032, increases in water prices are needed to hold usage to levels that are compatible with almost no expansion in dam capital. By 2032, the average value placed by the community on extra water supply is about $1.1 per kilolitre. This is sufficient to justify the costs of expanding supply through the installation of new dams.

Because of our lumpiness assumption, there is a sharp increase in water availability in the period after dam expansion (2034). This causes a fall in prices.

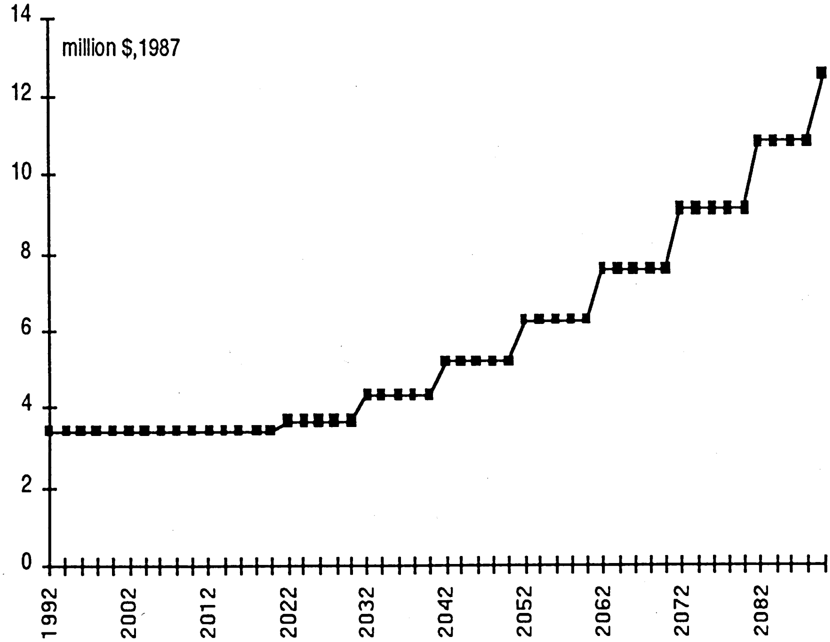

Figure 2: Dam Capital

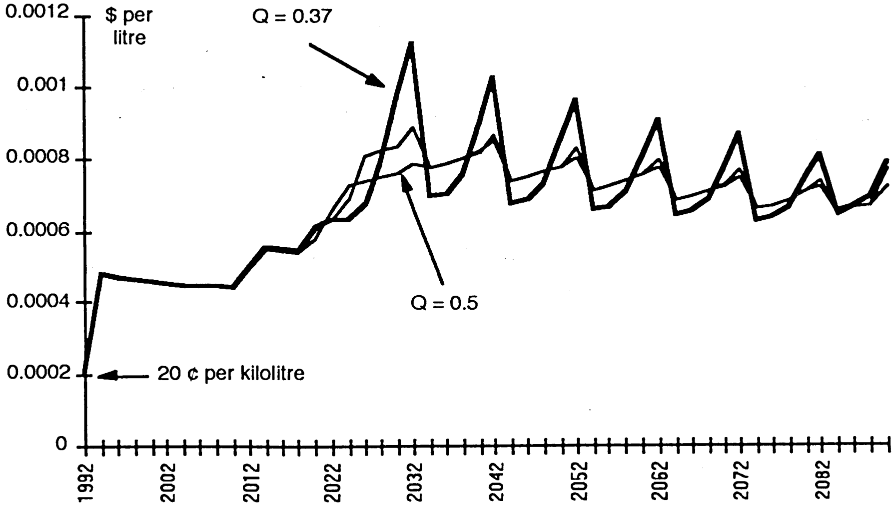

Figure 3: Average Water Price: Effect of Q Safety Factor

As the water-demand curve continues its rightward migration, the water authority inhibits usage growth through price increases until water from the next dam expansion becomes available in 2044.

One curious feature of Figure 3 is the apparent negative trend - the peaks and the troughs gradually get lower. This is a general equilibrium effect reflecting a gradual reduction in the share of water services in GDP and, correspondingly, in the opportunity cost of inputs used by the water authority.

Figures 3 and 4 give WAOP results for different values of Q. As Q is increased the path of water stocks moves lower - smaller water stocks are required to support any given path of sales. It is also true that the path of dam capital (K) moves lower. However, the differences are quite small making it inconvenient to show them on a diagram. For example, when Q=0.37, K(2032)=4.34. When Q=0.50, K(2032)=4.28. For Melbourne, the main problem is in providing capture capacity not storage capacity (the Melbourne technology has a high α/β ratio). Thus, as we increase Q (reducing required storage levels) we generate only minor reductions in requirements for dam capital - dam capital is still required in its capture role. In work on Sydney where the α/β ratio is much lower, we found that increases in Q allow quite large reductions in K. In Sydney, plenty of water is captured. The problem is to find a space to store it.

An interesting aspect of Figure 3 is the price-smoothing effect of increases in Q. Higher values of Q allow the water authority to improve the allocation of water sales across the 10-year dam cycle. With a high value of Q, the water authority can run down water stocks towards the end of the 10-year dam cycle (Figure 4) when water is scarce and build them up at the beginning of the next cycle when water is plentiful. With a low value of Q, this process is inhibited by the need to maintain large water stocks at the end of the cycle. With a low Q, increased sales at the end of the cycle would be possible only with large stocks.

UNCERTAINTY IN STREAMFLOW: THE RISKINESS OF THE WAOP STRATEGIES

The WAOP was solved under conditions of certainty. In particular, we ignored variations in climatic conditions and assumed that new water becomes available in year s at the annual rate of $\beta K(s)$. In reality, streamflow in Melbourne's catchment areas is highly uncertain and over the last 100 years there have been several long droughts. For example, in each of the ten years 1979 to 1988, streamflow was below average. In half of those years it was less than 75 per cent of average and in 1982 it was less than 35 per cent.

In setting a value for β (the water yield per unit of capital) we tried to adopt an attitude to climatic risk similar to that of the water authority. The authority has estimated that with its present dam assets (3.419 in our units) it could meet a constant annual demand of 650 gigalitres for 100 years with almost no chance (an acceptably low risk) of running out of water. Consequently we set β at 650/3.419, i.e., β represents a safe annual yield per unit of dam capital. As explained already, we also built a safety margin into the WAOP by insisting that the pricing and investment strategy be formulated so that if the net availability of new water were to follow the path $\beta K(s)$, then water usage in any year s, X(s), would be less than a fraction Q (0.37 in our base-case) of the year's opening water stocks, W(s).

Despite our seemingly conservative choices for the values of β and Q, and general agreement with our other parameter settings and assumptions, the initial reaction of Melbourne water officials to our results has been that

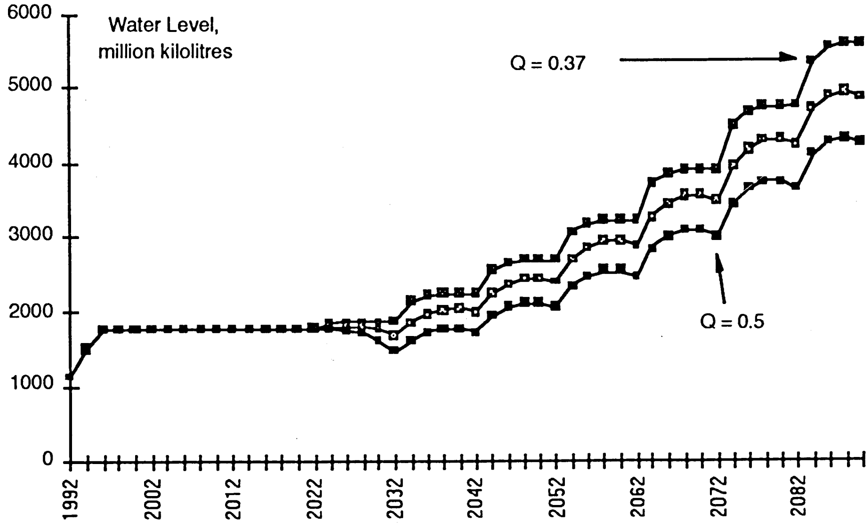

Figure 4: Effect of Q Safety Factor on Stored Water

implementation of strategies derived from the WAOP (especially the delaying of significant expansion of dam capital until 2032) would expose Melbourne to unacceptable risks of drought-related water shortages. In this section, we assess these risks for WAOP strategies computed with different values of Q. Suitable data for doing this were given to us by the Melbourne water authority. These are shown partially in Table 2. The complete data set is available from the authors.

Table 2
Simulated Water Harvest and Usage, 1913-90, Gigalitres

Year	Water Level[a] (W)	Streamflow[b] (S)	Demand[c] (D)	Unused[d] (U)	Evaporation (E)
1913	1480	1270	668	440	19
1914	1623	649	674	220	29
⋮					
1990	1064	1241	667	434	26
Mean	1542	1193	650	525	22

(a) Year's opening stock. The maximum possible level for W (i.e., W_{max}) is 1773. This occurs in 9 years.

(b) Refers to potentially harvestable water in rivers in catchment areas. Variations in streamflow reflect variations in weather.

(c) Underlying demand (including leakages), X(s), is set at 650. Actual demand, D(s), varies from 650, reflecting variations in weather conditions.

(d) This consists of streamflow which is not taken into dams and of dam overflow.

Statistical analysis of data on streamflow, demand, evaporation and unused water

The water authority data set is itself derived from a model, but reflects real historical variations in weather. It shows how water supply and usage would have varied over the 78 years 1913-90 if the same system of dams etc. as exists *today* had been in place all of that time. A steady base level demand of 650 gigalitres is assumed. Superimposed on this flat underlying scenario are the actual variations in rainfall and temperature from 1913-90. The water authority's model translates these weather variations into demand and supply variations. On the demand side their model incorporates observed past correlations between weather and usage, whilst the supply side derives from a very detailed hydraulic description of the entire Melbourne catchment, storage and reticulation systems. The path of water stocks is computed from the identity

$$W(s+1) = W(s) + S(s) - D(s) - U(s) - E(s). \qquad (3)$$

Table 3 shows the results we obtained from some statistical modelling of the water authority data set. In regression (i), we assume that streamflow is lognormally distributed. We found that it exhibits positive first-order serial correlation, consistent with the tendency for Melbourne to experience sequences of drought years. (The K appearing in this equation is constant in the water authority data and plays no role in the regression analysis. Its role is in the Monte Carlo simulations discussed below.) In regression (ii), we found demand to be negatively correlated with streamflow – low streamflow is associated with long, hot, garden-thirsty summers. (X is constant in the regression analysis.) Similarly in regression (iii) we found evaporation to be negatively correlated with streamflow.

In modelling the 'unused' part of streamflow, we were guided by a metaphor — see Figure 5. The funnel represents the catchment area of the Melbourne system, the barrel its storage capacity. The stem of the funnel has only limited flow capacity – because flow capacity is costly, and so it makes sense to cater for

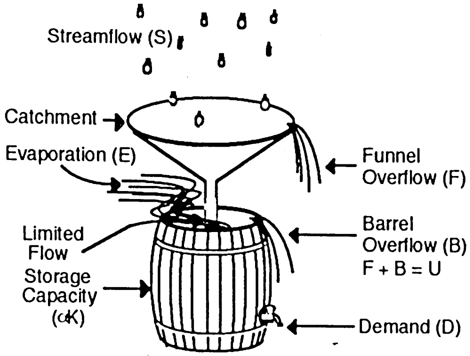

Figure 5: Water Harvesting and Storage Technology

Table 3
Regression and Monte Carlo Simulation Model

(a) Variables

S	Streamflow
D	Demand for water, includes leakages
X	Underlying demand for water, i.e., demand under average weather conditions
W	Stock of water at the beginning of the year
U	Unused water
W_{max}	Dam storage capacity
H	Funnel overflow plus "non-full" year barrel overflow
K	Dam capital
E	Evaporation

(b) Regression equations

The water authority data are generated assuming constant underlying demand at 650 and a constant dam capital stock at *today's* level. Thus, in fitting the regression equations, we set X at 650 in all years and K at 3.419.

(i) $\ln(S/K) = \alpha_1 + \alpha_2 \ln[(S/K)_{-1}] + \varepsilon_1$ Streamflow regression

$\alpha_1 = 4.764\ (7.22)\quad \alpha_2 = 0.179\ (1.57)$

$sd(\varepsilon_1) = 0.33\quad R^2 = 0.03\quad DW = 1.91$

(ii) $\ln(D/X) = \beta_1 + \beta_2 \ln(S/K) + \varepsilon_2$ Demand regression

$\beta_1 = 0.384\ (7.14)\quad \beta_2 = -0.066\ (-7.16)$

$sd(\varepsilon_2) = 0.027\quad R^2 = 0.41\quad DW = 1.78$

(iii) $\ln(E/K) = \gamma_1 + \gamma_2 \ln(S/K) + \varepsilon_3$ Evaporation regression

$\gamma_1 = 3.74\ (11.32)\quad \gamma_2 = -0.32\ (-5.71)$

$sd(\varepsilon_3) = 0.165\quad R^2 = 0.30\quad DW = 2.08$

(iv) $H/K = a + b(S/K) + c(S/K)^2 + d(W/K) + \varepsilon_4$ Regression explaining part of 'unused'

Tobit Estimates:

$a = -41.19\ (-2.41)$	$b = 0.0474\ (0.71)$
$c = 0.00090\ (11.55)$	$d = 0.119\ (5.20)$
$sd(\varepsilon_4) = 15.51\ (11.77)$	$R^2 = 0.99$

(c) Identities to complete the Monte Carlo simulation model

(v) $U = \mathrm{Max}(W - W_{max} + S - D - E;\ H)$ Total unused

(vi) $W_{+1} = W + S - D - E - U$ Water accounting identity

(vii) $W_{max} = 518.6K$ Dam storage capacity

[In our data base $W_{max} = 1773$ and $K = 3.419$. Thus we set the parameter α (defined in Table 1) at 518.6.]

less than the maximum possible flow that the funnel might generate. During prolonged periods of intense rain, therefore, the funnel overflows, even if the barrel is not full. Working with this metaphor, we developed the following model of 'unused':

$$U(s) = F(s) + B(s) \tag{4}$$

where

$$F(s) = g_1[S(s)] + \varepsilon_f(s) \; ; \; \varepsilon_f \sim N(0,\sigma_f^2) \; , \tag{5}$$

$$B(s) = B_1(s) + g_2[W(s)] + \varepsilon_b \; ; \; \varepsilon_b \sim N(0,\sigma_b^2) \; , \tag{6}$$

and

$$B_1(s) = 0 \text{ if } W(s+1) < W_{max} \; . \tag{7}$$

Equation (4) is an identity stating that unused is the sum of funnel overflow and barrel overflow. Equation (5) links funnel overflow to streamflow. Equations (6) and (7) express barrel overflow as the sum of two components. The first (B_1) is zero in years in which the dam is not full at the end of the year. The other component (g_2) allows for the possibility that although the dam was not full at the end of the year, it was full during the year. We relate this possibility to the opening water stock, W(s).

For the purpose of estimating the parameters of the functions g_1 and g_2, we manipulated (3) - (7) into a standard format for a Tobit model with censored data (Greene, 1991, pp.565-6):

$$U(s) = H(s) \text{ if } H(s) > U_{min}(s) \; ,$$

$$U(s) = U_{min}(s) \text{ if } H(s) \le U_{min}(s) \; ,$$

where H is the unobservable variable defined by

$$H(s) = g_1[S(s)] + g_2[W(s)] + \varepsilon \; , \qquad \varepsilon \sim N(0,\sigma_{f+b}^2) \; ,$$

and

$$U_{min}(s) = -W_{max}(s) + W(s) + S(s) - D(s) - E(s) \; .$$

After some experimentation, we specified g_1 as quadratic and g_2 as linear, obtaining the results given in regression equation (iv) in Table 3. Having estimated the behaviour of H, we can use equation (v) to calculate the behaviour of U.

Monte Carlo analysis of the riskiness of WAOP strategies

We used the system of equations (i) - (vii) in Table 3 to generate many scenarios of water supply and usage, each scenario being for a century. For all scenarios, the paths of dam capital (K(s), s=0,...,100) and of underlying demand (X(s), s=1,...,100) were given, together with the initial values of streamflow, S(0), and water stock, W(1). To generate a scenario, we started by drawing for each of the 100 years 4 normal deviates - one for each regression in Table 3 - with variances estimated from the regressions. Then we calculated the path of the water stock recursively as follows. With the drawn values for $\varepsilon_i(1)$, i=1,...,4 and with given values for S(0)/K(0), K(1), X(1) and W(1), equations (i) - (iv) yield values for S(1), D(1), E(1) and H(1). From (vii) we can calculate $W_{max}(1)$. U(1) can now be calculated from (v) and W(2) from (vi). With values in place for S(1), K(1), K(2), X(2) and W(2), and with drawn values for $\varepsilon_i(2)$, i=1,...,4, we can now calculate W(3) and so on. Results from 30,000 century-long scenarios calculated in this way are shown in Table 4.

The first 10,000 scenarios mimic the water authority's calculations in Table 2. That is, we assumed X(s)=650 and K(s)=3.419 for all s. We simply imposed alternative weather patterns through our drawings for the ε_i's. We found that 0.45 per cent of centuries were marked by one or more years when water stocks ran out altogether. According to the water authority, the last 18 per cent of

Table 4

Monte Carlo Results

Levels	Fixed Demand and Capital (X=650, K=3.419)		Demand and Capital Paths derived from WAOP with Q= 0.37		Demand and Capital Paths derived from WAOP with Q= 0.5	
	Years	Centuries	Years	Centuries	Years	Centuries
Ran out of water	107	45	69	32	82	35
Below 18% full (safe minimum)	567	251	223	114	431	219
Below 50% full	23,182	6,272	14,479	4,136	25,776	5,818
Below 75% full	272,711	9,995	171,108	9,919	315,196	9,999
Total	1,000,000	10,000	1,000,000	10,000	1,000,000	10,000

water in the dams is of poor quality: 2.51 per cent of centuries experienced one or more years when water levels fell to this unpalatable level. Since the authority considers 650 gigalitres to be the present system's safe yield, we deduced that it considers the risks indicated in the first panel of Table 4 to be acceptably low. We use these levels of risk as a standard against which to measure the strategies suggested by the WAOP.

In the second 10,000 scenarios, we set the paths of X(s) and K(s) according to the WAOP solution computed with Q=0.37. In this case 0.32 per cent of centuries were marked by empty dams and 1.14 per cent of centuries experienced one or more years when water levels fell to the 18 per cent mark. We conclude that, contrary to the water authority's fears, the WAOP strategy with Q=0.37 surpasses the safety standards regarded by the authority as reasonable.

In the third set of scenarios, X(s) and K(s) followed the paths given by the WAOP with Q=0.5. The higher setting for Q is equivalent to a relaxation of the safety constraint that we imposed to allow for unanticipated variations in weather. Thus, we would expect an increase in the riskiness of the WAOP strategy. With Q=0.5, 0.35 per cent of centuries had empty dams and 2.19 per cent experienced the 18 per cent level: riskier than the Q=0.37 strategy but still bettering the water authority's safety standards. The higher level of Q offers the community two benefits: water prices fluctuate less, and slightly less dam capital is needed.

CONCLUSION

Our paper has fallen into two parts. The first describes a multiperiod, general equilibrium model of water pricing and investment. Initially we solved the model by formulating it as a single-entity, linear programming problem.

Recently we have worked with an alternative formulation consisting of a set of equations depicting utility-maximizing behaviour by consumers and purely competitive behaviour by the industrial sector and by entities concerned with activities such as dam operation and sewage treatment. Relative to our earlier LP approach, we found that application of Newton-Raphson techniques to the equations of this multi-entity competitive formulation gave large gains in computational efficiency.

To illustrate the competitive formulation, we derived the equations describing the behaviour of the dam-operating entity. In so doing, we drew attention to an *ad hoc* constraint imposed in our model to guard against water shortage during droughts. Even with this constraint imposed, the optimal plan for Melbourne indicated by our model was regarded as risky by the Melbourne water authority.

In the second part of the paper we measured risks of running short of water under different price and investment strategies. Water authority data were used in a statistical analysis of weather-induced variations in streamflow, demand and lost or wasted water. Monte Carlo simulations were then used to find what probability of shortage was implied by the authority's estimate of the 'safe yield' of a unit of dam capital. More Monte Carlo simulations measured the corresponding probabilities associated with optimal price/investment strategies derived from our model. We found that the model-derived strategies involved risk levels lower than those apparently acceptable to the water authority.

There are many ways in which our research could be extended. Some will involve applications of our existing model and solution programs while others will require further modelling and programming developments.

Projects in the first category include various sensitivity analyses. For example, it would be interesting to allow in our Monte Carlo experiments not only for streamflow uncertainty but also for parameter uncertainty. This could be introduced by variations in parameter values consistent with the ranges of uncertainty suggested by the t-statistics in our regression equations.

Another sensitivity project is to analyse the implications of a sharp drop in streamflow caused by a forest fire. Chapman *et. al* (1991) have estimated that a major fire such as that of 1939 could reduce the average annual yield of the Melbourne catchment area by about 15 per cent for up to 100 years.

In the category of extensions requiring further modelling and programming is an analysis of optimal reactive policies. Our aim would be the derivation of optimal policies taking account of the fact that policies can be revised if water stocks run low. This would take us into the realm of stochastic dynamic programming requiring, perhaps, thousands of solutions of our full general equilibrium model. Recognition of this possibility is part of the explanation of our decision to look beyond our initial LP approach for a more computationally efficient method of solving our model.

Acknowledgments

The authors are indebted to officers of Melbourne Water for data, suggestions and encouragement. Chris Panayi, Rob Lewis, Hugh Duncan, Jaya Jayasuriya and Howard Rose were particularly helpful. None of these people, nor their institution should be held responsible for any aspect of our analysis and conclusions.

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