AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

CANTER

9309

Department of Economics
UNIVERSITY OF CANTERBURY
CHRISTCHURCH, NEW ZEALAND

ISSN 1171-0705

LIBRARY
WITHDRAWN
JUL 28 1993
GIANNINI FOUNDATION OF
AGRICULTURAL ECONOMICS

TESTING FOR SERIAL INDEPENDENCE IN ERROR COMPONENTS MODELS: FINITE SAMPLE RESULTS

John P. Small

Discussion Paper

No. 9309

This paper is circulated for discussion and comments. It should not be quoted without the prior approval of the author. It reflects the views of the author who is responsible for the facts and accuracy of the data presented. Responsibility for the application of material to specific cases, however, lies with any user of the paper and no responsibility in such cases will be attributed to the author or to the University of Canterbury.

Department of Economics, University of Canterbury
Christchurch, New Zealand

Discussion Paper No. 9309

June 1993

TESTING FOR SERIAL INDEPENDENCE IN ERROR COMPONENTS MODELS: FINITE SAMPLE RESULTS

John P. Small

TESTING FOR SERIAL INDEPENDENCE IN ERROR COMPONENTS MODELS : FINITE SAMPLE RESULTS

John P. Small
Department of Economics
University of Canterbury

June 1993

Abstract

A popular class of tests for simple autoregressive processes is considered in the context of the error components model previously discussed by Revanker (1980) and King (1982). We show that the expected value of all such test statistics is further from the rejection region in this model, relative to the classical model. More importantly, as the degree of positive autocorrelation becomes very strong, the power of each test must decline to its level of significance, irrespective of the data.

1. INTRODUCTION

In earlier issues of this Review, Revanker (1980) and King (1982) disagreed over the advisability of using a Durbin Watson (DW) test (Durbin & Watson (1950)) in a linear regression model with disturbances comprised of two independent components, one of which is autocorrelated. The discussion concerned a first order autoregressive (AR(1)) process but applies equally well to any simple[1] AR(p) scheme. Revanker (1980) observed that the asymptotic relationship between the DW test statistic d, and the AR(1) parameter p, in such a model is given by plim $d < 2(1-\rho\lambda)$ for some constant λ satisfying $0 \leq \lambda \leq 1$. He concluded that the DW test is asymptotically biased towards the null hypothesis in this model. King (1982), however, noted that there is no bias if the null is true (i.e. $\rho = 0$) and showed that the DW test is approximately the best invariant test in the neighborhood of the null hypothesis.

In this note we show that the presence of an additional component in an otherwise standard AR(1) process moves d further from the rejection region, on average, when the alternative hypothesis is true. This result applies to a group of tests related to the DW test and to one sided alternatives in either direction. It is further shown that as ρ approaches unity the power of each test must approach its true size so that no power function can be monotonic in ρ in an error components model of this form. Graphs depicting power functions are presented to illustrate these results.

2. THE MODEL AND TESTS

The linear regression model

(1) $$y = X\beta + w$$

is used, where y is $n \times 1$, X is an $n \times k$ non-stochastic matrix of full

rank, β is a $k \times 1$ vector of parameters and w is an $n \times 1$ disturbance vector satisfying

$$w_t = u_t + v_t, \qquad t = 1, \ldots, n.$$

Here, $u_t = \rho u_{t-1} + \varepsilon_t$ with $|\rho| < 1$, $v = (v_1,\ldots,v_n)' \sim N(0,\sigma_v^2 I_n)$, $\varepsilon = (\varepsilon_1,\ldots,\varepsilon_n)' \sim N(0,\sigma_\varepsilon^2 I_n)$ and u_t, v_t are independent.

Following Revanker (1980) we define the variance of the regression disturbances as $\sigma_w^2 = \sigma_u^2 + \sigma_v^2$, and the variance ratio $\lambda = \dfrac{\sigma_u^2}{\sigma_w^2}$. The covariance matrix of w is now given by

$$\sigma_w^2 \Sigma(\rho,\lambda) = \sigma_u^2 \begin{bmatrix} \lambda^{-1} & \rho & \rho^2 & \cdots & & \rho^{n-1} \\ \rho & \lambda^{-1} & \rho & & & \\ \rho^2 & \rho & \lambda^{-1} & & & \vdots \\ & & & \ddots & & \\ \vdots & & & & \ddots & \\ \rho^{n-1} & \rho^{n-2} & \cdots & \rho^2 & \rho & \lambda^{-1} \end{bmatrix}$$

Observe that $o \le \lambda \le 1$ so that the correlation between adjacent w_t's, $\lambda\rho$, is less than would occur without the presence of v_t. In all other respects Σ is identical to the standard covariance matrix arising from an AR(1) process.

There are several exact tests of $H_0 : \rho = 0$ vs $H_a: \rho > 0$ which have desirable power properties in standard regression models. We consider a class of these which reject H_0 for small values of a statistic with the general form

$$r = \frac{\hat{w}' A \hat{w}}{\hat{w}' \hat{w}}$$

where $\hat{w}$ is the vector of OLS residuals from the estimation of (1). This class includes the DW test, King's (1981) alternative DW test, the Berenblut-Webb (1973) test (BW), which is based on results of Kadiyala

(1970), and a related point-optimal test (King (1985)). These tests are distinguished by the particular non-stochastic nxn matrix A which each uses. For the DW test A is a tridiagonal matrix with the leading diagonal comprising two's except for the top-left and bottom right elements which are ones; all off diagonal entries are -1. King's alternative DW test has an identical A matrix except that all leading diagonal entries are twos. The Berenblut-Webb test uses a matrix B which is the DW matrix A with only the top left element changed to a two and defines

$$A = B - B\,X\,(X'BX)^{-1}X'B.$$

The A matrix for the point-optimal test is the same as that for the Berenblut-Webb test except that B is replaced by the inverse of the covariance matrix of an AR(1) process with ρ chosen as some mid-range value (0.5 and 0.75 are often used).

3. POWER FUNCTIONS

To consider the power of the tests, rewrite r as a function of the population disturbances.

$$r = \frac{w'MAMw}{w'Mw}$$

where $M = I_n - X(X'X)^{-1}X'$. The power of each test is given by

$$pr\left[(r<r^*)\,|\,E(ww')\right]$$

for some α% size critical value r^*. Standard manipulations (eg. Koerts & Abrahamse (1969)) can be used to write test power as

$$\mathrm{pr}\left[\left((r-r^*)<0 \,|\, \Sigma(\rho,\lambda)\right)\right] = \mathrm{pr}\left[w'M(A-r^*I_n)Mw < 0\right]$$

$$= \mathrm{pr}\left[\sum_{j=1}^{n} \lambda_j \chi_j^2 < 0\right]$$

where the λ_j's are the eigenvalues of $M(A-r^*I_n)M\Sigma(\rho,\lambda)$ and the χ_j^2's are independent central chi-square variates with one degree of freedom each.

To analyse the effect of the uncorrelated error component, v_t, on test power define

$$\Omega(\rho) = \Sigma(\rho,\lambda) - \Delta$$

where $\Delta = \mathrm{diag}(\delta)$ and $\delta = \frac{1-\lambda}{\lambda} > 0$. The matrices $\Omega(\rho)$ and $\Sigma(\rho,\lambda)$ now define the covariances between the w_t when v_t is absent and present, respectively. The following theorem compares the power of each test under each scenario, for a given (finite sample) design matrix.

Theorem 1

When $E(ww')$ is given by $\Sigma(\rho,\lambda)$ rather than $\Omega(\rho)$, the average value of the test statistic r is increased, when testing against H_a: $\rho > 0$.

Proof

Let $S = M(A-r^*I_n)M$ and define the ij^{th} element of S by s_{ij}. Consider the first moment of $(r-r^*)$ which is given by $E(w'Sw) = \mathrm{tr}(S(E(ww')))$. We must compare $\mathrm{tr}(S\Sigma)$ with $\mathrm{tr}(S\Omega)$.

$$\mathrm{tr}(S\Sigma) = \mathrm{tr}(S(\Omega+\Delta))$$

$$= \mathrm{tr}S\Omega + \sum_{i=1}^{n} s_{ii}\delta$$

$$= \mathrm{tr}S\Omega + \delta\ \mathrm{tr}\ (S)$$

Observe that $t_r(S) = E(r - r^*)|_{\rho=0}$ and recall that $E(r)|_{p=0} > r^*$. Thus $\mathrm{tr}(S) > 0$ and $\mathrm{tr}(S\Sigma) > \mathrm{tr}(S\Omega)$. #

At least on average, therefore, the probability of rejecting H_0 is reduced as λ decreases, which occurs as σ_v^2 becomes large relative to σ_u^2. When the negative alternative $H_a^-:\rho<0$ is used, H_0 is rejected for $r > r^*$. Thus $E(r)|_{\rho=0} < r^*$ and $tr(S) < 0$; the average value of r is therefore reduced and the powers of all tests considered are again lower than would occur if σ_v^2 were zero.

It is clear that the standard exact tests against AR(1) alternatives are less powerful when a second, independent, component is present in the error term. To establish the magnitude of this phenomenon the exact power of each test was evaluated numerically under a variety of data conditions. These evaluations were performed with Davies' (1980) algorithm in the SHAZAM (1993) package.

Figures 1 and 2 are representative of the numerical results obtained with all data matrices.[2] In figure 1, the X matrix is 60 × 3 and comprises an intercept, a linear time trend and a series of drawings from the uniform [0, 10] distribution. Figure 2 shows the power of the BW test using a 20 × 3 matrix in which the regressors are an intercept, a linear trend and drawings from the N(30,4) distribution.

It is apparent from both figures that when $\lambda \neq 1$ the power functions of these tests converge to some small value as ρ approaches unity. This phenomenon is clarified in the following result.

Theorem 2

When $E(ww')$ is given by $\Sigma(\rho,\lambda)$ the limiting power of the DW test as $\rho \to 1$ is the same as the true size of the test, provided the regression has an intercept.

Proof

As $\rho \to 1$, $\Sigma(\rho,\lambda) \to V + \Delta$ where V is a matrix of ones and Δ is defined above. Recall from above that the power of the test depends on the eigenvalues of $S\Sigma(\rho,\lambda)$, being the vector γ which satisfies

$$\gamma z = S\Sigma z \quad \text{for some non-null vector } z$$

or,

$$\gamma z = S(V+\Delta)z.$$

Now when an intercept is present, $MV = SV = 0$ so that

$$\gamma z = S\Delta z$$

$$= \Delta\Sigma z$$

So

$$\frac{\gamma}{\delta} z = Sz.$$

The effect of δ is to scale each eigenvalue by the same factor which does not affect the rejection probability. This probability depends only on the eigenvalues of S and is therefore equal to the true size of the test. #

The above result extends readily to all other tests in the class under consideration by using results of Small (1993).

4. CONCLUSION

The presence of an additional, independent, component in the error term of a standard linear regression model has severe consequences for the powr of a popular class of exact tests for serial independence. Although the size of the tests is entirely robust, the power function of each test must eventually return to the true size, dramatically reducing power against strong positive autocorrelation. These results strongly suggest that the standard exact tests for AR(p) errors are unreliable in an error components model.

REFERENCES

Berenblut, I.I. and G.I. Webb, (1973), A new test for autocorrelated errors in the linear regression model, *Journal of the Royal Statistical Society*, B, 35, 33-50.

Davies, R.B., (1980), The distribution of a linear combination of chi-square random variables, *Applied Statistics*, 29, 323-333.

Durbin, J and G.S. Watson, (1950), Testing for serial correlation in least squares regression I, *Biometrika*, 37, 409-428.

Kadiyala, K.R., (1970), Testing for the independence of regression disturbances, *Econometrica*, 38, 97-117.

King, M.L., (1981), The alternative Durbin-Watson test: an assessment of Durbin and Watson's choice of test statistic, *Journal of Econometrics*, 17, 51-66.

King, M.L., (1982), Testing for a serially correlated component in regression disturbances, *International Economic Review*, 23, 577-582.

King, M.L., (1985), A point-optimal test for autoregressive disturbances, *Journal of Econometrics*, 27, 21-37.

Koerts, J. and A.P.J. Abrahamse, (1969), On the theory and application of the general linear model, *Rotterdam University Press*, Rotterdam.

Revanker, N.S. (1980), Analysis of regressions containing serially correlated and serially uncorrelated error components, *International Economic Review*, 21, 185-199.

SHAZAM Econometrics Computer Program, Users Reference Manual, Version 7.0, (1993) McGraw-Hill, New York.

Small, J.P., (1993), The limiting power of point-optimal autocorrelation tests, forthcoming in Communications in Statistics, Theory and Methods.

FOOTNOTES

1. That is, to any AR process in which only one parameter is non-zero.
2. In both figures power against one sided alternatives in either direction are shown. In all cases, the significance level is 5%.

Figure 1

Power of DW Test in Error Components Model
Uniform Data; n=60

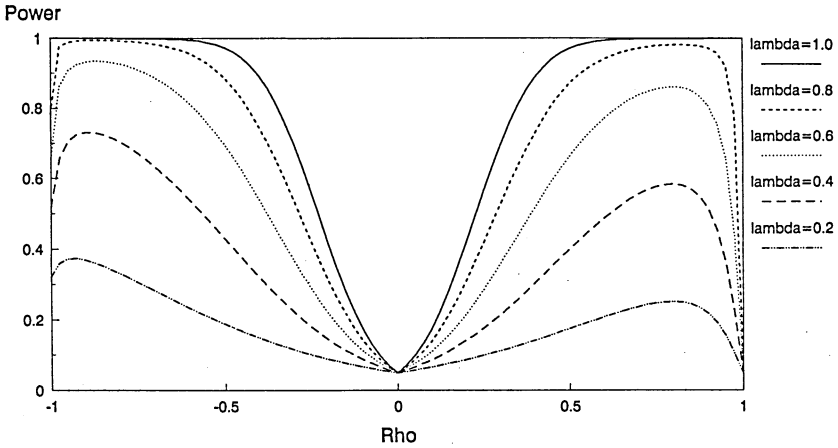
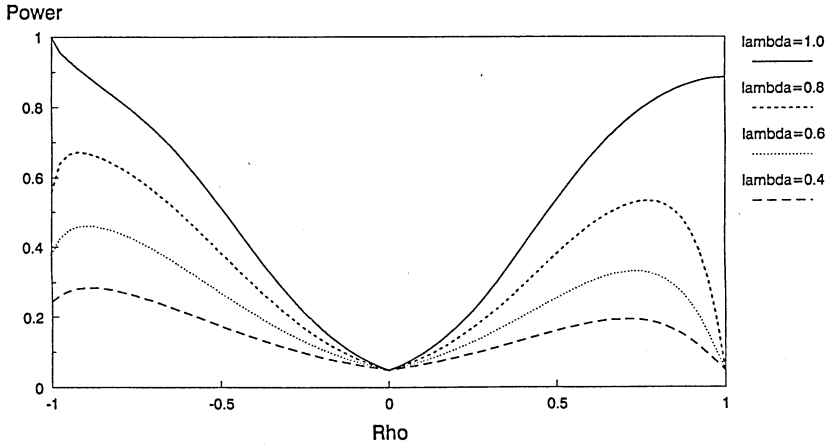


Figure 2

Power of BW Test in Error Components Model
Normal Data; n=20



LIST OF DISCUSSION PAPERS*

No. 8901 Testing for Financial Buffer Stocks in Sectoral Portfolio Models, by P. Dorian Owen.

No. 8902 Provisional Data and Unbiased Prediction of Economic Time Series by Karen Browning and David Giles.

No. 8903 Coefficient Sign Changes When Restricting Regression Models Under Instrumental Variables Estimation, by David E. A. Giles.

No. 8904 Economies of Scale in the New Zealand Electricity Distribution Industry, by David E. A. Giles and Nicolas S. Wyatt.

No. 8905 Some Recent Developments in Econometrics: Lessons for Applied Economists, by David E. A. Giles.

No. 8906 Asymptotic Properties of the Ordinary Least Squares Estimator in Simultaneous Equations Models, by V. K. Srivastava and D. E. A. Giles.

No. 8907 Unbiased Estimation of the Mean Squared Error of the Feasible Generalised Ridge Regression Estimator, by V. K. Srivasatva and D. E. A. Giles.

No. 8908 An Unbiased Estimator of the Covariance Matrix of the Mixed Regression Estimator, by D. E. A. Giles and V. K. Srivastava.

No. 8909 Pre-testing for Linear Restrictions in a Regression Model with Spherically Symmetric Disturbances, by Judith A. Giles.

No. 9001 The Durbin-Watson Test for Autocorrelation in Nonlinear Models, by Kenneth J. White.

No. 9002 Determinants of Aggregate Demand for Cigarettes in New Zealand, by Robin Harrison and Jane Chetwyd.

No. 9003 Unemployment Duration and the Measurement of Unemployment, by Manimay Sengupta.

No. 9004 Estimation of the Error Variance After a Preliminary-Test of Homogeneity in a Regression Model with Spherically Symmetric Disturbances, by Judith A. Giles.

No. 9005 An Expository Note on the Composite Commodity Theorem, by Michael Carter.

No. 9006 The Optimal Size of a Preliminary Test of Linear Restrictions in a Mis-specified Regression Model, by David E. A. Giles, Offer Lieberman, and Judith A. Giles.

No. 9007 Inflation, Unemployment and Macroeconomic Policy in New Zealand: A Public Choice Analysis, by David J. Smyth and Alan E. Woodfield.

No. 9008 Inflation — Unemployment Choices in New Zealand and the Median Voter Theorem, by David J. Smyth and Alan E. Woodfield.

No. 9009 The Power of the Durbin-Watson Test when the Errors are Heteroscedastic, by David E. A. Giles and John P. Small.

No. 9010 The Exact Distribution of a Least Squares Regression Coefficient Estimator After a Preliminary t-Test, by David E. A. Giles and Virendra K. Srivastava.

No. 9011 Testing Linear Restrictions on Coefficients in a Linear Regression Model with Proxy variables and Spherically Symmetric Disturbances, by Kazuhiro Ohtani and Judith A. Giles.

No. 9012 Some Consequences of Applying the Goldfeld-Quandt Test to Mis-Specified Regression Models, by David E. A. Giles and Guy N. Saxton.

No. 9013 Pre-testing in a Mis-specified Regression Model, by Judith A. Giles.

No. 9014 Two Results in Balanced-Growth Educational Policy, by Alan E. Woodfield.

No. 9101 Bounds on the Effect of Heteroscedasticity on the Chow Test for Structural Change, by David Giles and Offer Lieberman.

No. 9102 The Optimal Size of a Preliminary Test for Linear Restrictions when Estimating the Regression Scale Parameter, by Judith A. Giles and Offer Lieberman.

No. 9103 Some Properties of the Durbin-Watson Test After a Preliminary t-Test, by David Giles and Offer Lieberman.

No. 9104 Preliminary-Test Estimation of the Regression Scale Parameter when the Loss Function is Asymmetric, by Judith A. Giles and David E. A. Giles.

No. 9105 On an Index of Poverty, by Manimay Sengupta and Prasanta K. Pattanaik.

No. 9106 Cartels May Be Good For You, by Michael Carter and Julian Wright.

No. 9107 Lp-Norm Consistencies of Nonparametric Estimates of Regression, Heteroskedasticity and Variance of Regression Estimate when Distribution of Regression is Known, by Radhey S. Singh.

(Continued on next page)

No. 9108 Optimal Telecommunications Tariffs and the CCITT, by Michael Carter and Julian Wright.

No. 9109 Price Indices : Systems Estimation and Tests, by David Giles and Ewen McCann.

No. 9110 The Limiting Power of Point Optimal Autocorrelation Tests, by John P. Small.

No. 9111 The Exact Power of Some Autocorrelation Tests When the Disturbances are Heteroscedastic, by John P. Small.

No. 9112 Some Consequences of Using the Chow Test in the Context of Autocorrelated Disturbances, by David Giles and Murray Scott.

No. 9113 The Exact Distribution of R^2 when the Disturbances are Autocorrelated, by Mark L. Carrodus and David E. A. Giles.

No. 9114 Optimal Critical Values of a Preliminary Test for Linear Restrictions in a Regression Model with Multivariate Student-t Disturbances, by Jason K. Wong and Judith A. Giles.

No. 9115 Pre-Test Estimation in a Regression Model with a Misspecified Error Covariance Matrix, by K. V. Albertson.

No. 9116 Estimation of the Scale Parameter After a Pre-test for Homogeneity in a Mis-specified Regression Model, by Judith A. Giles.

No. 9201 Testing for Arch-Garch Errors in a Mis-specified Regression, by David E. A. Giles, Judith A. Giles, and Jason K. Wong.

No. 9202 Quasi Rational Consumer Demand — Some Positive and Normative Surprises, by John Fountain.

No. 9203 Pre-test Estimation and Testing in Econometrics: Recent Developments, by Judith A. Giles and David E. A. Giles.

No. 9204 Optimal Immigration in a Model of Education and Growth, by K-L. Shea and A. E. Woodfield.

No. 9205 Optimal Capital Requirements for Admission of Business Immigrants in the Long Run, by K-L. Shea and A. E. Woodfield.

No. 9206 Causality, Unit Roots and Export-Led Growth: The New Zealand Experience, by David E. A. Giles, Judith A. Giles and Ewen McCann.

No. 9207 The Sampling Performance of Inequality Restricted and Pre-Test Estimators in a Mis-specified Linear Model, by Alan T. K. Wan.

No. 9208 Testing and Estimation with Seasonal Autoregressive Mis-specification, by John P. Small.

No. 9209 A Bargaining Experiment, by Michael Carter and Mark Sunderland.

No. 9210 Pre-Test Estimation in Regression Under Absolute Error Loss, by David E. A. Giles.

No. 9211 Estimation of the Regression Scale After a Pre-Test for Homoscedasticity Under Linex Loss, by Judith A. Giles and David E. A. Giles.

No. 9301 Assessing Starmer's Evidence for New Theories of Choice: A Subjectivist's Comment, by John Fountain.

No. 9302 Preliminary-Test Estimation in a Dynamnic Linear Model, by David E. A. Giles and Matthew C. Cunneen.

No. 9303 Fans, Frames and Risk Aversion: How Robust is the Common Consequence Effect? by John Fountain and Michael McCosker.

No. 9304 Pre-test Estimation of the Regression Scale Parameter with Multivariate Student-t Errors and Independent Sub-Samples, by Juston Z. Anderson and Judith A. Giles

No. 9305 The Exact Powers of Some Autocorrelation Tests When Relevant Regressors are Omitted, by J. P. Small, D. E. Giles and K. J. White.

No. 9306 The Exact Risks of Some Pre-Test and Stein-Type Regression Estimators Under Balanced Loss*, by J. A. Giles, D. E. A. Giles, and K. Ohtani.

No. 9307 The Risk Behavior of a Pre-Test Estimator in a Linear Regression Model with Possible Heteroscedasticity under the Linex Loss Function, by K. Ohtani, D. E. A. Giles and J. A. Giles.

No. 9308 Comparing Standard and Robust Serial Correlation Tests in the Presence of Garch Errors, by John P. Small.

No. 9309 Testing for Serial Independence in Error Components Models: Finite Sample Results, by John P. Small.

* Copies of these Discussion Papers may be obtained for $4 (including postage, price changes occasionally) each by writing to the Secretary, Department of Economics, University of Canterbury, Christchurch, New Zealand. A list of the Discussion Papers prior to 1989 is available on request.