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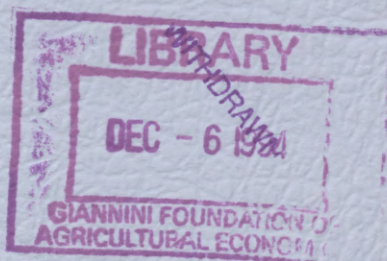
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**ARE PEOPLE REALLY RISK SEEKING
WHEN FACING LOSSES?**

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Dean Morris

Discussion Paper

No. 9407

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December 1994

**ARE PEOPLE REALLY RISK SEEKING
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**John Fountain, Michael McCosker
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Are People Really Risk Seeking When Facing Losses?

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November 1994

Abstract: An experiment and operational subjective Bayesian statistical methods are used to investigate the relation between risk attitudes in the loss domain and framing effects. We find that subjects avoid pure increases in risk when such risks are transparent, that there is little or no correlation between risk attitudes in frames that alternately mask and make transparent pure increases in risk, and that analysing risk attitudes when prospects are presented as lists of prizes and probabilities overstates the likelihood of risk seeking in the loss domain. In general GEUT fails to predict better than a naive theory holding a uniform prior and Bayesian updating. The one exception is in a frame (viewed marginally) where costs of acquiring and processing information are low.

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I Introduction

This paper reports on an experiment designed to help understand relationships between framing effects and the hypothesis that individuals are risk seeking when facing losses. A recent contribution to this literature, Kagel et al (1990), hereafter KMB, investigated the latter hypothesis. KMB presented subjects with common ratio gambles involving losses that are mean preserving spreads (MPS) of one another and found a significant number to be risk seeking. Their results replicate others established in the literature and are viewed as statistically significant deviations from Expected Utility (EU) theory.

However, KMB's gambles, as most others in the literature on this question (see Holt and Davis (1993) or Sugden (1987) for recent general overviews), are presented to subjects (framed) as lists of probabilities and prizes. The subjects in the experiment were not explicitly informed, if even they had been able to comprehend, that the gambles they were choosing between were MPS's of one another. To be sure, simple calculations can be performed by the subjects in the experiment to alert them to this logical fact, but other evidence on framing (see the Bell Raiffa Tversky (1988) symposia) strongly suggests that people do not typically perform such "routine" information processing tasks.

These observations lead to several interesting questions:

- 1 • Will people be risk seeking in the face of gambles involving losses when MPS's can be readily detected?
- 2 • How are risk attitudes in frames where detecting MPS's is difficult correlated with risk attitudes in frames where MPS's are more readily transparent?
- 3 • How well can existing theories predict choices in these kinds of situations? In particular are there systematic tendencies more predictable than by "chance", as alleged by KMB and others?

Briefly, our answer to these questions are: not as much as you might expect (for #1), uncorrelated (for #2), and not very well, in fact usually no better than by chance (for #3). Our answers are based on an experiment that replicated parts of KMB's experiment and on an analysis of the data using operational subjective statistical methods (Lad(1993))¹. The following four sections of the paper explain the experimental design and the statistical methodology employed, present the experimental results and statistical inferences that can coherently be made, and assess the predictive ability of Generalized Expected Utility Theories (GEUT) theories. A short summary and interpretation concludes the paper.

¹The statistical methods employed in the paper are interesting in their own right as they offer the promise of powerful, coherent methods to analyse experimental data and assess competing theories.

II The Experiment

Three of the pairs of gambles used in the KMB experiment were selected for our experiment, as shown in the upper half of Figure I.C. Each of the three binary decision situations, labelled E1, E2, and E3, involves a choice between two gambles with the same mean loss, with the upper right choice being an MPS of the corresponding lower left choice. Subjects² were asked to assess their preferences for these gambles³ in two separate rounds one week apart. In Round 1 the gambles were presented in two ways or frames, first as simple prospects, i.e. lists of probabilities and corresponding prizes, as in Figure I.A (the **standard prospect frame**), and second, as lists of expected wealth levels and probabilities of worst outcomes, as in Figure I.B (the **MPS transparent frame**). Figures I.A and I.B present logically equivalent information, although some mental effort is required to discover this. During Round 1 each one of the three decision situations E1, E2, and E3 was presented to each subject in each of the two frames.⁴

Figure I.A: Round 1 questions: the standard prospect frame

Possible outcomes are	N gamble label	M gamble label
Best=\$0 loss,		
Middle=\$14 loss,		
Worst=\$20 loss		
Probability of \$0 loss	37%	10%
Probability of \$14 loss	0%	90%
Probability of \$20 loss	63%	0%

Figure I.B: Round 1 questions: the MPS transparent frame

Possible outcomes are	N* gamble label	M* gamble label
Best=\$0 loss,		
Middle=\$14 loss,		
Worst=\$20 loss		
Prob of worst outcome	63%	0%
Expected Loss	\$12.60	\$12.60

The MPS transparent frame makes it immediately apparent that the two gambles have the same mean, and almost immediately apparent that one gamble is a mean preserving spread of the other (there are only 3 outcomes and the probability in one tail is given). Of course, the MPS frame also offers a simple (low effort cost) lexicographic evaluation strategy for assessing gambles: since the gambles are readily seen to have the same expected loss, select the gamble with the lower probability of a worst outcome, here a risk averse choice. We chose this frame deliberately to see if it would in fact induce risk aversion in situations where past research suggests risk seeking.

In Round 2 of the experiment, held one week after Round 1, the subjects were involved in a 3/4 hour discussion explaining the unit probability triangle, framing effects, mean preserving spreads,

²Third year micro economic students at the University of Canterbury who had passed a second year microeconomics course that used Hal Varian's *Intermediate Microeconomics* as a text.

³The gambles are hypothetical and not played for "real" monetary outcomes. We are just beginning to analyse the results of repeating our experiment where subjects are endowed with \$50 prior to the experiment.

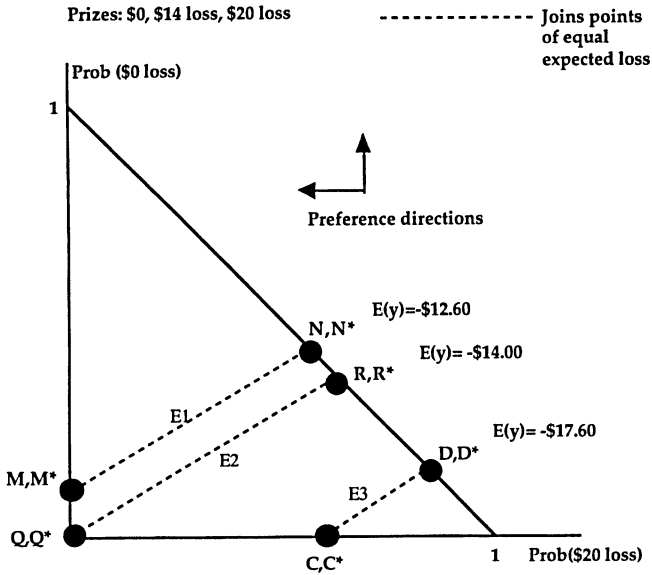
⁴Each decision situation E1...E3 was presented in two frames in Round 1. The 6 situation/frame pairings were scrambled with 4 other similar questions, with each question taking up one page in the booklet. The experimental design of Round 1 neither explicitly encouraged nor discouraged the subjects from comparing different decision situations on different pages, but the booklet form we used (deliberately) increased the effort required to discover the equivalence of gamble pairs across frames.

theoretical concepts of risk seeking and risk averse attitudes, and how GEUT theories predict certain patterns of choice in the probability triangle. Subjects were encouraged to ask questions, and did so freely. Moreover, in Round 2 their own choices from Round 1 were presented to them as private information in a chart as in Figure I.C, along with the full probability triangle in that Figure. This information was available and used extensively in the discussion.

The subjects were then asked to take a second look at the gambles and choose their more preferred option, in light of their (possibly) new understanding of the Round 1 decision situations. In Round 2 information acquisition and processing costs were virtually zero for detecting both the logical equivalence between gambles and also that the gambles were mean preserving spreads of one another. We call Round 2 the **Integrated frame** in the experiment as it integrates a wide range of the relevant bits of information one would like to have to make 'fully informed', sensible choices about these gambles, including information on earlier 'blind' binary choices.

Figure I.C: Round 2 : the Integrated frame

Prizes: \$0, \$14 loss, \$20 loss



Your answers to last week's questionnaire are included in the following table.

Choice pair	Prob of \$0 loss	Prob of \$14 loss	Prob of \$20 loss	Expected \$ Loss $E(y)$	Situation labels in Figure 1	Last week's choice	Choice now ????
Choice situation between M and N (or M^* and N^* reframed)							
M, M^*	10%	90%	0%	$E(y) = \$12.60$ loss	M vs N		
N, N^*	37%	0%	63%	$E(y) = \$12.60$ loss	$M^* \text{ vs } N^*$		
Choice situation between Q and R (or Q^* and R^* reframed)							
Q, Q^*	0%	100%	0%	$E(y) = \$14.00$ losses	Q vs R		
R, R^*	30%	0%	70%	$E(y) = \$14.00$ loss	$Q^* \text{ vs } R^*$		
Choice situation between C and D (or C^* and D^* reframed)							
C, C^*	0%	40%	60%	$E(y) = \$17.60$ loss	C vs D		
D, D^*	12%	0%	88%	$E(y) = \$17.60$ loss	$C^* \text{ vs } D^*$		

Statistical Methodology

There are three decision situations in three separate frames. Thus the data for an experimental subject "i" is an 9-tuple $X_i = [(E1_{i1}, E2_{i1}, E3_{i1}), (E1_{i2}, E2_{i2}, E3_{i2}), (E1_{i3}, E2_{i3}, E3_{i3})]$, where $E_{ijk} = 1$ or 0 according as the choice on question E1, E2, E3 in the prospect, MPS, and integrated frame (second index) is risk averting or risk seeking. We use the notation $X_N = \{X_1, X_2, \dots, X_N\}$ to describe a possible sequence of observations of 9-tuples X_i in an experiment involving N subjects. A histogram, $s_k(X_N)$, $k=1, 2, \dots, R$ corresponding to any sequence of $X_N = \{X_1, X_2, \dots, X_N\}$ is defined in the natural way as the sum or count of the number of observations X_i in the sequence which lie in each of R categories. The maximum number of categories is equal to 512, one for each of the $2^9=512$ logically possible outcomes from the experiment. We use the notation $s_1^*, s_2^*, \dots, s_R^*$ to denote the histograms derived from the sequence of actual observations X_1, X_2, \dots, X_m in an experiment with m observations, and s_1, s_2, \dots, s_R to denote category sums for yet to be observed sequences of observations $X_{N-m} = \{X_{m+1}, X_{m+2}, \dots, X_N\}$.

Scientific activity is concerned with making inferences, coherent conditional probability assessments, about yet to be observed sequences of observations $X_{N-m} = \{X_{m+1}, X_{m+2}, \dots, X_N\}$, or their associated histograms s_1, s_2, \dots, s_R , having observed other data sequences, summarised by their histograms $s_1^*, s_2^*, \dots, s_R^*$. We base our predictive inferences on equation (1), which has the form of a Polya distribution with parameters $(\alpha_1, \dots, \alpha_R)$. Equation (1) specifies the complete joint predictive probability distribution over events (yet to be observed histograms) we are interested in. It is derived from applying the theory of operational subjective statistical procedures to a fundamental representation theorem of de Finetti on infinitely extendible and exchangeable data sequences (Lad (1993)). Exchangeability in our experimental context means simply that data sequences with the same histograms are regarded as equally likely. The parameters $(\alpha_1, \dots, \alpha_R)$ arise from applying a Dirichlet mixing distribution to a multinomial as outlined briefly in the Appendix.

$$(1) \quad P[s_1, s_2, \dots, s_R \mid s_1^*, s_2^*, \dots, s_R^*] = \frac{(N-m)!}{s_1! s_2! \dots s_R!} \frac{\Gamma[\sum_{j=1}^R \alpha_j + s_j^*] \cdot \prod_{j=1}^R \Gamma[\alpha_j + s_j^* + s_j]}{\Gamma[(N-m) + \sum_{j=1}^R \alpha_j + s_j^*] \cdot \prod_{j=1}^R \Gamma[\alpha_j + s_j^*]}$$

A useful way to think about the choice of parameters $(\alpha_1, \dots, \alpha_R)$ in (1) is to imagine we have no experimental evidence available ($m=0$), and we wish predict marginally only one trial out in the experiment ($N-m=1$). In the case equation (1) reduces to.

$$(2) \quad P((s_1=0), (s_2=0), \dots, (s_j=1) \dots (s_R=0) \mid 0, 0, \dots, 0) = \frac{\alpha_j}{\sum_{k=1}^R \alpha_k}$$

(2) shows clearly that the *relative* sizes of the α_j 's indicate the relative *prior probability* of an outcome in category j .

As experimental evidence accumulates in the form of histograms ($s_1^*, s_2^* \dots s_R^*$), coherent inferences about the next trial of the experiment change. Keeping $N-m=1$, so predicting out in the future one trial at a time, but having observed the histogram $s_1^*, s_2^* \dots s_R^*$, (1) reduces to (3), a multiple category version of Laplace's law of succession for binary outcome events.

$$(3) \quad P((s_1=0), (s_2=0), \dots (s_j=1) \dots (s_R=0) | s_1^*, s_2^* \dots s_R^*) = \frac{\alpha_j + s_j^*}{\sum_{k=1}^R (\alpha_k + s_k^*)}$$

The probability assessments in (3) will be called *predictive probabilities*.

Predictive probabilities change with observational data s_k^* at a rate determined by the size of the $\alpha_j + s_j^*$ and $\sum_{k=1}^R (\alpha_k + s_k^*)$. Since $\sum_{k=1}^R s_k^* = m$, the number of observations, the larger is $\sum_{k=1}^R \alpha_k$ the slower will be the rate of change of predictive probabilities with respect to any given change in observations in category k , s_k^* . Consequently $\sum_{k=1}^R \alpha_k$ is known as the *strength* of belief (prevision). A specification of prior probabilities that is held strongly states that it takes a large number of observations to change these prior beliefs. In short, beliefs held weakly change priors rapidly with new data, while beliefs held strongly change priors less rapidly as new data arises.

From equation (3) changes in $(\alpha_1, \alpha_2, \dots, \alpha_R)$ have precisely the same (marginal) impact on predictive probabilities as do changes in observational data $s_1^*, s_2^* \dots s_R^*$. Thus the selection of parameters $(\alpha_1, \alpha_2, \dots, \alpha_R)$ can be calibrated in terms of "observational" equivalents in each category R . For example, if $\sum_{k=1}^R \alpha_k$ increases by n and the corresponding histogram sum (number of observations) decreases by n the predictive probability of equation (3) remains unchanged. Thus specifying $(\alpha_1, \alpha_2, \dots, \alpha_R)$ with a strength of n is like saying your prior beliefs are worth (have the same weight in your probability assessments as) n observations.

In this paper we restrict our theoretical interest to making inferences "in the small", i.e. about the outcome of next trial of the experiment, on the basis of theories. Theories predict patterns in the data $X_N = \{X_1, X_2, \dots, X_N\}$. For example, GEUT in its expected utility version (EU) predicts an X_i with either all 1's (risk averse), or all 0's (risk seeking), that is, consistent risk attitudes across and within frames. GEUT in its Fanning Out (FO) version (see Holt and Davis, 447, for a succinct explanation of FO theories) predicts any one of the following patterns $(E1_{ik}, E2_{ik}, E3_{ik}) = (1, 1, 1), (0, 0, 0), (1, 1, 0)$ or $(1, 0, 0)$ within any specific frame, and, being a frame invariant theory, the same pattern in all frames simultaneously. Thus the EU version of GEUT predicts two specific 9 tuples and the FO

version four specific 9-tuples. EU is "nested" in FO. A simple frame sensitive⁵ theory is the lexicographic decision strategy induced by the MPS transparent frame. It leads one to predict the risk averse choice pattern $(E1, E2, E3) = (1, 1, 1)$ in the MPS transparent frame.

No theory predicts perfectly. For the purposes of making predictions however we assert four theories to cover a broad range of plausible beliefs. We distinguish two versions of GEUT.

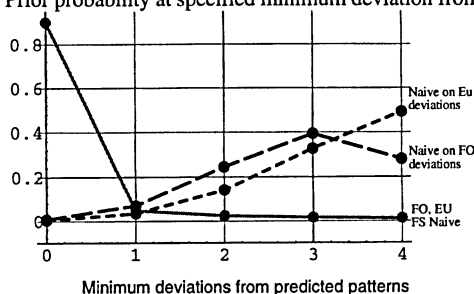
- 1• **EU** places 90% prior probability (equally distributed) on the two outcomes predicted by expected utility theory (all risk averse or all risk seeking) and the remaining 10% prior probability is spread in inverse proportion to the minimum deviation from one of the EU patterns.
- 2• **FO** places 90% probability equally on the four patterns consistent with Fanning Out of indifference curves, with the remaining 10% spread in inverse proportion to the minimum number of deviations from an FO pattern.
- 3• **Naive** asserts equal prior probability for any outcome on any one trial of the experiment.
- 4• **FS Naive** (Frame Sensitive Naive) asserts a 90% probability (equally distributed) on patterns with all risk averse choices in the MPS frame with the remaining 10% spread in inverse proportion to the minimum deviations from its predicted patterns.

Each theory is assessed for a range of strengths ("observational equivalents") from 1 to 100. Figure III.A illustrates the prior probability being asserted in each theory on the space of the number of deviations from predicted patterns within that theory.⁶ By a deviation is meant a difference between the 9-tuple X_i predicted by the theory and the 9-tuple X_i actually observed. The figure shows graphically the basic idea that some theories predicts certain patterns with high prior probabilities and patterns that differ from these theories with lower prior probabilities. It also illustrates the prior chances being assessed for getting deviations from the predicted patterns of the two versions of GEUT, EU and FO, by chance, i.e., under the Naive theory, where any one 9-tuple is as likely as any other 9-tuple.

⁵Prospect theory is one of the few theories of choice that does not assume frame invariance. Prospect theory is not examined in the current paper. It will be compared with GEUT on a decision situation by decision situation basis in a forthcoming paper by the authors.

⁶The maximum deviation from an EU or FO predicted pattern is 4 while the maximum deviation from an FS Naive predicted pattern is 3, so the graph of the GEUT prior theoretical distribution is similar in form to the FS Naive theory.

Figure III.A: Prior probability at specified minimum deviation from predicted pattern⁷



The acid test of a theory is its ability to predict relative to other theories. A systematic way of assessing the predictive performance of theories is provided by the theory of proper scoring rules (Lad (1993, Ch.6)). If an unknown quantity X can take on possible values $\{x_1, \dots, x_K\}$ a theory can be viewed as asserting knowledge about X in the form of a distribution function (Q_1, \dots, Q_K) from the K dimensional unit simplex where Q_i is the probability of the event $(X=y_i)$. With the convention that $(X=y_i) = 1$ if $X=y_i$ and 0 otherwise, the log scoring rule is given by:

$$(4) \quad S(X, Q_1, \dots, Q_K) = \sum_{i=1}^K (X=y_i) * \text{Ln}(Q_i)$$

We interpret Equation (4) as the sequential score in the small for a theory's predictive probability distribution at any one point in time. Before the first trial of an experiment a predictive probability assessment $f_x(X_1=x_i)$ is made for the outcome of that first trial X_1 (the prior in standard Bayesian terminology). An outcome y_1 is observed and a score $S_1 = \text{Ln}(f_x(X_1=y_1))$ calculated. If the theory predicted that outcome with a high probability it gets a high score, otherwise it gets a low score. Since the log of a fraction is negative, the score in this case can be interpreted as a penalty⁸. The predictive probability is then updated via here Bayes rule to $f_x(X_2=x_i | X_1=y_1)$. The second trial occurs, with outcome y_2 observed and a score $S_2 = \text{Ln}(f_x(X_2=y_2 | X_1=y_1))$ calculated. Continuing in this way the cumulative score for a theory after a sequence of m observations (y_1, y_2, \dots, y_m) is:

$$(4) \quad \begin{aligned} \sum_{i=1}^m S_i &= \sum_{i=1}^m \text{Ln}(f_x(X_i=y_i | X_{i-1}=y_{i-1}, X_{i-2}=y_{i-2}, \dots)) \\ &= \text{Ln}(f_x(X_m=y_m, X_{m-1}=y_{m-1}, \dots, X_1=y_1)) \end{aligned}$$

⁷A count of the logically possible outcome vectors at specified minimum deviation from predicted pattern yields:

Minimum Deviations	0	1	2	3	4
from EUT	2	18	72	168	252
from FO	4	36	126	202	144

⁸The log function, being increasing and concave, has the property that the penalty increases the less accurate the prediction, both in total and at the margin. E.g., the score for a 0.2 probability is -1.6 and for a 0.9 probability is -0.11

The second equality holds because the sum of the logs is the log of the product, and the product⁹ in this case is just one way of factoring a joint pdf into a product of conditional pdfs. The log scoring rule has a total score that is independent of the order in which the observations arrive¹⁰, which is particularly appropriate for our purposes.

⁹To assess scores over the entire outcome space, set R , in equation (1) the number of categories, equal to K , the number of possible outcomes, and remove the multinomial coefficient. By exchangeability, all sequences with the same category sum are equally likely. The multinomial coefficient in equation (1) simply counts the number of such sequences.

¹⁰The log scoring rule is also a proper scoring rule. If an agent asserting a theory (Q_1, \dots, Q_K) personally holds (P_1, \dots, P_K) as his/her own probability assessments a proper scoring rule assures that the expected score viewed as a function of (Q_1, \dots, Q_K) , where the expectation is taken with respect to (P_1, \dots, P_K) , is maximised by choosing $(Q_1, \dots, Q_K) = (P_1, \dots, P_K)$. A proper scoring rule encourages honest revelation of personal probability assessments if expected scores matter to the agent. The log scoring rule is discussed in Buehler (1971) and Lad (1993) Ch. 6

III Analysis of Data from the Experiment

III.A Single Frame Analysis: Predictive Probabilities for Risk Seeking

The histograms of the experimental data in Table A.2 make it clear that the extent of risk seeking in the loss domain (RS indicates the number of risk seeking choices) when choosing between gambles that are mean preserving spreads of one another depends significantly on the framing of decisions.

Table A.2 Histogram for the number of risk seeking choices (RS) in the loss domain, by frame

	RS=0	RS=1	RS=2	RS=3
Prospect Frame	1	10	4	4
MPS transparent Frame	16	1	2	0
Integrated Frame	10	3	4	2

The main points to take from this histogram are that

- in the prospect frame a large proportion, 18 out of 19, make at least 1 risk seeking choice (consistent with KMB and other results in the literature)
- this pattern is virtually reversed in the MPS transparent frame, with 16 out of 19 displaying no risk seeking behaviour whatsoever
- in the integrated frame approximately half of the subjects make at least 1 risk seeking choice, but the pattern is skewed towards risk averting behaviour

What predictive inferences can coherently be drawn from this data on the extent of risk seeking in yet to be observed experimental results? Figure A.1 illustrates the prior probability distributions on the space of the number of risk seeking choices within any one frame for the four theories we are assessing. Clearly our four theories cover a wide range of prior beliefs over outcomes on this space. Predictive probabilities for risk seeking are shown in Figures A.3 through A.4 for four theorists who hold their beliefs with a similar degree of strength¹¹.

¹¹Our choice for reporting purposes is for a moderate strength equal to 20, but the same qualitative conclusions hold for all of the strengths we investigated.

Figure A.1: Prior probability¹² on number of risk seeking choices within each frame

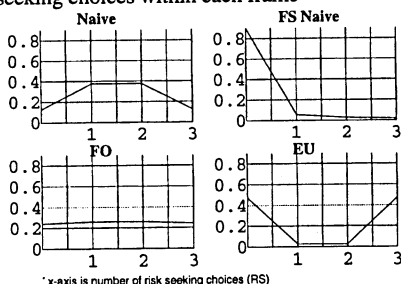


Figure A.2 Predictive probabilities for risk seeking : **prospect frame**

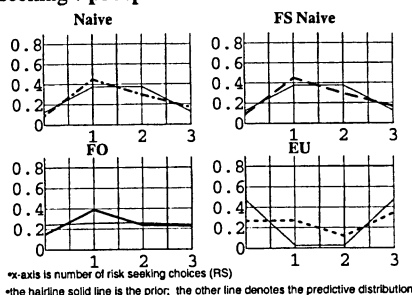


Figure A.3 Predictive probabilities for risk seeking : **MPS transparent frame**

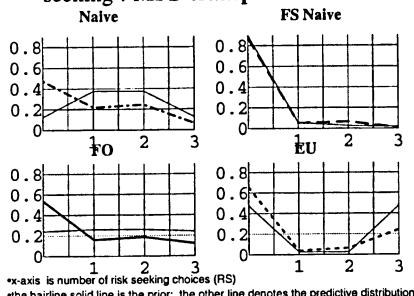
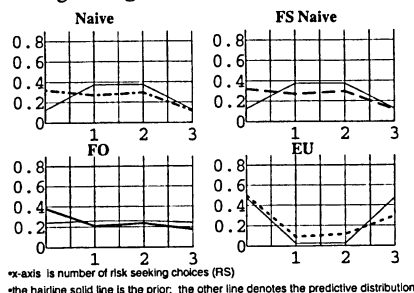


Figure A.4 Predictive probabilities for risk seeking : **integrated frame**



In the *prospect frame* predictive probabilities for no risk seeking are relatively low, 10% to 25%, depending on particular theoretical beliefs (see Figure A.2). Notice from Figure A.2 that the experimental data induces theorists with FO or Naive type priors to only slightly revise their prior probability assessments, giving a bit less weight to there being all risk averse behaviour (RS=0) and a bit more to making 1 risk seeking choice out of 3 possible in the experiment. A comparably committed theorist with EU priors on the other hand has to make a more substantial revision of their prior beliefs about these two values of risk seeking. These results are consistent with the earlier KMB experiment (and others in the literature) in this respect : *when alternatives are presented as*

¹²The prior predictive probabilities on number of risk seeking choices (RS) within a frame are shown in the table below. The prior predictive distribution for FS Naive and Naive theories are identical in all frames except the MPS frame.

	RS=0	RS=1	RS=2	RS=3
EU	0.474	0.026	0.026	0.474
FO	0.241	0.259	0.259	0.241
Naive	0.125	0.375	0.375	0.125
FS Naive	0.900	0.054	0.027	0.018

simple prospects the chances of observing risk seeking in the loss domain are assessed as high (for a wide variety of prior beliefs).

By way of contrast, however, when alternatives are presented in the MPS transparent frame (Figure A.3) predictive probabilities for no risk seeking are relatively high, from 50% to 90%, depending on particular theoretical beliefs. Notice that experimental data induces the FO, EU and Naive theories to each make a substantial upwards revision to their prior probability assessments of observing pure risk averting behaviour ($RS=0$). The frame sensitive version of the Naive theory (FS Naive) essentially maintains the 90% chance it initially assigned to all risk averting in this frame.

In the integrated frame, where frame distortions are explained and made obvious and where information acquisition and processing costs for translating between frames are low, the chances of no risk seeking behaviour are between 32% and 50%, depending on particular theoretical beliefs. These figures are lower than in the MPS frame but higher than in the prospect frame. Figure A.4 plots the relevant prior and predictive probabilities. As for the MPS transparent frame, the experimental data induces all theories except EU to each make substantial upwards revision to the probability of observing pure risk averting behaviour ($RS=0$) relative to the prior predictions¹³. Simply put, *risk seeking in the loss domain becomes less likely, and significantly so, when the nature of the alternatives as mean preserving spreads is made transparent using either the MPS transparent frame or the Integrated frame.*

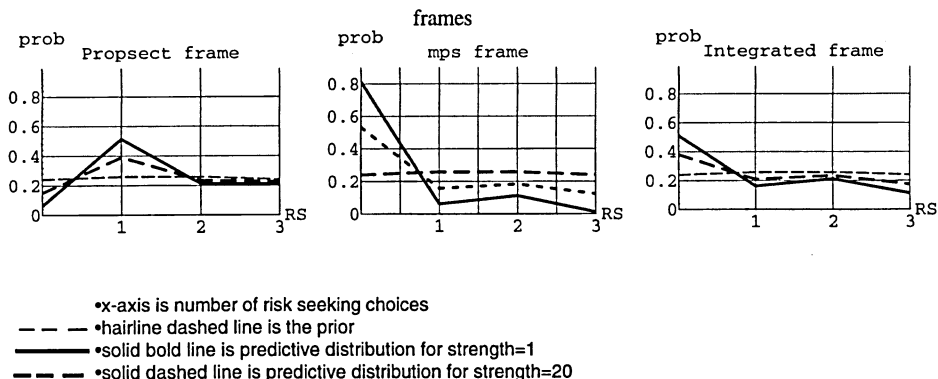
Table A.3 summarises this analysis by presenting the prior and predictive probabilities for risk seeking (the event $RS \geq 1$) in each frame for each of the four theories. The two GEUT theories start out by asserting frame invariance for their predictions about risk seeking, but after coherent revision via Bayes rule the message is clear - predictive probabilities about risk seeking vary significantly across frames. Figure A.5 illustrates the point more dramatically for FO versions of GEUT. It is apparent from the graph that learning from the data via Bayes rule leads to two qualitatively distinct predictive probability distributions - one for the prospect frame and another for the MPS and Integrated frames.

¹³EU initially assigns a high chance to this event, so the only substantial revision for EU is downward on consistent risk seeking behaviour ($RS=3$).

Table A.3 Prior and Predictive probabilities for risk seeking ($RS \geq 1$) in the loss domain, by frame and theory type (*priors in small font*)

	EU		FO		Naive		FS Naive	
Prospect Frame	53%	73%	76%	85%	88%	91%	88%	91%
Integrated Frame	53%	50%	76%	62%	88%	68%	88%	68%
MPS transparent Frame	53%	35%	76%	47%	88%	53%	10%	13%

Figure A.5: FO predictive probabilities on number of risk seeking choices for three strengths, all 3



Both the histograms and the analysis of predictive probabilities viewing each frame marginally strongly suggest that conventional experimental results (such as KMB) overstate the case¹⁴ for risk seeking in the loss domain when choosing between mean preserving spreads. The problem is that these experiments choose a frame for presenting their questions (the prospect frame) that, *relative to other frames*, masks important information about the alternatives, namely that they are mean preserving spreads of one another. In frames where this relationship is made more transparent - i.e. when relative information acquisition and processing costs of detecting such significant relationships between alternatives are low - the chances assessed for risk seeking in the face of losses decreases substantially, across a wide range of theoretical beliefs.

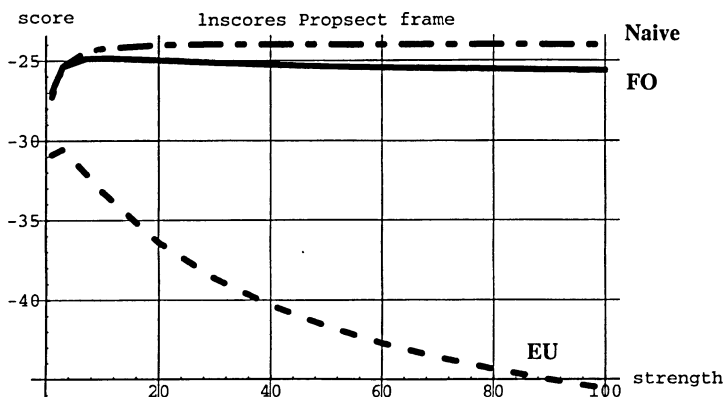
¹⁴Measured by the coherent posterior inferences they suggest for risk seeking in the loss domain.

III.B Single Frame Analysis: Log Scores for Predicting Risk Seeking

We now ask how predictable risk seeking over such gambles is viewing each frame on its own (i.e. marginally). In particular the question we want to ask in this section is whether risk seeking within a given frame is any more predictable by GEUT theories than "by chance"?

Figure B.1 presents the log scores for the predictions of various theories about the number of risk seeking choices in *the prospect frame* for a range of strength levels¹⁵. Using this scoring rule to assess relative predictive power of alternative theories, a simple, strongly held naive theory scores better at predicting the number of risk seeking choices in the prospect frame than either version of GEUT, FO or EU. GEUT simply does not predict choices between mean preserving spreads in the prospect frame as well as a uniform prior updated by Bayes rule (Laplace's Law of succession). It is encouraging, however, that the scores for FO do not decline significantly with increases in strength so that, unlike the case for EU, being strongly committed to FO does not decrease predictive power of that theory in the prospect frame.

Figure B.1 Log scores for predicting the number of risk seeking choices in the prospect frame



This inference appears to be contrary to KMB's claim that

...a little under half the responses violate expected utility theory, and of these, 71.4% of the deviations correspond to fanning out rather than fanning in, *far more than one would expect on the basis of chance factors alone.* (Kagel et al (1991) p918 italics added)

¹⁵As a guide to interpreting the magnitudes of the scores, take the natural log of the ratio of the total score to the number of observations (here 19). This yields a fraction which can be interpreted as the average probability of a successful prediction. For example, a score of -25 would have resulted from predicting what actually occurred on the 19 trials with probability 25% on average. The corresponding probability for a score of -60 is 4%. (the order of magnitudes of scores for joint frame predictions, eg Figure D.1)

The null hypothesis that KMB actually reject is that choices in these types of questions are essentially based on Expected Utility theory and that errors from the patterns predicted by Expected Utility theory on pairs of questions (not all questions simultaneously) are identically and independently distributed with probability 1/2. In contrast, our EU hypothesis is predicting the **total** number of risk seeking choices over three questions in three frames and is not merely expressing beliefs about errors over **pairs** of questions (in any one frame). Moreover, errors in the sense of deviations from EU predictions are viewed exchangeably in our theories, not independently¹⁶. However, the KMB null hypothesis is similar in spirit¹⁷ to the EU hypothesis we analyse (90% chance on predicted patterns of expected utility theory) with a strength approaching infinity, since such beliefs imply predictive probabilities for errors that are virtually unaffected by past data, i.e. independence. As the curve for EU in Figure B.1 suggests however, such theories will score very poorly indeed, much worse than any of the theories we examine.

In effect our analysis agrees with KMB, namely that the Fanning Out version of GEUT does predict better than theories based on independent (extremely strongly held) "chances", but it also goes beyond theirs, comparing the predictive power of GEUT with other theories about "chance" where exchangeability, not independence, is assumed. In essence we replace the KMB null hypothesis of an independent Bernoulli process generating the data with an assertion¹⁸ that scientific beliefs about whatever process is generating the data can best be predicted using Equation (1), an exchangeable Polya stochastic process, for a suitable choice of parameters. In this other sense of "chance" the FO version of GEUT does not predict as well as a simple theory of errors based on chance factors alone in the Prospect frame.

In the MPS frame matters are reversed however (see Figure B.2). Ignore the curve for FS Naive for the moment and focus on the curves for the log scores of the other 3 theories. The major points to take from Figure B2 are that, in the MPS frame,

- weakly held versions of GEUT both predict better than by "chance" , but strongly held versions of GEUT predict worse than by "chance"
- weakly held FO predicts better than EU but for more strongly held theories EU predicts better than FO.
- weakly held versions of GEUT predict better than strongly held versions of GEUT¹⁹.

¹⁶See Lad(1993) for a lucid discussion of the difference between regarding events exchangeably and regarding these events independently in the context of scientific investigation. Exchangeability is also well explained in Diaconis(1977)

¹⁷A more exact comparison would involve scoring theories on the patterns of errors from Expected Utility theory on pairs of questions, frame by frame. In our preliminary investigations of this issue the hypothesis of independent errors distributed 50/50 over the two possible error patterns scored abysmally low, but the Naive Laplacean law of succession still predicts better than GEUT. This analysis is still only preliminary however.

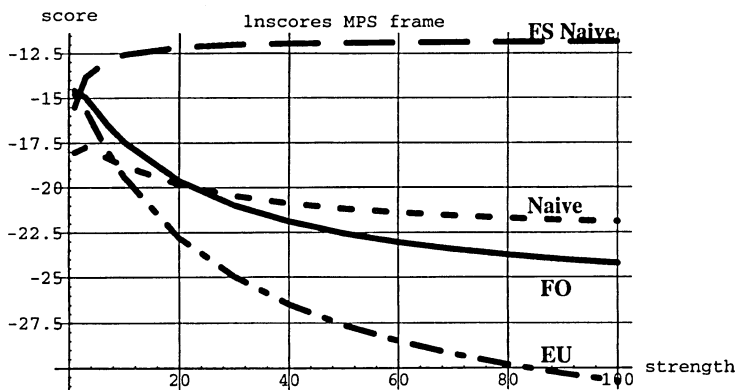
¹⁸A philosophical note here is a propos. Chance in our investigation is an assertion made by someone that is coherent (in accord with the "laws" of probability) about the quantitative extent of their uncertainty concerning what is going to be happening in unknown (to that person) experiments. Classical hypothesis testing is based on the view that chance is a property of reality where reality is ultimately determined by some true but unknown data generating mechanism.

¹⁹The last point is important to note because many people feel that unless there is "enough data" one can't really say anything sensible (statistically "significant") about how good or bad a theory is. From the subjectivist viewpoint we are

In the MPS frame there is persistent risk aversion (see the histograms in Table A.2). Both GEUT theories assign high prior probabilities to this event, and so receive high scores. But there is also enough variation in observed choices so that holding steadfastly to the theory's initial predicted patterns rather than revising predictions quickly in line with the changing data leads to poor aggregate predictive performance. Strongly held GEUT based beliefs simply are not being revised fast enough to be able to successfully predict the actual, changing pattern of risk seeking behaviour in the data.

For comparison purposes Figure B.2 also plots the scores of the naive theory sensitive to the simple framing effect in the MPS transparent frame. FS Naive clearly does better at predicting outcomes within the MPS transparent frame than either version of GEUT. It is also the only theory (of those we examine) where strongly held beliefs (here 90% on pure risk aversion) score better than weakly held theories. Also, in this frame and with the FS Naive theory, one achieves better predictive performance by holding steadfastly to the theory's predicted patterns rather than modifying beliefs rapidly as experimental evidence accumulates. In short, in the MPS frame, the best predictions come from using a strongly held simple theory that incorporates the obvious induced framing effect rather than relying on GEUT in either form.

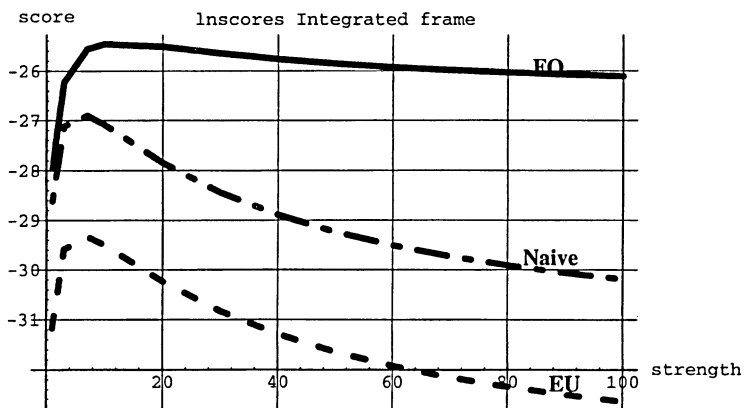
Figure B.2 Log scores for predicting the number of risk seeking choices in the MPS transparent frame



adopting in this paper such an attitude amounts to holding one's beliefs strongly and, as Figure B.2 shows, strongly held beliefs simply do not predict as well as weakly held beliefs.

Reassuringly, GEUT can claim a predictive success over naive chance theories in predicting the amount of risk seeking within the Integrated frame. This is evident from Figure B.3 where the highest score achieved is with the FO version of GEUT held with a moderate (strength=20) degree of conviction. Simply put, when information processing costs of detecting mean preserving spreads in sets of gambles are low, the Fanning Out version of GEUT does indeed predict the amount of risk seeking better than by chance. Once again it is encouraging that the scores for strongly held FO convictions do not tail off rapidly (unlike the case for EU). Moreover weakly held versions of FO, where the data are allowed to "speak for themselves" (in accord with Bayes law) do not score particularly well in predicting the number of risk seeking choices in the integrated frame. A moderately held FO theory with a little bit of data is better than just the data, but holding too tenaciously to the theory's predicted patterns decreases predictive performance.

Figure B.3 Log scores for predicting the number of risk seeking choices in the integrated frame



III.C *Joint Frame Analysis: Predictive Probabilities for Risk Seeking*

In only one frame out of three (the low transaction cost integrated frame) have we seen that GEUT theories predict better than by chance, suitably interpreted. These predictions have, however, all been marginal, rather than joint. An interesting question to ask is whether people are consistently risk averse or risk seeking, across several frames for the same decision situation. Does a frame change, holding the underlying binary decision constant, induce people to switch their risk attitude? In particular is risk seeking in the prospect frame (a relatively high information processing cost frame) correlated with risk seeking in either the integrated or mps transparent frames (the low information processing cost frames)? A naturally related question is whether theories like GEUT can predict risk attitudes across frames and across a number of decision situations²⁰ better or worse than predicting by chance.

Table C.1 shows the experimental data for 3 sets of pairs of frames. An entry in row i column j of the table for a particular frame pair is the count of the number of individuals in the experiment who made i risk seeking choices in the row frame and j risk seeking choices in the column frame. The main points to take from these histograms are that:

- In pairs of frames involving the MPS transparent frame the histograms are markedly asymmetric, with a dominant number of choices along the 0 risk seeking choices axis in the MPS frame.
- Of the 18 who made at least 1 risk seeking choice in the prospect frame, 15 switch to pure risk averting behaviour in the MPS frame.
- The main diagonal in all 3 frame pairings is noticeably small, indicating very little interframe consistency in choice.
- Except in the MPS/integrated frame pairing there is a conspicuous absence of clustering of observations in any of the main and cross diagonal "corners" of the histogram, clustering which would indicate strong positive or negative correlation between the number of risk seeking choices in different frames
- The prospect/integrated pairing indicates that there is some persistence of risk seeking behaviour across frames as 9 out of 19 had at least one ($RS \geq 1$) risk seeking choice jointly in both frames, however another 9 out of 19 switched to pure risk averting behaviour in the integrated frame after showing some risk seeking behaviour in the prospect frame.

²⁰Current statistical practice, exemplified in KMB, is simply to do a series of repeated marginal hypothesis tests on pairs of questions. Our statistical methods permit us to examine predictions on three questions and in three frames simultaneously.

Table C.1 Histograms of number of risk seeking choices in pairs of frames

		Prospect frame		# risk seeking choices	
MPS frame		0	1	2	3
# risk seeking choices	0	1	8	3	4
	1	0	1	0	0
	2	0	1	1	0
	3	0	0	0	0

		Prospect frame		# risk seeking choices	
Integrated frame		0	1	2	3
# risk seeking choices	0	1	4	2	3
	1	0	3	0	0
	2	0	3	1	0
	3	0	0	1	1

		MPS frame		# risk seeking choices	
Integrated frame		0	1	2	3
# risk seeking choices	0	10	0	0	0
	1	3	0	0	0
	2	2	1	1	0
	3	1	0	1	0

What predictive inferences can coherently be drawn from this data? Figure C.2 and C.3 illustrate relevant aspects of the theories on the space of two-frame risk seeking choices. Figure C.2 shows the prior probability distributions on the space of the number of risk seeking choices within pairs of frame for the four theories we are assessing²¹. Our four theories cover a wide range of prior beliefs over outcomes on this space. Notice that both GEUT theories are frame invariant, predicting diagonal elements (interframe consistency in risk seeking) with relatively high probability: EU either all risk seeking or all risk averse, FO about equal probability for all main diagonals. Only FS Naive attributes any substantial weight to off diagonal elements, namely RS=0 in the MPS frame.

Figure C.3 plots the conditional expectation (regression lines) of the number of risk seeking choices in one frame given the number of risk seeking choices in the prospect frame²² for the four prior distributions in C.2. The GEUT theories are frame invariant and therefore predict that, by and large²³, risk seeking or risk averse choices cluster together across frames. That is, people who take a risk

²¹The prior predictive distribution for FS Naive and Naive theories are identical in all frame pairs except those involving the MPS transparent frame. The actual conditional expectations for each theory, given the number of risk seeking choices (RS) in the Prospect frame are:

	EU	FO	Naive	FS Naive
RS=0	0.0386	0.063	1.5	0.163
RS=1	0.89	1.035	1.5	0.163
RS=2	2.109	1.964	1.5	0.163
RS=3	2.961	2.937	1.5	0.163

²²Actually, given any other frame, not just the prospect frame, for all theories except FS Naive.

²³After allowing for the small probabilities assigned to deviations from the predicted patterns of each GEUT theory.

seeking attitude x times in the prospect frame are expected (by someone holding a GEUT theory) to take about x risk seeking choices in another frame. By contrast, the regression lines for both Naive theories expresses the view that risk attitudes in one frame are completely uncorrelated with risk attitudes in any other frame. Moreover, FS Naive asserts that no matter whether someone makes 0, 1, 2, or 3 risk seeking choices in the prospect frame they are expected to switch to being risk averters (expected value of RS, the number of risk seeking choices, is 0.16) in the MPS transparent frame.

Figure C.2 Prior distributions for joint (two frame) risk seeking

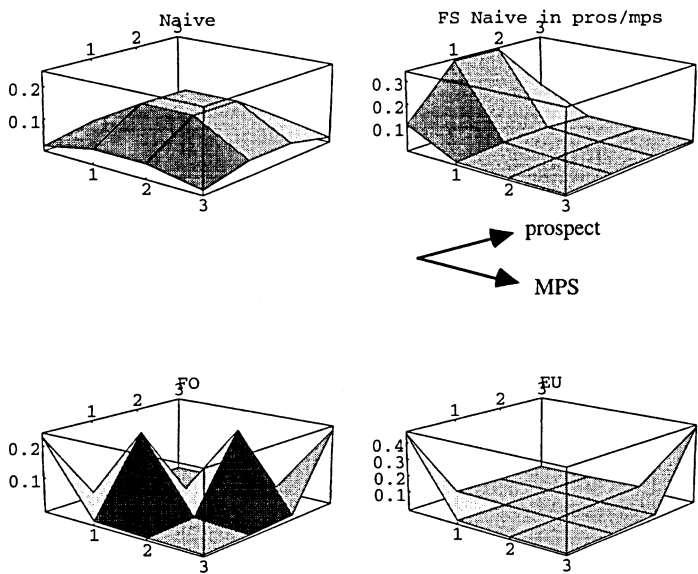
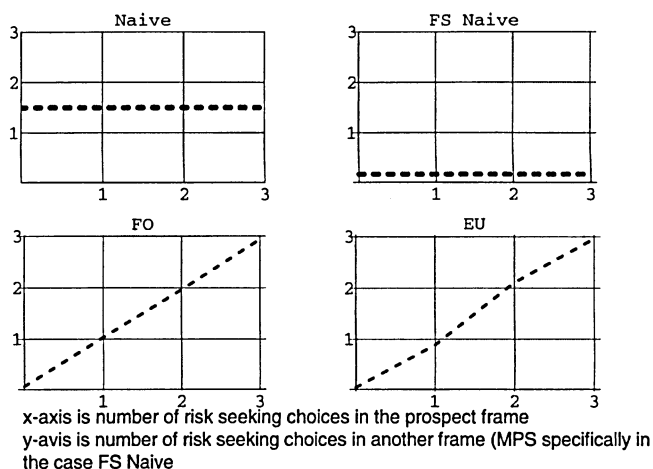


Figure C.3 Prior conditional expectations for risk seeking choices in one frame given the number of risk seeking choices in the prospect frame²⁴.



²⁴The prior predictive distribution for FS Naive and Naive theories are identical in all frame pairs except those involving the MPS transparent frame. The actual conditional expectations for each theory are:

EU	FO	Naive	FS Naive
0.0386	0.063	1.5	0.163
0.89	1.035	1.5	0.163
2.109	1.964	1.5	0.163
2.961	2.937	1.5	0.163

Figure C.4 shows the predictive distributions for each of the four theories (strength=20) in the two frame pairs that involve the prospect frame. The only pattern that stands out is the FS Naive theory. FS Naive concentrated most of its prior probability along the RS=0 axis in the MPS frame and continues to do so after coherent revision. There are no other obvious patterns except that the predictive probabilities of each theory, both GEUT and Naive, are undergoing substantial revision by the experimental data. This is not surprising since the data histograms in Figure C.1 bear little resemblance to the prior distributions of GEUT and Naive. The prior expectations of these frame insensitive theories simply do not fit the data viewed jointly rather than marginally.

In section III, Table A.3, we discovered that both GEUT theories and Naive increased their predictive probabilities for at least some ($RS \geq 1$) risk seeking within each frame viewed marginally, particularly so for FO which suggested an 85% predictive probability of this event for strength=20. However, viewed jointly, the predictive probability of joint risk seeking ($RS \geq 1$) in two frames simultaneously is greatly reduced, as demonstrated in Table C.2. This reduction relative to prior prediction is not unexpected for frame pairs involving the MPS frame, but it is somewhat surprising in the Prospect/Integrated frames. Comparing the figures in Table C.2 and C.3 the chances of finding simultaneous risk seeking in the prospect frame and the integrated frame are only around 49% to 70% as compared to the 73% to 90% in the prospect frame alone (Table A.3).

Table C.2 Prior and Predictive (st=1) probabilities for risk seeking ($RS \geq 1$) in the loss domain, by frame pairs and theory type (*priors in small font*)

	EU		FO		Naive		FS Naive	
Prospect/MPS Frames	51%	34%	75%	46%	77%	47%	9%	12%
Prospect/Integrated Frames	51%	49%	75%	70%	77%	62%	77%	62%
MPS/Integrated Frames	51%	34%	75%	46%	77%	47%	9%	12%

This finding is important. The integrated frame is a decision situation in which the costs of information processing are extremely low relative to those costs in the prospect frame. We expect people to make relatively sensible decisions under circumstances where they can see clearly, and with little cost, what is going on. But the literature, in its recognition of the likelihood (high predictive probability in our terminology) of some risk seeking in the loss domain, overstates the case for risk seeking by focusing marginally on the prospect frame. For example, KMB's claim that the results of their experiment on risk seeking in the loss domain can stand as stylised facts around which to fit new theories is suspect at best. This recognition of a marginally viewed fact ignores what individuals will tend to do when they are (more) fully informed about the nature of the decision situation they face. And the posterior chances of them doing some risk seeking in both situations simultaneously are around the same as tossing a fair coin, 50%, hardly enough for a stylised fact.

This conclusion is reinforced by examining the conditional expectations of Figure C.5. The solid line indicates the predictive conditional expectation of the number of risk seeking choices in the frame "y" given the number of risk seeking choices in frame "x" for a strength=1. The dash-dot line gives the corresponding information for strength=20. The dashed line is the prior conditional expectation .

In the prospect/MPS frame pairing (Fig. C.5a) the number of risk seeking choices in the prospect frame is not a good indicator of the number of risk seeking choices in the MPS frame. In fact, there is a slight negative correlation in all weakly held theories²⁵ in the increase from 2 to 3 risk seeking choices in the prospect frame. In the prospect/integrated frame pairing (Fig. C.5b) the conditional expectations are revised downwards, and particularly strongly when the number of risk seeking choices in the prospect frame is 3. Evidently someone who is entirely risk seeking in the prospect frame is not likely to be so in the integrated frame. Again there is a slight negative correlation for all weakly held theories. In the MPS integrated frame pairing (Fig. C.5c) the Round 1 choices in the MPS frame are used as the conditioning variable (x-axis). The figures indicate that given x risk seeking choices in the MPS frame, slightly more than x risk seeking choices are expected in the integrated frame²⁶. These regressions suggest strongly that the extent of risk seeking in the prospect frame is not well correlated with risk seeking in the other two frames. Moreover, the number of risk seeking choices in the prospect frame tends to overstate the number of risk seeking choices one would expect in the other two frames.

²⁵The next section establishes that weakly held theories do better at predicting simultaneous choices across frames than strongly held theories

²⁶The lower triangular array of the data histogram in Table C.1 confirms this pattern.

Figure C.4 Predictive distributions for joint (two frame) risk seeking:

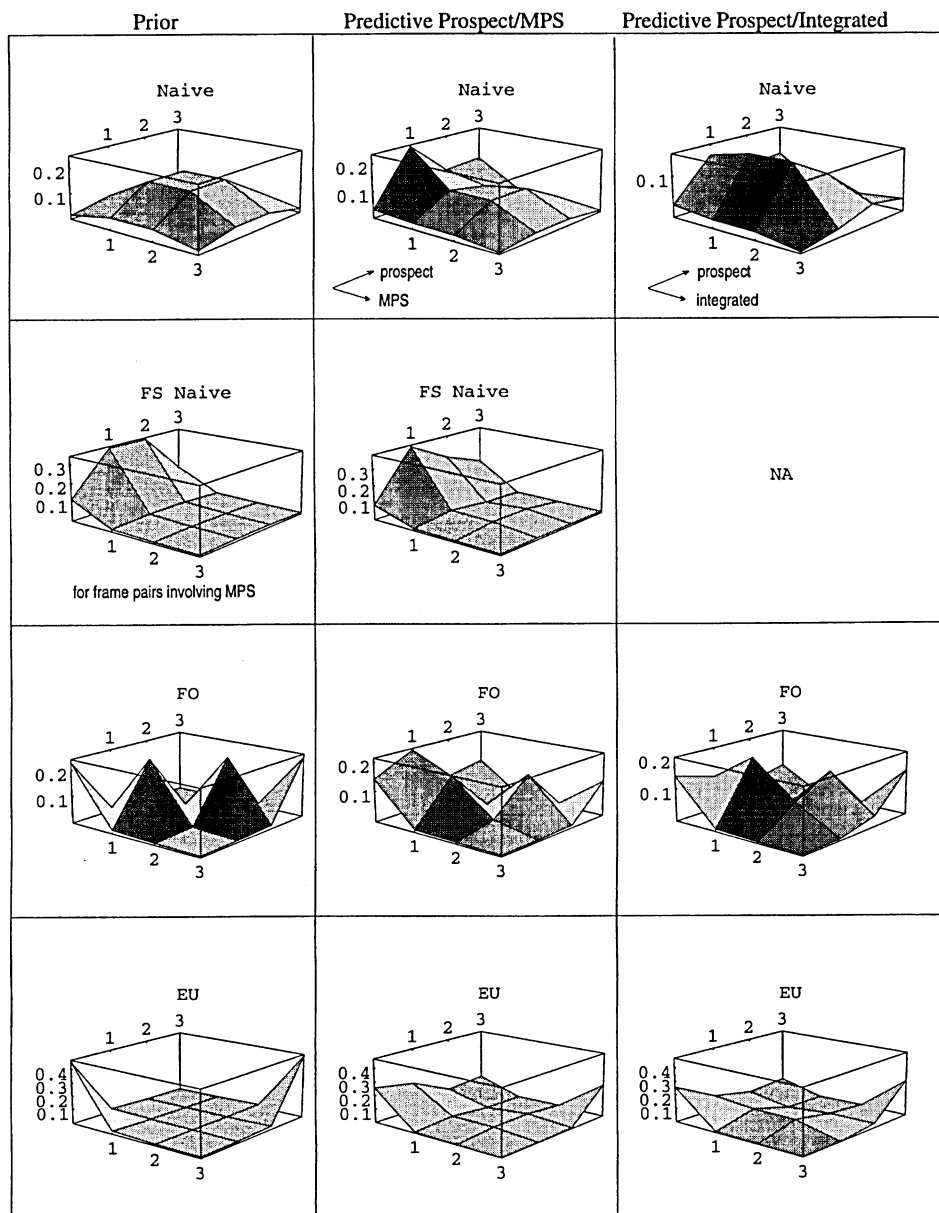


Figure C.5a Conditional expectations for RS in the MPS frame given RS in the prospect frame

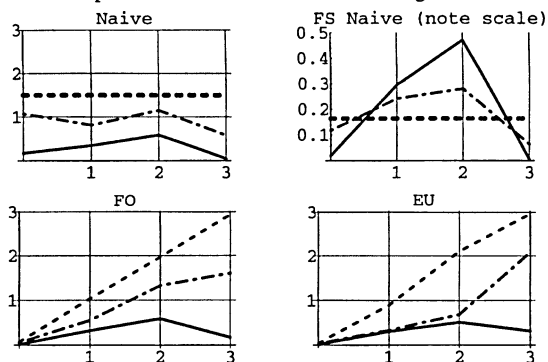


Figure C.5b Conditional expectations for RS in the Integrated frame given RS in the prospect frame

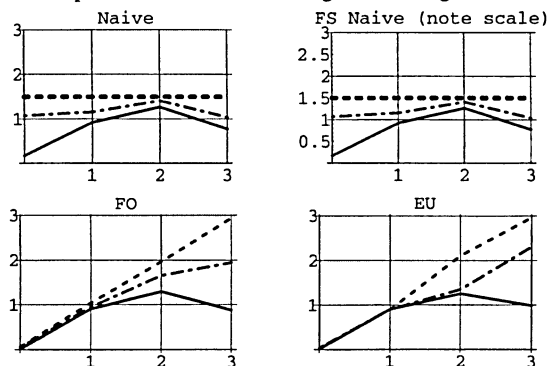
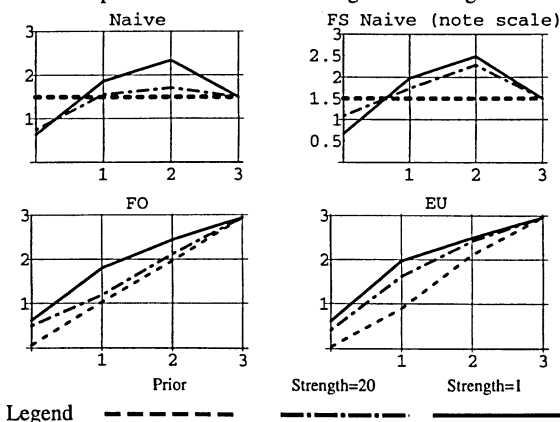


Figure C.5c Conditional expectations for RS in the Integrated frame given RS in the MPS frame



III.D Joint Frame Analysis: Log Scores for predictions of Risk Seeking

In view of the relatively poor predictive performance of GEUT within single frames, it will come as no surprise that the theories perform no better than simple Naive theories when trying to predict jointly across frames. In fact, the absolute scores predicting simultaneously across frames are much poorer than trying to predict within any one frame viewed marginally. GEUT theories predict consistency (frame invariance) in risk attitudes across frames yet it is not the kind of consistency the people in the experiment are generating (see the data in the histograms of Table C.1). Figures D.1 through D.3 present the results of the log scores for the four theories in three frame pairings.

The best overall score was for the frame sensitive Naive theory with a moderate strength (10) in the prospect/MPS frame, and this same theory predicts better than the others (at some strength) in two out of three frame pairings. However, in the MPS/integrated frame (Figure D.3) the simple naive theory has the best scores, so that frame invariance is not necessarily a defect in the predictive performance of a theory. Any way you look at it, joint choices across frames are more predictable by "chance" than by GEUT in either form. This result is disturbing since one might have hope that the predictive superiority of GEUT(FO) in the integrated frame viewed marginally (see Figure B.3) might have more than compensated for its predictive inferiority in the MPS frame.

In general, weakly held theories (down to strength=10) predict better than strongly held theories when trying to predict risk seeking behaviour in two frames simultaneously. To predict well it is necessary to learn rapidly from the data in these joint frame situations, unlike the case in the single frame situations where strongly held GEUT FO predicts almost as well as weakly held versions.

Figure D.1 Scores in the prospect/integrated frame pair

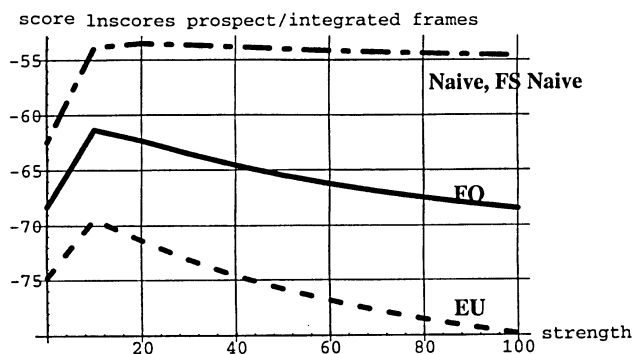


Figure D.2 Scores in the prospect/MPS frame pair

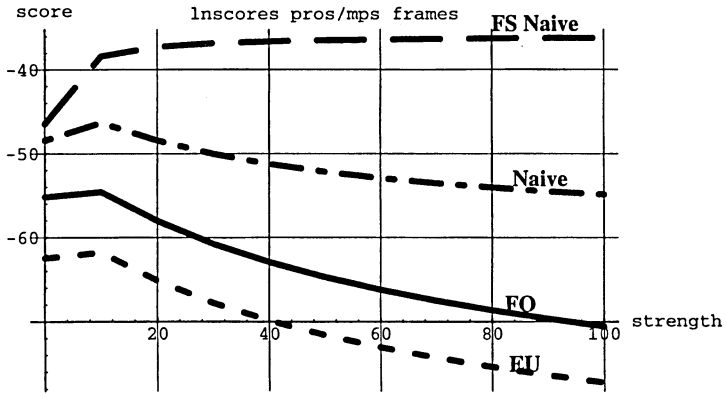
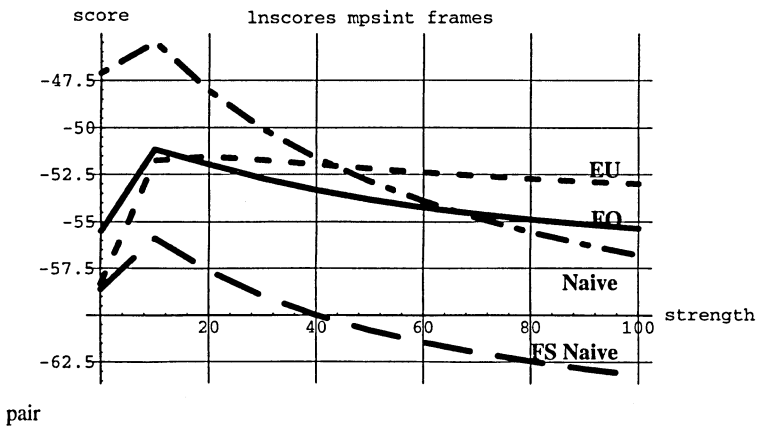


Figure D.3 Scores in the integrated/MPS frame pairing



IV Summary

Our experiment and statistical analysis has shown that the extent and likelihood of risk seeking in the face of losses where the gambles are mean preserving spreads of one another is markedly frame dependent. The three frames we have used vary significantly in the information they make readily available to subjects about the alternatives in the decision situations. The standard way of presenting gambles in the literature as lists of prizes and corresponding probabilities creates a framing effect that leads to larger amounts of risk seeking than one would expect if the decision situations were framed in ways that lowered the costs for individuals of acquiring and processing information. Risk attitudes in such a low cost-of-information frames were found to be uncorrelated with risk attitudes in the standard prospect frame for a wide range of theoretical beliefs.

Moreover, GEUT theories generally do not have any more predictive power than a naive uniform prior updated by Bayes rule (Laplace's law of succession). This is especially true when trying to predict risk attitudes simultaneously across several frames - there is just too much interframe inconsistency for a frame invariant theory like GEUT to predict well. The revision of frame invariant beliefs in GEUT's prior probabilities towards frame dependent predictive probabilities in the joint frame analysis sends a clear message: the observational data is suggesting that GEUT needs revision to take account of framing effects if it is going to be able to predict simultaneous choices in several frames better than by chance (suitably interpreted).

Even within frames, GEUT does not predict systematically any better, and is usually worse, than by chance. A notable exception to the above pattern is that the FO version of GEUT predicted relatively well in the integrated frame where information acquisition and processing costs are low, even though it's predictive performance in the prospect frame is poor. Moreover, that predictive power tapered off only slowly with strength. One can hold a FO theory with conviction and still predict better than by chance (viewing frames marginally) when information costs are low. This is encouraging. Moreover it makes one slightly suspicious of claims that GEUT in it's FO version is not good at predicting the majority of deviations from EU (KMB,919), since those claims are based on experiments conducted only in the prospect frame where information costs are comparatively high. Besides, it is relative, not absolute performance that counts, in predicting²⁷, a question KMB and most other experimentalists sidestep.

The cynical amongst us might draw the conclusion that it is the experiment itself, namely the hypothetical character of the gambles, not GEUT theory, that is responsible for these results. Perhaps the statistical analysis is only confirming what many already believe, namely that experiments based on hypothetical decisions really don't tell us much - i.e. our theories won't predict very well,

²⁷This paper has only compared GEUT's predictive ability with a naive chance theory. Our current research involves a comparison of GEUT with Prospect Theory using the same experimental data and statistical methodology..

probably no better than by chance, in such situations. We are currently running the experiment again with individuals playing real gambles from an endowment of \$50 to check this hypothesis out.

Appendix:

Whatever one thinks about the credibility of expected utility theory, fanning out, risk aversion, framing, etc., we presume that almost everyone would regard the sequence of observations from an experiment like ours involving N subjects, $X_N = \{X_1, X_2, \dots, X_N\}$, *exchangeably*. Exchangeability is a restriction on one's personal probability assessment of sequences of possible experimental results $X_N = \{X_1, X_2, \dots, X_N\}$. It means that, if a particular sequence of experimental results $X'_N = \{X'_1, X'_2, \dots, X'_N\}$ yields a histogram $s_j(X'_N)$ $j=1, 2, \dots, R$, where R is the (finite) number of possible values each X'_j can take, one would assert equal probabilities to *any* individual sequence of experimental results $\{X_1, X_2, \dots, X_N\}$ yielding the same histogram. Exchangeability seems eminently sensible in the context of our experiment where there is no information on individual subjects that can be correlated with their individual responses.

Exchangeability has a very powerful implications for coherent personal probability assessments for possible data sequences $X_N = \{X_1, X_2, \dots, X_N\}$. According to de Finetti's representation theorem, Lad (1993, Ch 5, pp 62-64)²⁸, if we regard the sequence X_1, X_2, \dots, X_N as exchangeable and if our subjective probability distribution is infinitely exchangeably extendible then :

A• The histogram $s_1^*, s_2^*, \dots, s_R^*$ corresponding to the observed sequence X_1, X_2, \dots, X_m is a sufficient statistic for any coherent inference about the remaining $N-m$ quantities in the sequence X_1, X_2, \dots, X_N .

B• One's personal probability distribution for an observable sequence $X_n = \{X_1, X_2, \dots, X_n\}$, for any choice of n observations from N , can be written as a mixture multinomial:

$$(A1) \quad P[X_1, X_2, \dots, X_n] = \int_0^1 \dots \int_0^1 \prod_{j=1}^R \theta_j^{s_j(X_n)} d_1 \dots d_{R-1} M(\theta_1 \dots \theta_{R-1})$$

where $s_j(X_n)$ $j=1, 2, \dots, R$, is the histogram for X_n , $(\theta_1 \dots \theta_{R-1})$ is a vector of parameters and $M((\theta_1 \dots \theta_{R-1}))$ is a mixing distribution. The parameters θ_j in equation (1) are the imagined "long run" proportions of observations that fall in category j in an infinitely extended sequence of observations X_N .

C• Using the natural conjugate form of mixing function for (A1), a Dirichlet distribution with parameters $(\alpha_1, \alpha_2, \dots, \alpha_R)$, the conditional distribution of the category sums s_1, s_2, \dots, s_R for the remaining $N-m$ observations from X_N , given a histogram $s_1^*, s_2^*, \dots, s_R^*$ of observations on m of them, is distributed $\text{Polya}(N-m, \alpha_1 + s_1^*, \alpha_2 + s_2^*, \dots, \alpha_R + s_R^*)$; i.e.

$$(A2) \quad P[s_1, s_2, \dots, s_R \mid s_1^*, s_2^*, \dots, s_R^*] = \frac{(N-m)!}{s_1! s_2! \dots s_R!} \frac{\prod_{j=1}^R \alpha_j + s_j^* \cdot \prod_{j=1}^R \Gamma[\alpha_j + s_j^* + s_j]}{\Gamma[(N-m) + \sum_{j=1}^R \alpha_j + s_j^*] \cdot \prod_{j=1}^R \Gamma[\alpha_j + s_j^*]}$$

While we presume that almost everyone will regard sequences of observations exchangeably, we are in no way implying that different people will make the same probability assessments for sequences of observations. Equations (A1) and (A2) permit us to distinguish between theoretical views that assess the probability of histograms of data differently through a choice of the mixing distribution M .

²⁸This theorem is discussed in Good (1975), Ch. 4 and Diaconis (1977)

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