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INTERIOR POINT ALGORITHM FOR SOLVING FARM RESOURCE ALLOCATION PROBLEM

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Abstract: *This paper introduces interior point algorithm as an alternative approach to simplex algorithm for solving farm resource allocation problem. The empirical result of interior point algorithm is compared with that of the simplex algorithm. It goes further to address a profit maximization problem. The result revealed several relevant patterns. Results of the interior point algorithm is similar to that of the simplex algorithm. Findings indicated that in both algorithms, the farm is to produce peppers, wheat which is irrigated and weeded manually, hire additional month of labour, and also purchase urea and muriate fertilizer to realize a similar amount of profit. Additionally, both algorithms suggested that practicing crop rotation where beans, if grown, should be altered with wheat cannot be possible since no beans will be grown. The Simplex algorithm saves 39 iterations over Interior Point algorithm in solving the farm resource allocation problem. The findings demonstrate that the interior point algorithm offers a useful alternative to the simplex algorithm when addressing farm resource allocation problem.*

Keywords: *Linear Programming, Simplex Algorithm, Interior Point Algorithm, Farm Resource Allocation*
(JEL code: *D24, D57, C61, C63, C67*)

INTRODUCTION

Farm resource allocation has been given considerable attention in the literature. Numerous authors have addressed the issue of farm resource allocation using linear programming technique. Specifically, most of these studies employ the simplex algorithm in addressing the farm resource allocation problem. For example, Majeke and Majeke (2010) applied linear programming technique to address the farm resource allocation problem among small-scale commercial farmers in Zimbabwe. Majeke (2013) applied linear programming technique to address optimum combination of crop farm enterprises of small-scale farm in Marondera, Zimbabwe. Also, Majeke, Majeke, Mufandaedza and Shoko (2013) modelled a small farm livelihood system using linear programming in Bindura, Zimbabwe. Moreover, Wankhade and Lunge (2012) applied linear programming to address allocation of agricultural land to the major crops of Saline track. The concept of farm resource allocation is highly relevant to farmers. Knowing the optimal resource mix can lead to increase crop productivity, increased purchasing power, increase farm income, maximise consumer surplus and subsequently decrease food insecurity. The fundamental question this paper seeks to answer is whether there is an alternative programming technique for farm resource allocation whose performance is similar to the simplex method. Having alternative programming technique is extremely relevant to researchers because it will provide a means of validating the results of the simplex programming technique.

GAP

Although numerous studies have examined farm resource allocation problem using the simplex algorithm, an alternative approach to addressing the problem, namely the interior point algorithm have remained relatively understudied. Though the interior point algorithm developed by Karmarkar (1984) offers an alternative approach to solving the farm resource allocation problem, very little work has been done with regard to its application to the farm resource allocation problem. Tomlin (1989) notes that interest in the empirical applications of interior point method is arouse by claims that it improves upon the performance of the Simplex method. Similarly, Hoffman et al (1953) notes that there are competing methods to the Simplex method. A fundamental research question which remains unaddressed with regard to the farm resource allocation problem is that can alternative approaches such as the interior point algorithm be used to validate the results of the Simplex method? Additionally, can the interior point method improve upon the performance of the Simplex method when it is applied to solve the farm resource allocation problem? In order to address these issues, this paper compares the estimation results of the Simplex and Interior Point algorithm to demonstrate their usefulness in solving the farm resource allocation problem.

MATERIALS AND METHODS

Linear Programming (LP)

Linear programming is a special case of mathematical programming. When the mathematical representation uses linear functions exclusively, (that is, both the objective function and the constraints are all in a linear form), we have a linear programming model. Mathematically, it is of the form:

Optimise (maximise or minimise):

$$z - c_1x_1 + c_2x_2 + \ldots + c_nx_n = 0$$

Subject to structural constraints:

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n (\geq)(\leq) b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n (\geq)(\leq) b_2$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n (\geq)(\leq) b_m$$

$$x_1, x_2, \ldots x_n \geq 0$$

In performing the simplex algorithm, the objective function $z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$ is rewritten as $z - c_1x_1 - c_2x_2 - \ldots - c_nx_n = 0$. This is referred to as row 0.

Basic Assumptions of Linear Programming

Several assumptions underlie or are implicit in linear programming problems. They include:

1. Linearity: There exist linear relationships between the output of each product and the total quantity of each resource consumed.
2. Additivity: Activities contribution and consumption are additive.
3. Non-negativity: The values of the activities cannot be negative.
4. Proportionality: The level of activity is proportional to the contribution as well as consumption of resources.
5. Fixed external factors: This implies that the external environment is assumed not to vary.
6. Certainty: This presupposes that all values and quantities are known.
7. Single objective function: There can only be one objective in a particular problem, either to maximise profit or to minimise cost not both.

Components of a Linear Programming Problem

Below are the various components that make up an LP model:

1. Decision variables
2. Objective function
3. Constraints
4. Non-negativity constraints/sign restrictions

Simplex Algorithm

The Simplex algorithm is used to solve LP problems involving two or more decision variables. There are no theoretical restrictions placed on the number of decision variables or constraints in a linear problem. It utilizes the property of an LP problem of having an optimal solution only at the corner point of the feasible solution space. It systematically generates corner point solutions and evaluates them for optimality. It stops when an optimal solution is found. Steps involved in the simplex algorithm are as follows (Bronson and Naadimuthu, 1997):

Step 1: Convert the LP to standard form. In converting LP to standard form, we convert all inequality constraints to equality constraints. To convert a "≤" constraint to equality "=" constraint, we add a slack variable, and a "≥" constraint to equality constraint we subtract a surplus (excess) variable. Slack variables are the fictitious variables which indicate how much of a particular resource remains unused in any solution and surplus (excess) variables are the fictitious variables which indicate additional amount of a particular resource needed in any solution.

Step 2: Obtain a basic feasible solution (BFS) (if possible) from the standard form. Notably, a basic variable (BV) refers to variables having positive values in a basic feasible solution; this variable has a coefficient of 1 in only one of the constraints and zero in the row 0 and remaining constraints, and non-basic variable (NBV) also refers to variables which are set equal to zero, so as to define a corner point.

Step 3: Determine whether the current BFS is optimal.

Step 4: If the current BFS is not optimal, then, determine which NBV should become a BV and which BV should become a NBV to find a new BFS with a better objective function value. This is done by determining the entering variable and the outgoing variable. An entering variable is a variable we choose to find new BV from a current basic feasible solution that is not optimal. We choose the entering variable (in a maximization problem) to be the non-basic variable with the most negative coefficient in row 0 (ties may be broken in an arbitrary fashion). Similarly, we choose the entering variable (in a minimization problem) to be the non-basic variable with the most non-negative (positive) coefficient in row 0 (ties may be broken in an arbitrary fashion). Outgoing variable refers to the variable with the smallest non-negative ratio (to find the ratios, divide the right hand side of the constraint by the coefficient of the entering variable, wherever possible). It is also called the pivot term/element. The constraint with the smallest ratio wins the ratio test.

Step 5: Use the Elementary Row Operations (EROs) to find the new BFS with the better objective function value. Repeat step 3 through step 5 until an optimal solution is found.

Interior Point Algorithm

Interior point algorithms (also referred to as barrier algorithms) are a certain class of algorithms that solves linear and nonlinear convex optimization problems. An interior point method is a linear or nonlinear programming method (Forsgren et al. 2002) that achieves optimization by going through the middle of the solid defined by the problem rather than around its surface. Current efficient implementations are mostly based on a predictor-corrector technique (Mehrotra, 1992), where the Cholesky decomposition of the normal equation or LDL^T factorization of the symmetric indefinite system augmented system is used to perform Newton's method (together with some heuristics to estimate the penalty parameter). All current interior point methods implementations rely heavily on very efficient code for factoring sparse symmetric matrices. The simplex algorithm of linear programming finds the optimal solution by starting at the origin and moving along adjacent corner points of the feasible region. Narendra Karmarkar in 1984 introduced the Karmarkar's algorithm for solving linear programming problems that reaches a best solution by traversing the interior of the feasible region.

Consider a Linear Programming problem in matrix form:

Minimize: $z = C^T \boldsymbol{X}$
Subject to: $\boldsymbol{AX = 0}$
$\boldsymbol{1X = 1}$
$\boldsymbol{X \geq 0}$

where $\boldsymbol{X}$ is a column vector of size n; $\boldsymbol{C}$ is an integer column vector of size n; **1** is a unit row vector of size n; $\boldsymbol{A}$ is an integer matrix of size $(m \times n)$; $n \geq 2$.

In addition, assume the following two conditions:

1. $X_0 = (1/n, \ldots, 1/n)$ is a feasible solution.
2. Minimum $z = 0$

Steps involved in using Karmarkar's algorithm as simplified by Bronson and Naadimuthu (1997) as outlined below:

Preliminary Step:

$$k = 0$$

$$\boldsymbol{X}_0 = (1/n, \ldots, 1/n)^T$$

$$r = 1/\sqrt{n(n-1)}$$

$$\alpha = (n-1)/3n$$

Iteration k:

a. Define the following:

i. $\boldsymbol{Y}_0 = \boldsymbol{X}_0$
ii. $\boldsymbol{D}_k = diag\{\boldsymbol{X}_k\}$, which is the diagonal matrix whose diagonal consists of the elements of.
iii. $\boldsymbol{P} = \begin{pmatrix} \boldsymbol{AD_k} \\ \boldsymbol{1} \end{pmatrix}$
iv. $\overline{\boldsymbol{C}} = \boldsymbol{C}^T \boldsymbol{D}_k$

b. Compute the following:

i. $\boldsymbol{C_P} = [\boldsymbol{I} - \boldsymbol{P}^T(\boldsymbol{PP}^T)^{-1}\boldsymbol{P}]\overline{\boldsymbol{C}}^T$
Note: If $\boldsymbol{C_P} = 0$, any feasible solution becomes an optimal solution. Stop.
ii. $\boldsymbol{Y}_{new} = \boldsymbol{Y}_0 - \alpha r \frac{\boldsymbol{C_P}}{\|\boldsymbol{C_P}\|}$
iii. $\boldsymbol{X_{k+1}} = (\boldsymbol{D}_k \boldsymbol{Y}_{new})/(\boldsymbol{1D}_k \boldsymbol{Y}_{new})$
iv. $z = \boldsymbol{C}^T \boldsymbol{X_{k+1}}$
v. $k = k + 1$
vi. Repeat iteration k until the objective function (z) value is less than a prescribed tolerance ϵ.

Repeat iteration k until the objective function (z) value is less than a prescribed tolerance .

Linear Programming Formulation

A farmer who grows beans, peppers and wheat on his plots wishes to find the combination of crops that maximizes his total profit. The gross profit per hectare of beans is \$1800, \$2300 for peppers and \$1500 for wheat. He has 10 hectares of land and \$1200 of capital. He and the members of his family are able to spend 16.5 months on farming activities. Additionally, he can use his mules for 11 months. The labour requirements per hectare of beans, peppers, and wheat are 1.87, 2.64, and 1.42 respectively. Mules and capital requirements per hectare of beans, peppers, and wheat are 1.27, 1.45, 1.25 and \$100, \$220, \$12 respectively. The farmer can choose between weeding his wheat by hand (weeding manually) or by application of weedicide. The use of weedicide reduces the total labour requirement per hectare from 1.42 to 0.98 months, but increases the total capital requirement from \$12 to \$52. The farm has 3 hectares of irrigated land and 7 hectares of rain fed land. Wheat can be grown on the irrigated land, and reaches a gross profit of \$1700 under irrigated conditions. The farmer has the option of hiring additional labour at a monthly wage of \$50. Moreover, a hectare of wheat grown on the farm requires 25kg of nitrogen and 10kg of potash. The available fertilizers are urea (46% nitrogen), compound fertilizer (16% nitrogen and 10% potash), and muriate of potash (30% potash). A kilogram of urea, compound fertilizer, and muriate of potash cost \$1.23, \$1.78, and \$0.89 respectively. The application of fertilizers increases the gross profit of wheat by \$200. The farmer adheres to crop rotation where beans, if grown, must be altered with wheat. The problem above is expressed in linear programming form as shown in Table 1.

Table 1: The Basic Model for the Problem Above

Activities	Beans (ha)	Peppers (ha)	Irrigated Wheat Weeded Manually (ha)	Rain Fed Wheat Weeded Manually (ha)	Irrigated Wheat Weeded by Weedicide (ha)	Rain Fed Wheat Weeded by Weedicide (ha)	Hire Labour (months)	Buy Urea (kg)	Buy Compound (kg)	Buy Muriate (kg)		
Objective Values, Max	1800	2300	1900	1700	1900	1700	-50	-1.23	-1.78	-0.89		
Constraints											Inequality	Capacity
Rotation requirement (ha)	1	0	-1	-1	-1	-1	0	0	0	0	≤	0
Dry land (ha)	1	1	0	1	0	1	0	0	0	0	≤	7
Irrigated land (ha)	0	0	1	0	1	0	0	0	0	0	≤	3
Labour (months)	1.87	2.64	1.42	1.42	0.98	0.98	-1	0	0	0	≤	16.5
Nitrogen (kg)	0	0	25	25	25	25	0	-0.46	-0.16	0	≤	0
Potash (kg)	0	0	10	10	10	10	0	0	-0.10	-0.30	≤	0
Mules (months)	1.27	1.45	1.25	1.25	1.25	1.25	0	0	0	0	≤	11
Capital ($)	100	220	12	12	52	52	0	0	0	0	≤	1200

RESULTS AND DISCUSSION

The results from the Simplex algorithm to the farmer's problem presented in Table 2 reveal that the strategies for this farm as specified in the model are to produce 5.31056 ha of peppers, 2.63975 ha of irrigated wheat to be weeded manually, no beans, no irrigated wheat weeded by weedicide and no rain fed wheat. In addition, the farm has to purchase 143.46476 kg of urea, 87.99172 kg of muriate, no compound fertilizer, and hire 1.26832 months of labour to realize a total profit of $16911.60. These results suggest that practicing crop rotation where beans, if grown, should be altered with wheat cannot be possible since no beans will be grown (Table 2).

Similarly, the results from the Interior point algorithm to the farmer's problem presented in Table 2 shows that the strategies for this farm as specified in the model are to produce 5.310190 ha of peppers, 2.639690 ha of irrigated wheat to be weeded manually, no beans, no irrigated wheat weeded by weedicide and no rain fed wheat. In addition, the farm has to purchase 143.4603 kg of urea, 87.98551 kg of muriate, no compound fertilizer, and hire 1.268401 months of labour to realize a total profit of $16910.66. This is consistent with the results from the simplex algorithm. Similarly, the results suggest that practicing crop rotation where beans, if grown, should be altered with wheat cannot be possible since no beans will be grown.

It has been observed that the Interior Point algorithm takes 45 iterations while the Simplex algorithm takes only 6 iterations to find an optimum solution to the problem. Thus Simplex algorithm saves 39 iterations over Interior Point algorithm while solving the problem (Table 2).

Table 2: The Optimal Solution to the Problem

	Simplex Algorithm	**Interior Point Algorithm**
Optimal Value	16911.60	16910.66
Activities		
Beans	0.00000	2.076755e-08
Peppers	5.31056	5.310190e+00
Irrigated Wheat Weeded Manually	2.63975	2.639690e+00
Rain Fed Wheat Weeded Manually	0.00000	3.762265e-09
Irrigated Wheat Weeded by Weedicide	0.00000	1.161979e-07
Rain Fed Wheat Weeded by Weedicide	0.00000	2.490331e-09
Hire Labour	1.26832	1.268401e+00
Buy Urea	143.46476	1.434603e+02
Buy Compound	0.00000	4.157138e-06
Buy Muriate	87.99172	8.798551e+01
Number of Iteration	6	45

These results establish the superiority of the simplex method over the interior point method in solving the farm resource allocation problem. This is consistent with Hoffman et al (1953) whose experiments established the superiority of the simplex method over its competitors. Similarly, Tomlin (1989) notes that the interior point method complements rather than duplicates or supersedes the efficiency of the simplex method on the problems it handles well.

Resource Utilization

As shown in Table 3 under the simplex algorithm, 5.31056 ha of dry land and 2.63975 ha of irrigated land are used up while 1.68944 ha and 0.36025 ha are unused respectively. All the months of labour, months of mule's labour and capital are utilised. Similarly, the results from the Interior point algorithm suggest that 5.31019 ha of dry land and 2.63969 ha of irrigated land are used up while 1.68981 ha and 0.36031 ha are unused respectively. All the months of labour, months of mule's labour and capital are utilised. This is consistent with the results from the simplex algorithm. If months of labour, months of mule's labour and capital could be increased, more land could be utilised, thus, increasing total profit.

Table 3: Resource Utilization

Resources	Available	Usage		Left Over	
		Simplex Algorithm	Interior Point Algorithm	Simplex Algorithm	Interior Point Algorithm
Dry land (ha)	7.00000	5.31056	5.31019	1.68944	1.68981
Irrigated land (ha)	3.00000	2.63975	2.63969	0.36025	0.36031
Labour (months)	16.50000	16.50000	16.50000	0.00000	0.00000
Mules (months)	11.00000	11.00000	11.00000	0.00000	0.00000
Capital ($)	1200.00000	1200.00000	1200.00000	0.00000	0.00000

CONCLUSION

This paper proposes the interior point method as an alternative to the simplex method for solving the farm resource allocation problem. A comparison of the results from the interior point method and simplex method indicate that the estimates obtained in the alternative methods are similar.

On the basis of the interior point method and simplex algorithm the farm is to produce peppers, wheat which is irrigated and weeded manually, hire additional month of labour, and also purchase urea and muriate fertilizer to realize a similar amount of profit. Furthermore, both algorithms suggested that practicing crop rotation where beans, if grown, should be altered with wheat cannot be possible since no beans will be grown. In summary, this paper has demonstrated that the interior point method offers an alternative and a useful approach to solving farm resource allocation problem.

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