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Abstract

A popular class of tests for simple autoregressive processes is considered in the context of the error components model previously discussed by Revanker (1980) and King (1982). We show that the expected value of all such test statistics is further from the rejection region in this model, relative to the classical model. More importantly, as the degree of positive autocorrelation becomes very strong, the power of each test must decline to its level of significance, irrespective of the data.

1. INTRODUCTION

In earlier issues of this Review, Revanker (1980) and King (1982) disagreed over the advisability of using a Durbin Watson (DW) test (Durbin & Watson (1950)) in a linear regression model with disturbances comprised of two independent components, one of which is autocorrelated. The discussion concerned a first order autoregressive (AR(1)) process but applies equally well to any simple[1] AR(p) scheme. Revanker (1980) observed that the asymptotic relationship between the DW test statistic d, and the AR(1) parameter p, in such a model is given by plim $d < 2(1-\rho\lambda)$ for some constant λ satisfying $0 \leq \lambda \leq 1$. He concluded that the DW test is asymptotically biased towards the null hypothesis in this model. King (1982), however, noted that there is no bias if the null is true (i.e. $\rho = 0$) and showed that the DW test is approximately the best invariant test in the neighborhood of the null hypothesis.

In this note we show that the presence of an additional component in an otherwise standard AR(1) process moves d further from the rejection region, on average, when the alternative hypothesis is true. This result applies to a group of tests related to the DW test and to one sided alternatives in either direction. It is further shown that as ρ approaches unity the power of each test must approach its true size so that no power function can be monotonic in ρ in an error components model of this form. Graphs depicting power functions are presented to illustrate these results.

2. THE MODEL AND TESTS

The linear regression model

(1) $$y = X\beta + w$$

is used, where y is $n \times 1$, X is an $n \times k$ non-stochastic matrix of full rank, β is a $k \times 1$ vector of parameters and w is an $n \times 1$ disturbance vector satisfying

$$w_t = u_t + v_t, \qquad t = 1, \ldots, n.$$

Here, $u_t = \rho u_{t-1} + \varepsilon_t$ with $|\rho| < 1$, $v = (v_1,\ldots,v_n)' \sim N(0,\sigma_v^2 I_n)$, $\varepsilon = (\varepsilon_1,\ldots,\varepsilon_n)' \sim N(0,\sigma_\varepsilon^2 I_n)$ and u_t, v_t are independent.

Following Revanker (1980) we define the variance of the regression disturbances as $\sigma_w^2 = \sigma_u^2 + \sigma_v^2$, and the variance ratio $\lambda = \dfrac{\sigma_u^2}{\sigma_w^2}$. The covariance matrix of w is now given by

$$\sigma_w^2 \Sigma(\rho,\lambda) = \sigma_u^2 \begin{bmatrix} \lambda^{-1} & \rho & \rho^2 & \cdots & & \rho^{n-1} \\ \rho & \lambda^{-1} & \rho & & & \\ \rho^2 & \rho & \lambda^{-1} & & & \vdots \\ & & & \ddots & & \\ \vdots & & & & \ddots & \\ \rho^{n-1} & \rho^{n-2} & \cdots & \rho^2 & \rho & \lambda^{-1} \end{bmatrix}$$

Observe that $0 \le \lambda \le 1$ so that the correlation between adjacent w_t's, $\lambda\rho$, is less than would occur without the presence of v_t. In all other respects Σ is identical to the standard covariance matrix arising from an AR(1) process.

There are several exact tests of $H_0 : \rho = 0$ vs $H_a : \rho > 0$ which have desirable power properties in standard regression models. We consider a class of these which reject H_0 for small values of a statistic with the general form

$$r = \frac{\hat{w}' A \hat{w}}{\hat{w}' \hat{w}}$$

where $\hat{w}$ is the vector of OLS residuals from the estimation of (1). This class includes the DW test, King's (1981) alternative DW test, the Berenblut-Webb (1973) test (BW), which is based on results of Kadiyala

(1970), and a related point-optimal test (King (1985)). These tests are distinguished by the particular non-stochastic $n \times n$ matrix A which each uses. For the DW test A is a tridiagonal matrix with the leading diagonal comprising two's except for the top-left and bottom right elements which are ones; all off diagonal entries are -1. King's alternative DW test has an identical A matrix except that all leading diagonal entries are twos. The Berenblut-Webb test uses a matrix B which is the DW matrix A with only the top left element changed to a two and defines

$$A = B - B X (X'BX)^{-1}X'B.$$

The A matrix for the point-optimal test is the same as that for the Berenblut-Webb test except that B is replaced by the inverse of the covariance matrix of an AR(1) process with ρ chosen as some mid-range value (0.5 and 0.75 are often used).

3. POWER FUNCTIONS

To consider the power of the tests, rewrite r as a function of the population disturbances.

$$r = \frac{w'MAMw}{w'Mw}$$

where $M = I_n - X(X'X)^{-1}X'$. The power of each test is given by

$$pr\left[(r<r^*)\,|\,E(ww')\right]$$

for some $\alpha\%$ size critical value r^*. Standard manipulations (eg. Koerts & Abrahamse (1969)) can be used to write test power as

$$\mathrm{pr}\left[\left((r-r^*)<0\,|\,\Sigma(\rho,\lambda)\right)\right] = \mathrm{pr}\left[w'M(A-r^*I_n)Mw < 0\right]$$

$$= \mathrm{pr}\left[\sum_{j=1}^{n} \lambda_j\chi_j^2 < 0\right]$$

where the λ_j's are the eigenvalues of $M(A-r^*I_n)M\Sigma(\rho,\lambda)$ and the χ_j^2's are independent central chi-square variates with one degree of freedom each.

To analyse the effect of the uncorrelated error component, v_t, on test power define

$$\Omega(\rho) = \Sigma(\rho,\lambda) - \Delta$$

where $\Delta = \mathrm{diag}(\delta)$ and $\delta = \frac{1-\lambda}{\lambda} > 0$. The matrices $\Omega(\rho)$ and $\Sigma(\rho,\lambda)$ now define the covariances between the w_t when v_t is absent and present, respectively. The following theorem compares the power of each test under each scenario, for a given (finite sample) design matrix.

Theorem 1

When $E(ww')$ is given by $\Sigma(\rho,\lambda)$ rather than $\Omega(\rho)$, the average value of the test statistic r is increased, when testing against H_a: $\rho > 0$.

Proof

Let $S = M(A-r^*I_n)M$ and define the ij^{th} element of S by s_{ij}. Consider the first moment of $(r-r^*)$ which is given by $E(w'Sw) = tr(S(E(ww')))$. We must compare $tr(S\Sigma)$ with $tr(S\Omega)$.

$$tr(S\Sigma) = tr(S(\Omega+\Delta))$$

$$= trS\Omega + \sum_{i=1}^{n} s_{ii}\delta$$

$$= trS\Omega + \delta\ tr\ (S)$$

Observe that $t_r(S) = E(r - r^*)|_{\rho=0}$ and recall that $E(r)|_{p=0} > r^*$. Thus $tr(S) > 0$ and $tr(S\Sigma) > tr(S\Omega)$. #

At least on average, therefore, the probability of rejecting H_0 is reduced as λ decreases, which occurs as σ_v^2 becomes large relative to σ_u^2. When the negative alternative $H_a^-:\rho<0$ is used, H_0 is rejected for $r > r^*$. Thus $E(r)|_{\rho=0} < r^*$ and $tr(S) < 0$; the average value of r is therefore reduced and the powers of all tests considered are again lower than would occur if σ_v^2 were zero.

It is clear that the standard exact tests against AR(1) alternatives are less powerful when a second, independent, component is present in the error term. To establish the magnitude of this phenomenon the exact power of each test was evaluated numerically under a variety of data conditions. These evaluations were performed with Davies' (1980) algorithm in the SHAZAM (1993) package.

Figures 1 and 2 are representative of the numerical results obtained with all data matrices.[2] In figure 1, the X matrix is 60 × 3 and comprises an intercept, a linear time trend and a series of drawings from the uniform [0, 10] distribution. Figure 2 shows the power of the BW test using a 20 × 3 matrix in which the regressors are an intercept, a linear trend and drawings from the N(30,4) distribution.

It is apparent from both figures that when $\lambda \neq 1$ the power functions of these tests converge to some small value as ρ approaches unity. This phenomenon is clarified in the following result.

Theorem 2

When $E(ww')$ is given by $\Sigma(\rho,\lambda)$ the limiting power of the DW test as $\rho \to 1$ is the same as the true size of the test, provided the regression has an intercept.

Proof

As $\rho \to 1$, $\Sigma(\rho,\lambda) \to V + \Delta$ where V is a matrix of ones and Δ is defined above. Recall from above that the power of the test depends on the eigenvalues of $S\Sigma(\rho,\lambda)$, being the vector γ which satisfies

$$\gamma z = S\Sigma z \quad \text{for some non-null vector } z$$

or,

$$\gamma z = S(V+\Delta)z.$$

Now when an intercept is present, MV = SV = 0 so that

$$\gamma z = S\Delta z$$

$$= \Delta\Sigma z$$

So

$$\frac{\gamma}{\delta}z = Sz.$$

The effect of δ is to scale each eigenvalue by the same factor which does not affect the rejection probability. This probability depends only on the eigenvalues of S and is therefore equal to the true size of the test. #

The above result extends readily to all other tests in the class under consideration by using results of Small (1993).

4. CONCLUSION

The presence of an additional, independent, component in the error term of a standard linear regression model has severe consequences for the powr of a popular class of exact tests for serial independence. Although the size of the tests is entirely robust, the power function of each test must eventually return to the true size, dramatically reducing power against strong positive autocorrelation. These results strongly suggest that the standard exact tests for AR(p) errors are unreliable in an error components model.

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FOOTNOTES

1. That is, to any AR process in which only one parameter is non-zero.
2. In both figures power against one sided alternatives in either direction are shown. In all cases, the significance level is 5%.

Figure 1

Power of DW Test in Error Components Model
Uniform Data; n=60

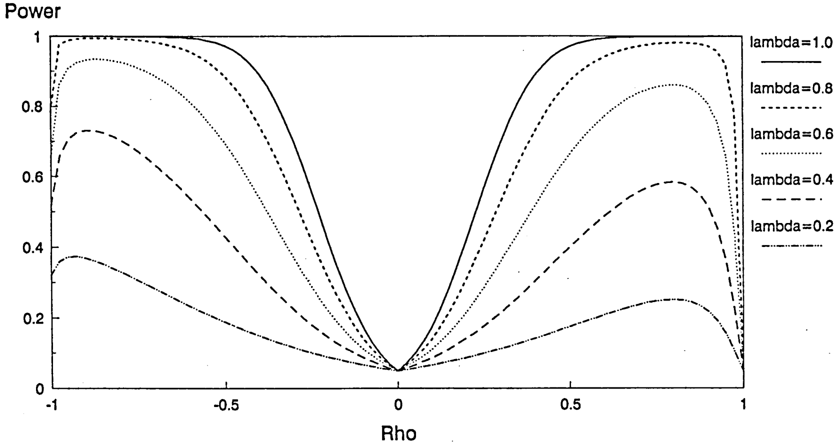
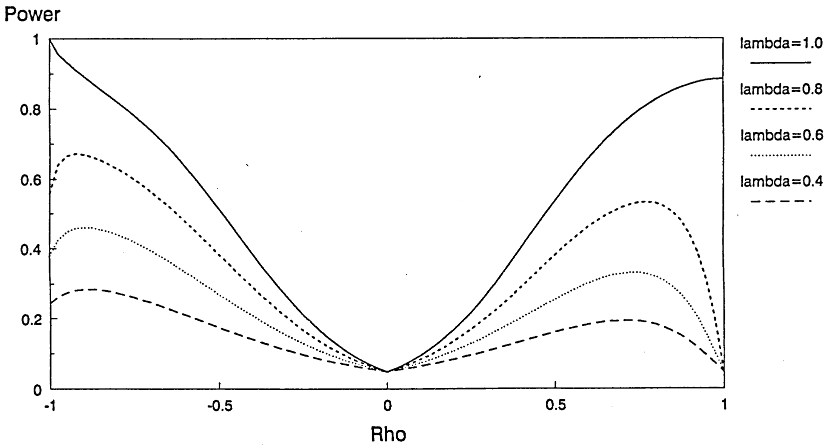


Figure 2

Power of BW Test in Error Components Model
Normal Data; n=20



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