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With Applications to the New Zealand Aggregate Consumption Function[1]

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ABSTRACT

This study addresses the theory and application of periodic integration and cointegration. A strategy for testing for periodic integration, originally proposed by Osborn *et al.* (1988) is further developed and, along with the Vector of Quarters test (Franses (1994)), is applied to aggregate New Zealand consumption and disposable income data. Consumption is found to be possibly periodically integrated while income appears to be seasonally integrated. Where appropriate, the bootstrap technique is used to simulate critical values for these tests. A representation for a bivariate periodic cointegration model, including its associated error correction mechanism, is derived. However, because of limited degrees of freedom, a parsimonious approximation is proposed. This is applied to the New Zealand aggregate consumption function and its results briefly compared to those from a non-periodic cointegration model. Neither model is found to be superior in all respects.

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I. INTRODUCTION

Incorporating seasonality into the cointegration framework has been a logical extension although it has been achieved in two quite different ways. Seasonal integration/cointegration (see Dickey *et al.* (1984), Hylleberg *et al.* (1990), Engle *et al.* (1993)) allows for unit roots at seasonal frequencies as well as at the conventional zero frequency. In contrast, Osborn *et al.* (1988) have introduced the concept of periodic integration, which generalises the orthodox analysis by allowing for periodically varying coefficients. It is the development and application of this idea that forms the main focus of this paper.

As is the case in traditional integration/cointegration analysis, we must begin by considering the univariate properties of the data. In Section II, the concept of periodic integration is addressed, and some methods of testing for it are explored.

Once the univariate attributes of the series have been determined, the next step is to analyse the relationship between them and test for cointegration. Section III considers periodic cointegration and its application in modelling the aggregate New Zealand consumption function. A representation for a periodic cointegration model is developed and its results compared with those obtained from a non-periodic model. A conclusion is provided in Section IV.

II. PERIODIC INTEGRATION

A simple quarterly integrated process can be written as

$$(1-L^4)x_t = \psi_q + \varepsilon_t \tag{1}$$

where L is the lag operator and ψ_q a constant that can take different values each quarter.

Expanding the polynomial in (1) yields

$$(1-L)(1+L)(1+iL)(1-iL)x_t = \psi_q + \varepsilon_t \tag{2}$$

(see Hylleberg *et al.* (1990)). Thus, when the data is observed quarterly, a seasonally integrated process has four unit roots. One of these is a unit root the zero frequency, whilst the other three, -1, i, and -i, are termed the seasonal unit roots. The root -1 corresponds to the biannual cycle and the roots i, -i correspond to the quarter and three quarter frequencies. A process of this nature may be denoted by $x_t \sim I(1,1,1,1)$.

Now consider the periodic AR(1) model

$$x_t = \phi_q x_{t-1} + \psi_q + \varepsilon_t \tag{3}$$

where ϕ_q, ψ_q vary according to which quarter the observation is in. Here, the seasonality enters through the periodically varying coefficients rather than through a direct dependence of x_t on its annual lag x_{t-4}. Osborn (1987) shows that the process in (3) is stationary if $\left|\prod_{q=1}^{4} \phi_q\right| < 1$ (see also Franses (1994)). When this product is equal to one, the data is non-stationary: this case is termed a periodic unit root.

More formally, Osborn *et al.* (1988) consider

$$\begin{aligned} \delta_q x_t &= x_t - \phi_q x_{t-1} \\ &= (1-\phi_1 L)x_t \end{aligned} \tag{4}$$

and define x_t to be a periodically integrated of order one, or $x_t \sim PI(1)$, if and only if

(i) $\prod_{q=1}^{4} \phi_q = 1$, i.e. x_t is non-stationary

(ii) $\delta_q x_t$ is stationary.

The conventional I(1) process is a special case of periodic integration, occurring when all ϕ_q are equal to one.

Previous research by Osborn *et al.* (1988), Franses and Kloek (1991) and Franses (1994) has found that consumption in the U.K., Japan and Australia each follows a non-stationary process like (3).

TESTING AT SEASONAL FREQUENCIES

Hylleberg *et al.* (1990) have derived a test (herein referred to as HEGY), which is able to test for, and discriminate between, unit roots at the zero and seasonal frequencies. When the data is collected quarterly, it involves estimating the equation

$$\Delta_4 x_t = \pi_1 z_{1t-1} + \pi_2 z_{2t-1} + \pi_3 z_{3t-1} + \pi_3 z_{3t-2} + \psi_q + \sum_{i=1}^{p} \beta_i \Delta_4 x_{t-i} + u_t \qquad (5)$$

where $z_{1t} = (1 + L + L^2 + L^3)x_t$

$z_{2t} = -(1 - L + L^2 - L^3)x_t$

$z_{3t} = -(1 - L^2)x_t$

and test the hypotheses

H_0: $\pi_1 = 0$ vs H_A: $\pi_1 < 0$

H_0: $\pi_2 = 0$ vs H_A: $\pi_2 < 0$

H_0: $\pi_3 = \pi_4 = 0$ vs H_A: π_3 and/or $\pi_4 \neq 0$

These will indicate whether the series has statistically significant unit roots at the zero, biannual and ¼ and ¾ frequencies respectively.[1] The critical values for the HEGY test are tabulated in Hylleberg *et al.* (1990).

This test and the ADF test for a unit root at the zero frequency do not accommodate a periodically integrated process. However, if they are applied to data which is PI(1), the result obtained depends on the amount of variation in the periodic autoregressive parameters, the ϕ_q's. It is intuitively obvious that I(1) will be the conclusion when the ϕ_q's are all close to one. A Monte Carlo experiment by Franses (1994) considers the properties of the HEGY test when the deviations are moderately large. It is found that, even when ϕ_q ranges between 0.8 and 1.25, the test will still conclude I(1) with probability 0.75.[2] It is unlikely that an economic time series would possess parameters which varied by more than this, especially if the modelling is conducted using logarithms of the data.

To test explicitly for periodic integration, two tests are used: the Periodic OCSB test (Osborn *et al.* (1988)) and the Vector of Quarters test (Franses (1994)).

Test 1: Periodic OCSB[3]

Consider a quarterly series x_t, which can be decomposed into four series X_{qT}, where X_{qT} contains the observations of x_t that fall in quarter q of year T.

Firstly, the four equations

$$
\begin{aligned}
X_{1T} &= \psi_1 + \phi_1 X_{4T-1} + u_{1T} \\
X_{2T} &= \psi_2 + \phi_2 X_{1T} \quad + u_{2T} \\
X_{3T} &= \psi_3 + \phi_3 X_{2T} \quad + u_{3T} \\
X_{4T} &= \psi_4 + \phi_4 X_{3T} \quad + u_{4T}
\end{aligned} \qquad (6)
$$

are estimated by OLS and the residual standard deviation, $\hat{\sigma}_q$, is obtained for each quarter. Scaling the data by $1/\hat{\sigma}_q$ then has the effect of minimising any seasonal variations in the error variance, enabling OLS to be a more efficient estimator.

The full periodic AR(1) model is now

$$x_t^* = \sum_{q=1}^{4} \psi_q D_{qt}^* + \sum_{q=1}^{4} \phi_q D_{qt}^* x_{t-1} + \varepsilon_t \qquad t=1,2,\dots,n \qquad (7)$$

where - D_{qt}^* is a dummy variable, taking the value $1/\hat{\sigma}_q$ in quarter q and zero otherwise.

$$- x_t^* = \sum_{q=1}^{4} D_{qt}^* x_{t^*}$$

If x_t is found to be non-stationary then, by analogy to the ADF test, it would be prudent to estimate the following.

$$\Delta x_t^* = \sum_{q=1}^{4} \psi_q D_{qt}^* + \sum_{q=1}^{4} (\phi_q - 1) D_{qt}^* x_{t-1} + \varepsilon_t \qquad (8)$$

and test the hypothesis

$$H_0 : \phi_1\phi_2\phi_3\phi_4 = 1 \quad \text{vs} \quad H_A : \phi_1\phi_2\phi_3\phi_4 \neq 1.$$

If the null cannot be rejected, then we conclude that x_t has a periodic unit root.

In testing the null hypothesis of a unit root (at the zero frequency) Dickey and Fuller (1981) find some Monte Carlo evidence to support a likelihood ratio test (LR) over the Wald (W) and Lagrange Multiplier (LM) tests. Whilst this can only be indicative of the properties of a similar test in the periodic framework, it is at least enough to suggest that the LR test could be no worse than W and LM.

The Wald test also has some appeal due to the fact that it avoids the estimation of the restricted model, although the Wald test statistic is not independent of the way its restriction is written. For example, H_0 could be written as

$$H_0 : \phi_1\phi_2\phi_3 = \frac{1}{\phi_4}$$

yielding a different derivative and therefore a different value of the test statistic.[4] Consequently, $H_0 : \phi_1\phi_2\phi_3\phi_4 = 1$ has been consistently used in all applications and simulations of the Wald test.

Although both the LR and Wald test statistics have asymptotic distributions which are chi-squared under the null hypothesis, their finite sample distributions are likely to differ. Therefore, in order to obtain more accurate results from these tests, we have simulated their distributions using the bootstrap (see Efron (1979)).

If the LR and Wald tests yield a non-rejection of the null, implying that there is a periodic unit root, we must then test for the presence of other unit roots, both at the zero and seasonal frequencies. This is accomplished by estimating (8) under the restriction $\phi_1\phi_2\phi_3\phi_4 = 1$, and applying the ADF and HEGY tests to the residuals. It is also desirable to bootstrap the critical values for these tests since the prior estimation of the period model will alter their distribution.[5] If the null hypotheses of unit roots in the periodic residuals are rejected, then we accept that the series x_t is periodically integrated of order 1.

The conventional I(1) process is a special case of periodic integration. It is therefore possible to test whether x_t conforms to this special case using a conventional F-test on the unrestricted version of (8). The hypothesis is

$$H_0 : \phi_1 = \phi_2 = \phi_3 = \phi_4 = 1 \quad \text{vs} \quad H_A : \phi_q \neq 1 \text{ for some } q = 1,2,3,4.$$

The distribution of this statistic is also non-standard because of the unit root that is present under the null hypothesis and because of its dependence on the result of the previous W and LR tests. Consequently, the bootstrapping technique has been used to simulate its distribution.

Test 2: Vector of Quarters (VQ)

Consider the periodic AR(1) model $(1 - \phi_q L)x_t = \psi_q + u_t$ which has a vector of quarters representation

$$\Phi(L)X_T = \Psi + U_T \qquad T = 1,2,\dots,n/4 \tag{9}$$

where $X_T = [X_{1T}, X_{2T}, X_{3T}, X_{4T}]'$ and $\Phi(L) = \begin{bmatrix} 1 & 0 & 0 & -\phi_1 L \\ -\phi_2 & 1 & 0 & 0 \\ 0 & -\phi_3 & 1 & 0 \\ 0 & 0 & -\phi_4 & 1 \end{bmatrix}$.

U_T is a vector of error terms and Ψ a vector of constants. Following Franses (1994) we decompose $\Phi(L)$ as $\Phi(L) = A_0 - A_1 L$ so that (9) becomes $A_0 X_t = A_1 X_{T-1} + \Psi + U_T$

Rearranging and subtracting X_{T-1} from both sides yields

$$\Delta X_T = \Pi X_{T-1} + \Pi_0 + \Omega_T \tag{10}$$

where

$$\Pi = \left(A_0^{-1} A_1 - I_4\right)$$

$$= \begin{bmatrix} -1 & 0 & 0 & \phi_1 \\ 0 & -1 & 0 & \phi_1\phi_2 \\ 0 & 0 & -1 & \phi_1\phi_2\phi_3 \\ 0 & 0 & 0 & \phi_1\phi_2\phi_3\phi_4 - 1 \end{bmatrix}$$

If x_t is periodically integrated (including the I(1) special case) then, by definition, the system (10) will contain one unit root. This implies that the linear combinations of X_T that are non-stationary span a space of dimension one. Since the space of all linear combinations is four dimensional, there must exist a three dimensional space containing stationary linear combinations of X_T. This is the cointegration space and it has dimension equal to the rank of Π. From the above it can be seen that, when x_t is PI(1), Π contains a row of zero's and therefore has rank three; indicating the existence of three linearly independent cointegrating vectors.

Recall that the $\Delta = 1 - L$ filter, when applied to X_T, is equivalent to a fourth difference of the original series x_t, which implies that there are four unit roots in the system (i.e. it is seasonally integrated). However, the periodically integrated system in (10) contains only one unit root, so is therefore over-differenced. The three cointegrating vectors can be thought of as corrections for this over-differencing.

When x_t is seasonally integrated, and so has unit roots at the zero and all seasonal frequencies, ΔX_T will be stationary and Π will be the null matrix[6]. This implies that there is no cointegration between the seasons, with each quarter evolving independently of the others, giving rise to the phenomenon known as "summer becoming winter" (see Hylleberg *et al.* (1990)). However, if the components of X_T are stationary, then the cointegration space has dimension four and Π is of full rank. In this case, there are no unit roots in the system and the process is stationary.

Thus determining the rank of Π will give insights into the integration properties of the series x_t. This can be accomplished using the Johansen Maximum Likelihood (JML) technique[7] (Johansen and Juselius (1990)). We can write (10) in its Error Correction Mechanism (ECM) form

$$\Delta X_T = \alpha\beta' X_{T-1} + \Pi_0 + \Omega_T \tag{11}$$

where β is a 4×r matrix containing the r cointegrating vectors and α is a 4×r weighting matrix. Note that because all linear combinations of the cointegrating vectors are also cointegrating vectors, α and β are not uniquely identified. Consequently it is only possible to estimate the spaces that are spanned by the columns of α and β, rather than being able to estimate two distinct matrices.

The JML method seeks to find the r linear combinations of X_{T-1} that have the largest correlation with ΔX_T, after correcting for the deterministic component Π_0. The number of non-zero eigenvalues is then equal to r, the dimension of the cointegration space. It can be shown that the columns of β contain the eigenvectors corresponding to the r significant eigenvalues (see Johansen and Juselius (1990)). Two likelihood ratio statistics are used to determine r. The trace statistic

$$Q_1(p) = -\frac{n}{4}\sum_{i=p+1}^{4} \log\,(1-\lambda_i) \tag{12}$$

tests the hypothesis H_0: $r \leq p$ vs H_A: $r > p$ where λ_i is the i'th largest eigenvalue of the canonical correlations matrix. If the null hypothesis cannot be rejected, then the tests suggest that there are at most p cointegrating vectors. The maximal eigenvalue statistic can be used to test H_0: $r < p$ vs H_A: $r = p$ and is given by

$$Q_2(p) = -\frac{n}{4} \log (1 - \lambda_p) \tag{13}$$

where λ_p is the p'th largest eigenvalue.

The statistics in (12) and (13) are not asymptotically chi-squared, but rather are multivariate versions of the Dickey-Fuller distribution. Asymptotic critical points are tabulated in Johansen and Juselius (1990), whilst Monte Carlo simulations by Franses (1994) have yielded some finite sample values.

Now consider a matrix of cointegrating vectors

$$H = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

From the definition of cointegration, $H'X_T$ must be stationary i.e.

$$H'X_T = \begin{bmatrix} X_{2T} - X_{1T} \\ X_{3T} - X_{2T} \\ X_{4T} - X_{3T} \end{bmatrix}$$

This implies that, when the cointegrating vectors are of the form in H, a first difference is appropriate to restore x_t to stationarity, i.e. $x_t \sim I(1)$. Thus a test of the hypothesis that β=H enables discrimination between periodic and fixed parameter unit roots.

Since any linear combination of cointegrating vectors is also a cointegrating vector, the restriction β=H can be written as $\beta = H\varphi$, where φ is some $r \times r$ matrix. This facilitates the formation of the likelihood ratio statistic

$$Q_3(r) = N \sum_{i=1}^{r} \log \left(\frac{1-\xi_i}{1-\eta_i} \right) \tag{14}$$

where ξ_i is the i'th largest eigenvalue of the canonical correlations matrix evaluated under the null hypothesis that $\beta = H\varphi$. $Q_3(r)$ is asymptotically $\chi^2_{(3)}$.

Depending on the number of cointegrating vectors that are found, there exist other H matrices which can be used to test hypotheses about the nature of the system's unit roots. For example, if one cointegrating vector is detected, it means that there could be unit roots at three out of the possible four frequencies. The above LR test can be used to detect which frequencies contain the unit roots and whether they are periodic.[8] However it is unlikely that a quarterly economic time series would be generated by such a process, as it would imply that either a $(1+L+L^2+L^3)$ or a $(1-L+L^2-L^3)$ filter, or some periodic generalisation, would be required to induce stationarity. A data generating process of this nature has no obvious economic validity and so non-stationary series should generally be transformed using either a first or an annual difference or the corresponding periodic filter.

A short Monte Carlo study of the properties of the VQ test has been conducted by Franses (1994). In comparison with the HEGY test, he concludes that its performance is satisfactory when the data generating process is stationary and when it contains four unit roots. In the case where there is one unit root, both periodic and non-periodic, a VQ test (nominal size 10%) often falsely finds that there are two, only giving the correct result on about 50% of occasions. Consequently, Franses recommends conducting the trace test at the 20% level of significance and the LR tests of linear restrictions on the cointegrating vectors at the 1% level.

RESULTS

The data that is used in this project emanates from the Reserve Bank of New Zealand and consists of two quarterly series; household's real consumption of non-durables and services and household's real disposable income[9]. The sample of 111 observations spans the second quarter 1965 to the fourth quarter 1992. All analysis is conducted on the logarithms of the series. Figure 1 provides a graph of the data. The Periodic OCSB and VQ tests have been applied to these two series, with the results summarised in tables 1 and 2.

Figure 1

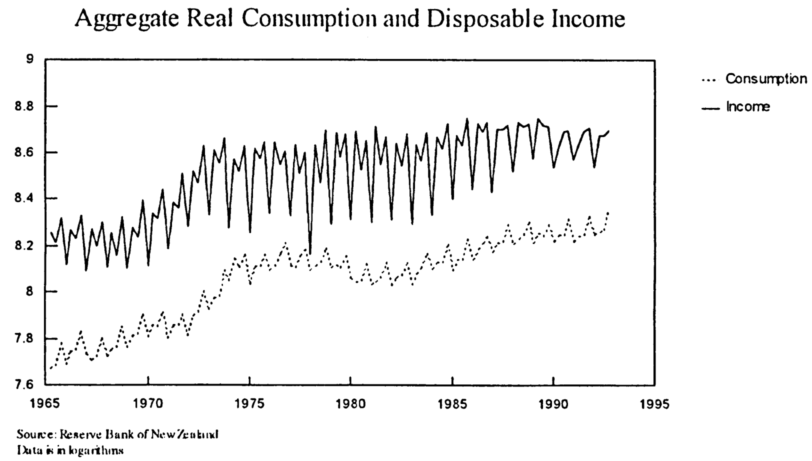

Table 1: Periodic OCSB Test

Hypothesis Tested	Consumption			Income			Tabulated Critical Values[c]
	Test Statistics[a]	Critical Values[b]		Test Statistics[a]	Critical Values[b]		
		20%	10%		20%	10%	10%
$\phi_1\phi_2\phi_3\phi_4=1$							
Wald	2.784	5.64	8.21	12.680**	5.64	8.16	2.71
LR	2.810	4.75	6.37	8.757**	4.75	6.35	2.71
F-test: $\phi_q=1\ \forall q$	1.754*	1.74	2.17	NA	NA	NA	2.07
u_t non-stationary:							
ADF (p=4)[d]	-4.302**	-1.49	-2.07	NA	NA	NA	-1.62
HEGY (p=5)[d]							
Zero Frequency	-3.714**	-2.50	-2.83	NA	NA	NA	-1.61
1/2 Frequency	-3.649**	-2.75	-3.11	NA	NA	NA	-1.57
1/4 and 3/4 Freq.	12.393**	6.93	8.32	NA	NA	NA	4.89

[a] * denotes significance at 20% and ** significance at 10%.

[b] All critical values in this column have been simulated using the bootstrap technique programmed in the SHAZAM econometrics package.

[c] This column contains the 10% critical values that the researcher would typically use, in the absence of simulation techniques such as the bootstrap. It highlights the need to understand and know the properties of a test statistic before applying it.

[d] A constant was found to be insignificant and was therefore not used in the formation of these statistics.

Table 2: Vector of Quarters Test

Hypothesis Tested	Test Statistics[a]		Critical Values[b]		
	Consumption	Income	20%	10%	1%
Trace: $r \leq 0$	89.577***	65.774**	47.10	51.59	64.33
$r \leq 1$	44.916***	20.370	27.69	31.22	40.98
$r \leq 2$	15.472*	5.894	13.99	16.56	23.70
$r \leq 3$	2.875	0.644	4.93	6.70	12.09
LR Test: $x_t \sim I(1)$	13.585***	NA	4.64	6.25	11.34

[a] * denotes significance at 20%, ** significance at 10% and *** significance at 1%.

[b] The trace test critical values originate from Franses (1994) and the LR test values from the Chi-Square tables. Recall that Franses suggests conducting the trace test at 20% and the LR test at 1%.

For the consumption series, both the Wald and LR tests cannot reject the hypothesis that $\phi_1\phi_2\phi_3\phi_4=1$, implying that it is valid to estimate the univariate process with this periodic restriction imposed. The residuals of this regression are then tested for unit roots using the ADF and HEGY tests. All hypotheses are rejected, suggesting that u_t is stationary and c_t is PI(1). The F-test on the hypothesis that there is no periodicity in the c_t process is rejected at 20%, but not at 10%.

The VQ test tends to support these results although its conclusion is marginal on the hypothesis that there are at least two unit roots in the series. However, following Franses (1994) recommendation of using a significance level of 20% for the trace test implies that consumption is PI(1)[10]. Thus it seems reasonable to conclude that c_t contains one unit root and that it may be valid to restrict this to be non-periodic. The LR test of this non-periodic hypothesis is rejected at 1%. Because of this ambiguity about whether consumption is periodic or not, both the periodic and non-periodic representations are considered when determining an appropriate cointegrating relationship (see Section III).

When applied to the income series, both the Wald and the LR tests lead to a rejection of the null hypothesis that there is a periodic unit root. The VQ test indicates that, at the 20% significance level, there are three unit roots in y_t. However, a data generating process involving three unit roots is unlikely (see Section II). Given this, and results obtained by Harrison (1993), who applied the HEGY and other seasonal integration tests to the same data set and concluded that y_t contained four unit roots, we proceed under the assumption that $y_t \sim I(1,1,1,1)$.

III. PERIODIC COINTEGRATION

Before estimating a periodic cointegration model, it is helpful to first derive a form for the periodic ECM. Recall from equation (9) that if c_t is PI(1), then it can be written as

$$\Phi_c(L)C_T = U_{CT} \tag{15}$$

where

$$\Phi_c(L) = \begin{bmatrix} 1 & 0 & 0 & -\phi_1 L \\ -\phi_2 & 1 & 0 & 0 \\ 0 & -\phi_3 & 1 & 0 \\ 0 & 0 & -\phi_4 & 1 \end{bmatrix}$$

and U_{CT} is stationary. Now consider another series y_t which contains s unit roots ($1 \leq s \leq 4$), one of which is at the zero frequency.[11] The vector of quarters corresponding to y_t is transformed to stationarity by a matrix polynomial $\Phi_y(L)$. Define a vector $X_T = [C_T' \ Y_T']'$ and write

$$\Phi(L)X_T = U_T \tag{16}$$

where

$$\Phi(L) = \begin{bmatrix} \Phi_c(L) & 0 \\ 0 & \Phi_y(L) \end{bmatrix}$$

As U_T is stationary it can be expressed as an infinite moving average process. Thus equation (16) has a multivariate Wold representation of the following form

$$\Phi(L)X_T = B(L)\varepsilon_T \tag{17}$$

For $\Phi(L)$ to be invertible, the roots of $\det(\Phi(L)) = 0$ must all lie outside the unit circle

$$\det(\Phi(L)) = \begin{vmatrix} \Phi_c(L) & 0 \\ 0 & \Phi_y(L) \end{vmatrix}$$

$$= |\Phi_c(L)|\ |\Phi_y(L)|$$

$$= (1-L)\ (1-L)^s$$

$$= (1-L)^{s+1}$$

Since these roots are clearly one, and therefore on the unit circle, the polynomial $\Phi(L)$ cannot be inverted. However, the matrix formula $\Phi^{-1}(L) = \dfrac{\Phi^a(L)}{|\Phi(L)|}$ where $\Phi^a(L)$ is the adjoint of $\Phi(L)$, can be utilised to write (17) as

$$X_T = \frac{\Phi^a(L)}{|\Phi(L)|} B(L) \varepsilon_T \qquad (18)$$

It is possible to define $\Phi^a(L)$ in terms of a partitioned matrix

$$\Phi^*_{cy}(L) = \begin{bmatrix} \Phi^*_c(L) & 0 \\ 0 & \Phi^*_y(L) \end{bmatrix} \qquad (19)$$

such that

$$\Phi^a(L) = (1-L)^s\ \Phi^*_{cy}(L) \qquad (20)$$

Thus, (18) can be written as

$$X_T = \frac{(1-L)^s}{(1-L)^{s+1}}\ \Phi^*_{cy}(L)\ B(L)\ \varepsilon_T$$

A simple rearrangement yields

$$(1-L)X_T = D(L)\varepsilon_T \tag{21}$$

where $D(L) = \Phi^*_{cy}(L)B(L)$. This is now in a convenient Wold representation form and is analagous to the one used in Granger's Representation Theorem (Engle and Granger (1987)), to obtain a cointegration model.

The determinant of D(L) is given by

$$\begin{aligned} \det(D(L)) &= \left|\Phi^*_{cy}(L)\right| \, |B(L)| \\ &= \left|\Phi^*_c(L)\right| \left|\Phi^*_y(L)\right| \, |B(L)| \\ &= (1-L)^3 \, (1-L)^{4-s} \, |B(L)| \\ &= (1-L)^{7-s} \, |B(L)| \end{aligned} \tag{22}$$

From Engle and Granger (1987), det(D(L)) is equal to $(1-L)^r d(L)$, where r is the cointegrating rank and d(L) a scalar lag polynomial. This means that, in this case, there are at least 7-s independent cointegrating relationships between the elements of X_T. Applying the 1-L filter to all eight components of X_T implicitly assumes that the system contains eight unit roots, when in fact there are no more than s+1, since c_t is PI(1) and therefore has one unit root, whilst y_t contains s unit roots. Thus, the 7-s cointegrating vectors can be thought of as corrections for the over-differencing (see VQ test above). If |B(L)| has a factor 1-L, then there is an extra cointegrating vector and c_t and y_t are periodically cointegrated.[12]

The only difference between the modified Wold representation in (21) and the corresponding representation in the conventional analysis is that, in this case, $D(0) \neq I$. This is because of the contemporaneous relationships within the vectors of quarters C_T and Y_T. However, since it requires only that D(0) has full rank[13], Granger's Representation Theorem can still be used to obtain a periodic error correction mechanism of the form

$$A^{*}(L)(1-L)X_{T} = -\alpha\beta' X_{T-1} + d(L)\epsilon_{T} \tag{23}$$

where $A^{*}(L)$ is a matrix polynomial with $A^{*}(0) \neq I_N$.

The system in (23) is unable to be efficiently estimated in this form because there are eight components in X_T and, with only one observation per annum, there will be few, if any, degrees of freedom. An alternative strategy is to use this form of the cointegrating relationship and some restrictions on the components of $A^{*}(L)$ to reduce the system to an estimable specification. There are two main cases of interest; $y_t \sim PI(1)$[14], i.e. s=1 and $y_t \sim I(1,1,1,1)$, i.e. s=4.

If y_t contains one unit root and c_t and y_t are periodically cointegrated, then the system in (23) will contain seven cointegrating vectors; six of which will account for the over-differencing and one which will cointegrate the two series. The equilibrium relations will be of the following form[15]

$$\begin{aligned} &C_{4T} = \phi_4 C_{3T}, \quad C_{3T} = \phi_3 C_{2T}, \quad C_{2T} = \phi_2 C_{1T} \\ &Y_{4T} = \theta_4 Y_{3T}, \quad Y_{3T} = \theta_3 Y_{2T}, \quad Y_{2T} = \theta_2 Y_{1T} \\ &C_{1T} = \beta_1 Y_{1T} \end{aligned} \tag{24}$$

where ϕ_i, θ_j, and β_1 are scalar constants. It can be shown that this is a linear combination of, and is therefore equivalent to

$$\begin{gathered} C_{4T} = \phi_4 C_{3T}, \quad C_{3T} = \phi_3 C_{2T}, \quad C_{2T} = \phi_2 C_{1T} \\ C_{4T} = \beta_4 Y_{4T}, \quad C_{3T} = \beta_3 Y_{3T}, \quad C_{2T} = \beta_2 Y_{2T}, \quad C_{1T} = \beta_1 Y_{1T} \\ \text{where } \beta_i = \frac{\beta_{i-1}\phi_i}{\theta_i} \quad i=2,3,4 \end{gathered} \tag{25}$$

These can then be converted back to the quarterly notation, and the cointegrating relationships written as

$$c_t = \beta_q y_t \,, \quad c_t = \phi_q c_{t-1}$$

where $\phi_1 \phi_2 \phi_3 \phi_4 = 1$. The ECM in (23) now simplifies to

$$a_1^*(L)(1-L^4)x_t = -\alpha_q \begin{pmatrix} c_{t-4} - \beta_q y_{t-4} \\ c_{t-4} - \phi_q c_{t-5} \end{pmatrix} + d(L)\varepsilon_t$$

$$= -\alpha_{q1}(c_{t-4} - \beta_q y_{t-4}) - \alpha_{q2}(c_{t-4} - \phi_q c_{t-5}) + d(L)\varepsilon_t \qquad (26)$$

where $x_t=[c_t \;\; y_t]'$, with the restriction that the parameters in $a_1^*(L)$ are non-periodic.[16] The second error correcting term is important, as it imposes the condition that both of the series have data generating processes containing one unit root. Excluding it implies that there is no long-run relationship between the quarters, thus mis-specifying the model.[17] The Periodic ECM used by Franses and Kloek (1991) does not contain this term.

However if y_t is seasonally integrated, and thus has four unit roots, there will be four linearly independent cointegrating vectors in the full system; three because $(1-L)C_T$ is over-differenced and one to cointegrate c_t and y_t. Since the zero frequency unit root is the one that is common between c_t and y_t, it is at this frequency that the cointegration occurs. Consequently, it is necessary to filter out the seasonal roots from y_t using a moving sum filter (see Hylleberg *et al.* (1990)). The equilibrium relationships are

$$C_{4T} = \phi_4 C_{3T} \,, \quad C_{3T} = \phi_3 C_{2T} \,, \quad C_{2T} = \phi_2 C_{1T}$$
$$C_{4T} = \beta_4 (Y_{4T} + Y_{3T} + Y_{2T} + Y_{1T})/4 \qquad (27)$$

which can be rearranged as

$$c_t = \beta_q \bar{y}_t \tag{28}$$

where $\bar{y}_t = \frac{1}{4}(1+L+L^2+L^3)y_t$ and $\beta_q = \dfrac{\beta_{q+1}}{\phi_q}$

Under the assumption that $A^*(L)$ is non-periodic, the system (23) reduces to

$$a_4^*(L)(1-L^4)x_t = -\alpha\beta'\bar{x}_{t-4} + d(L)\varepsilon_t \tag{29}$$

where $\bar{x}_t=[c_t, \bar{y}_t]'$ and $\beta=[1, -\beta_q]$. The first equation in (29) is

$$g_c(L)\Delta_4 c_t = \mu_q - \alpha_q\left(c_{t-4} - \beta_q\bar{y}_{t-4}\right) + g_y(L)\Delta_4 y_t + d(L)\varepsilon_{t-4} \tag{30}$$

where $g_c(L)$ and $g_y(L)$ are lag polynomials and μ_q is a periodically varying constant[18]. This representation of the periodic cointegration model is very similar to the one proposed by Franses and Kloek (1991).

Testing the validity of the cointegrating relationship involves estimation of the long run process

$$c_t = \beta_1 D_{1t}\bar{y}_t + \beta_2 D_{2t}\bar{y}_t + \beta_3 D_{3t}\bar{y}_t + \beta_4 D_{4t}\bar{y}_t + u_t \tag{31}$$

The hypothesis of a periodic unit root in the errors can then be tested using the tests outlined in Section II above. It is important to test the full quarterly series u_t for a unit root rather than the four annual u_{qt} series. This is because c_t and y_t each contain one unit root, so u_t should be tested for the presence of one unit root, not four as is implied by a test on each of the four quarters.

It has been shown by Inder (1993) that in the case of conventional cointegration, the most efficient method for estimating the long run parameters of a cointegrating process is to apply OLS to the unrestricted ECM. We have assumed that a similar result holds in the periodic case and have thus estimated the full cointegration model using this method.

To determine the appropriate degree of the polynomials $g_c(L)$ and $g_y(L)$, the "general to specific" methodology has been adopted; beginning with a relatively large number of lags and successively eliminating those which are insignificant. However, one difficulty with this approach is a problem that is frequently ignored, namely pre-testing. Each time a decision is made to include or exclude a lag, all ensuing analysis is dependent on that decision. Since it is based on a random statistic, then the decision itself is random so that the distributions of the "t-statistics" will change every time a test is conducted and its outcome is acted upon. There is no easy solution to this problem but it is important to be aware of it. A sensible strategy for the researcher is to be conservative, as it is usually better include an insignificant variable than exclude a significant one.

A PERIODIC COINTEGRATION MODEL

Taking income to be I(1,1,1,1) and consumption to be PI(1), the appropriate periodic cointegration model is given in equation (23). To test whether this is a valid cointegrating relationship, the residuals are tested for a periodic unit root using the periodic OCSB and VQ tests[19].

Table 3: Periodic OCSB Test

Hypothesis Tested		Test Statistics[a]	Critical Values[b]		Tabulated Critical Values[c]
			20%	10%	10%
$\phi_1\phi_2\phi_3\phi_4=1$					
	Wald	6.646	5.65	8.19	2.71
	LR	4.988	4.73	6.34	2.71

[a] * denotes significance at 20% and ** significance at 10%.
[b] All critical values in this column have been simulated using the bootstrap technique.
[c] This column contains the 10% critical values that the researcher would typically use, in the absence of simulation techniques such as the bootstrap. It highlights the need to understand and know the properties of a test statistic before applying it.
[d] A constant was found to be insignificant and was therefore not used in the formation of these statistics.

Table 4: Vector of Quarters Test

Hypothesis Tested		Test Statistics[a]	Critical Values[b]		
			20%	10%	1%
Trace: $r \leq 0$		82.126***	47.10	51.59	64.33
	$r \leq 1$	29.379*	27.69	31.22	40.98
	$r \leq 2$	15.257*	13.99	16.56	23.70
	$r \leq 3$	4.158	4.93	6.70	12.09

[a] * denotes significance at 20%, ** significance at 10% and *** significance at 1%.
[b] The trace test critical values originate from Franses (1994) and the LR test values from the Chi-Square tables. Recall that Franses suggests conducting the trace test at 20% and the LR test at 1%.

Following the procedure outlined in Franses and Kloek (1991), we have tested the hypotheses that α_q and β_q of equation (30) are non-periodic. A conventional F-test is used to determine whether $\alpha_q=\alpha$ for all q, with the calculated F-statistic[20] being 1.34. Johansen (1991) showed that this statistic has an asymptotic χ^2 distribution and therefore we cannot reject[21] the null hypothesis at the 10% (and 20%) level of significance. To test $\beta_q=\beta$ for all q, Franses and Kloek suggest estimating an "all encompassing" model, containing a periodic error correcting term and a non-periodic one. Since the coefficients of these terms are required to be negative, their significance can be tested using a one sided t-test. Applying this recommendation yielded a positive coefficient on the non-periodic error term, so the null hypothesis that it is zero cannot be rejected and consequently it should be excluded.[22] Thus, we believe that the β_q parameters are periodic, which implies that consumption is PI(1) rather than I(1). The final representation for the periodic ECM is

$$\begin{aligned}\Delta_4\hat{c}_t = & \underset{(10.9)}{0.69}\,\Delta_4 c_{t-1} - \underset{(-2.51)}{0.22}\,\Delta_4 c_{t-4} + \underset{(2.87)}{0.23}\,\Delta_4 c_{t-5} + \underset{(3.15)}{0.13}\,\Delta_4\bar{y}_{t-2} - \underset{(-1.89)}{0.088}\,\Delta_4\bar{y}_{t-5} \\ & - \underset{(-4.84)}{0.25}\left(c_{t-4} - \hat{\beta}_q\bar{y}_{t-4}\right) + \underset{(2.59)}{0.038}\,GST_t \qquad SSE = 0.03770\end{aligned} \tag{32}$$

where $\hat{\beta}_1 = 0.947$, $\hat{\beta}_2 = 0.951$, $\hat{\beta}_3 = 0.950$, $\hat{\beta}_4 = 0.959$. The numbers in brackets are the reported t-statistics. An F-test on whether the intercept was constant over all quarters gave a statistic of 0.95 with a p-value 44%, so the null hypothesis could not be rejected. Since the analysis has been conducted in logarithms, the $\hat{\beta}_q$ coefficients are the estimated quarterly elasticities of consumption with respect to annual income. The variable GST_t is included to account for the Goods and Services Tax that was introduced in New Zealand in 1986. It takes the value 1 in the period just prior to the introduction of GST, −1 in the period immediately after it and zero otherwise. This covers the transfer of spending that occurred as people brought forward purchases to avoid the price increases resulting from of the tax.

If, following the results of the earlier univariate tests, we believed that c_t was I(1), an appropriate strategy would be to transform y_t into an I(1) series by applying the $(1+L+L^2+L^3)$ filter, and estimate a conventional cointegration model. The ADF test on the residuals of the cointegrating regression was applied and the null hypothesis of no cointegration is rejected[23]. To determine the lag length, the same methodology as for the periodic model was used, but with a nominal size of 20% on each test. The following model resulted

$$\begin{aligned}\Delta \hat{c}_t = {} & \underset{(0.4)}{0.12}\,\Delta c_{t-2} + \underset{(2.9)}{0.25}\,\Delta c_{t-4} - \underset{(-1.6)}{0.29}\,\Delta \bar{y}_{t-1} + \underset{(1.2)}{0.22}\,\Delta \bar{y}_{t-2} - \underset{(-2.0)}{0.33}\,\Delta \bar{y}_{t-5} \\ & - \underset{(-3.8)}{0.18}\left(c_{t-1} - 0.98\,\bar{y}_{t-1}\right) + \underset{(2.5)}{0.034}\,GST_t - \underset{(-0.82)}{0.098}\,D_{1t} + \underset{(-0.23)}{0.028}\,D_{2t} \\ & + \underset{(-0.21)}{0.026}\,D_{3t} + \underset{(0.051)}{0.062}\,D_{4t} \qquad SSE = 0.03106\end{aligned} \tag{33}$$

where D_{1t}, D_{2t}, D_{3t} and D_{4t} are quarterly dummy variables.

Plotting the residuals of these models reveals that both exhibit some outliers in the mid 1970's which can probably be attributed to the aftermath of the 1973 oil shock. In addition, a sequential Chow test detects some parameter instability at this time. One could attempt to rectify these problems by applying a dummy variable to account for each of the relevant observations. However, this approach creates a perfect fit for that observation and so has same the effect as would deleting it, thus, corrupting the estimation of the dynamics. Consequently, specific dummies have not been included.

If seasonality is ignored then the regression of c_t on y_t produces an estimated elasticity of 0.77. The ADF test for cointegration then yields a statistic of -1.77, which, when compared to the 10% critical value[24] of -2.91, indicates that there is no cointegration.[25] Consequently, if the seasonal properties of the data are not properly considered, one could falsely conclude that consumption and income are not cointegrated.

IV. CONCLUSION

The contending periodic and non-periodic models for the aggregate New Zealand consumption function have been examined. Inference on the periodicity of the cointegrating parameters implied that there was a periodic relationship between consumption and income, and estimation of this relationship yielded what appears to be an adequate model. However, imposing the restriction that the model is non-periodic also leads to a plausible specification which, with a smaller sum of squared residuals, fits the data better. Though this statistic alone should not be relied upon as a criterion for model selection, it does suggest that, in this case, it may be better to exclude the periodicity completely than to include it and try to estimate the model under certain restrictions. This is not to say, however, that the estimated periodic cointegration model is of no use, since such things as forecasting performance have not been considered. Consequently, there is no need at this stage to conclude that one model is superior to the other.

This study has also raised a number of issues which require further research. Firstly, the power properties of the VQ and Periodic tests are largely unknown and investigation of them would assist practitioners in determining the univariate characteristics of their data. Another area which requires more research is the estimation of periodic cointegration models and, in particular, the magnitudes of the trade-offs that occur when restrictions are imposed to reduce the number of parameters to be estimated.

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ENDNOTES

1. Acceptance of a unit root at these frequencies requires that both π_3 and π_4 are zero (see Hylleberg *et al.* (1990)). Consequently, it is appropriate to conduct a joint test.

2. A nominal size of 5% was used in this simulation.

3. This procedure has been developed from the simple strategy used by Osborn *et al.* (1988).

4. For the periodic integration tests, the calculated Wald statistics exhibited a 20% variation, depending on how the hypothesis was expressed.

5. Osborn *et al.* (1988) did not simulate critical values for the ADF and HEGY tests when used in this context. Rather, they used the original tabulated values. As can be seen from the results in Tables 1 and 2, the simulated values are markedly different from the tabulated ones.

6. This is analogous to the case in the ADF test, where $\gamma=0$ when there is a unit root (see equation (2.3.1)).

7. Bewley *et al.* (1994) have presented Monte Carlo evidence that a Box-Tiao procedure (see Box and Tiao (1977)) can, in some cases, have better properties than JML. A comparison of their performance in the VQ framework could be a productive topic for further research.

8. For a full description of the other cases, see Franses (1994).

9. In the Reserve Bank database, these are encoded CPNDX and CPSX for the two components of the consumption series and YDX for the disposable income series.

10. The results of the maximal eigenvalue test are identical to those obtained using the trace statistic and are therefore not tabulated here.

11. Since c_t has only a zero frequency unit root, y_t must also have a unit root at this frequency for there to be cointegration. This is because cointegration can occur only between series with unit roots at common frequencies.

12. Note that there cannot be more than 8-s linearly independent cointegrating vectors in the system i.e. the cointegration space must have dimension no greater than 8-s and the space containing all non-stationary linear combinations dimension no less than s. This is because y_t contains s-1 seasonal unit roots, which cannot be cointegrated out and, even if the zero frequency components are cointegrated, one of these roots must remain in the system; thus leaving s unit roots in total.

13. This condition is satisfied because $\det(D(L))=(1-L)^r d(L)$ has no roots at zero, i.e. $\det(D(0))\neq 0$.

14. Note that this includes the I(1) special case.

15. An alternative to this is the common trends method (Franses (1993b)), which uses the Box-Tiao procedure (Box and Tiao (1977)) to find the most non-stationary linear combinations of C_T and Y_T, which are then tested for cointegration.

16. This assumption is necessary because it reduces the number of coefficients to be estimated by 75%, and is therefore likely to produce large efficiency gains. If one begins with eight lags on each of the variables, then it means that 16 lag coefficients need to be estimated, rather than 64 if they are unrestricted.

17. The only reason that this second error correcting term involves c_t rather than y_t is because we have chosen to normalise on c_t.

18. Throughout this section, in order to avoid complicating the mathematics, we have assumed that the mean, and any other deterministic components, have been subtracted out.

19. The lag lengths were determined by sequential search, beginning with eight lags and eliminating the longest insignificant lag based on conventional t statistics.

20. The insignificant lags were removed before evaluating this statistic. However, the result does not change if the test is done prior to the tests on the lag coefficients, with the F-statistic in that case being 1.06.

21. We did not bootstrap the critical values for this test because of the complicated lag structure. However, the large p-value means that we can still be confident that the null cannot be rejected.

22. An alternative strategy would be to estimate two models, one with a periodic error correcting term and the other with a non-periodic one, and use a likelihood ratio test to discriminate between them.

23. The ADF statistic with five augmentation lags was -3.07, the lag length was determined by increasing the lag lengths until the residuals' autocorrelation and partial autocorrelation functions revealed no significant values.

24. This critical value was obtained from Engle and Yoo (1987).

25. It should be noted that the non-seasonal cointegrating regression includes a constant, whereas the seasonal and periodic ones do not.

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