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Abstract

We study the exact power of the Goldfeld-Quandt test in a linear regression model with errors which are both heteroscedastic and autocorrelated. The test is not robust to this form of mis-specification, but is less sensitive to autocorrelation in smaller samples.

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1. Introduction

This paper reports on an exploratory study of the robustness of the Goldfeld and Quandt (GQ) (1965) test of homoscedasticity of linear regression model errors to relaxation of the standard assumption of serially independent errors. Several previous papers have examined the sensitivity of the GQ test to its underlying assumptions. These include Giles and Saxton (1993) who focus on the appropriate number of omitted central observations when relevant regressors have been excluded from the model, Evans (1992) who studies the true size of the test under various non-normal error distributions, and Epps and Epps (1977) who address the consequences of serial correlation using a very limited Monte Carlo experiment.

The results presented below use the exact power function of the GQ test with a variety of data types. We find that the test is not robust to the presence of autocorrelation.

2. The Model

We use the standard linear regression model

$$y = X\beta + u, \qquad u \sim N(0,\eta^2 V)$$

where y is $(T\times1)$ and X is $(T\times K)$, non-stochastic and of full rank. We allow V to reflect a combination of stationary first-order autoregressive (AR(1)) errors and multiplicative heteroscedasticity according to the form:

$$V = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & & \cdots & \rho^{T-1}\sigma_1\sigma_T \\ \rho\sigma_1\sigma_2 & \sigma_2^2 & & & \rho^{T-2}\sigma_2\sigma_T \\ \rho^2\sigma_1\sigma_3 & \rho\sigma_2\sigma_3 & \sigma_3^2 & \ddots & \vdots \\ \vdots & & & \ddots & \\ \rho^{T-1}\sigma_1\sigma_T & & & \cdots & \sigma_T^2 \end{bmatrix}$$

where ρ is the AR(1) parameter and $\sigma_t^2 = X_{jt}^{\alpha}$, the parameter α being adjusted to control the degree of heteroscedasticity. Application of the GQ test proceeds by sorting the data so that the regressor thought to be inducing heteroscedasticity is increasing. After omitting c central observations[1], separate regressors are run over the remaining sub-samples and the GQ statistic is formed as the ratio of the resulting sums of squared errors. Following Harvey and Phillips (1974) we define $u^{*\prime} = (u_1'u_2')$ and $M_i = I - X_i(X_i'X_i)^{-1}X_i'$ $(i = 1,2)$ where subscripts refer to the first and second sub-samples. Defining $M_1^* = \begin{bmatrix} M_1 & 0 \\ 0 & 0 \end{bmatrix}$ and $M_2^* = \begin{bmatrix} 0 & 0 \\ 0 & M_2 \end{bmatrix}$ allows us to write the GQ test statistic as $g = (u^{*\prime}M_2^*u^*)/(u^{*\prime}M_1^*u^*)$. The power of the test can now be written[2] as

$$\Pr(g \geq f^*) = \Pr\left(\sum_{j=1}^{T-c} \lambda_j z_j^2 \leq 0 \right)$$

where λ_j's are the eigenvalues of $(f^*M_1^* - M_2^*)V^*$, $E(u^*u^{*\prime}) = V^*$, and the z_j^2's are each independent central $\chi^2_{(1)}$. Several algorithms are capable of evaluating probabilities of this form, such as those by Imhof (1961), Davies (1980) or Lieberman (1994).[3]

3. Design of the Study

The exact power of the GQ test was evaluated using five data sets in an effort to reveal the more general consequences of AR(1) errors in a variety of contexts. The matrices, each of which included an intercept, were: X1 comprising the annual income and price data from Durbin and Watson's (1951) "spirits" example; X2 comprising the quarterly Australian Consumer's Price Index and its lag; X3 and X4 which contain a lognormal (2.2, 19.6) and a uniform [1,10] variable respectively and X5 comprising a linear trend and a normal (5,1.5) variable.[4]

A small (T=21) and moderate (T=69) sample was used with each design matrix and all tests were conducted at the 5% significance level. Several positive values of ρ were used and the degree of heteroscedasticity, measured by $h = (\sigma_T^2/\sigma_1^2)$ ranged from 1 up to 50. We used Davies' algorithm within the SHAZAM (1993) package for all computations.

The power function in the limit as $\rho \to 1$ was also studied[5] and found to be degenerate regardless of the presence of an intercept; i.e. the limiting power of the GQ test as $\rho \to 1$ must be either zero or unity[6] for $h \geq 1$, and $\alpha \neq 2$..

4. Results

The true size of the GQ test is typically larger than its nominal level when autocorrelation is present. The effect is generally stronger in larger samples, with true sizes of 20% being evident in figures 3 and 4. This size distortion makes power comparisons difficult[7] but some conclusions can be drawn from figure 1 for example. Here the power of the GQ test is unambiguously lower for $h > 10$ when $\rho > 0$, as the size of the test is larger but the power is lower, relative to the $\rho = 0$ power curve. For values of $h < 10$, a larger rejection probability under the alternative

($h > 1$) is obtained, but only at the cost of also rejecting more frequently under the null hypothesis ($h = 1$), so that direct comparison cannot be made.

5. Conclusion

We have shown that the GQ test is not robust to the presence of AR(1) errors when the covariance matrix is of the form given by V^*. This concurs with the only other work on this topic by Epps and Epps (1977). We have also shown that size distortion is more pronounced in larger samples, and that sensitivity to autocorrelation occurs across a range of data types. The covariance matrix we used is similar to that used by Small (1994) to investigate the converse of this problem. That study found a group of exact AR(1) tests to be reasonably robust to heteroscedasticity for moderate degrees of autocorrelation.

Work in progress includes investigating the effect of omitting observations from locations other than the centre of the re-ordered sample, and the merits of particular orderings of tests for serial independence and homoscedasticity.

Footnotes

* We wish to thank David Giles and Judith Giles for helpful comments on this paper. Remaining errors or omissions are our responsibility.

1. Harvey & Phillips (1974) suggest that c should be chosen so that the remaining sub-sample degrees of freedom are (equal and) approximately one third of the full sample.
2. See Koerts and Abrahamse (1971) for example.
3. Davies' algorithm can additionally handle non-central z_j^2's, while Lieberman's method is based on a saddle-point expansion which avoids the need to compute the λ_j's.
4. These data have been used in several similar studies such as Evans (1992).
5. The methodology used for this is outlined by Krämer and Zeisel (1990).
6. The sign of the only non-zero eigenvalue of $(f^*M_1^*-M_2^*)V^*$ uniquely determines whether the limiting power is zero or unity.
7. In theory, one could adjust the critical values so that all power curves begin at the same size. In practice, this is not possible.

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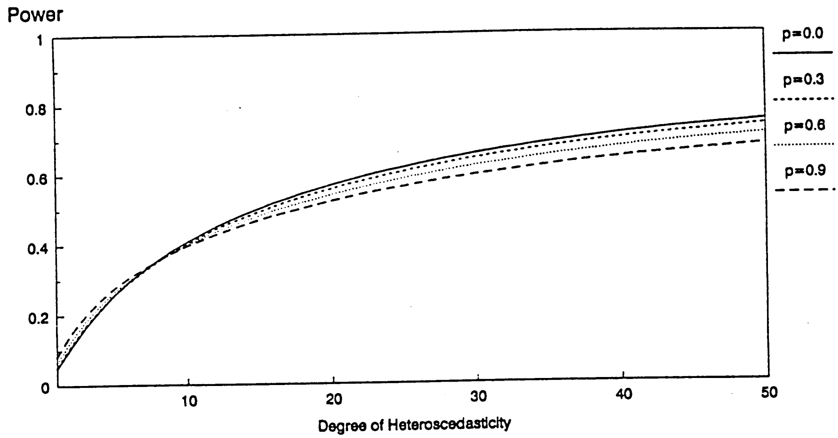

Figure 1: Power of the GQ Test; Uniform Data (X4); T=21

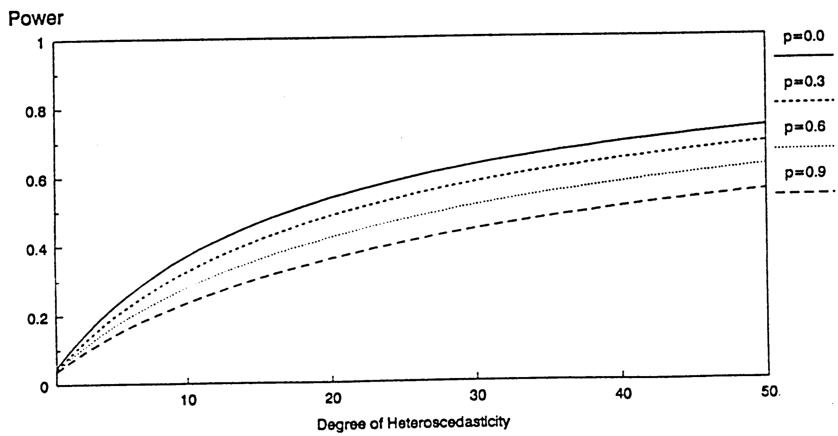

Figure 2: Power of the GQ Test; Spirits Data (X1); T=21

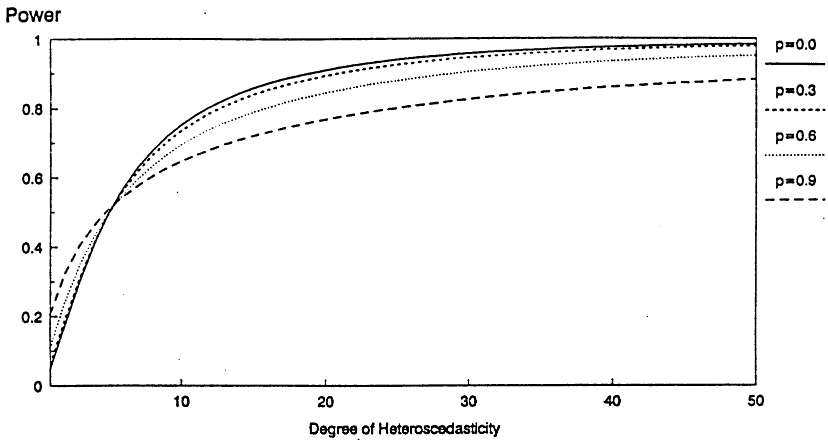

Figure 3. Power of the GQ Test; Lognormal Data (X3); T=69

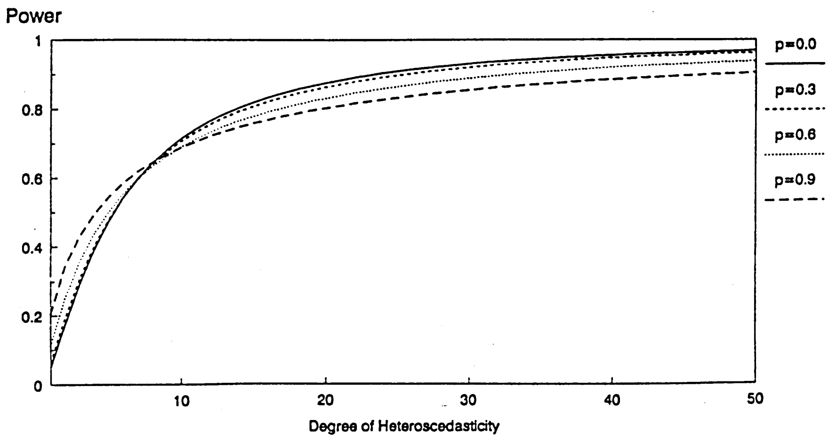

Figure 4. Power of the GQ Test; Normal Data (X5); T=69

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