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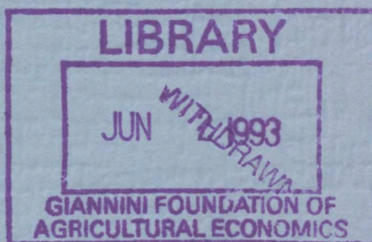
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THE EXACT RISKS OF SOME PRE-TEST
AND STEIN-TYPE REGRESSION ESTIMATORS
UNDER BALANCED LOSS

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Discussion Paper

No. 9306

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April 1993

ABSTRACT

In regression analysis we are often interested in using an estimator which is "precise" and which simultaneously provides a model with "good fit". In this paper we consider the risk properties of several estimators of the regression coefficient vector under "balanced" loss. This loss function (Zellner (1991)) reflects both of the above attributes. Under a particular form of balanced loss, we derive the predictive risk of the pre-test estimator which results after a test for exact linear restrictions on the coefficient vector. The corresponding risks of a Stein-rule and positive-part Stein-rule estimators are also established. The risks based on loss functions which allow only for estimation precision, or only for goodness of fit, are special cases of our results, and we draw appropriate comparisons. In particular, we show that some of the well-known results under (quadratic) precision-only loss are *not* robust to our generalisation of the loss function.

KEY WORDS: Linear regression; shrinkage estimators; estimation precision; goodness of fit; conditional inference

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1. INTRODUCTION

In regression analysis we are often concerned both with the precision with which the coefficient vector is estimated, and with the "goodness of fit" of the overall model. However, typically this dual interest has not been taken into account in the construction of estimators in this context, or in the formulation of the loss function when comparing estimators' performances on the basis of "risk".

This has motivated Zellner (1991) to suggest the (quadratic) "Balanced" Loss Function (BLF), which explicitly takes account of *both* goodness of fit *and* estimation precision. Zellner analyses several standard estimation problems, including the linear regression model, and develops estimators which are optimal relative to a BLF. Rodrigues and Zellner (1992) extend this analysis by generalising the loss function still further, and consider a further application. In this paper we use the quadratic BLF as the basis for examining two somewhat related regression problems.

First, we investigate the consequences of adopting a common "pre-testing" strategy when estimating the regression coefficient vector. Specifically, we consider the case where *either* (unrestricted) Ordinary Least Squares (OLS) or Restricted Least Squares (RLS) is used, depending on the outcome of a prior test of the validity of certain exact linear restrictions on the coefficients. The BLF risks of the OLS and RLS estimators are compared with that of the Pre-Test Estimator (PTE), and the results are shown to differ from their well known counterparts based on the standard (quadratic) precision-only loss function (e.g., Judge and Bock (1978)).

Second, we consider Stein-Rule (SR) and Positive-part Stein Rule (PSR) estimators for the regression coefficient vector (e.g., Stein (1956) and James and Stein (1961), and Baranchik (1964) and Stein (1966) respectively), using (quadratic) BLF risk as the basis of comparison with OLS estimation. It is

found that the standard risk-dominance results that are associated with the usual quadratic precision-only loss generalise naturally to the BLF case, provided that the shrinkage factor is chosen appropriately. The appropriate such choice differs from that under precision-only loss.

By considering the properties of these regression estimators under the BLF we add to our understanding of the robustness of certain key results to the choice of performance measure. Other recent examples of this type of investigation have focused on the use of the asymmetric LINEX loss function (Varian (1974) and Zellner (1986)), of which quadratic loss is a special case. Recent LINEX risk evaluations in the regression context include those of Giles and Giles (1993a,b), Ohtani *et al.* (1993) and the references cited therein.

Section 2 introduces the model and notation more formally, and the "pre-test" estimation problem is discussed and analysed in Section 3. Section 4 deals with the SR and PSR estimators, and some general discussion is given in Section 5. Proofs of the main analytic results appear in Appendices A and B.

2. MODEL AND NOTATION

We are concerned with estimating the coefficients in the linear regression model

$$y = X\beta + \epsilon \quad (1.1)$$

where $\epsilon \sim N(0, \sigma^2 I)$, X is non-stochastic, of full rank, and $(T \times k)$, β is $(k \times 1)$ and y and ϵ are $(T \times 1)$.

If $\bar{\beta}$ is some estimator of β , and $L(\bar{\beta}, \beta)$ is a loss function which takes zero value if $\bar{\beta} = \beta$ and is otherwise strictly positive, then the risk associated with $\bar{\beta}$ is

$$R(\bar{\beta}) = E \left[L(\bar{\beta}, \beta) \right] \quad (1.2)$$

where the expectation is taken over the sample space.

For example, if we have quadratic loss, then

$$L_Q(\bar{\beta}, \beta) = (\bar{\beta} - \beta)' A (\bar{\beta} - \beta) \quad (1.3)$$

where A is a non-stochastic matrix of weights. Choosing $A = S = X'X$, it is well known that the risk associated with using $X\bar{\beta}$ as a *predictor* of $E(y|X)$ is the same as the risk of using $\bar{\beta}$ as an *estimator* of β if the regressors are orthonormal. We shall exploit this idea to make the results below independent of the X data.

The loss function in (1.3) focuses only on estimator (or predictor) *precision*. Zellner's (1991) BLF is a generalisation of (1.3) to

$$L_B(\bar{\beta}, \beta) = w(y - X\bar{\beta})'(y - X\bar{\beta}) + (1-w)(\bar{\beta} - \beta)' S(\bar{\beta} - \beta) \quad (1.4)$$

where $w \in [0,1]$ is non-stochastic, and the first term in (1.4) clearly allows for "goodness of fit".

3. PRE-TEST ESTIMATION

Suppose that in estimating (1.1), we are also interested in the possibility that β should be constrained. In particular, suppose that, potentially, β satisfies $R\beta = r$, where R and r are known and non-stochastic, R is $(m \times k)$ and of rank m, and r is $(m \times 1)$. Such restrictions are encountered frequently in applied regression analysis. In particular, this formulation allows for "zero" restrictions on one or more elements of β . So, the estimation of the regression model after a prior test for the significance of regressors is covered by our analysis here.

If the restrictions in question are valid, there is an incentive to impose them and use the Restricted Least Squares estimator (which is also the restricted maximum likelihood estimator) of β . Ignoring them in this case, and using the Ordinary Least Squares (unrestricted maximum likelihood) estimator will result in inefficient estimation. On the other hand, if the restrictions are invalid, their imposition will result in an RLS estimator

which is biased, but it may have smaller MSE than OLS (e.g., see Toro-Vizcarrondo and Wallace (1968)).

Clearly, there is an incentive to (pre-) test the hypothesis, $H_0: R\beta = r$ (vs. $H_A: R\beta \neq r$) and choose the RLS or OLS estimator according to the outcome of this test. In the special case of a single zero restriction on one coefficient when $k = 2$, this problem was first considered by Bancroft (1944), who derived the bias of the least squares pre-test estimator of the other coefficient. The (quadratic) risk properties of the above general pre-test estimator of β are discussed, for example, by Judge and Bock (1978), the choice of an optimal significance level for the pre-test is considered by Brook (1976), and much of the recent pre-testing literature associated with inference in the context of the regression model is surveyed by Giles and Giles (1993c).

A uniformly most powerful invariant size- α test of H_0 against H_A is based on the statistic $f = \left[(r - R\tilde{\beta})' (RS^{-1}R')^{-1} (r - R\tilde{\beta}) \right] / (m\tilde{\sigma}^2)$, where $\tilde{\beta} = S^{-1}X'y$ is the OLS estimator of β in (1.1) and $\tilde{\sigma}^2 = (y - X\tilde{\beta})'(y - X\tilde{\beta}) / (T - k)$. The statistic f is non-central F-distributed with m and $\nu (= T - k)$ degrees of freedom, and non-centrality parameter $\lambda = (r - R\beta)' (RS^{-1}R')^{-1} (r - R\beta) / (2\sigma^2)$. We reject H_0 if $f > c$, where $\int_0^c p(f)df = (1 - \alpha)$ and $p(f)$ is central F with m and ν degrees of freedom. The associated pre-test estimator of β is $\hat{\beta} = I_{[0,c]}(f)\beta^* + I_{(c,\infty)}(f)\tilde{\beta} = \tilde{\beta} + (\beta^* - \tilde{\beta})I_{[0,c]}(f)$, where $\beta^* = \tilde{\beta} + S^{-1}R'[RS^{-1}R']^{-1}(r - R\tilde{\beta})$ is the RLS estimator of β , and $I_{[a,b]}(f)$ is an indicator function which is unity if $f \in [a,b]$, and zero otherwise.

While the risks of $\tilde{\beta}$, β^* and $\hat{\beta}$ are well documented under quadratic (precision-only) loss (and Giles (1993) considers the corresponding problem under absolute error loss), our purpose here is to re-consider the situation under the more general BLF in (1.5). Under such a loss structure, the risks of these three estimators are given in the following theorem.

Theorem 3.1

Under the quadratic BLF, (1.5), the risks of $\tilde{\beta}$, β^* and $\hat{\beta}$ are given by

$$R(\tilde{\beta}) = \sigma^2 [wv + (1-w)k] \quad (3.1)$$

$$R(\beta^*) = R(\tilde{\beta}) + \sigma^2 [m(2w-1)+2\lambda] \quad (3.2)$$

$$R(\hat{\beta}) = R(\tilde{\beta}) + \sigma^2 \left[(m(2w-1)+4\lambda(1-w))P_2 + 2\lambda(2w-1)P_4 \right] \quad (3.3)$$

where $P_i = \Pr. \left[F'_{(m+i, \nu; \lambda)} < \left(cm/(m+i) \right); i = 0, 1, \dots \right]$ and $F'_{(n_1, n_2; \delta)}$ is non-central F with n_1 and n_2 degrees of freedom, and non-centrality parameter δ .

Proof

See Appendix A.

Remark 3.1

Clearly, $R(\beta^*) = R(\tilde{\beta})$ when $\lambda = \lambda_0 = m(1-2w)/2$, and $\lambda_0 = 0$ if $w = 0.5$. For $w > (\geq) 0.5$, $\tilde{\beta}$ risk-dominates β^* for all $\lambda \geq (>) 0$. For $w < 0.5$, $R(\beta^*) < R(\tilde{\beta})$ for $\lambda < \lambda_0$.

Remark 3.2

Clearly, $\tilde{\beta}$ risk-dominates $\hat{\beta}$ and $\hat{\beta}$ risk-dominates β^* for all $\lambda \geq (>) 0$ if $w > (\geq) 0.5$.

When $w < 0.5$, the situation is ambiguous, as is typified in¹ Figure 1, which corresponds to the traditional quadratic loss situation.² In that particular case ($w = 0$) it is known that $R(\hat{\beta}) = R(\tilde{\beta})$ at $\lambda = \lambda_1$, where $\lambda_1 \in [m/4, m/2]$ (e.g., see Judge and Bock (1978, p.73)). These bounds on the intersection of $R(\tilde{\beta})$ and $R(\hat{\beta})$ are readily generalised when $w \in [0, 0.5]$, and a BLF is used.

Corollary 3.1

When $w \leq 0.5$, the intersection of $R(\tilde{\beta})$ and $R(\hat{\beta})$ occurs at $\lambda = \lambda^*$, which is bounded by $m(1-2w)/4 \leq \lambda^* \leq m(1-2w)/2$.

Proof

First,

$$\Delta = R(\tilde{\beta}) - R(\hat{\beta}) = \sigma^2 \left\{ \left[m(1-2w) - 2\lambda \right] P_2 + (P_4 - P_2) 2\lambda(1-2w) \right\}$$

and as $P_2 > P_4$, a sufficient condition for $\Delta \leq 0$ is $\lambda \geq m(1-2w)/2$, if $w \leq 0.5$.

Second,

$$\Delta = \sigma^2 \left\{ \left[m(1-2w) - 4\lambda \right] P_2 + 4\lambda w(P_2 - P_4) + 2\lambda P_4 \right\},$$

and as $P_2 > P_4$, a sufficient condition for $\Delta \geq 0$ is $\lambda \leq m(1-2w)/4$, if $w \leq 0.5$.

These results are illustrated further in Figures 2 to 4. The well known result associated with this particular pre-test estimation problem, namely that pre-testing can never be the best of the three strategies considered, and may be the worst (as in Figure 1) is clearly negated if goodness of fit is given sufficient weight in the loss function (*i.e.*, if $w \geq 0.5$). Then, pre-testing is always "second-best" to simply applying OLS with no prior testing of H_0 , and naively applying the RLS estimator is the worst of the three strategies in terms of BLF risk. It is also clear from these figures that when $w \geq 0.5$ the "optimal" choice of pre-test critical value is $c = 0$, which corresponds to OLS estimation. Finally, when $0 < w < 0.5$ the "optimal" choice of c (under a mini-max regret criterion) will be less than that suggested by³ Brook (1976).

4. STEIN-RULE ESTIMATION

A Stein-rule estimator of β in (1.1) is

$$b_{SR} = \left[1 - p(y - X\tilde{\beta})'(y - X\tilde{\beta})/\tilde{\beta}'S\tilde{\beta} \right] \tilde{\beta} \quad (4.1)$$

where $p \in \left[0, 2(k-2)/(\nu+2) \right]$, for $k \geq 3$. It is well known that $R(b_{SR}) < R(\tilde{\beta})$ under quadratic loss for $k \geq 3$, and that the "optimum" choice of p is $p^* = (k-2)/(\nu+2)$, in terms of maximising the risk-gain in using b_{SR} over $\tilde{\beta}$.

Further, the positive-part Stein-rule estimator,

$$b_{PSR} = \max \left[0, 1 - p(y - X\tilde{\beta})'(y - X\tilde{\beta})/\tilde{\beta}'S\tilde{\beta} \right] \tilde{\beta} \quad (4.2)$$

is known to risk-dominate b_{SR} under the same conditions in the case of quadratic loss.

To facilitate our analysis of b_{SR} and b_{PSR} under the BLF, it is convenient to define a different pre-test estimator of β , namely

$$\hat{b} = I_{(a/c^*, \infty)}(f^*)(1 - a/f^*)\tilde{\beta} \quad (4.3)$$

where I is a conventional indicator function; $f^* = \left[(\tilde{\beta}'S\tilde{\beta})/(\tilde{k}\sigma^2) \right]$; $a = (p\nu/k)$; c^* is the critical value for the pre-test of $H_0: X\beta = 0$ vs $H_A: X\beta \neq 0$; and $f^* \sim F'_{(k, \nu; \theta)}$, where $\theta = \beta'S\beta/2\sigma^2$. Then,

- (i) $\hat{b} = \tilde{\beta}$; if $p = 0$.
- (ii) $\hat{b} \rightarrow \hat{b}_{SR}$; if $c^* \rightarrow \infty$.
- (iii) $\hat{b} = \hat{b}_{PSR}$; if $c^* = 1$.

Theorem 4.1

Under the quadratic BLF, (1.5), the risk of \hat{b} is given by

$$R(\hat{b}) = \sigma^2 \left\{ w(\nu+k) + 2\theta + 2 \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} \left[(1-2w)g_d(0,1) + p^2 g_d(2,1) \right. \right. \\ \left. \left. + (1-w) \left[2p\theta g_{d+1}(1,0) - 2\theta g_{d+1}(0,0) - 2p g_d(1,1) \right] \right] \right\} \quad (4.4)$$

where

$$g_{\tau}(i, j) = \Gamma\left(\frac{k}{2} + \tau + j - i\right) \Gamma\left(\frac{\nu}{2} + i\right) / \left(\Gamma\left(\frac{k}{2} + \tau\right) \Gamma\left(\frac{\nu}{2}\right) \right) \\ \times \left(1 - I_{c_0} \left(\frac{k}{2} + \tau + j - i, \frac{\nu}{2} + i \right) \right), \quad i, j = 0, 1, 2, \dots,$$

$I_{c_0}(\dots)$ is the incomplete beta function and $c_0 = p/(p+c^*)$.

Proof

See Appendix B.

Remark 4.1

Clearly, $R(\hat{b}) = R(\tilde{\beta})$ when $p = 0$.

Corollary 4.1

When $c^* \rightarrow \infty$, $R(\hat{b}) \rightarrow R(b_{SR})$, which is given by

$$R(b_{SR}) = \sigma^2 \left\{ \nu w + k(1-w) + p\nu(\nu+2) \left[p - 2(1-w)(k-2)/(\nu+2) \right] E \left\{ 1/\chi_{(k; \theta)}^2 \right\} \right\} \quad (4.5)$$

Proof

When $c^* \rightarrow \infty$, $c_0 = 0$ and $I_{c_0}(\dots) = 0$. Then, (4.4) becomes

$$R(\hat{b} | c \rightarrow \infty) = \sigma^2 \left\{ \nu \left(w + 2p(w-1) \right) + k(1-w) \right. \\ \left. + p^2 \nu(\nu+2) \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} (k+2d-2)^{-1} + 4(1-w)\theta p \nu \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} (k+2d)^{-1} \right. \\ = \sigma^2 \left\{ \nu \left(w + 2p(w-1) \right) + k(1-w) \right. \\ \left. + p^2 \nu(\nu+2) E \left\{ 1/\chi_{(k; \theta)}^2 \right\} + 4(1-w)\theta p \nu E \left\{ 1/\chi_{(k+2; \theta)}^2 \right\} \right\}, \quad (4.6)$$

as $E[1/\chi_{(J; \theta)}^2] = \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} (J+2d-2)^{-1}$, $J = 3, 4, \dots$ (Bock *et al.* (1984)).

Further, (Judge and Bock (1978, p.202))

$$2\theta E\left[1/\chi_{(k+2;\theta)}^2\right] = 1-(k-2)E\left[1/\chi_{(k;\theta)}^2\right]$$

and so using this relationship in (4.6) gives (4.5) directly.

Remark 4.3

We can write (4.5) as

$$R(b_{SR}) = R(\tilde{\beta}) + \sigma^2 p \nu (\nu+2) \left[p-2(1-w)(k-2)/(\nu+2) \right] E\left[1/\chi_{(k;\theta)}^2\right] \quad (4.7)$$

which implies that b_{SR} risk-dominates $\tilde{\beta}$ for all $p \in \left[0, 2(1-w)(k-2)/(\nu+2)\right]$, $k \geq 3$.

Remark 4.4

Using (4.7) it is straightforward to show that the maximum risk gain in using b_{SR} over $\tilde{\beta}$ occurs when $p^* = (1-w)(k-2)/(\nu+2)$.

Remark 4.5

When $w = 0$, (4.5) becomes

$$R(b_{SR}|w=0) = \sigma^2 \left\{ k + p \nu (\nu+2) \left[p-2(k-2)/(\nu+2) \right] \right\} E\left[1/\chi_{(k;\theta)}^2\right]$$

which is the usual expression for $R(b_{SR})$ under quadratic loss. (See, for example, Judge and Bock (1978, p.188)).

Remark 4.6

When $w = 1$, $p^* = 0$, and $b_{SR} = b_{PSR} = \tilde{\beta}$.

These results are illustrated in Figures 5 and 6 where $p = p^*$. Clearly, if the value of p which is optimal when $w = 0$ is used instead when $w \neq 0$, then there is no guarantee that b_{SR} and b_{PSR} will risk-dominate $\tilde{\beta}$. This is illustrated in Figures 7 and 8.

5. DISCUSSION

In this paper we have examined the risk properties of several estimators of the coefficient vector in the standard linear regression model under the so-called "balanced" loss function. This loss structure takes account of goodness of fit, as well as estimation precision.

In looking at this estimation problem after a preliminary test for the validity of a set of exact linear restrictions on the regression coefficients, we find that the well known risk results which hold under conventional (precision-only) quadratic loss also hold under a balanced loss function as long as less than 50% weight is given to goodness of fit. On the other hand, if less than 50% weight is given to estimation precision in the loss function, then the strategies of pre-testing or applying the restricted maximum likelihood estimator are strictly dominated by the strategy of simply applying the unrestricted maximum likelihood estimator. This, of course, reflects the fact that here the maximum likelihood and least squares estimators coincide, and the underlying goodness of fit component in the loss function is quadratic in form.

We have also shown that the unrestricted maximum likelihood estimator is itself inadmissible relative to balanced loss, unless all of the weight is given to goodness of fit. This result is established by considering both Stein-rule and positive-part Stein-rule estimators of the coefficient vector (which coincide with the maximum likelihood estimator when estimation precision gets zero weight in the loss function). Our results show that if at least 50% weight (w) is given to goodness of fit then the optimal estimator among those considered in this paper is the positive-part Stein-rule estimator with a shrinkage factor of $p^* = (1-w)(k-2)/(v+2)$, where k is the number of regressors, and v is the degrees of freedom. It is never preferable to

to pre-test. This is a natural generalisation of the corresponding conventional result under quadratic loss. Moreover, our findings tentatively suggest a case for the positive-part estimator when $w < 0.5$.

Clearly, the balanced loss function proposed by Zellner (1991), and applied by Rodrigues and Zellner (1992), has many interesting applications. The use of a penalty function which allows for both the goodness of fit of the model, as well as the precision of parameter estimation, has considerable appeal in regression analysis. Work in progress by the authors explores some of the associated pre-test issues in various directions.

APPENDIX A: PROOF OF THEOREM 3.1

$$R(\tilde{\beta}) = wE\left[(y-X\tilde{\beta})'(y-X\tilde{\beta})\right] + (1-w)E\left[(\tilde{\beta}-\beta)'S(\tilde{\beta}-\beta)\right]$$

where $S = X'X$, $\tilde{\beta} = S^{-1}X'y$ and $0 \leq w \leq 1$. Now, $E\left[(y-X\tilde{\beta})'(y-X\tilde{\beta})\right] = \sigma^2\nu$, $\nu = T-k$, as $(y-X\tilde{\beta})'(y-X\tilde{\beta})/\sigma^2 = u'Mu/\sigma^2 \sim \chi^2_{(\nu)}$ where $M = I-X(X'X)^{-1}X'$. Further, $E\left[(\tilde{\beta}-\beta)'S(\tilde{\beta}-\beta)\right] = \sigma^2k$ (see, for example, Judge and Bock (1978), Giles and Giles (1993c)). Equation (3.1) then follows directly.

$$R(\beta^*) = wE\left[(y-X\beta^*)'(y-X\beta^*)\right] + (1-w)E\left[(\beta^*-\beta)'S(\beta^*-\beta)\right]$$

where $\beta^* = \tilde{\beta} + S^{-1}R'[RS^{-1}R']^{-1}(r-R\tilde{\beta})$. As $(y-X\beta^*)'(y-X\beta^*)/\sigma^2 \sim \chi^2_{(\nu+m;\lambda)}$ where $\lambda = (R\beta-r)'(RS^{-1}R')^{-1}(R\beta-r)/(2\sigma^2)$, we have that $E\left[(y-X\beta^*)'(y-X\beta^*)\right] = \sigma^2(\nu+m+2\lambda)$. It is also straightforward to show that $E\left[(\beta^*-\beta)'S(\beta^*-\beta)\right] = \sigma^2(k-m+2\lambda)$ (see, for example, Judge and Bock (1978), Giles and Giles (1993c)), and so $R(\beta^*)$ follows.

Finally,

$$R(\hat{\beta}) = wE\left[(y-X\hat{\beta})'(y-X\hat{\beta})\right] + (1-w)E\left[(\hat{\beta}-\beta)'S(\hat{\beta}-\beta)\right]. \quad (A.1)$$

Let $\hat{u} = y-X\hat{\beta}$ and $\tilde{u} = y-X\tilde{\beta}$, so that

$$\begin{aligned} \hat{u}'\hat{u} &= \tilde{u}'\tilde{u} - 2\tilde{u}'X(\beta^*-\tilde{\beta})I_{[0,c]}(f) + (\beta^*-\tilde{\beta})'S(\beta^*-\tilde{\beta})I_{[0,c]}(f) \\ &= \tilde{u}'\tilde{u} + (\beta^*-\tilde{\beta})'S(\beta^*-\tilde{\beta})I_{[0,c]}(f) \end{aligned}$$

as $\tilde{u}'X = 0$. Then,

$$\begin{aligned} E\left[(y-X\hat{\beta})'(y-X\hat{\beta})\right] &= E(\hat{u}'\hat{u}) \\ &= E(\tilde{u}'\tilde{u}) + E\left[(\beta^*-\tilde{\beta})'S(\beta^*-\tilde{\beta})I_{[0,c]}(f)\right]. \end{aligned} \quad (A.2)$$

Now,

$$\begin{aligned} E\left[(\beta^*-\tilde{\beta})'S(\beta^*-\tilde{\beta})I_{[0,c]}(f)\right] &= E\left[I_{[0,c]}(f)(r-R\tilde{\beta})'(RS^{-1}R')^{-1}(r-R\tilde{\beta})\right] \\ &= \sigma^2(mP_2+2\lambda P_4) \end{aligned} \quad (A.3)$$

using Lemma 1 of Clarke *et al.* (1987), with

$$P_i = \Pr. \left[F'_{(m+i, \nu; \lambda)} \leq \left(cm/(m+i) \right), \quad i = 0, 1, 2, \dots \right]$$

Further, from Judge and Bock (1978) for example,

$$E \left[(\hat{\beta} - \beta)' S (\hat{\beta} - \beta) \right] = \sigma^2 \left[k + (4\lambda - m) P_2 - 2\lambda P_4 \right]. \quad (\text{A.4})$$

The desired result follows by substituting (A.2), (A.4) and $E(\tilde{u}'\tilde{u}) = \sigma^2 \nu$ into (A.1).

APPENDIX B: PROOF OF THEOREM 4.1

The SR estimator of β is

$$b_{SR} = (1 - p\tilde{u}'\tilde{u}/\tilde{\beta}'S\tilde{\beta})\tilde{\beta}$$

where p is a constant such that $0 \leq p \leq 2(1-w)(k-2)/(\nu+2)$ for $k \geq 3$ and the PSR is

$$b_{PSR} = \max \left[0, 1 - p\tilde{u}'\tilde{u}/\tilde{\beta}'S\tilde{\beta} \right] \tilde{\beta}.$$

Let $a = p\nu/k$ and $f^* = (\tilde{\beta}'S\tilde{\beta}/k)/(\tilde{u}'\tilde{u}/\nu)$ so that

$$b_{PSR} = I_{(a, \infty)}(f^*)(1 - a/f^*)\tilde{\beta}$$

where $I_{(a, \infty)}(f^*) = 0$ if $f^* \leq a$ and 1 otherwise. To derive the risk of the SR and PSR estimators we define a PTE of β

$$\hat{b} = I_{(a/c^*, \infty)}(f^*)(1 - a/f^*)\tilde{\beta}$$

where c^* is the critical value for the pre-test of $H_0: X\beta = 0$ vs $H_A: X\beta \neq 0$ (see, for example, Judge and Bock (1978, p.184) and Ohtani (1993)). Then, f^*

$\sim F'_{(k, \nu; \theta)}$ where $\theta = \beta'S\beta/(2\sigma^2)$ and note that

- (i) $\hat{b} = \tilde{\beta}$ when $p = 0$,
- (ii) $\hat{b} \rightarrow b_{SR}$ when $c^* \rightarrow \infty$,
- (iii) $\hat{b} = b_{PSR}$ when $c^* = 1$.

So,

$$\begin{aligned} R(\hat{b}) &= wE\left[(y-X\hat{\beta})'(y-X\hat{\beta})\right] + (1-w)E\left[(\hat{b}-\beta)'S(\hat{b}-\beta)\right] \\ &= wT_1 + (1-w)T_2. \end{aligned} \quad (B.1)$$

We consider first T_2 :

$$\begin{aligned} T_2 &= E\left[(\hat{b}-\beta)'S(\hat{b}-\beta)\right] \\ &= E(\hat{b}'S\hat{b}) - 2\beta'SE(\hat{b}) + \beta'S\beta. \end{aligned} \quad (B.2)$$

Now,

$$\begin{aligned} E(\hat{b}'S\hat{b}) &= E\left\{I_{(a/c^*, \omega)}(f^*)\tilde{\beta}'S\tilde{\beta} - 2pI_{(a/c^*, \omega)}(f^*)(\tilde{u}'\tilde{u})\right. \\ &\quad \left.+ p^2I_{(a/c^*, \omega)}(f^*)(\tilde{u}'\tilde{u})^2(\tilde{\beta}'S\tilde{\beta})^{-1}\right\} \\ &= E\left\{I_{(a/c^*, \omega)}(f^*)\tilde{\beta}'S\tilde{\beta} - 2pI_{(a/c^*, \omega)}(f^*)\left(\frac{\tilde{u}'\tilde{u}}{\tilde{\beta}'S\tilde{\beta}}\right)\tilde{\beta}'S\tilde{\beta}\right. \\ &\quad \left.+ p^2I_{(a/c^*, \omega)}(f^*)\left(\frac{\tilde{u}'\tilde{u}}{\tilde{\beta}'S\tilde{\beta}}\right)^2(\tilde{\beta}'S\tilde{\beta})\right\}. \end{aligned} \quad (B.3)$$

To evaluate this expression further we require the following Lemma:

Lemma 1.

For $i, j = 0, 1, 2, \dots$; $k > 2(i-j)$,

$$\begin{aligned} &E\left\{I_{(a/c^*, \omega)}(f^*)(\tilde{u}'\tilde{u}/\tilde{\beta}'S\tilde{\beta})^i(\tilde{\beta}'S\tilde{\beta})^j\right\} \\ &= (2\sigma^2)^j \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} g_d(i, j) \end{aligned}$$

where

$$g_d(i, j) = \frac{\Gamma(\frac{k}{2} + d + j - i)\Gamma(\frac{\nu}{2} + i)}{\Gamma(\frac{k}{2} + d)\Gamma(\frac{\nu}{2})} \left(1 - I_x\left(\frac{k}{2} + d + j - i, \frac{\nu}{2} + i\right)\right),$$

$x = p/(p+c^*)$ and $I(\dots)$ is the incomplete beta function.

Proof

Let $\gamma = \tilde{u}'\tilde{u}/\sigma^2 \sim \chi^2_{(v)}$ and $\tau = \tilde{\beta}'S\tilde{\beta}/\sigma^2 \sim \chi^2_{(k;\theta)}$. Then,

$$\begin{aligned} & E \left[I_{(a/c^*, \omega)}^{(f^*)} (\tilde{u}'\tilde{u}/\tilde{\beta}'S\tilde{\beta})^i (\tilde{\beta}'S\tilde{\beta})^j \right] \\ &= E \left[I_{(a/c^*, \omega)}^{(f^*)} (\gamma/\tau)^i (\tau\sigma^2)^j \right] \\ &= E \left[I_{(p/c^*, \omega)} (\tau/\gamma) (\gamma/\tau)^i (\tau\sigma^2)^j \right] \\ &= \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} (\sigma^2)^j \left[2^{(k+\nu)/2+d} \Gamma\left(\frac{k}{2}+d\right) \Gamma\left(\frac{\nu}{2}\right) \right]^{-1} \\ & \times \int \int_{(\tau/\gamma) > (p/c^*)} \gamma^{\nu/2+i-1} \tau^{j-i+k/2+d-1} \exp\left(-(\gamma+\tau)/2\right) d\gamma d\tau, \end{aligned} \quad (B.4)$$

as γ and τ are independently distributed quadratic forms.

Let $A_d = e^{-\theta} \theta^d (\sigma^2)^j \left[2^{(k+\nu)/2+d} \Gamma\left(\frac{k}{2}+d\right) \Gamma\left(\frac{\nu}{2}\right) d! \right]^{-1}$, $z_1 = (\tau/\gamma)$ and $z_2 = \gamma$, so that (B.4) is

$$\begin{aligned} & \sum_{d=0}^{\infty} A_d \int_{p/c^*}^{\infty} \int_0^{\infty} z_2^{\nu/2+i-1} (z_1 z_2)^{k/2+d-1+j-i} \exp\left(-\frac{z_1 z_2 + z_2}{2}\right) (z_2) dz_2 dz_1 \\ &= \sum_{d=0}^{\infty} A_d \int_{p/c^*}^{\infty} \int_0^{\infty} z_2^{(k+\nu)/2+j+d-1} z_1^{k/2+d-1+j-i} \\ & \times \exp\left(-\frac{(1+z_1)z_2}{2}\right) dz_2 dz_1. \end{aligned} \quad (B.5)$$

Now, let $z_3 = (1+z_1)z_2/2$ and $z_1 = z_1$. Then (B.5) is

$$\begin{aligned} & \sum_{d=0}^{\infty} A_d \int_{p/c^*}^{\infty} z_1^{k/2+d-1+j-i} (1+z_1)^{-[(k+\nu)/2+j+d]} \\ & \times 2^{(k+\nu)/2+j+d} dz_1 \int_0^{\infty} z_3^{(k+\nu)/2+j+d-1} \exp(-z_3) dz_3 \\ &= \sum_{d=0}^{\infty} A_d \Gamma\left(\frac{\nu+k}{2}+j+d\right) \int_{p/c^*}^{\infty} z_1^{k/2+d-1+j-i} (2/(1+z_1))^{(k+\nu)/2+j+d} dz_1. \end{aligned}$$

Finally, let $z_4 = z_1/(1+z_1)$ and $x = p/(p+c^*)$, so that (B.6) is

$$\begin{aligned} & \sum_{d=0}^{\infty} A_d \Gamma\left(\frac{\nu+k}{2}+j+d\right) 2^{(k+\nu)/2+j+d} \int_x^1 z_4^{k/2+d+j-i-1} (1-z_4)^{\nu/2+i-1} dz_4 \\ &= \sum_{d=0}^{\infty} A_d \Gamma\left(\frac{\nu+k}{2}+j+d\right) 2^{(k+\nu)/2+j+d} B\left(\frac{k}{2}+d+j-i, \frac{\nu}{2}+i\right) \\ & \times \left(1-I_x\left(\frac{k}{2}+d+j-i, \frac{\nu}{2}+i\right)\right) \end{aligned}$$

where $B(\dots)$ is the beta function. The lemma then follows directly.

Applying Lemma 1 repeatedly to (B.3) we have

$$E(\hat{b}' S \hat{b}) = 2\sigma^2 \sum_{d=0}^{\infty} \frac{e^{-\theta d}}{d!} \left\{ g_d(0,1) - 2p g_d(1,1) + p^2 g_d(2,1) \right\}. \quad (\text{B.6})$$

To complete the expression for T_2 we need $SE(\hat{b})$.

$$\begin{aligned} S E(\hat{b}) &= S E \left[I_{(a/c^*, \infty)}(f^*) \tilde{\beta} - I_{(a/c^*, \infty)}(f^*) (a/f^*) \tilde{\beta} \right] \\ &= S \int \int_{\bar{R}} \tilde{\beta} p(\tilde{\beta}) p(\tilde{u}' \tilde{u}) d(\tilde{\beta}) d(\tilde{u}' \tilde{u}) \\ & \quad - S p \int \int_{\bar{R}} \tilde{\beta} (\tilde{u}' \tilde{u} / \tilde{\beta}' S \tilde{\beta}) p(\tilde{\beta}) p(\tilde{u}' \tilde{u}) d\tilde{\beta} d(\tilde{u}' \tilde{u}) \end{aligned} \quad (\text{B.7})$$

where \bar{R} is the region, $\bar{R} = \left\{ (\tilde{\beta}, \tilde{u}' \tilde{u}) \mid \tilde{\beta}' S \tilde{\beta} / \tilde{u}' \tilde{u}' > p/c^* \right\}$ and $p(\tilde{\beta})$ and $p(\tilde{u}' \tilde{u})$ are the probability density functions of $\tilde{\beta}$ and $\tilde{u}' \tilde{u}$ respectively. We require the following Lemma to proceed further.

Lemma 2

Let $Q(i)$ be defined by

$$Q(i) = \int \int_{\bar{R}} \tilde{\beta} (\tilde{u}' \tilde{u} / \tilde{\beta}' S \tilde{\beta})^i p(\tilde{\beta}) p(\tilde{u}' \tilde{u}) d\tilde{\beta} d(\tilde{u}' \tilde{u}), \quad i = 0, 1, 2, \dots$$

Then

$$S Q(i) = S\beta \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} g_{d+1}(i, 0) .$$

Proof

Let

$$\begin{aligned} G(i) &= E \left[I_{(a/c^*, \omega)}^{(f^*)} (\tilde{u}' \tilde{u} / \tilde{\beta}' S \tilde{\beta})^i \right] \\ &= \int_{\bar{R}} \int (\tilde{u}' \tilde{u} / \tilde{\beta}' S \tilde{\beta})^i p(\tilde{\beta}) p(\tilde{u}' \tilde{u}) d\tilde{\beta} d(\tilde{u}' \tilde{u}) \\ &= \int_{\bar{R}} \int (\tilde{u}' \tilde{u} / \tilde{\beta}' S \tilde{\beta})^i \left[(2\pi)^{k/2} |\sigma^2 S^{-1}|^{\frac{1}{2}} \right]^{-1} \\ &\quad \times \exp \left[-(\tilde{\beta} - \beta)' S (\tilde{\beta} - \beta) / (2\sigma^2) \right] p(\tilde{u}' \tilde{u}) d\tilde{\beta} d(\tilde{u}' \tilde{u}) \end{aligned}$$

as $\tilde{\beta} \sim N(\beta, \sigma^2 S^{-1})$. Then,

$$\begin{aligned} \frac{\partial G(i)}{\partial \beta} &= \int_{\bar{R}} \int (\tilde{u}' \tilde{u} / \tilde{\beta}' S \tilde{\beta})^i \left[(2\pi)^{k/2} |\sigma^2 S^{-1}|^{\frac{1}{2}} \right]^{-1} \\ &\quad \times \left\{ (S\tilde{\beta}' / \sigma^2) \exp \left(-\frac{1}{2\sigma^2} (\tilde{\beta} - \beta)' S (\tilde{\beta} - \beta) \right) \right. \\ &\quad \left. - (S\beta / \sigma^2) \exp \left(-\frac{1}{2\sigma^2} (\tilde{\beta} - \beta)' S (\tilde{\beta} - \beta) \right) \right\} p(\tilde{u}' \tilde{u}) d\tilde{\beta} d(\tilde{u}' \tilde{u}) \\ &= (S / \sigma^2) \int_{\bar{R}} \int \tilde{\beta}' (\tilde{u}' \tilde{u} / \tilde{\beta}' S \tilde{\beta})^i p(\tilde{\beta}) p(\tilde{u}' \tilde{u}) d\tilde{\beta} d(\tilde{u}' \tilde{u}) - (S\beta / \sigma^2) G(i) \\ &= (S / \sigma^2) Q(i) - (S\beta / \sigma^2) G(i) . \end{aligned} \tag{B.8}$$

Further,

$$G(i) = \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} g_d(i, 0) \tag{B.9}$$

and $\partial \theta / \partial \beta = S\beta / \sigma^2$ so that using (B.9) we have

$$\begin{aligned}
\frac{\partial G(i)}{\partial \beta} &= -(S\beta/\sigma^2) \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} g_d(i,0) \\
&+ (S\beta/\sigma^2) \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} g_{d+1}(i,0) \\
&= -(S\beta/\sigma^2)G(i) + (S\beta/\sigma^2) \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} g_{d+1}(i,0) .
\end{aligned} \tag{B.10}$$

Equating (B.8) and (B.10) gives the required result.

Using Lemma 2 with $i = 0$ and $i = 1$ in (B.7) we have

$$SE(\hat{b}) = S\beta \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} g_{d+1}(0,0) - pS\beta \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} g_{d+1}(1,0)$$

so that

$$\begin{aligned}
T_2 &= 2\sigma^2 \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} \left\{ g_d(0,1) - 2pg_d(1,1) + p^2 g_d(2,1) \right\} \\
&- 2\beta' S\beta \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} \left\{ g_{d+1}(0,0) - pg_{d+1}(1,0) \right\} + \beta' S\beta \\
&= 2\sigma^2 \left\{ \theta + \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} \left[g_d(0,1) - 2pg_d(1,1) + p^2 g_d(2,1) \right. \right. \\
&\quad \left. \left. - 2\theta g_{d+1}(0,0) + 2\theta pg_{d+1}(1,0) \right] \right\} .
\end{aligned} \tag{B.11}$$

Finally, we require

$$\begin{aligned}
T_1 &= E(y - X\hat{b})'(y - X\hat{b}) \\
&= E(\tilde{u}'\tilde{u}) + E(\tilde{\beta}'S\tilde{\beta}) - E\left[\tilde{\beta}'S\tilde{\beta}I_{(a/c^*, \omega)}(f^*) \right] \\
&+ E\left[\tilde{\beta}'S\tilde{\beta}I_{(a/c^*, \omega)}(f^*)(a^2/f^{*2}) \right] .
\end{aligned} \tag{B.12}$$

As $(\tilde{u}'\tilde{u}/\sigma^2) \sim \chi^2_{(\nu)}$ and $(\tilde{\beta}'S\tilde{\beta}/\sigma^2) \sim \chi^2_{(k; \theta)}$, it follows that $E(\tilde{u}'\tilde{u}) = \sigma^2\nu$ and $E(\tilde{\beta}'S\tilde{\beta}) = \sigma^2(k+2\theta)$.

Further, using Lemma 1

$$E\left(\tilde{\beta}' S \tilde{\beta} I_{(a/c^*, \omega)}(f^*)\right) = 2\sigma^2 \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} g_d(0,1)$$

and

$$E\left(\tilde{\beta}' S \tilde{\beta} I_{(a/c^*, \omega)}(f^*)(a^2/f^{*2})\right) = 2\sigma^2 p^2 \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} g_d(2,1)$$

so that (B.12) becomes

$$T_1 = \sigma^2 \left[\nu + k + 2\theta - 2 \sum_{d=0}^{\infty} \frac{e^{-\theta} \theta^d}{d!} \left(g_d(0,1) - p^2 g_d(2,1) \right) \right]. \quad (\text{B.13})$$

Substituting (B.11) and (B.13) into (B.1) and rearranging terms gives the required result.

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FOOTNOTES

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1. The evaluations for the figures in this paper were undertaken with FORTRAN code written by the authors, incorporating routines from Press *et al.* (1986), and executed on a VAXstation 4000 under VMS 5.5. In all of the figures $\sigma^2 = 1$, which results in no loss of generality.
2. In Figures 1-4, two choices of c are considered: $c = 4.35$ ($\alpha = 5\%$), and $c = 1.895$ ($\alpha = 18.5\%$). The latter choice is Brook's (1976) "optimal" c in the case of quadratic loss.
3. That is, regardless of the value of m or ν , the "optimal" value of c will lie between zero and two in value, approximately.

FIGURE 1
 RISKS UNDER BALANCED LOSS FUNCTION
 ($w = 0.0, m = 1, v = 20, k = 3$)

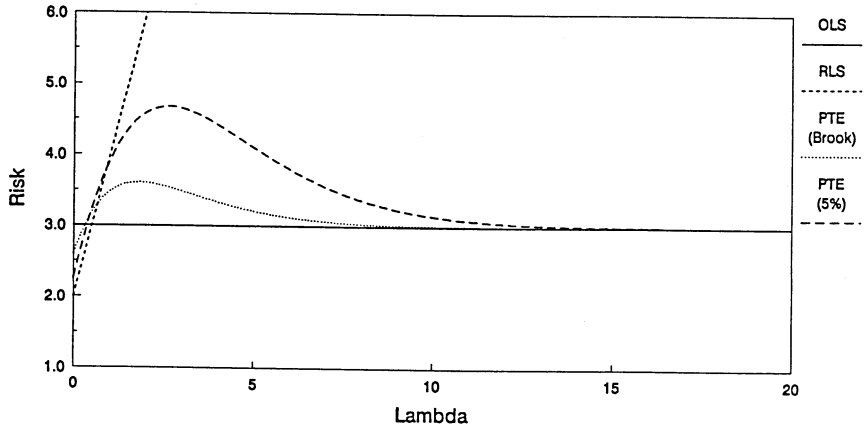


FIGURE 2
 RISKS UNDER BALANCED LOSS FUNCTION
 ($w = 0.5, m = 1, v = 20, k = 3$)

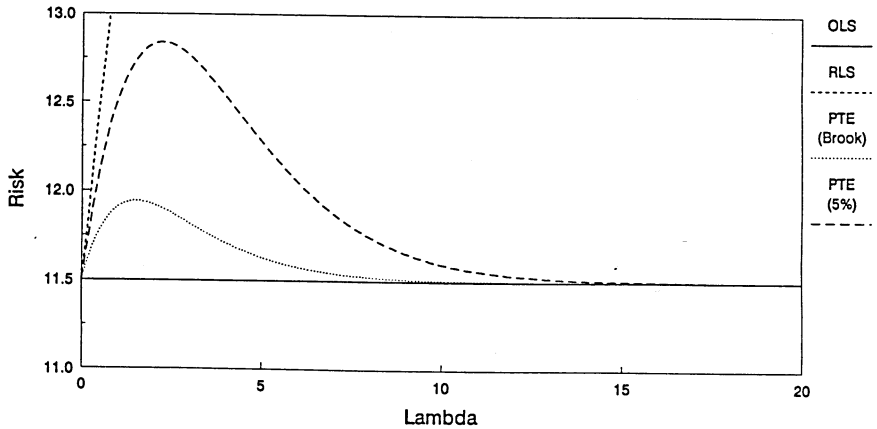


FIGURE 3
 RISKS UNDER BALANCED LOSS FUNCTION
 ($w = 0.75, m = 1, v = 20, k = 3$)

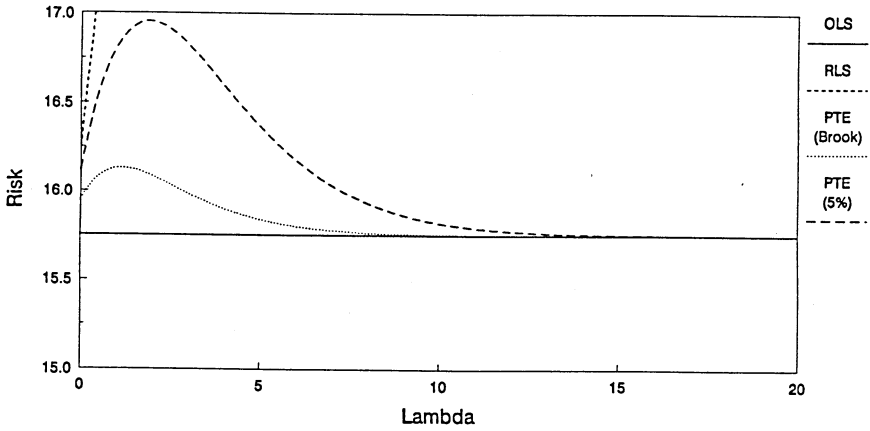


FIGURE 4
 RISKS UNDER BALANCED LOSS FUNCTION
 ($w = 1.0, m = 1, v = 20, k = 3$)

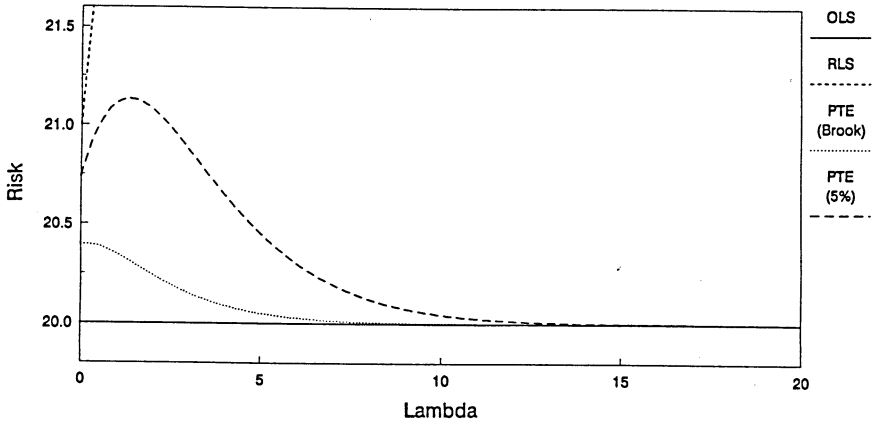


FIGURE 5
RISKS UNDER BALANCED LOSS FUNCTION
 ($w = 0.0, k = 3, v = 20, p = 0.045045$)

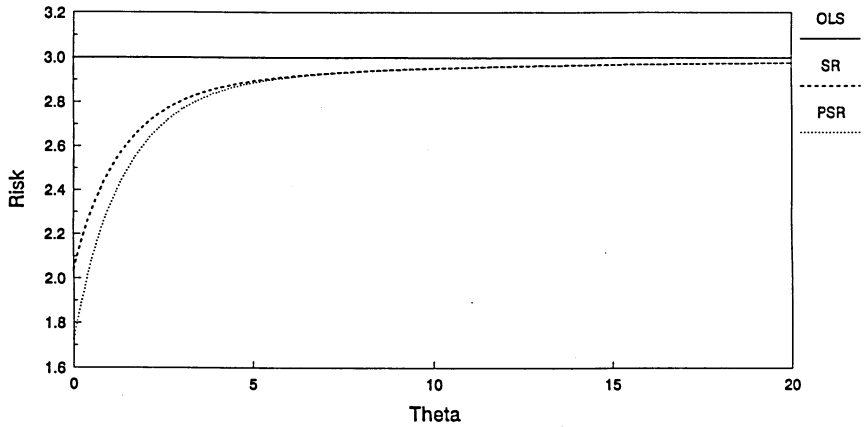


FIGURE 6
RISKS UNDER BALANCED LOSS FUNCTION
 ($w = 0.75, k = 3, v = 20, p = 0.01136363$)

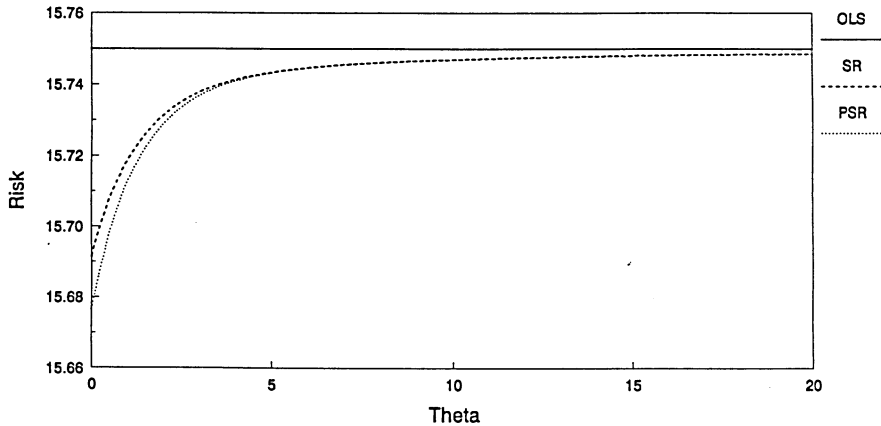


FIGURE 7
RISKS UNDER BALANCED LOSS FUNCTION
 ($w = 0.75, k = 3, v = 20, p = 0.045045$)

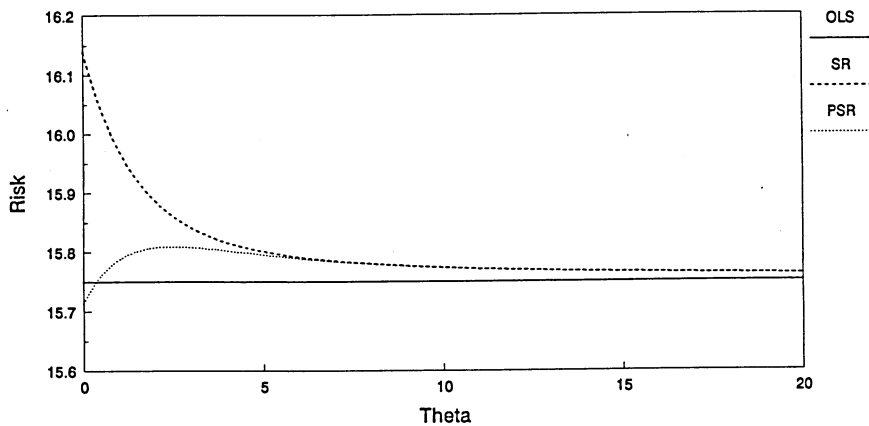
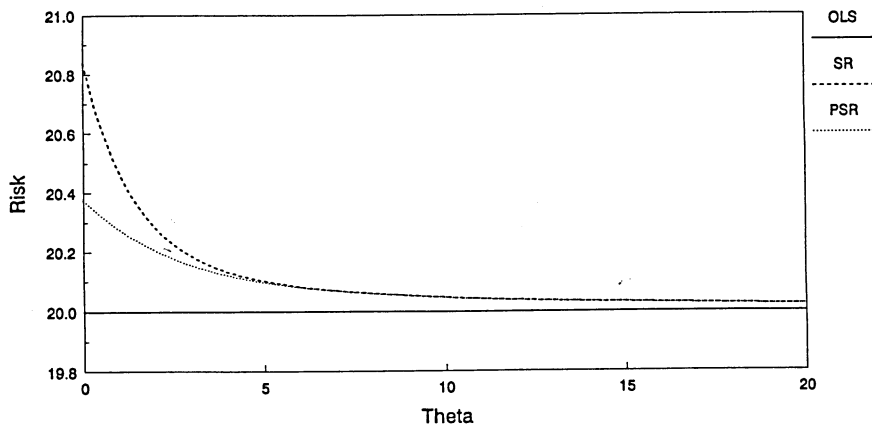


FIGURE 8
RISKS UNDER BALANCED LOSS FUNCTION
 ($w = 1.0, k = 3, v = 20, p = 0.045045$)



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