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Department of Economics
UNIVERSITY OF CANTERBURY
CHRISTCHURCH, NEW ZEALAND

ISSN 1171-0705

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Discussion Paper

No. 9307

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April, 1993

Abstract

In this paper, using the asymmetric LINEX loss function, we examine the risk performance of the ordinary least squares estimator (OLSE), two-stage Aitken estimator (2SAE) and pre-test estimator (PTE) after a pre-test for homoscedasticity in a linear regression model with possible heteroscedasticity. It is shown that the 2SAE is dominated by the PTE with the critical value of unity not only under the quadratic loss function but also under the asymmetric LINEX loss function.

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1. Introduction

In applied regression analysis using economic data, the assumption of homoscedasticity is often violated. If inequality of the error variances between two sample periods is suspected, we may conduct a pre-test for homoscedasticity prior to the estimation of the regression coefficients. We may use the ordinary least squares estimator (OLSE) if the null hypothesis of homoscedasticity is accepted in the pre-test, but we may use the two-stage Aitken estimator (2SAE) if it is rejected. Then, the resulting estimator is a pre-test estimator (PTE) after the pre-test for homoscedasticity.

Taylor (1978) derives the exact moments of the 2SAE and examines the performance of the mean squared error (MSE) of the 2SAE. (Since the 2SAE is unbiased, its MSE and variance are the same.) Greenberg (1980) and Ohtani and Toyoda (1980) examine the MSE performance of the PTE. (See also, for example, Mandy (1984), Yancey *et al.* (1984) and Adjibolosoo (1991) for related studies.) In particular, Ohtani and Toyoda (1980) show that the optimal critical value of the pre-test is unity in the sense of minimizing the average relative MSE. They also show that the 2SAE is strictly dominated by the PTE with the critical value of unity if the alternative hypothesis in the pre-test is one-sided.

The existing literature on the OLSE, 2SAE and PTE in a heteroscedastic linear model uses a quadratic loss function. When a quadratic loss function is used, the same penalty is imposed for both positive and negative estimation errors of the same magnitude as it is symmetric about the origin. However, positive (or negative) estimation error may be more serious than negative (or positive) estimation error of the same magnitude in practical situations. If so, the use of the quadratic loss function is inappropriate. Varian (1974) proposes the very useful asymmetric LINEX loss function and

Zellner (1986) uses the LINEX loss function to compare the risk functions of several estimators.

Recently, using the LINEX loss function, Giles and Giles (1993a,b) examine the risk performances of pre-test estimators of the error variance after a pre-test for linear restrictions on the regression coefficients, or after a pre-test for error variance homogeneity. They show that these pre-test estimators can be strictly dominated by the unrestricted estimator when a negative estimation error is deemed to be much more serious than a positive estimation error in the construction of the loss function. This contrasts with the results obtained under quadratic loss, as then the unrestricted estimator of the error variance is strictly dominated by the pre-test estimator if the critical value is chosen appropriately. (See, for example, Ohtani (1988) and Giles (1991).)

In this paper, we examine the risk performance of the OLSE, 2SAE and PTE in a linear regression model with possible heteroscedasticity when the asymmetric LINEX loss function is used. The model and estimators are presented in section 2 and the risk function of the PTE is derived in section 3. It is shown that the necessary condition for the risk function of the PTE to have extrema is that the critical value of the pre-test is zero, unity or infinity. Numerical evaluations in section 4 show that the 2SAE is dominated by the PTE with the critical value of unity, not only under the quadratic loss function, but also under the asymmetric LINEX loss function. Some final comments appear in section 5.

2. The Model and Estimators

We consider the following heteroscedastic linear regression model:

$$\begin{aligned} y_1 &= X_1\beta + \varepsilon_1, \\ y_2 &= X_2\beta + \varepsilon_2, \end{aligned} \qquad (1)$$

where, for $i = 1$ and 2, y_i is an $n_i \times 1$ vector of observations on the dependent variable, X_i is an $n_i \times k$ matrix of non-stochastic regressors, β is a $k \times 1$ vector of common regression coefficients, and ε_i is an $n_i \times 1$ vector of error terms distributed as $N(0, \sigma_i^2 I_{n_i})$. We assume that X_1 and X_2 are of rank k ($<n_i$), and ε_1 and ε_2 are mutually independent.

The OLSE of β is

$$b = (X_1'X_1+X_2'X_2)^{-1}(X_1'y_1+X_2'y_2) , \tag{2}$$

and the 2SAE is

$$\hat{\beta} = \left[(1/s_1^2)X_1'X_1 + (1/s_2^2)X_2'X_2\right]^{-1}\left[(1/s_1^2)X_1'y_1 + (1/s_2^2)X_2'y_2\right] , \tag{3}$$

where $s_i^2 = (y_i-X_ib_i)'(y_i-X_ib_i)/\nu_i$, $\nu_i = n_i-k$ and $b_i = (X_i'X_i)^{-1}X_i'y_i$.

We assume that the null hypothesis in the pre-test is H_0: $\sigma_1^2 = \sigma_2^2$ and the alternative hypothesis is H_A: $\sigma_1^2 > \sigma_2^2$. As shown in Greenberg (1980), the one-sided alternative appears, for example, in a reparameterized version of an error component model. The test statistic s_1^2/s_2^2 is F-distributed with ν_1 and ν_2 degrees of freedom under the null hypothesis.

The PTE for β is

$$\hat{\beta}^* = I_{(0,d)}(s_1^2/s_2^2)b + I_{[d,\infty)}(s_1^2/s_2^2)\hat{\beta} , \tag{4}$$

where d is a critical value of the pre-test, and $I_{(0,d)}(s_1^2/s_2^2) = 1$ if $0 < s_1^2/s_2^2 < d$ and $I_{(0,d)}(s_1^2/s_2^2) = 0$ if $s_1^2/s_2^2 \geq d$. $I_{[d,\infty)}(s_1^2/s_2^2)$ is defined similarly.

Let P be a $k \times k$ matrix with full rank such that

$$P'X_1'X_1P = I_k ,$$

$$P'X_2'X_2P = \Lambda_k ,$$

where Λ_k is the diagonal matrix whose diagonal elements, $\lambda_1, \lambda_2, \ldots, \lambda_k$ (>0), are the roots of the polynomial

$$|X_2'X_2 - \lambda X_1'X_1| = 0.$$

Denoting $Z_i = X_i P$ $(i = 1,2)$ and $\gamma = P^{-1}\beta$, model (1) is reparameterized as

$$y_i = Z_i\gamma + \varepsilon_i, \qquad i = 1,2. \tag{5}$$

The OLSE, 2SAE and PTE for γ are, respectively,

$$\hat{c} = (I_k+\Lambda_k)^{-1}(Z_1'y_1+Z_2'y_2) , \tag{6}$$

$$\hat{\gamma} = \left[(1/s_1^2)I_k+(1/s_2^2)\Lambda_k\right]^{-1}\left[(1/s_1^2)Z_1'y_1+(1/s_2^2)Z_2'y_2\right] , \tag{7}$$

$$\hat{\gamma}^* = I_{(0,d)}(s_1^2/s_2^2)\hat{c} + I_{[d,\infty)}(s_1^2/s_2^2)\hat{\gamma} , \tag{8}$$

where $s_i^2 = (y_i-Z_ic_i)'(y_i-Z_ic_i)/\nu_i = (y_i-X_ib_i)'(y_i-X_ib_i)/\nu_i$, $c_1 = Z_1'y_1$ and $c_2 = \Lambda_k^{-1}Z_2'y_2$.

Denoting the j-th column vector of Z_i as Z_{ij} and the j-th element of γ as γ_j $(i = 1,2,\ j = 1,2,\ldots,k)$, the OLSE, 2SAE and PTE for γ_j are

$$\hat{c}_j = \left[1/(1+\lambda_j)\right](Z_{1j}'y_1+Z_{2j}'y_2) , \tag{9}$$

$$\hat{\gamma}_j = \left[s_1^2s_2^2/(s_2^2+\lambda_j s_1^2)\right]\left[(1/s_1^2)Z_{1j}'y_1+(1/s_2^2)Z_{2j}'y_2\right] , \tag{10}$$

$$\hat{\gamma}_j^* = I_{(0,d)}(s_1^2/s_2^2)\hat{c}_j + I_{[d,\infty)}(s_1^2/s_2^2)\hat{\gamma}_j . \tag{11}$$

3. Risk Functions

As the PTE reduces to the OLSE when $d\to\infty$ and to the 2SAE when $d = 0$, the risk functions of the OLSE and 2SAE can be derived from that of the PTE. Thus, it is sufficient to derive the risk function of the PTE. Hereafter, we consider the estimators of individual coefficients in the reparameterized model (5).

The asymmetric LINEX loss function for $\hat{\gamma}_j^*$ is of the form

$$L(\Delta_j) = \exp(a\Delta_j) - a\Delta_j - 1 , \tag{12}$$

where

$$\Delta_j = (\hat{\gamma}^*_j - \gamma_j)/\sigma_1$$

is the relative estimation error, and the parameter 'a' determines the asymmetry of $L(\Delta_j)$ about the origin. If a is positive, positive estimation error is more serious than negative estimation error of the same magnitude, and *vice versa*. If a^j ($j \geq 3$) is close to zero, the loss function is approximately quadratic.

Using the Taylor series expansion, the risk function of $\hat{\gamma}^*_j$ with the LINEX loss function is

$$R(\hat{\gamma}^*_j) = E\left[L(\Delta_j)\right] = E\left[\sum_{i=0}^{\infty} (a\Delta_j)^i/i! - a\Delta_j - 1\right]$$

$$= \sum_{i=2}^{\infty} a^i E\left[(\hat{\gamma}^*_j - \gamma_j)^i\right]/(\sigma_1^i i!) \ . \qquad (13)$$

The i-th moment of $\hat{\gamma}^*_j - \gamma_j$ is given by the following lemma:

Lemma 1.

The odd and even-ordered moments of $\hat{\gamma}^*_j - \gamma_j$ are:

$$E\left[(\hat{\gamma}^*_j - \gamma_j)^{2m+1}\right] = 0 \ , \qquad (14)$$

$$E\left[(\hat{\gamma}^*_j - \gamma_j)^{2m}\right] = \left[(2\sigma_2^2)^m / \left\{\pi(1+\lambda_j)^{2m}\right\}\right] I_{d*}(\nu_1/2, \nu_2/2)$$

$$\times \sum_{q=0}^{m} {}_{2m}C_{2q} \lambda_j^{m-q} \theta^q \Gamma(q+1/2)\Gamma(m-q+1/2)$$

$$+ \left[(2\sigma_2^2)^m / \left\{\pi B(\nu_1/2, \nu_2/2)\right\}\right]$$

$$\times \sum_{q=0}^{m} {}_{2m}C_{2q} \lambda_j^{m-q} \theta^{2m-q} \Gamma(q+1/2)\Gamma(m-q+1/2)$$

$$\times \nu_1^{2q} \nu_2^{2(m-q)} J_{d*}(\nu_1, \nu_2, m, q, \theta, \lambda_j) \ , \qquad (15)$$

where $\theta = \sigma_1^2/\sigma_2^2 \geq 1$, $\Gamma(.)$ is the gamma function, $B(.,.)$ is the beta function, $I_{d*}(.,.)$ is the incomplete beta function, $d* = \nu_1 d/(\nu_1 d+\nu_2\theta)$, ${}_aC_b = a!/\left[b!(a-b)!\right]$, a, b = 0,1,2,..., and

$$J_{d*}(\nu_1,\nu_2,m,q,\theta,\lambda_j) = \int_{d*}^{1} t^{\nu_1/2+2(m-q)-1}$$

$$\times (1-t)^{\nu_2/2+2q-1}\left[\nu_1(1-t)+\theta\lambda_j\nu_2 t\right]^{-2m} dt .$$

Proof.

See the Appendix.

Substituting (14) and (15) into equation (13) and noting that $\sigma_1^2 = \theta\sigma_2^2$, we have:

Theorem 1.

The risk function of $\hat{\gamma}_j^*$ is

$$R(\hat{\gamma}_j^*) = \sum_{m=1}^{\infty}\left[a^{2m}/(2m)!\right]\left[2^m/\left\{\pi(1+\lambda_j)^{2m}\right\}\right] I_{d*}(\nu_1/2,\nu_2/2)$$

$$\times \sum_{q=0}^{m} {}_{2m}C_{2q}\lambda_j^{m-q}\ \theta^{q-m}\Gamma(q+1/2)\Gamma(m-q+1/2)$$

$$+ \sum_{m=1}^{\infty}\left[a^{2m}/(2m)!\right]\left[2^m/\left\{\pi B(\nu_1/2,\nu_2/2)\right\}\right]$$

$$\times \sum_{q=0}^{m} {}_{2m}C_{2q}\lambda_j^{m-q}\ \theta^{m-q}\Gamma(q+1/2)\Gamma(m-q+1/2)$$

$$\times \nu_1^{2q}\nu_2^{2(m-q)} J_{d*}(\nu_1,\nu_2,m,q,\theta,\lambda_j) . \qquad (16)$$

To examine the extrema of this risk function, the following lemma is useful:

Lemma 2.

By straightforward differentiation, we obtain:

$$\partial I_{d*}(\nu_1/2,\nu_2/2)/\partial d$$

$$= \nu_1^{\nu_1/2}\nu_2^{\nu_2/2}\theta^{\nu_2/2}d^{\nu_1/2-1}(\nu_1 d+\nu_2\theta)^{-(\nu_1+\nu_2)/2}/B(\nu_1/2,\nu_2/2)\ , \qquad (17)$$

$$\partial J_{d*}(\nu_1,\nu_2,m,q,\theta,\lambda_j)/\partial d$$

$$= -\nu_1^{\nu_1/2-2q}\nu_2^{\nu_2/2+2(q-m)}\theta^{\nu_2/2+2(q-m)}d^{\nu_1/2+2(m-q)-1}$$

$$\times\ (\nu_1 d+\nu_2\theta)^{-(\nu_1+\nu_2)/2}(1+\lambda_j d)^{-2m}\ . \qquad (18)$$

So, using Lemma 1 and Lemma 2, we obtain:

$$\partial E\left[(\hat{\gamma}_j^*-\gamma_j)^{2m}\right]/\partial d$$

$$= \left[(2\sigma_2^2)^m\nu_1^{\nu_1/2}\nu_2^{\nu_2/2}\theta^{\nu_2/2}d^{\nu_1/2-1}\right]\left[\pi B(\nu_1/2,\nu_2/2)\right]^{-1}$$

$$\times\ (\nu_1 d+\nu_2\theta)^{-(\nu_1+\nu_2)/2}\sum_{q=0}^{m}{}_{2m}C_{2q}\lambda_j^{m-q}\theta^q\Gamma(q+1/2)\Gamma(m-q+1/2)$$

$$\times\ \left[(1+\lambda_j)^{-2m}-d^{2(m-q)}(1+\lambda_j d)^{-2m}\right]\ . \qquad (19)$$

From (19), we see that if $d = 0$, 1 or ∞, then the first derivatives of all even-ordered moments of $\hat{\gamma}_j^* - \gamma_j$ are zero for $\nu_1 \geq 3$. Also, as Lemma 1 shows that all odd-ordered moments of $\hat{\gamma}_j^* - \gamma_j$ are zero, we have:

Theorem 2.

For $\nu_1 \geq 3$, the necessary condition for the risk function of $\hat{\gamma}_j^*$ to have extrema is $d = 0$, 1, or ∞.

Using the quadratic loss function, Ohtani and Toyoda (1980) show that the 2SAE is strictly dominated by the PTE with d = 1. Although Theorem 2 gives only the necessary condition, it indicates that their result may still hold when the asymmetric LINEX loss function is used. As further exact analysis of the risk function seems difficult, we compare the risk functions of the OLSE, 2SAE and PTE numerically in the next section.

4. Numerical Evaluations

The parameter values used in the numerical evaluations are; (ν_1,ν_2) = (10,10), (10,30), (20,20), (30,10), (30,30); $\lambda_j = 1.0$, $j = 1,2,,\ldots,k$; $d = 0$ (OLSE), $d_{0.05}$ (critical value corresponding to the size 0.05), 1.0, ∞ (2SAE); a = 0.1, 1, 3, 5; θ = various values. As the risk function given in (16) does not include the term a^{2m+1}, it is sufficient to examine positive values of a. The integral in the expression for J_{d*} is evaluated by Simpson's rule with 200 equal subdivisions, and the infinite series in (16) converges with a convergence tolerance of 10^{-15}.

Some typical results are shown in Figures 1 to 3. These graphs depict relative risk, defined as $R(\hat{\gamma}_j^*)/R(\hat{\gamma}_j)$. Thus, the relative risk is less than unity if the PTE has smaller risk than the 2SAE, and the relative risk of the 2SAE is unity. Note that the horizontal axis measures $1/\theta$, and not θ itself.

From the figures, we see the following. When a = 0.1 (Figure 1), as expected, the risk performance of the estimators is similar to the case of quadratic loss. The 2SAE is dominated by the PTE with d = 1, and the relative risk of the PTE with d = 1 increases monotonically towards unity from below as θ increases. Figures 2 and 3 show that the PTE with d = 1 still dominates the 2SAE even if the asymmetry of the loss function is large. This supports our conjecture that Ohtani and Toyoda's (1980) result is extended to the case of this asymmetric loss function. In the context of estimating the error variance after pre-tests for linear restrictions on the regression coefficients, or for error variance homogeneity, Giles and Giles (1993a,b) show that the result obtained under quadratic loss need not be robust to the change in the loss function. Thus, our result contrasts with theirs.

As 'a' increases, the point of intersection of the relative risks of the OLSE and 2SAE shifts to the right. This indicates that the region of θ where the risk of the OLSE is smaller than that of the 2SAE increases as the degree of asymmetry increases. We find a similar tendency for the point of intersection of the relative risks of the PTE with $d_{0.05}$ and 2SAE. However, the point of intersection of the relative risks of the OLSE and the PTE with $d = 1$ is not so sensitive to the increase in a. Also, when $a = 5$, the relative risk of the PTE with $d = 1$ is U-shaped. This is not observed when the loss is quadratic.

5. Conclusions

In this paper we have extended the analysis of an important pre-testing problem to the case where the loss function is asymmetric. Estimating the coefficients of a regression model after testing for possible heteroscedasticity across sub-samples is common practice, and often the researcher faces a greater implicit loss by (say) over-estimating certain coefficients than by under-estimating them, or *vice versa*. When the loss function is quadratic (and hence symmetric), it is well known that the strategy of testing for homoscedasticity, and then using either the least squares or two-stage Aitken estimator accordingly, strictly dominates the strategy of applying the latter estimator without such a test. We have shown that this result is robust to asymmetry (in either direction) in the loss function, and that the potential risk gains increase with the degree of such asymmetry. Whilst least squares estimation can still be the preferred strategy if the degree of heteroscedasticity is mild, the risk associated with this strategy is unbounded under extreme heteroscedasticity, whether the loss function is symmetric or not. Work in progress by the authors considers the robustness of these results to other choices of loss function outside the LINEX family.

Appendix: Proof of Lemma 1

Define $u_{1j} = c_{1j} - \gamma_j$ and $u_{2j} = c_{2j} - \gamma_j$, which are independently distributed as $N(0, \sigma_1^2)$ and $N(0, \sigma_2^2/\lambda_j)$, respectively. Also, let $w = s_1^2/(\theta s_2^2)$, where $\theta = \sigma_1^2/\sigma_2^2$. Then, w is F-distributed with ν_1 and ν_2 degrees of freedom.

Using the above notation, $\hat{\gamma}_j^* - \gamma_j$ reduces to

$$I_{(0,d)}(\theta w)(u_{1j}+\lambda_j u_{2j})/(1+\lambda_j) + I_{[d,\infty)}(\theta w)(u_{1j}+\theta\lambda_j w u_{2j})/(1+\theta\lambda_j w). \tag{A.1}$$

As $I_{(0,d)}(\theta w)\times I_{[d,\infty)}(\theta w) = 0$, the i-th moment of $\hat{\gamma}_j^* - \gamma_j$ is

$$\begin{aligned} E\left[(\hat{\gamma}_j^* - \gamma_j)^i\right] &= E\left[I_{(0,d)}(\theta w)\left\{(u_{1j}+\lambda_j u_{2j})/(1+\lambda_j)\right\}^i\right] \\ &\quad + E\left[I_{[d,\infty)}(\theta w)\left\{(u_{1j}+\lambda_j \theta w u_{2j})/(1+\theta\lambda_j w)\right\}^i\right] \\ &\equiv E_1 + E_2. \end{aligned} \tag{A.2}$$

First, we evaluate the first term in (A.2) denoted as E_1. Using the binomial expansion, E_1 is

$$E_1 = E\left[I_{(0,d)}(\theta w)(1+\lambda_j)^{-i} \sum_{r=0}^{i} {}_iC_r u_{1j}^r (\lambda_j u_{2j})^{i-r}\right] . \tag{A.3}$$

As all odd-ordered moments of u_{ij} and u_{2j} are zero, and u_{ij} and u_{2j} are independent we have

$$E[u_{1j}^{r} u_{2j}^{i-r}] = 0$$

for all odd i. (If i and r are odd, i-r is even. Also, if i is odd and r is even, i-r is odd. Thus, E_1 is zero for all odd i.) Noting that $E[u_{1j}^{r}] = 0$ for all odd r, (A.3) reduces to

$$(1+\lambda_j)^{-2m} \sum_{q=0}^{m} {}_{2m}C_{2q} \lambda_j^{2(m-q)} E\left[I_{(0,d)}(\theta w) u_{1j}^{2q} u_{2j}^{2(m-q)}\right] , \qquad \text{(A.4)}$$

with i = 2m and r = 2q.

Let $v_1 = u_{1j}^2/\sigma_1^2$ and $v_2 = \lambda_j u_{2j}^2/\sigma_2^2$, so that v_1 and v_2 are χ^2-distributed with one degree of freedom. As v_1, v_2 and w are mutually independent, the expectation in (A.4) is

$$E\left[I_{(0,d)}(\theta w)(\sigma_1^2 v_1)^q (\sigma_2^2 v_2/\lambda_j)^{m-q}\right]$$

$$= (\sigma_1^2)^q (\sigma_2^2/\lambda_j)^{m-q} E_w\left[I_{(0,d/\theta)}(w)\right] E_{v_1}[v_1^q] E_{v_2}[v_2^{m-q}] , \qquad \text{(A.5)}$$

where $E_X[X]$ denotes the expectation of the random variable X.

It is easy to see that

$$E_{v_i}[v_i^f] = 2^f \Gamma(f+1/2)/\Gamma(1/2) , \qquad \text{(A.6)}$$

$$E_w\left[I_{(0,d/\theta)}(w)\right] = I_{d*}(\nu_1/2, \nu_2/2) , \qquad \text{(A.7)}$$

where $I_{d*}(\nu_1/2,\nu_2/2)$ is the incomplete beta function and $d* = \nu_1 d/(\nu_1 d + \nu_2 \theta)$. Substituting (A.6) and (A.7) in (A.5), and then substituting (A.5) in (A.4), E_1 is finally written as

$$E_1 = \left[2^m (\sigma_2^2)^m / \left\{\pi (1+\lambda_j)^{2m}\right\}\right] I_{d*}(\nu_1/2, \nu_2/2) \sum_{q=0}^{m} {}_{2m}C_{2q} \lambda_j^{m-q} \theta^q$$

$$\times\ \Gamma(q+1/2)\Gamma(m-q+1/2). \tag{A.8}$$

Next, we evaluate the second term in (A.2) denoted as E_2. Using the binomial expansion, E_2 is

$$E_2 = E\left[I_{[d,\infty)}(\theta w)(1+\theta\lambda_j w)^{-i} \sum_{r=0}^{i} {}_iC_r u_{1j}^r(\theta\lambda_j w u_{2j})^{i-r}\right] . \tag{A.9}$$

By the same reason as in the derivation of E_1, E_2 is zero for all odd i. Thus, all odd-ordered moments of $\hat{\gamma}_j^* - \gamma_j$ are zero. Putting i = 2m and r = 2q, (A.9) reduces to

$$\sum_{q=0}^{m} {}_{2m}C_{2q}(\theta\lambda_j)^{2(m-q)} E\left[I_{[d,\infty)}(\theta w)u_{1j}^{2q}u_{2j}^{2(m-q)}w^{2(m-q)}\right.$$

$$\left.\times\ (1+\theta\lambda_j w)^{-2m}\right]$$

$$= \sum_{r=0}^{m} {}_{2m}C_{2q}(\theta\lambda_j)^{2(m-q)}(\sigma_1^2)^q(\sigma_2^2/\lambda_j)^{m-q}$$

$$\times\ E_w\left[I_{[d/\theta,\infty)}(w)\left[w^{2(m-q)}/(1+\theta\lambda_j w)^{2m}\right]\right]E_{v_1}[v_1^q]E_{v_2}[v_2^{m-q}]. \tag{A.10}$$

The first expectation (over w) in (A.10) is

$$\int_{d/\theta}^{\infty}\left[w^{2(m-q)}/(1+\theta\lambda_j w)^{2m}\right]\left[\nu_1^{\nu_1/2}\nu_2^{\nu_2/2}/B(\nu_1/2,\nu_2/2)\right]$$

$$\times\ w^{\nu_1/2-1}(\nu_1 w+\nu_2)^{-(\nu_1+\nu_2)/2}dw . \tag{A.11}$$

Making use of the change of variable, $t = \nu_1 w/(\nu_1 w+\nu_2)$, (A.11) reduces to

$$\left[\nu_1^{2q}\nu_2^{2(m-q)}/B(\nu_1/2,\nu_2/2)\right]J_{d*}(\nu_1,\nu_2,m,q,\theta,\lambda_j) \tag{A.12}$$

where

$$J_{d*}(\nu_1,\nu_2,m,q,\theta,\lambda_j) = \int_{d*}^{1} t^{\nu_1/2+2(m-q)-1}(1-t)^{\nu_2/2+2q-1}$$

$$\times\ \left[\nu_1(1-t)+\theta\lambda_j\nu_2 t\right]^{-2m}dt .$$

Substituting (A.6) and (A.12) in (A.10), E_2 is finally written as

$$E_2 = \left[2^m(\sigma_2^2)^m / \left\{\pi B(\nu_1/2,\nu_2/2)\right\}\right] \sum_{q=0}^{m} {}_{2m}C_{2q}\,\theta^{2m-q}\lambda_j^{m-q}$$

$$\times\ \nu_1^{2q}\nu_2^{2(m-q)}\Gamma(q+1/2)(\Gamma(m-q+1/2)J_{d*}(\nu_1,\nu_2,m,q,\theta,\lambda_j)\ . \qquad (A.13)$$

Substituting (A.8) and (A.13) in (A.2), we obtain (15) in the text.

References

Adjibolosoo, S.B-S.K., 1991, On choosing the level of significance for the Goldfeld and Quandt heteroskedasticity pretesting, *Communications in Statistics-Simulation and Computation* 20, 437-447.

Giles, J.A., 1991, Pre-testing for linear restrictions in a regression model with spherically symmetric disturbances, *Journal of Econometrics* 50, 377-398.

Giles, J.A. and D.E.A. Giles, 1993a, Preliminary-test estimation of the regression scale parameter when the loss function is asymmetric, *Communications in Statistics - Theory and Methods*, forthcoming.

Giles, J.A. and D.E.A. Giles, 1993b, Risk of a heteroscedasticity pre-test estimator of the regression scale under LINEX loss, *Journal of Statistical Planning and Inference*, forthcoming.

Greenberg, E., 1980, Finite sample moments of a preliminary test estimator in the case of possible heteroscedasticity, *Econometrica* 48, 1805-1813.

Mandy, D.M., 1984, The moments of a pre-test estimator under possible heteroscedasticity, *Journal of Econometrics* 25, 29-33.

Ohtani, K. 1988, Optimal levels of significance of a pre-test in estimating the disturbance variance after the pre-test for a linear hypothesis on coefficients in a linear regression, *Economics Letters* 28, 151-156.

Ohtani, K. and T. Toyoda, 1980, Estimation of regression coefficients after a preliminary test for homoscedasticity, *Journal of Econometrics* 12, 151-159.

Taylor, W.E., 1978, The heteroscedastic linear model: Exact finite sample results, *Econometrica* 46, 663-675.

Varian, H.R., 1974, A Bayesian approach to real estate assessment, in S.E. Fienberg and A. Zellner (eds.), *Studies in Bayesian Econometrics and Statistics in Honor of Leonard J. Savage,* North-Holland, Amsterdam, 195-208.

Yancey, T.A., G.G. Judge and S. Miyazaki, 1984, Some improved estimators in the case of possible heteroscedasticity, *Journal of Econometrics* 25, 133-150.

Zellner, A., 1986, Bayesian estimation and prediction using asymmetric loss functions, *Journal of the American Statistical Association* 81, 446-451.

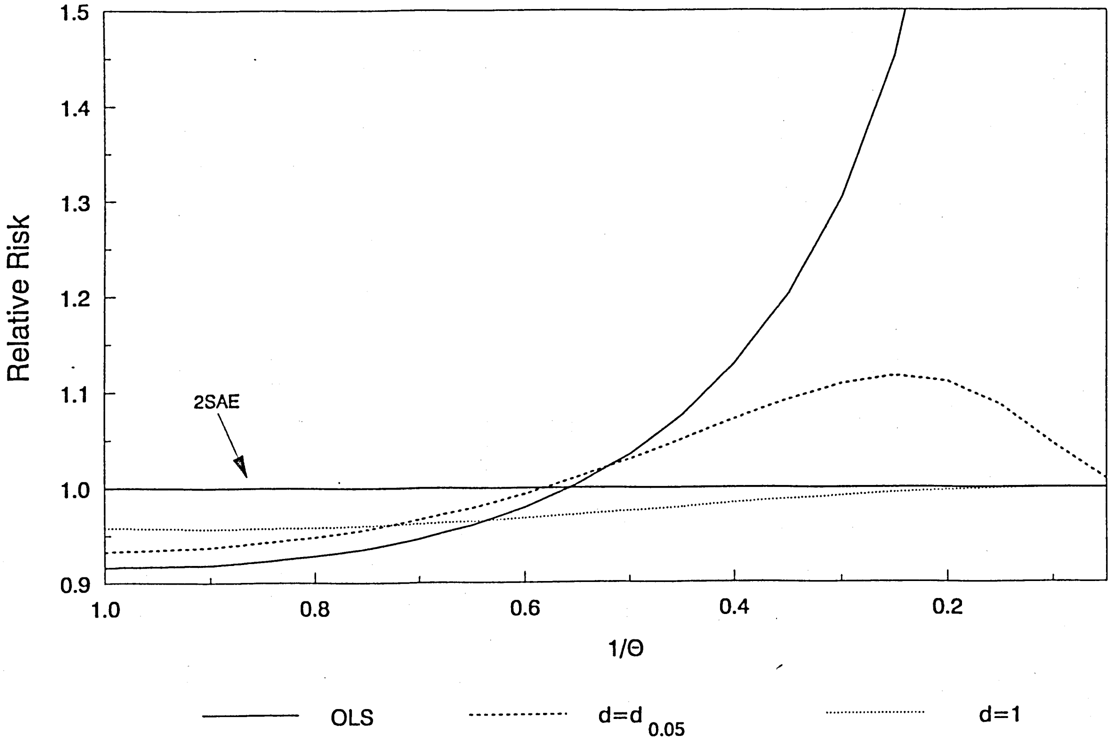

Figure 1. Relative risk functions for $\nu_1 = \nu_2 = 10$ when $a = 0.1$.

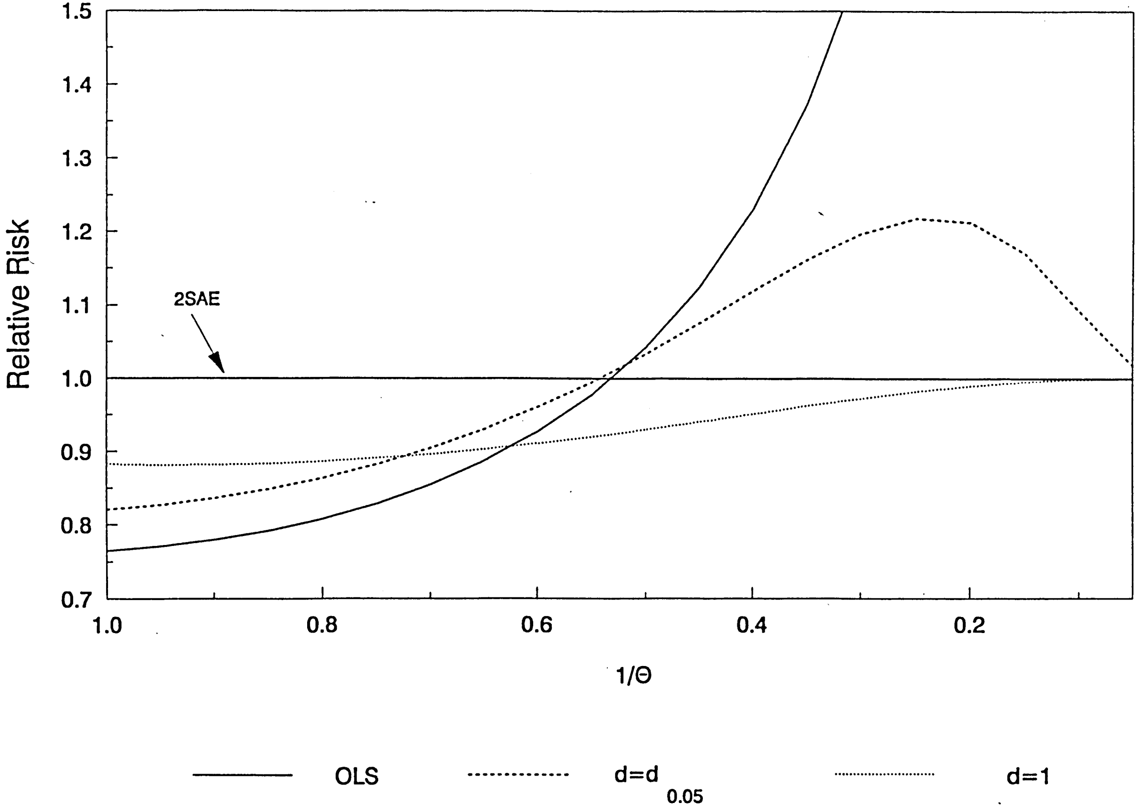

Figure 2. Relative risk functions for $\nu_1 = \nu_2 = 10$ when a = 3.

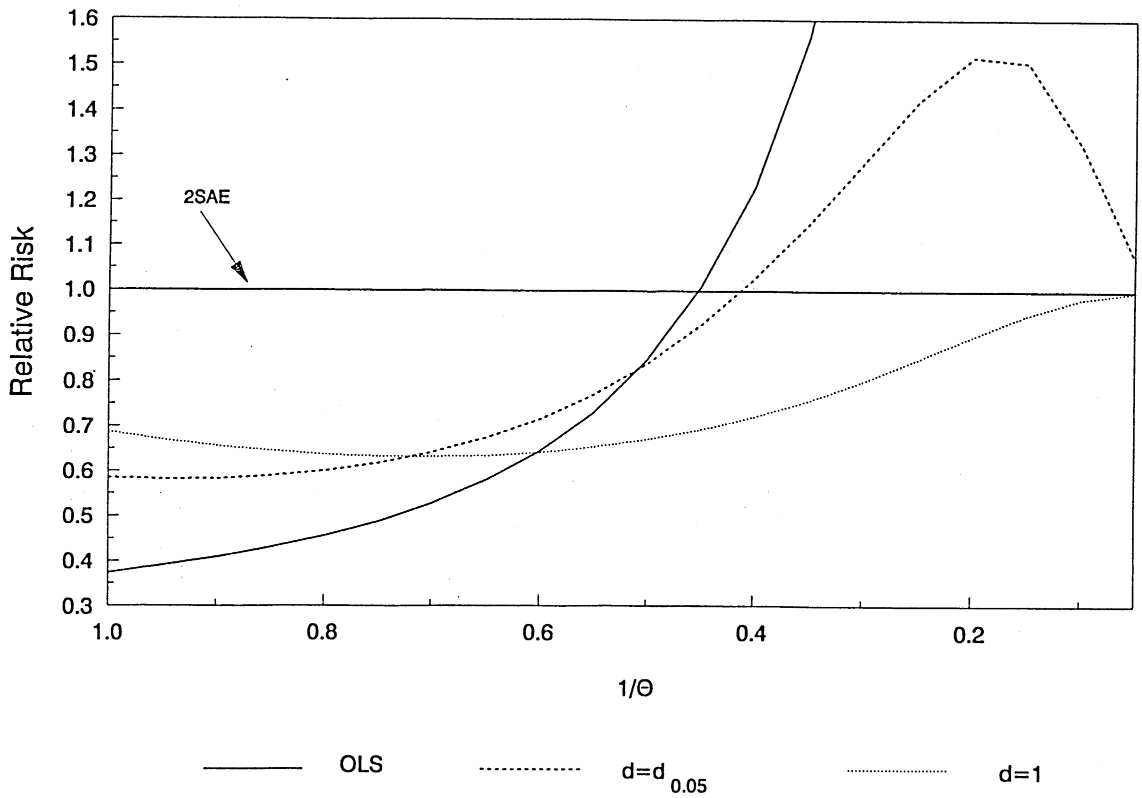

Figure 3. Relative risk functions for $\nu_1 = \nu_2 = 10$ when $a = 5$.

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