Approximation Properties and Estimation of the Translog Production Function with Panel Data

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Abstract

This paper provides a general theoretical and methodological framework for the estimation of the translog production function using panel data. The estimation is carried out using generalised least squares estimator (random-effects model) under different maintained hypotheses for the underlying production technology. Our empirical results indicate that imposing a priori restrictions such as homotheticity, homogeneity or separability on the production structure are not palatable and should rather remain a testable hypothesis within the estimation framework.

Key Words: *flexible functional forms, structure of production, translog, panel data*

Introduction

In the last thirty years considerable effort has been devoted towards specifying general forms of the production or cost function. These forms are amenable to econometric estimation, consistent with the properties of the input requirement set and contain most of the implied economic effects. The criteria for choosing the appropriate functional form are largely dependent upon the purpose of the particular analysis. The most important group of selection criteria refers to the consistency of production technology with the theoretical properties[1]. This concept is probably fundamental to the selection among alternative functional forms and as pointed out by Fuss *et al.*, (1978) maintained hypotheses are not themselves tested as part of the analysis but are assumed to be true. The choice of a functional form immediately implies that some hypotheses are maintained while others remain testable. Popular concerns about potential maintained hypotheses within economic models include *homogeneity*, *homotheticity* and *separability*. Thus, it is advantageous in specifying a functional form for applied production analysis to have estimable relationships that place relatively few restrictions on the technology.

Recent advances in developing new functional forms have been dominated by efforts to conceive the so-called *flexible* forms, and different technical definitions of flexibility have arisen as a result of these pursuits. *Local flexibility*, which is also called *Diewert flexibility*, implies that a functional form represents a *second-order Taylor series expansion* or a *second-order differential approximation* of an arbitrary function at a particular point (Fuss *et al.*, 1978; Chambers, 1988). The locally flexible form places no restrictions on the value of the function or its first and second derivatives at the point of approximation. On the other hand, *global flexibility* is preferred to local flexibility in that second-order restrictions are everywhere absent (Gallant, 1981).

This is very important since locally flexible forms can impose large and unknown restrictions away from the unknown point of expansion. Thus, a variety of flexible functional forms have been introduced for empirical studies without imposing *a priori* restrictions on functional additivity and separability. Tables 1 and 2 present the alge-

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braic form along with the relevant properties of the most important and widely applied flexible functional forms. Noteworthy reference material for these functions and some of their properties can be found in among others Diewert (1971), Christensen *et al.*, (1973), Fuss *et al.*, (1978), Denny (1974) and in De Janvry (1972).

However, the global approximation properties of the above functional forms are usually not known[2]. The notions of approximation that relied upon are local in nature; either a point approximation to a function or a second order Taylor series expansion. Neither are truly global, and approximations based on them cannot be very exact for a wide range of observations. Most of the applied empirical studies in economics have dealt with local approximation of production or cost functions. These developments were based mainly on a quadratic expression of the general linear model since it can approximate, in two general senses, any arbitrary twice continuously differentiable function. Moreover, the number of the independent parameters is not restrictive, in the sense that it does not impose many *a priori* restrictions on the economic phenomena being measured[3].

However, by far the most widely used in applied production analysis has been the translog function. Hence, in this paper we will concentrate on the translog production function. Most of the flexible functional forms and their associated demand functions are, like the translog, linear in their parameters. Thus, the extension to other functional forms will, in many cases be direct. The prime purpose of the paper is to compare the description of the production technology as that is summarised by the production elasticities, the returns to scale, the substitution possibilities and the bias in technology produced by alternative formulations of the translog production function model when different assumptions are maintained. The rest of the paper is organised as follows: first, the theoretical underpinnings associated with the properties and estimation procedures of the translog production function are discussed; the data set and results of different assumptions concerning the structure of the underlying technology are set out and interpreted; and finally, some conclusions are drawn.

The Translog Production Function Model

Specification

The translog functional form is conceptually simple and imposes no *a priori* restrictions on the structure of technology. Its flexibility circumvents the problem of over-restriction and allows a more general specification of the model since it can represent any underlying arbitrary structure of production at the chosen point of approximation. Hence, it has been widely applied in empirical analysis. Specifically, the translog production function has been utilised to examine input substitution (e.g. Berndt and Christensen, 1973), separability and aggregation (e.g. Denny and Fuss, 1977), technical change and productivity growth (e.g. May and Denny, 1979) and productive efficiency (e.g. Greene, 1980; Kalirajan, 1990).

The translog production function in a panel data[4] context, assuming that a time-trend representation of technical change may be *non-neutral* and *scale-augmenting*, has the following form (e.g. Baltagi and Griffin, 1988):

$$ln\, y_{it} = \beta_0 + \sum_{j=1}^{J} \beta_j \, ln\, x_{jit} + \frac{1}{2}\sum_{j=1}^{J}\sum_{k=1}^{J} \beta_{jk} \, ln\, x_{jit} \, ln\, x_{kit} + \gamma_1 t + \frac{1}{2}\gamma_2 t^2 + \sum_{j=1}^{J} \alpha_j \, ln\, x_{jit}\, t \quad (1)$$

where, $i=1, 2, ..., N$ are the cross-section units; $t=1, 2, ..., T$ are the time periods; $j,k=1, 2, ..., J$ are the applied inputs; lny_{it} is the logarithm of the output of the *ith* individual in period *t*; lnx_{jit} is the logarithm of the *jth* input applied of the *ith* individual in period *t*; *t* is the familiar time-trend that serves as a proxy of technical change and; β, γ and α are the parameters to be estimated. A number of restrictions are required for the estimation of the above equation.

First, an identifying restriction of *symmetry* is necessary which require that $\beta_{jk}=\beta_{kj}$ for all j, k (Young's theorem). Hence the above expression of the production structure has one *neutral-scale* parameter (β_0), j+2 *first-order* parameters (β_j, γ_1, γ_2) and (j+1)(j/2)+j *second-order* parameters (β_{jk}, α_j).

Second, *monotonicity* requires the marginal products of inputs to be positive, that is, $MP_{jit} = \partial y_{it} / \partial x_{jit} > 0$. In the context of the translog production function the marginal product of input j is obtained by multiplying the logarithmic marginal product with the average product of input j. Thus the monotonicity condition holds for the translog specification if the following equation is positive.

$$MP_{jit} = \frac{\partial y_{it}}{\partial x_{jit}} = \frac{y_{it}}{x_{jit}} \cdot \frac{\partial \ln y_{it}}{\partial \ln x_{jit}} = \frac{y_{it}}{x_{jit}} \cdot \left(\beta_j + \sum_{k=1}^{J} \beta_{jk} \ln x_{kit} + \alpha_j t \right) > 0 \qquad (2)$$

Since both y and x_j are positive numbers, monotonicity depends on the sign of the term in parenthesis. Assuming that markets are competitive and factors of production are paid their marginal products, the above term is the same as the input j's share of total output, S_j (Chung, 1994):

Third, marginal products, apart from being positive should also be *decreasing* in inputs (law of diminishing marginal productivities) which implies that the following expression should hold:

$$\left(\beta_j + \sum_{k=1}^{J} \beta_{jk} \ln x_{kit} + \alpha_j t \right) \cdot \left(\beta_j - 1 + \sum_{k=1}^{J} \beta_{jk} \ln x_{kit} + \alpha_j t \right) < 0 \qquad (3)$$

Again decreasing marginal products depend on the nature of the parenthesised terms. Researchers should check the quantitative nature of these terms in both Eq. (2) and Eq. (3) *a posteriori* against the estimated parameters at each data point. However, both restrictions should hold at the point of approximation ($\rho=0$). At that point, *positive* marginal products for each input requires $\beta_j > 0 \, \forall j$. On the other hand, *diminishing* marginal productivities require $\beta_j (\beta_j - 1) < 0 \, \forall j$, which is true always if and only if $0 < \beta_j < 1 \, \forall j$ (Hatziprokopiou *et al.*, 1996).

Table 1: Selected Flexible Functional Forms and their First-Order Partial Derivatives

Function	Algebraic Form	First-Order Partial Derivatives
Generalised C-D	$\ln y = \alpha + \sum_i \sum_j \delta_{ij} \ln\left(\left(x_i + x_j\right)/2\right)$	$y \sum_j 2\beta_{ij} \Big/ \left(x_i + x_j\right)$
Generalised Leontief[1]	$y = \sum_i \sum_j \delta_{ij} x_i^{1/2} x_j^{1/2}$	$\sum_j \delta_{ij} x_i^{-1/2} x_j^{1/2}$
Translog[1]	$\ln y = \alpha + \sum_i \beta_i \ln x_i + \sum_i \sum_j \delta_{ij} \ln x_i \ln x_j$	$y\left[\beta_i + 2\sum_j \beta_{ij} \ln\left(x_j\right)\right] \Big/ x_i$
Generalised Quadratic	$y = \left[\sum_i \sum_j \beta_{ij} x_i^{\delta\gamma} x_j^{\delta(1-\gamma)}\right]^{\nu/\delta}$	$\nu x_i^{-1} y^{\frac{\nu-\delta}{\nu}} \left[\gamma \sum_j \beta_{ij} x_i^{\delta\gamma} x_j^{\delta(1-\gamma)} + (1-\gamma) \sum_j \beta_{ij} x_i^{\delta(1-\gamma)} x_j^{\delta\gamma}\right]$
Generalised Power	$y = \alpha \prod_i x_i^{f_i(\underline{x}_i)} \exp\left(g(x)\right)$	2

[1] Assuming $\delta_{ij} = \delta_{ji}$ for all i, j; [2] Differs according to the specified functions f(x) and g(x)

Table 2: Properties of Selected Flexible Functional Forms

Properties	Generalised C-D	Generalised Leontief[1]	Translog[1]	Generalised Quadratic	Generalised Power
$\partial y/\partial x_i$	U	UBN	U	UBN	U
Homogeneous	if $\sum_i \sum_j \delta_{ij} = 1$	yes	if $\sum_i \beta_i = 1, \sum_i \delta_{ij} = 0$	if $\nu = 1$	NG
Homothetic	yes	yes	if $\sum_i \delta_{ij} = 0$	yes	NG
Constant σ	no	NG	NG	NG	NG
Concave	NG	if $\delta_{ij} \geq 0$	if $\underline{\beta} \geq \underline{0},\ \delta_{ij} \geq 0$	if $\beta_{ij} \geq 0,\ 0 \leq \gamma \leq 1, \delta \leq 1, \nu \leq 1$	NG
Separable	yes	yes	yes	NG	NG
Subsumes	C-D	-	C-D	Generalised Leontief, CES	Transcedental

U: unrestricted sign, UBC: unrestricted in sign but constant, UBN: unrestricted but non-switching in sign, NG: not in general;[1] Sufficient but not necessary for local concavity

Alternatively, convexity of the isoquants can be interpreted requiring that the corresponding *bordered Hessian* matrix of the first and second-order partial derivatives be *negative semi-definite*, that is $d^2y \leq 0$ or $(-1)^J |\overline{H}_J| > 0$. Again researchers should ascertain that the Hessian matrix of the translog function is indeed negative semi-definite, which in turn implies that its diagonal elements $\left(\partial^2 y_{it} / \partial x_{jit}^2\right)$ are non-positive. In general, the convexity conditions are the natural extension of the second-order conditions for a minimum.

The Structure of Production

Given its flexible nature, the translog production function is *non-homothetic* and imposes no restrictions on production technology. For a *homothetic* production function the marginal rate of technical substitution is homogeneous of degree zero in inputs (see Chambers, 1988), which in turn in the context of translog specification requires that the following restriction holds: $\sum_k^J \beta_{jk} = 0$. If the above restrictions are imposed in Eq. (1) the *Kmenta* approximation of *Constant Elasticity of Substitution* (CES) function is obtained (Kim, 1992). Next, the translog production function is *homogeneous of degree* θ in inputs if: $\sum_j^J \beta_j = \theta, \sum_k^J \beta_{jk} = 0$ and $\sum_j^J \alpha_j = 0$.

Linear homogeneity or equivalently constant returns to scale are obtained if θ=1. Finally, the translog production function is *additively separable* if: $\beta_{jk} = 0 \, \forall j \neq k$, and *strongly separable* if (Fan, 1991): $\beta_{jk} = 0 \, \forall j,k$ which is equivalent with a Cobb-Douglas technology with input-biased technical change. Eq (1) reduces to a *Cobb-Douglas* technology with Hicks-neutral technical change if: $\beta_{jk} = 0, \alpha_j = 0$ and $\gamma_2 = 0$ $\forall j,k$. All the above hypotheses concerning the production technology can be tested using any of the available conventional statistical tests (Gujarati, 1995)[5].

The flexible nature of the translog function does not place *a priori* restrictions on the value of output elasticities, returns to scale, elasticities of substitution or technical change. First, production function estimates of *scale economies* can be obtained from the translog system as the sum of the marginal elasticities of output with regard to each input:

$$RTS(x_{jit}, t) = \sum_{j=1}^{J} \eta_{jit} = \sum_{j=1}^{J} \frac{\partial \ln y_{it}}{\partial \ln x_{jit}} = \sum_{j=1}^{J} \left(\beta_j + \sum_{k=1}^{J} \beta_{jk} \ln x_{kit} + \alpha_j t \right) \quad (4)$$

where, η_{jit} are the marginal elasticities of output (they depend on both the input levels and technology index) measuring the responsiveness of output to one per cent change in the use of *jth* input and are simply the logarithmic marginal product of the translog function. Further, the translog function facilitates analysis of *input substitution* by allowing input substitutability to vary with the quantity of inputs used. The standard approach is to calculate *Allen-Hicks* partial elasticities of substitution directly from the production function[6]. The relevant formula for the symmetric cross and own *elasticities of substitution* is given by (Berndt and Christensen, 1973; Humphrey and Moroney, 1975):

$$\sigma_{jk} = \sum_{f=1}^{F} \frac{H_f x_f \left|H_{jk}\right|}{x_j x_k \left|H\right|} \quad \forall j,k \tag{5}$$

where f=j+1 is the *order* of the bordered Hessian matrix, of the first and second partial derivatives, $\left|\mathrm{H}\right|$ is the *determinant* of the bordered Hessian matrix and $\left|\mathrm{H}_{\mathrm{jk}}\right|$ is the *cofactor* of H_{jk} in H. Inputs j and k are substitutes, independent or complements as σ_{jk} is greater, equal to or less than zero, respectively.

Finally, the elasticity of output with respect to time interpreted as the *primal rate of technical change* is defined as (Heshmati, 1996; Capalbo and Antle, 1988):

$$\left.\frac{d\, ln\, y_{it}}{dt}\right|_{dx_j=0} = \gamma_1 + \gamma_2 t + \sum_{j=1}^{J} \alpha_j \, ln\, x_{jit} \tag{6}$$

The above measure of technical change is both time and farm specific and varies with input use. It is usually non-negative. However, if farms face new regulations or the existing regulations are tightened, technical change can be negative (technical regress).

According to its effect on relative input utilisation the rate of technical change can be decomposed further into effects due to *pure* (or autonomous) and *biased* technical change (Wylie, 1991). The former shows the effect of technology accumulation *per se*, while the latter shows its effects through the use of various inputs, indicating changes in their productivity. In other words, the pure component is simply a *neutral-shift* effect on the production function that cannot be attributed to any particular input, while the biased component is a scale expansion effect affecting the efficiency in the use of various inputs. In terms of the above equation, the first two terms on the right hand side consist the pure component of technical change, while the last term is the biased component. In the *Hicksian* sense, technical change is input j using, neutral or saving as α_j is greater than, equal to or less than zero, respectively. Technical change is defined as *non-neutral* if the passage of time affects the marginal rate of technical substitution between inputs. However, if returns to scale are constant, factor-augmenting technical change with equal rates of augmentation is equivalent to Hicks-neutral technical change. Nevertheless, if constant returns to scale do not hold, then *Hicks-neutral* technical change requires that α_j is equal to zero for all j, and it is also a testable hypothesis within the translog specification (Kumbhakar and Hjalmarsson, 1995).

Data and Empirical Results

The translog production function model is applied to a panel data set of 125 olive-growing farms in Greece during the 1987-93 period for which a detailed discussion can be found in Tzouvelekas (1998). Following Zellner *et al*., (1966) we can assume that farmers maximise *expected* profits, are price takers and can sell all the product they choose to produce without affecting product price. Output price is known, as are input prices. One can justify this assumption in terms of a two-stage decision process where input choice decision is made prior to output decision. Thus, in the case that output is treated as endogenous, the maximisation problem of the farmer is to produce the amount of expected output that maximises expected profits.

The production function can be estimated under a single equation estimation[7] using iterative feasible generalised least squares (FGLS) as outlined in Greene (1993) which allow for time-invariant farm-specific attributes in the regression equation[8]. Besides being a lot of studies estimating the translog production function under different maintaining assumption for the underlying technology, these are using either time-series or cross-section data. There are only few (e.g. Thijssen, 1992) that are dealing with the problem in the context of panel data.

The dependent variable is the annual olive-oil production measured in kilograms. The aggregate inputs included as explanatory variables are: (a) total *labour*, comprising hired (permanent and casual), family and contract labour, measured in working hours; (b) *fertilisers*, including nitrogenous, phosphate, potash, complex and others, measured in kilograms; (c) *other cost* expenses, consisting of pesticides, fuel and electric power, irrigation taxes, depreciation, interest payments, fixed assets interest, taxes and other miscellaneous expenses, measured in drachmas (constant 1990 prices); (d) *land*, including only the share of farm's land devoted to olive-tree cultivation measured in stremmas (one stremma equals 1,000 m^2).

All data were *normalised* around the sample mean, prior to logarithmic transformation, to define the point of approximation. Normalising around the sample mean is performed by dividing by sample mean all observations. As stated by Friedlaender and Spady (1980, p. 207) "since the translog production function can be viewed as a Taylor series approximation around the point (1,1,...,1) or ln(1)=0, a number of important economic calculations are simplified at the point of approximation, which corresponds to a hypothetical farm whose values of inputs, technological conditions and generic outputs and qualities are those of the sample arithmetic mean". More importantly, unless the normalisation of the data around the sample mean does indeed take place, it is questionable whether the translog parameter estimates could provide sufficiently large *well-behaved* regions satisfying all *regularity* conditions so that the true underlying technology could be *approximated.* However, the translog function does not satisfy all regularity conditions globally. In fact, when at least one second-order parameter is different to zero $(\beta_{jk} \neq 0)$, there exist configurations of inputs such that *neither* monotonicity *nor* convexity are satisfied. This follows simply from the quadratic nature of the translog function.

Nevertheless, there are regions in input space where these conditions are satisfied. These well-behaved regions may be large enough so that the translog function can provide a good representation of relevant production technology. Evidence from simulation experiments by Wales (1977) indicates that translog estimates of important elasticities are fairly good if the regularity conditions are violated only for a small number of points in the data. For any set of parameters and input levels the monotonicity and convexity conditions can easily be *checked* and analysts should do so particularly at the point of approximation. In case at the point of approximation regularity conditions are not valid, then analysts should rather reconsider their model formulation than proceeding on any kind of inference for the production structure.

Table 3 presents the parameter estimates of the translog production function under different assumptions concerning the underlying technology. In parentheses are reported the corresponding corrected asymptotic standard errors[9]. Since the number of the estimated parameters in the context of translog functional specification is high one could expect *multicollinearity* to be a problem. However, in our data set multicollinearity is not severe as that was indicated using Kmenta (1986, p. 439) approach[10]. Moreover, the estimated Durbin-Watson statistics indicate that autocorrelation is not also a serious problem in this study (the relevant values within the 2.06-2.1 range in all five models).

For all models high values of adjusted R^2 in general indicate a good fit of the parameter estimates. Specifically, between 68 and 81 per cent of cross-section and time-series variation is explained by all five models, which is sufficiently satisfactory for a panel data context. The first-order coefficients are all highly significant, deviating considerably between the alternative models. The estimated first and second-order coefficients of time (which are a proxy of technical change) are also highly significant, indicating that technical change is strongly *non-neutral* and *scale augmenting*. It may also be noted that the second-order parameters associated with inputs (β_{ij}*'s*), which give information about input substitutability, show noticeable variation in estimates across different models.

In particular, the estimates for restrictive models are significantly different from those for non-homothetic model. Consequently, restrictive production models will produce biased substitution estimates. Strengthening further the above finding a Wald-test was conducted to examine whether the underlying production structure of the Greek olive-growing farms is characterised by non-homothetic or restrictive production models. The results reported in table 3 show that the null hypotheses of homotheticity, homogeneity, linear homogeneity and separabality (strong or additive) are not tenable at the 5% of the chi-squared distribution in favour of the unrestricted non-homothetic model.

At the point of approximation the translog production function models are all *well-behaved* satisfying all regularity conditions since all estimated β_j fall between zero and one. Strengthening further the above findings the marginal products of each input with regard to output were calculated at each data point (table 4 present the average values over years). The percentage frequency of positive marginal productivities is within the following range: labour 91.2-96.5 per cent, fertiliser 87.5-93.7 per cent, other cost inputs 82.4-88.9 per cent and land 94.3-98.7 per cent. Moreover, the *concavity* of the production function was checked by testing whether the bordered Hessian matrix of the first- and second-order partial derivatives is negative semi-definite at each data point. Indeed, for more than the 80.6 per cent of the observations included in the data set the above condition was satisfied, indicating diminishing marginal productivities in all models. Moreover, at the point of approximation the determinants of the principal minors of the bordered Hessian matrix alternate their sign. Consequently, translog production function estimates provide large enough well-behaved regions of the approximated underlying production technologies to fulfil neo-classical production theory regularity conditions of monotonicity and convexity in the neighbourhood covered by our data set of Greek olive-growing farms.

Since the coefficients of the translog production frontier do not have any direct interpretation, the elasticities of output, returns to scale, elasticities of substitution and the rate of technical change (both autonomous and biased) defined in the previous section were estimated. These measures are both time- and farm-specific and can be used in drawing inferences regarding the allocation of resources by farms, within the sample and over time. However, for preserving space only the average values over farms and time are reported in table 4. First, the estimated output elasticities with respect to each input show a divergent pattern across models. However, these figures also indicate that land had contributed the most to olive-oil production followed by labour, other capital inputs and fertilisers. This pattern remains unchanged between the alternative models.

On the other hand, the corresponding estimates of returns to scale although indicate decreasing returns according to the unrestricted and separable models, according to homogeneous model the production technology of the Greek farmers is characterised by increasing returns, while the homothetic model reveal a point estimate close to unity (constant returns to scale). Next, the estimated elasticities of substitution among inputs are all positive in all models as one may expect theoretically. The relevant point estimates exhibit considerable variations among models. This is arising from the differences in the magnitude of the second-order parameters across models.

Finally, the primal rate of technical change estimated over farms and time is also reported in table 4. The non-homothetic model exhibit the slowest rate of technical progress (1.090), while the homothetic model the highest (2.277) with all other model falling in between. Moreover, all models differ considerably in the bias of that technical progress. According to additive separable model the technological innovations were saving in the use of all inputs, whereas according to the unrestricted model technical change observed during the sample period was using against labour and fertilisers.

Conclusions

The flexible nature of the translog production function has been proven quite useful in applied production analysis to the extent that it is considered the most superior flexible functional form. This paper has provided a comprehensive discussion on the approximation properties of the translog function and a general framework for estimating production relationships under different assumptions of the underlying technology. Our empirical results indicate that imposing *a priori* restrictions such as homotheticity, homogeneity or separability on the production structure are not palatable and should be rather remain a testable hypothesis within the estimation framework. Clearly the use of a flexible form facilitates valuable insights into aggregate production behaviour.

However, no one can state that flexible forms represent a panacea for applied production analysis. Although widely used in a variety of empirical studies the last thirty years, they do have certain limitations that are being increasingly recognised. The fact that they limit the range of underlying technologies that can be characterised is not surprising since fundamental duality theory implies that any specification of a cost or production places some restrictions on that technology. In order for the technology to satisfy all properties of the production function presented in previous sections the technology must be consistent with *homotheticity*. Thus, they are characterised by isoclines rather than isoquants for which the marginal rate of technical substitution is constant, while the elasticities of scale are varying across them. Moreover, most of the developed flexible functional forms are very inflexible when representing separable technologies. This remains valid regardless of whether one is trying to represent the function of interest or a monotonic transformation of it. However, imposing separability on the function involves parametric restrictions that result in more restrictions than originally desired. Thus, it is not appropriate to call the resulting form flexible since there are not enough parameters left to depict the remaining distinctive effects.

Table 3: Parameter Estimates for Alternative Models (Non-Homothetic, Linear Homogeneous and Homogeneous)[1]

Par.[2]	UN	LHG	HG	HM	AS	SS
β_0	0.201 (0.051)	0.258 (0.045)	0.196 (0.046)	0.191 (0.046)	0.190 (0.047)	0.151 (0.042)
β_L	0.135 (0.018)	0.187 (0.024)	0.146 (0.026)	0.147 (0.026)	0.134 (0.025)	0.111 (0.016)
β_F	0.059 (0.029)	0.114 (0.039)	0.062 (0.041)	0.059 (0.041)	0.057 (0.039)	0.018 (0.013)
β_C	0.102 (0.015)	0.001 (0.197)	0.078 (0.021)	0.081 (0.021)	0.089 (0.089)	0.015 (0.007)
β_A	0.559 (0.064)	0.700 (0.051)	0.632 (0.054)	0.578 (0.048)	0.584 (0.057)	0.628 (0.043)
β_{LF}	-0.003 (0.005)	0.000 (0.007)	-0.001 (0.042)	-0.001 (0.006)	-	-
β_{LC}	0.002 (0.002)	0.001 (0.003)	0.004 (0.007)	0.004 (0.003)	-	-
β_{LA}	-0.003 (0.016)	-0.018 (0.014)	-0.012 (0.002)	-0.012 (0.004)	-	-
β_{LL}	0.010 (0.004)	0.017 (0.004)	0.010 (0.003)	0.010 (0.004)	0.009 (0.003)	-
β_{FC}	-0.002 (0.002)	-0.004 (0.003)	-0.002 (0.004)	-0.003 (0.003)	-	-
β_{FA}	0.001 (0.016)	-0.010 (0.014)	-0.004 (0.049)	-0.003 (0.004)	-	-
β_{FF}	0.007 (0.004)	0.013 (0.005)	0.007 (0.003)	0.007 (0.005)	0.005 (0.005)	-
β_{CA}	0.008 (0.007)	0.003 (0.007)	-0.007 (0.006)	-0.007 (0.008)	-	-
β_{CC}	0.008 (0.001)	-0.001 (0.001)	0.006 (0.005)	0.006 (0.002)	0.007 (0.002)	-
β_{AA}	-0.076 (0.036)	0.024 (0.005)	0.023 (0.007)	0.022 (0.006)	-0.030 (0.029)	
γ_1	0.019 (0.053)	0.062 (0.081)	0.046 (0.079)	0.042 (0.079)	0.042 (0.079)	0.039 (0.080)
γ_2	0.016 (0.028)	0.039 (0.042)	0.034 (0.080)	0.035 (0.041)	0.037 (0.041)	0.031 (0.042)
α_L	0.007 (0.012)	-0.007 (0.018)	-0.009 (0.018)	-0.006 (0.018)	-0.006 (0.018)	-0.003 (0.018)
α_F	0.001 (0.013)	0.004 (0.015)	-0.001 (0.001)	0.000 (0.015)	-0.001 (0.015)	0.004 (0.015)
α_C	-0.011 (0.006)	-0.011 (0.007)	-0.012 (0.014)	-0.018 (0.008)	-0.017 (0.008)	-0.017 (0.008)
α_A	-0.111 (0.040)	0.014 (0.004)	-0.024 (0.004)	-0.073 (0.047)	-0.080 (0.047)	-0.086 (0.047)
θ	-	1	0.918 (0.041)	-	-	-
$\overline{R}^2$	0.81	0.74	0.69	0.70	0.71	0.68
$\chi^2_{(k)}$		23.31 (k=6)	21.30 (k=5)	14.95 (k=4)	14.76 (k=6)	76.90 (10)

where, UN stands for the unrestricted non-homothetic translog model, LHG for linear homogeneous, HG for homogeneous, HM for homothetic, AS for additive separable and SS for strongly separable.

[1] in parentheses are the asymptotic standard errors (White, 1980; Greene, 1993); [2] where, L stands for labour, F for fertilisers, C for other costs, A for area and T for time.

Table 4: Estimated Output Elasticities, Returns to Scale and Technical Change

	UN[1]	LHG	HG	HM	AS	SS
Output Elasticities:						
Labour	0.113	0.152	0.173	0.168	0.116	0.111
Fertiliser	0.061	0.106	0.145	0.146	0.056	0.017
Other Cost	0.085	0.004	0.049	0.047	0.078	0.017
Area	0.582	0.738	0.670	0.628	0.603	0.640
RTS	0.841	1.000	1.036	0.989	0.846	0.786
Marginal Products:						
Labour	0.147	0.199	0.150	0.158	0.151	0.147
Fertiliser	5.499	9.663	0.518	5.356	4.491	1.507
Other Cost	14.310	13.005	10.414	11.861	13.298	15.297
Area	0.650	0.826	0.756	0.700	0.673	0.747
Elasticities of Substitution:						
σ_{LF}	1.548	1.428	1.825	1.325	1.332	1.663
σ_{LC}	1.210	1.324	1.425	1.236	1.245	1.478
σ_{LA}	1.021	1.004	1.122	1.001	1.058	1.362
σ_{FC}	1.420	1.258	1.624	1.298	1.325	1.458
σ_{FA}	1.102	1.136	1.269	1.175	1.365	1.745
σ_{CA}	1.531	1.465	1.732	1.421	1.574	1.685
Technical Change:						
Total	1.090	1.718	1.684	2.277	2.250	1.935
Autonomous	0.261	0.997	0.657	0.560	0.532	0.524
Biased	0.829	0.722	1.026	1.667	1.718	1.410

[1] where, UN stands for the unrestricted non-homothetic translog model, LHG for linear homogeneous, HG for homogeneous, HM for homothetic, AS for additive separable and SS for strongly separable.

Notes

[1] For a detailed discussion on the properties of the production or cost functions see Chambers, 1988 and/or Varian, 1992.

[2] For a comparison of the relative performance of alternative flexible functional forms see Griffin *et al*., (1987), Baffes and Vasavada (1989), Ornelas and Shumway (1993) and Shumway and Lim (1993).

[3] The best way to underline the meaning and the importance of flexibility is by contrast with a form that is not flexible, the Cobb-Douglas. The parameters of Cobb-Douglas cannot be chosen to satisfy either a second-order differential approximation or a second-order numerical approximation of an arbitrary function. In fact, Cobb-Douglas is a first-order approximation, since all cross-price derived demand elasticities equal parameters of the cost function that, by Shepard's lemma, also equals factor shares. Hence, first and second-order effects are *co founded* in the Cobb-Douglas function.

[4] Although we consider herein the panel data case, the analysis can be carried out in a single time-series or cross-section models.

[5] In the case of homotheticity the number of restrictions are j, for linear homogeneity $j+2$, for homogeneity $j+1$, for weak separability $(j+1)(j/2)-j$ and for strong separability $(j+1)(j/2)$.

[6] However, there are two ways to examine substitution elasticities within the primal side of the production. Another measure apart of the Allen-Hicks elasticities of substitution is the Hicks elasticity of *complementarity*, which measure the effect on the relative price of an input change of a change in the relative quantity of that input, holding the quantities of other inputs constant (for more details see Grant and Hamermesh, 1981).

[7] Alternatively as Berndt and Christensen (1973) have shown one can estimate the model using the set of factor demand equations. The system of equations can be jointly estimated using Zellner's (1962) seemingly unrelated regression (SUR) under the assumption of competitive output markets. Kim (1992) also suggested that one can use the set of inverse input demand equations, which do not rely on market behaviour and can accommodate both competitive and non-competitive output markets.

[8] For a review and summary of many possible models suggested in the literature for the estimation of panel data models see Judge *et al*., (1985, Ch. 13) and Hsiao (1986, Ch. 6).

[9] The estimates of the standard errors obtained from the econometric estimation of the production function are biased and inconsistent due to *heteroscedasticity* and thus, they can over- or under-estimate their correct counterparts (Heckman, 1979; Greene, 1981). Therefore the variance-covariance matrix was consistently estimated using White's (1980) *robust* estimator as outlined in Amemiya (1985, p. 370) and Heshmati (1994).

[10] According to Kmenta (1986) a simple measure of its degree can be obtained by regressing each of the explanatory variables on the remaining explanatory variables. Then the obtained coefficients of determination can be taken as a measure of the degree of multicollinearity in the sample. In our data set the various values of R^2 were as follows: 0.45 for labour, 0.19 for fertilisers, 0.24 for other cost inputs and 0.24 for land, indicating that multicollinearity is not high and consequently not a problem in the present study.

References

Amemiyia, T. (1985). Advanced Econometrics, Oxford UK: Basil Blackwell.

Baffes, J. and U. Vasavada (1989). On the Choice of Functional Forms in Agricultural Production Analysis, Applied Economics, 21: 1053-1061.

Baltagi, B.H. and J.M. Griffin (1988). A General Index of Technical Change, Journal of Political Economy, 96: 20-41.

Berndt, E.R. and L.R. Christensen (1973). The Translog Function and the Substitution of Equipment, Structures and Labour in US Manufacturing 1929-68, Journal of Econometrics, 1: 81-114.

Capalbo, S.M. and J.M. Antle (1988). Agricultural Productivity: Measurement and Explanation, Washington DC: Resource for the Future.

Christensen, L.R., Jorgenson, D.W. and L.J. Lau (1973). Transcendental Logarithmic Production Frontiers", Review of Economics and Statistics, 55: 28-45.

Chung, J.W. (1994). Utility and Production Functions, Cambridge MA: Backwell.

De Janvry, A. (1972). The Generalised Power Function", American Journal of Agricultural Economics, 54: 234-237.

Denny, M. (1974). The Relationship Between Forms of the Production Function, Canadian Journal of Economics, 7: 21-31.

Denny, M. and M. Fuss (1977). The Use of Approximation Analysis to Test for Separability and the Existence of Consistent Aggregates, American Economic Review, 67: 404-418.

Diewert, W.E. (1971). An Application of the Shepard Duality Theorem: A Generalised Leontief Production Function, Journal of Political Economy, 79: 481-507.

Fan, S. (1991). Effects of Technological Change and Institutional Reform on Production Growth in Chinese Agriculture, American Journal of Agricultural Economics, 73: 266-275.

Friedlaender, A.F. and R.H. Spady (1980). General Specifications of Costs and Technology: Freight Transport Regulation (Equity, Efficiency and Competition in the Rail and Trucking Industry), Cambridge MA: MIT Press.

Fuss, M., McFadden, D. and Y. Mundlak (1978). A Survey of Functional Forms in the Economic Analysis of Production, in Production Economics: A Dual Approach to Theory and Application, M. Fuss and D. McFadden (eds.), Amsterdam: North Holland.

Gallant, R.A. (1981). On the Bias in Flexible Functional Forms and an Essentially Unbiased Form, Journal of Econometrics, 15: 211-245.

Grant, J.H. and D.S. Hamermesh (1981). Labour Market Competition Among Youths, White Women and Others, Review of Economics and Statistics, 63: 354-360.

Greene, W.H. (1980). On the Estimation of a Flexible Frontier Production Model, Journal of Econometrics, 13: 27-56.

Greene, W.H. (1981). Sample Selection Bias as a Specification Error: Comment, Econometrica, 49: 795-798.

Griffin, R.G., Montgomery, J.M. and E.M. Rister (1987). Selecting Functional Form in Production Function Analysis, Western Journal of Agricultural Economics, 12: 216-227.

Gujarati, N.D. (1995). Basic Econometrics, New York: McGraw-Hill.

Hatziprokopiou, M., Karagiannis, G. and K. Velentzas (1996). Production Structure, Technical Change and Productivity Growth in Albanian Agriculture, Journal of Comparative Economics, 22: 295-310.

Heckman, J.J. (1979). Sample Selection Bias as a Specification Error, Econometrica, 46: 153-161.

Heshmati, A. (1994). Estimating Random Effects Production Function Models with Selectivity Bias: An Application to Swedish Crop Producers, Agricultural Economics, 11: 171-189.

Heshmati, A. (1996). On the Single and Multiple Time-Trends Representation of Technical Change, Applied Economic Letters, 3: 495-499.

Hsiao, C. (1986). Analysis of Panel Data, Cambridge: Cambridge University Press.

Humphrey, D.B. and J.R. Moroney (1975). Substitution Among Capital, Labour and Natural Products in American Manufacturing, Journal of Political Economy, 83: 57-82.

Judge, G.G., Hill, R.C., Griffiths, W.E., Lutkepohl, H. and T. Lee (1985). The Theory and Practice of Econometrics, 2nd ed., New York: John Wiley and Sons.

Kalirajan, K.P. (1990). On Measuring Economic Efficiency, Journal of Applied Econometrics, 5: 75-86.

Kim, H.Y. (1992). The Translog Production Function and Variable Returns to Scale, Review of Economics and Statistics, 74: 546-552.

Kmenta, J. (1986). Elements of Econometrics, New York: Macmillan Press.

Kumbhakar, S.C. and L. Hjalmarsson (1995). Decomposing Technical Change with Panel Data: An Application to the Public Sector, Scandinavian Journal of Economics, 97: 309-323.

May, J.D. and M. Denny (1979). Factor Augmenting Technical Progress and Productivity in US Manufacturing, International Economic Review, 20: 759-774.

Ornelas, F.S. and C.R. Shumway (1993). Multidimensional Evaluation of Flexible Functional Forms for Production Analysis, Journal of Agricultural and Applied Economics, 25: 106-118.

Shumway, C.R. and H. Lin (1993). Functional Form and US Agricultural Production Elasticities, Journal of Agricultural and Resource Economics, 18: 266-276.

Thijssen, G. (1992). A Comparison of Production Technology Using Primal and Dual Approaches: The Case of Dutch Dairy Farms, European Review of Agricultural Economics, 19: 49-65.

Tzouvelekas, V. (1998). Efficiency in Agricultural Production: The Case of Olive-Growing Farms in Greece, Ph.D. Dissertation, The University of Wales Aberystwyth.

Varian, H.R. (1992). Microeconomic Analysis, New York: W.W. Norton and Co.

Wales, T.J. (1977). On the Flexibility of Flexible Functional Forms, Journal of Econometrics, 5: 183-193.

White, H. (1980). A Heteroscedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroscedasticity, Econometrica, 48: 817-838.

Wylie, P.J. (1990). Scale-biased Technological Development in Canada's Industrialization, 1900-1929, Review of Economics and Statistics, 72: 219-27.

Zellner, A. (1962). An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias, Journal of the American Statistical Association, 57: 348-368.

Zellner, A., Kmenta, J. and J. Dreze (1966). Specification and Estimation of Cobb-Douglas Production Function Models, Econometrica, 34: 784-795.