Department of Economics
UNIVERSITY OF CANTERBURY
CHRISTCHURCH, NEW ZEALAND

ISSN 1171-0705

THE EXACT RISKS OF SOME PRE-TEST AND STEIN-TYPE REGRESSION ESTIMATORS UNDER BALANCED LOSS

J. A. Giles, D. E. A. Giles, and K. Ohtani

Discussion Paper

No. 9306

Department of Economics, University of Canterbury
Christchurch, New Zealand

Discussion Paper No. 9306

April 1993

THE EXACT RISKS OF SOME PRE-TEST AND STEIN-TYPE REGRESSION ESTIMATORS UNDER BALANCED LOSS

J. A. Giles, D. E. A. Giles and K. Ohtani

THE EXACT RISKS OF SOME PRE-TEST AND STEIN-TYPE REGRESSION ESTIMATORS UNDER BALANCED LOSS*

Judith A. Giles

David E.A. Giles

Department of Economics
University of Canterbury

and

Kazuhiro Ohtani

Faculty of Economics
Kobe University

April 1993

ABSTRACT

In regression analysis we are often interested in using an estimator which is "precise" and which simultaneously provides a model with "good fit". In this paper we consider the risk properties of several estimators of the regression coefficient vector under "balanced" loss. This loss function (Zellner (1991)) reflects both of the above attributes. Under a particular form of balanced loss, we derive the predictive risk of the pre-test estimator which results after a test for exact linear restrictions on the coefficient vector. The corresponding risks of a Stein-rule and positive-part Stein-rule estimators are also established. The risks based on loss functions which allow only for estimation precision, or only for goodness of fit, are special cases of our results, and we draw appropriate comparisons. In particular, we show that some of the well-known results under (quadratic) precision-only loss are *not* robust to our generalisation of the loss function.

KEY WORDS: Linear regression; shrinkage estimators; estimation precision; goodness of fit; conditional inference

CORRESPONDENCE: Dr Judith Giles, Department of Economics, University of Canterbury, Private Bag 4800, Christchurch, NEW ZEALAND.
(**INTERNET:** j.giles@csc.canterbury.ac.nz; **FAX:** 64-3-3642635; **VOICE:** 64-3-3642520)

1. INTRODUCTION

In regression analysis we are often concerned both with the precision with which the coefficient vector is estimated, and with the "goodness of fit" of the overall model. However, typically this dual interest has not been taken into account in the construction of estimators in this context, or in the formulation of the loss function when comparing estimators' performances on the basis of "risk".

This has motivated Zellner (1991) to suggest the (quadratic) "Balanced" Loss Function (BLF), which explicitly takes account of *both* goodness of fit *and* estimation precision. Zellner analyses several standard estimation problems, including the linear regression model, and develops estimators which are optimal relative to a BLF. Rodrigues and Zellner (1992) extend this analysis by generalising the loss function still further, and consider a further application. In this paper we use the quadratic BLF as the basis for examining two somewhat related regression problems.

First, we investigate the consequences of adopting a common "pre-testing" strategy when estimating the regression coefficient vector. Specifically, we consider the case where *either* (unrestricted) Ordinary Least Squares (OLS) *or* Restricted Least Squares (RLS) is used, depending on the outcome of a prior test of the validity of certain exact linear restrictions on the coefficients. The BLF risks of the OLS and RLS estimators are compared with that of the Pre-Test Estimator (PTE), and the results are shown to differ from their well known counterparts based on the standard (quadratic) precision-only loss function (*e.g.*, Judge and Bock (1978)).

Second, we consider Stein-Rule (SR) and Positive-part Stein Rule (PSR) estimators for the regression coefficient vector (*e.g.*, Stein (1956) and James and Stein (1961), and Baranchik (1964) and Stein (1966) respectively), using (quadratic) BLF risk as the basis of comparison with OLS estimation. It is found that the standard risk-dominance results that are associated with the usual quadratic precision-only loss generalise naturally to the BLF case, provided that the shrinkage factor is chosen appropriately. The appropriate such choice differs from that under precision-only loss.

By considering the properties of these regression estimators under the BLF we add to our understanding of the robustness of certain key results to the choice of performance measure. Other recent examples of this type of investigation have focused on the use of the asymmetric LINEX loss function (Varian (1974) and Zellner (1986)), of which quadratic loss is a special case. Recent LINEX risk evaluations in the regression context include those of Giles and Giles (1993a,b), Ohtani *et al.* (1993) and the references cited therein.

Section 2 introduces the model and notation more formally, and the "pre-test" estimation problem is discussed and analysed in Section 3. Section 4 deals with the SR and PSR estimators, and some general discussion is given in Section 5. Proofs of the main analytic results appear in Appendices A and B.

2. MODEL AND NOTATION

We are concerned with estimating the coefficients in the linear regression model

$$y = X\beta + \varepsilon \qquad (1.1)$$

where $\varepsilon \sim N(0,\sigma^2 I)$, X is non-stochastic, of full rank, and $(T\times k)$, β is $(k\times 1)$ and y and ε are $(T\times 1)$.

If $\bar{\beta}$ is some estimator of β, and $L(\bar{\beta},\beta)$ is a loss function which takes zero value if $\bar{\beta} = \beta$ and is otherwise strictly positive, then the risk associated with $\bar{\beta}$ is

$$R(\bar{\beta}) = E\left[L(\bar{\beta},\beta)\right] \qquad (1.2)$$

where the expectation is taken over the sample space.

For example, if we have quadratic loss, then

$$L_Q(\bar{\beta},\beta) = (\bar{\beta}-\beta)'A(\bar{\beta}-\beta) \tag{1.3}$$

where A is a non-stochastic matrix of weights. Choosing $A = S = X'X$, it is well known that the risk associated with using $X\bar{\beta}$ as a *predictor* of $E(y|X)$ is the same as the risk of using $\bar{\beta}$ as an *estimator* of β if the regressors are orthonormal. We shall exploit this idea to make the results below independent of the X data.

The loss function in (1.3) focuses only on estimator (or predictor) *precision*. Zellner's (1991) BLF is a generalisation of (1.3) to

$$L_B(\bar{\beta},\beta) = w(y-X\bar{\beta})'(y-X\bar{\beta}) + (1-w)(\bar{\beta}-\beta)'S(\bar{\beta}-\beta) \tag{1.4}$$

where $w \in [0,1]$ is non-stochastic, and the first term in (1.4) clearly allows for "goodness of fit".

3. PRE-TEST ESTIMATION

Suppose that in estimating (1.1), we are also interested in the possibility that β should be constrained. In particular, suppose that, potentially, β satisfies $R\beta = r$, where R and r are known and non-stochastic, R is $(m\times k)$ and of rank m, and r is $(m\times 1)$. Such restrictions are encountered frequently in applied regression analysis. In particular, this formulation allows for "zero" restrictions on one or more elements of β. So, the estimation of the regression model after a prior test for the significance of regressors is covered by our analysis here.

If the restrictions in question are valid, there is an incentive to impose them and use the Restricted Least Squares estimator (which is also the restricted maximum likelihood estimator) of β. Ignoring them in this case, and using the Ordinary Least Squares (unrestricted maximum likelihood) estimator will result in inefficient estimation. On the other hand, if the restrictions are invalid, their imposition will result in an RLS estimator which is biased, but it may have smaller MSE than OLS (*e.g.*, see Toro-Vizcarrondo and Wallace (1968)).

Clearly, there is an incentive to (pre-) test the hypothesis, H_0: $R\beta = r$ (*vs.* H_A: $R\beta \neq r$) and choose the RLS or OLS estimator according to the outcome of this test. In the special case of a single zero restriction on one coefficient when k = 2, this problem was first considered by Bancroft (1944), who derived the bias of the least squares pre-test estimator of the other coefficient. The (quadratic) risk properties of the above general pre-test estimator of β are discussed, for example, by Judge and Bock (1978), the choice of an optimal significance level for the pre-test is considered by Brook (1976), and much of the recent pre-testing literature associated with inference in the context of the regression model is surveyed by Giles and Giles (1993c).

A uniformly most powerful invariant size-α test of H_0 against H_A is based on the statistic $f = \left[(r-R\tilde{\beta})'(RS^{-1}R')^{-1}(r-R\tilde{\beta})\right]/(m\tilde{\sigma}^2)$, where $\tilde{\beta} = S^{-1}X'y$ is the OLS estimator of β in (1.1) and $\tilde{\sigma}^2 = (y-X\tilde{\beta})'(y-X\tilde{\beta})/(T-k)$. The statistic f is non-central F-distributed with m and ν (= T-k) degrees of freedom, and non-centrality parameter $\lambda = (r-R\beta)'(RS^{-1}R)^{-1}(r-R\beta)/(2\sigma^2)$. We reject H_0 if f > c, where $\int_0^c p(f)df = (1-\alpha)$ and p(f) is central F with m and ν degrees of freedom. The associated pre-test estimator of β is $\hat{\beta} = I_{[0,c]}(f)\beta^* + I_{(c,\infty)}(f)\tilde{\beta} = \tilde{\beta} + (\beta^*-\tilde{\beta})I_{[0,c]}(f)$, where $\beta^* = \tilde{\beta} + S^{-1}R'[RS^{-1}R']^{-1}(r-R\tilde{\beta})$ is the RLS estimator of β, and $I_{[a,b]}(f)$ is an indicator function which is unity if $f \in [a,b]$, and zero otherwise.

While the risks of $\tilde{\beta}$, β^* and $\hat{\beta}$ are well documented under quadratic (precision-only) loss (and Giles (1993) considers the corresponding problem under absolute error loss), our purpose here is to re-consider the situation under the more general BLF in (1.5). Under such a loss structure, the risks of these three estimators are given in the following theorem.

Theorem 3.1

Under the quadratic BLF, (1.5), the risks of $\tilde{\beta}$, β^* and $\hat{\beta}$ are given by

$$R(\tilde{\beta}) = \sigma^2\left[w\nu + (1-w)k\right] \tag{3.1}$$

$$R(\beta^*) = R(\tilde{\beta}) + \sigma^2\left[m(2w-1)+2\lambda\right] \tag{3.2}$$

$$R(\hat{\beta}) = R(\tilde{\beta}) + \sigma^2\left[\left(m(2w-1)+4\lambda(1-w)\right)P_2 + 2\lambda(2w-1)P_4\right] \tag{3.3}$$

where $P_i = \Pr.\left[F'_{(m+i,\nu;\lambda)} < \left(cm/(m+i)\right)\right]$; $i = 0,1,\ldots$ and $F'_{(n_1,n_2;\delta)}$ is non-central F with n_1 and n_2 degrees of freedom, and non-centrality parameter δ.

Proof

See Appendix A.

Remark 3.1

Clearly, $R(\beta^*) = R(\tilde{\beta})$ when $\lambda = \lambda_0 = m(1-2w)/2$, and $\lambda_0 = 0$ if $w = 0.5$. For $w > (\geq)\ 0.5$, $\tilde{\beta}$ risk-dominates β^* for all $\lambda \geq (>)\ 0$. For $w < 0.5$, $R(\beta^*) < R(\tilde{\beta})$ for $\lambda < \lambda_0$.

Remark 3.2

Clearly, $\tilde{\beta}$ risk-dominates $\hat{\beta}$ and $\hat{\beta}$ risk-dominates β^* for all $\lambda \geq (>)\ 0$ if $w > (\geq)\ 0.5$).

When $w < 0.5$, the situation is ambiguous, as is typified in[1] Figure 1, which corresponds to the traditional quadratic loss situation.[2] In that particular case ($w = 0$) it is known that $R(\hat{\beta}) = R(\tilde{\beta})$ at $\lambda = \lambda_1$, where $\lambda_1 \in [m/4, m/2]$ (*e.g.*, see Judge and Bock (1978, p.73)). These bounds on the intersection of $R(\tilde{\beta})$ and $R(\hat{\beta})$ are readily generalised when $w \in [0,0.5]$, and a BLF is used.

Corollary 3.1

When $w \le 0.5$, the intersection of $R(\tilde{\beta})$ and $R(\hat{\beta})$ occurs at $\lambda = \lambda^*$, which is bounded by $m(1-2w)/4 \le \lambda^* \le m(1-2w)/2$.

Proof

First,

$$\Delta = R(\tilde{\beta}) - R(\hat{\beta}) = \sigma^2\left\{\left[m(1-2w)-2\lambda\right]P_2 + (P_4-P_2)2\lambda(1-2w)\right\}$$

and as $P_2 > P_4$, a sufficient condition for $\Delta \le 0$ is $\lambda \ge m(1-2w)/2$, if $w \le 0.5$.

Second,

$$\Delta = \sigma^2\left\{\left[m(1-2w)-4\lambda\right]P_2 + 4\lambda w(P_2-P_4)+2\lambda P_4\right\} ,$$

and as $P_2 > P_4$, a sufficient condition for $\Delta \ge 0$ is $\lambda \le m(1-2w)/4$, if $w \le 0.5$.

These results are illustrated further in Figures 2 to 4. The well known result associated with this particular pre-test estimation problem, namely that pre-testing can never be the best of the three strategies considered, and may be the worst (as in Figure 1) is clearly negated if goodness of fit is given sufficient weight in the loss function (*i.e.*, if $w \ge 0.5$). Then, pre-testing is always "second-best" to simply applying OLS with no prior testing of H_0, and naively applying the RLS estimator is the worst of the three strategies in terms of BLF risk. It is also clear from these figures that when $w \ge 0.5$ the "optimal" choice of pre-test critical value is $c = 0$, which corresponds to OLS estimation. Finally, when $0 < w < 0.5$ the "optimal" choice of c (under a mini-max regret criterion) will be less than that suggested by[3] Brook (1976).

4. STEIN-RULE ESTIMATION

A Stein-rule estimator of β in (1.1) is

$$b_{SR} = \left(1-p(y-X\tilde{\beta})'(y-X\tilde{\beta})/\tilde{\beta}'S\tilde{\beta}\right)\tilde{\beta} \tag{4.1}$$

where $p \in \left[0,\ 2(k-2)/(\nu+2)\right]$, for $k \geq 3$. It is well known that $R(b_{SR}) < R(\tilde{\beta})$ under quadratic loss for $k \geq 3$, and that the "optimum" choice of p is $p^* = (k-2)/(\nu+2)$, in terms of maximising the risk-gain in using b_{SR} over $\tilde{\beta}$. Further, the positive-part Stein-rule estimator,

$$b_{PSR} = \max\left[0,\ 1-p(y-X\tilde{\beta})'(y-X\tilde{\beta})/\tilde{\beta}'S\tilde{\beta}\right]\tilde{\beta} \tag{4.2}$$

is known to risk-dominate b_{SR} under the same conditions in the case of quadratic loss.

To facilitate our analysis of b_{SR} and b_{PSR} under the BLF, it is convenient to define a different pre-test estimator of β, namely

$$\hat{b} = I_{(a/c^*,\infty)}(f^*)(1-a/f^*)\tilde{\beta} \tag{4.3}$$

where I is a conventional indicator function; $f^* = \left[(\tilde{\beta}'S\tilde{\beta})/(k\tilde{\sigma}^2)\right]$; $a = (p\nu/k)$; c^* is the critical value for the pre-test of H_0: $X\beta = 0$ *vs* H_A: $X\beta \neq 0$; and $f^* \sim F'_{(k,\nu;\theta)}$, where $\theta = \beta'S\beta/2\sigma^2$. Then,

(i) $\hat{b} = \tilde{\beta}$; if $p = 0$.

(ii) $\hat{b} \to \hat{b}_{SR}$; if $c^* \to \infty$.

(iii) $\hat{b} = \hat{b}_{PSR}$; if $c^* = 1$.

Theorem 4.1

Under the quadratic BLF, (1.5), the risk of $\hat{b}$ is given by

$$R(\hat{b}) = \sigma^2\left\{w(\nu+k) + 2\theta + 2\sum_{d=0}^{\infty}\frac{e^{-\theta}\theta^d}{d!}\left[(1-2w)g_d(0,1) + p^2g_d(2,1) + (1-w)\left(2p\theta g_{d+1}(1,0) - 2\theta g_{d+1}(0,0) - 2pg_d(1,1)\right)\right]\right\} \tag{4.4}$$

where

$$g_\tau(i,j) = \Gamma(\tfrac{k}{2}+\tau+j-i)\Gamma(\tfrac{\nu}{2}+i)\Big/\left(\Gamma(\tfrac{k}{2}+\tau)\Gamma(\tfrac{\nu}{2})\right)$$

$$\times\left(1-I_{c_0}(\tfrac{k}{2}+\tau+j-i,\tfrac{\nu}{2}+i)\right), \quad i,j = 0,1,2,\ldots,$$

$I_{c_0}(.,.)$ is the incomplete beta function and $c_0 = p/(p+c^*)$.

Proof

See Appendix B.

Remark 4.1

Clearly, $R(\hat{b}) = R(\tilde{\beta})$ when $p = 0$.

Corollary 4.1

When $c^* \to \infty$, $R(\hat{b}) \to R(b_{SR})$, which is given by

$$R(b_{SR}) = \sigma^2\left\{\nu w+k(1-w)+p\nu(\nu+2)\left[p-2(1-w)(k-2)/(\nu+2)\right]E\left(1/\chi'^2_{(k;\theta)}\right)\right\} \tag{4.5}$$

Proof

When $c^* \to \infty$, $c_0 = 0$ and $I_{c_0}(.,.) = 0$. Then, (4.4) becomes

$$R(\hat{b}|c\to\infty) = \sigma^2\left\{\nu\left(w+2p(w-1)\right) + k(1-w)\right.$$

$$+ p^2\nu(\nu+2)\sum_{d=0}^{\infty}\frac{e^{-\theta}\theta^d}{d!}(k+2d-2)^{-1} + 4(1-w)\theta p\nu\sum_{d=0}^{\infty}\frac{e^{-\theta}\theta^d}{d!}(k+2d)^{-1}$$

$$= \sigma^2\left\{\nu\left(w+2p(w-1)\right)+k(1-w)\right.$$

$$\left.+ p^2\nu(\nu+2)E\left(1/\chi'^2_{(k;\theta)}\right)+4(1-w)\theta p\nu E\left(1/\chi'^2_{(k+2;\theta)}\right)\right\}, \tag{4.6}$$

as $E[1/\chi^2_{(J;\theta)}] = \sum_{d=0}^{\infty}\frac{e^{-\theta}\theta^d}{d!}(J+2d-2)^{-1}$, $J = 3,4,\ldots$ (Bock *et al.* (1984)).

Further, (Judge and Bock (1978, p.202))

$$2\theta E\left(1/\chi^{2\prime}_{(k+2;\theta)}\right) = 1-(k-2)E\left(1/\chi^{2\prime}_{(k;\theta)}\right)$$

and so using this relationship in (4.6) gives (4.5) directly.

Remark 4.3

We can write (4.5) as

$$R(b_{SR}) = R(\tilde{\beta}) + \sigma^2 p\nu(\nu+2)\left[p-2(1-w)(k-2)/(\nu+2)\right]E\left(1/\chi^{2\prime}_{(k;\theta)}\right) \tag{4.7}$$

which implies that b_{SR} risk-dominates $\tilde{\beta}$ for all $p \in \left[0,\ 2(1-w)(k-2)/(\nu+2)\right]$, $k \geq 3$.

Remark 4.4

Using (4.7) it is straightforward to show that the maximum risk gain in using b_{SR} over $\tilde{\beta}$ occurs when $p^* = (1-w)(k-2)/(\nu+2)$.

Remark 4.5

When $w = 0$, (4.5) becomes

$$R(b_{SR}|w=0) = \sigma^2\left\{k+p\nu(\nu+2)\left[p-2(k-2)/(\nu+2)\right]E\left(1/\chi^{2\prime}_{(k;\theta)}\right)\right.$$

which is the usual expression for $R(b_{SR})$ under quadratic loss. (See, for example, Judge and Bock (1978, p.188)).

Remark 4.6

When $w = 1$, $p^* = 0$, and $b_{SR} = b_{PSR} = \tilde{\beta}$.

These results are illustrated in Figures 5 and 6 where $p = p^*$. Clearly, if the value of p which is optimal when $w = 0$ is used instead when $w \neq 0$, then there is no guarantee that b_{SR} and b_{PSR} will risk-dominate $\tilde{\beta}$. This is illustrated in Figures 7 and 8.

5. DISCUSSION

In this paper we have examined the risk properties of several estimators of the coefficient vector in the standard linear regression model under the so-called "balanced" loss function. This loss structure takes account of goodness of fit, as well as estimation precision.

In looking at this estimation problem after a preliminary test for the validity of a set of exact linear restrictions on the regression coefficients, we find that the well known risk results which hold under conventional (precision-only) quadratic loss also hold under a balanced loss function as long as less than 50% weight is given to goodness of fit. On the other hand, if less than 50% weight is given to estimation precision in the loss function, then the strategies of pre-testing or applying the restricted maximum likelihood estimator are strictly dominated by the strategy of simply applying the unrestricted maximum likelihood estimator. This, of course, reflects the fact that here the maximum likelihood and least squares estimators coincide, and the underlying goodness of fit component in the loss function is quadratic in form.

We have also shown that the unrestricted maximum likelihood estimator is itself inadmissible relative to balanced loss, unless all of the weight is given to goodness of fit. This result is established by considering both Stein-rule and positive-part Stein-rule estimators of the coefficient vector (which coincide with the maximum likelihood estimator when estimation precision gets zero weight in the loss function). Our results show that if at least 50% weight (w) is given to goodness of fit then the optimal estimator among those considered in this paper is the positive-part Stein-rule estimator with a shrinkage factor of $p^* = (1-w)(k-2)/(\nu+2)$, where k is the number of regressors, and ν is the degrees of freedom. It is never preferable to to pre-test. This is a natural generalisation of the corresponding conventional result under quadratic loss. Moreover, our findings tentatively suggest a case for the positive-part estimator when $w < 0.5$.

Clearly, the balanced loss function proposed by Zellner (1991), and applied by Rodrigues and Zellner (1992), has many interesting applications. The use of a penalty function which allows for both the goodness of fit of the model, as well as the precision of parameter estimation, has considerable appeal in regression analysis. Work in progress by the authors explores some of the associated pre-test issues in various directions.

APPENDIX A: PROOF OF THEOREM 3.1

$$R(\tilde{\beta}) = wE\left[(y-X\tilde{\beta})'(y-X\tilde{\beta})\right] + (1-w)E\left[(\tilde{\beta}-\beta)'S(\tilde{\beta}-\beta)\right]$$

where $S = X'X$, $\tilde{\beta} = S^{-1}X'y$ and $0 \leq w \leq 1$. Now, $E\left[(y-X\tilde{\beta})'(y-X\tilde{\beta})\right] = \sigma^2\nu$, $\nu = T-k$, as $(y-X\tilde{\beta})'(y-X\tilde{\beta})/\sigma^2 = u'Mu/\sigma^2 \sim \chi^2_{(\nu)}$ where $M = I-X(X'X)^{-1}X'$. Further, $E\left[(\tilde{\beta}-\beta)'S(\tilde{\beta}-\beta)\right] = \sigma^2k$ (see, for example, Judge and Bock (1978), Giles and Giles (1993c)). Equation (3.1) then follows directly.

$$R(\beta^*) = wE\left[(y-X\beta^*)'(y-X\beta^*)\right] + (1-w)E\left[(\beta^*-\beta)'S(\beta^*-\beta)\right]$$

where $\beta^* = \tilde{\beta} + S^{-1}R'[RS^{-1}R']^{-1}(r-R\tilde{\beta})$. As $(y-X\beta^*)'(y-X\beta^*)/\sigma^2 \sim \chi^{2\prime}_{(\nu+m;\lambda)}$ where $\lambda = (R\beta-r)'(RS^{-1}R')^{-1}(R\beta-r)/(2\sigma^2)$, we have that $E\left[(y-X\beta^*)'(y-X\beta^*)\right] = \sigma^2(\nu+m+2\lambda)$. It is also straightforward to show that $E\left[(\beta^*-\beta)'S(\beta^*-\beta)\right] = \sigma^2(k-m+2\lambda)$ (see, for example, Judge and Bock (1978), Giles and Giles (1993c)), and so $R(\beta^*)$ follows.

Finally,

$$R(\hat{\beta}) = wE\left[(y-X\hat{\beta})'(y-X\hat{\beta})\right] + (1-w)E\left[(\hat{\beta}-\beta)'S(\hat{\beta}-\beta)\right] . \qquad (A.1)$$

Let $\hat{u} = y-X\hat{\beta}$ and $\tilde{u} = y-X\tilde{\beta}$, so that

$$\hat{u}'\hat{u} = \tilde{u}'\tilde{u} - 2\tilde{u}'X(\beta^*-\tilde{\beta})I_{[0,c]}(f) + (\beta^*-\tilde{\beta})'S(\beta^*-\tilde{\beta})I_{[0,c]}(f)$$

$$= \tilde{u}'\tilde{u} + (\beta^*-\tilde{\beta})'S(\beta^*-\tilde{\beta})I_{[0,c]}(f)$$

as $\tilde{u}'X = 0$. Then,

$$E\left[(y-X\hat{\beta})'(y-X\hat{\beta})\right] = E(\hat{u}'\hat{u})$$

$$= E(\tilde{u}'\tilde{u}) + E\left[(\beta^*-\tilde{\beta})'S(\beta^*-\tilde{\beta})I_{[0,c]}(f)\right] . \qquad (A.2)$$

Now,

$$E\left[(\beta^*-\tilde{\beta})'S(\beta^*-\tilde{\beta})I_{[0,c]}(f)\right]$$

$$= E\left[I_{[0,c]}(f)(r-R\tilde{\beta})'(RS^{-1}R')^{-1}(r-R\tilde{\beta})\right]$$

$$= \sigma^2(mP_2+2\lambda P_4) \qquad (A.3)$$

using Lemma 1 of Clarke *et al.* (1987), with

$$P_i = Pr.\left[F'_{(m+i,\nu;\lambda)} \le \left(cm/(m+i)\right)\right], \quad i = 0,1,2,\ldots .$$

Further, from Judge and Bock (1978) for example,

$$E\left[(\hat{\beta}-\beta)' S(\hat{\beta}-\beta)\right] = \sigma^2\left[k+(4\lambda-m)P_2-2\lambda P_4\right]. \tag{A.4}$$

The desired result follows by substituting (A.2), (A.4) and $E(\tilde{u}'\tilde{u}) = \sigma^2\nu$ into (A.1).

APPENDIX B: PROOF OF THEOREM 4.1

The SR estimator of β is

$$b_{SR} = (1-p\tilde{u}'\tilde{u}/\tilde{\beta}'S\tilde{\beta})\tilde{\beta}$$

where p is a constant such that $0 \le p \le 2(1-w)(k-2)/(\nu+2)$ for $k \ge 3$ and the PSR is

$$b_{PSR} = \max\left[0,\ 1-p\tilde{u}'\tilde{u}/\tilde{\beta}'S\tilde{\beta}\right]\tilde{\beta} .$$

Let $a = p\nu/k$ and $f^* = (\tilde{\beta}'S\tilde{\beta}/k)/(\tilde{u}'\tilde{u}/\nu)$ so that

$$b_{PSR} = I_{(a,\infty)}(f^*)(1-a/f^*)\tilde{\beta}$$

where $I_{(a,\infty)}(f^*) = 0$ if $f^* \le a$ and 1 otherwise. To derive the risk of the SR and PSR estimators we define a PTE of β

$$\hat{b} = I_{(a/c^*,\infty)}(f^*)(1-a/f^*)\tilde{\beta}$$

where c^* is the critical value for the pre-test of H_0: $X\beta = 0$ *vs* H_A: $X\beta \ne 0$ (see, for example, Judge and Bock (1978, p.184) and Ohtani (1993)). Then, $f^* \sim F'_{(k,\nu;\theta)}$ where $\theta = \beta' S\beta/(2\sigma^2)$ and note that

(i) $\hat{b} = \tilde{\beta}$ when $p = 0$,

(ii) $\hat{b} \to b_{SR}$ when $c^* \to \infty$,

(iii) $\hat{b} = b_{PSR}$ when $c^* = 1$.

So,

$$R(\hat{b}) = wE\left[(y-X\hat{\beta})'(y-X\hat{\beta})\right] + (1-w)E\left[(\hat{b}-\beta)'S(\hat{b}-\beta)\right]$$

$$= wT_1 + (1-w)T_2 \ . \qquad \text{(B.1)}$$

We consider first T_2:

$$T_2 = E\left[(\hat{b}-\beta)'S(\hat{b}-\beta)\right]$$

$$= E(\hat{b}'S\hat{b})-2\beta'SE(\hat{b})+\beta'S\beta \ . \qquad \text{(B.2)}$$

Now,

$$E(\hat{b}'S\hat{b}) = E\Big\{I_{(a/c^*,\infty)}(f^*)\tilde{\beta}'S\tilde{\beta} - 2pI_{(a/c^*,\infty)}(f^*)(\tilde{u}'\tilde{u})$$

$$+ p^2 I_{(a/c^*,\infty)}(f^*)(\tilde{u}'\tilde{u})^2(\tilde{\beta}'S\tilde{\beta})^{-1}\Big\}$$

$$= E\left\{I_{(a/c^*,\infty)}(f^*)\tilde{\beta}'S\tilde{\beta}-2pI_{(a/c^*,\infty)}(f^*)\left(\frac{\tilde{u}'\tilde{u}}{\tilde{\beta}'S\tilde{\beta}}\right)\tilde{\beta}'S\tilde{\beta}\right.$$

$$\left.+ p^2 I_{(a/c^*,\infty)}(f^*)\left(\frac{\tilde{u}'\tilde{u}}{\tilde{\beta}'S\tilde{\beta}}\right)^2(\tilde{\beta}'S\tilde{\beta})\right\} \ . \qquad \text{(B.3)}$$

To evaluate this expression further we require the following Lemma:

Lemma 1.

For $i,j = 0,1,2,\ldots$; $k > 2(i-j)$,

$$E\left(I_{(a/c^*,\infty)}(f^*)(\tilde{u}'\tilde{u}/\tilde{\beta}'S\tilde{\beta})^i(\tilde{\beta}'S\tilde{\beta})^j\right)$$

$$= (2\sigma^2)^j \sum_{d=0}^{\infty} \frac{e^{-\theta}\theta^d}{d!}\, g_d(i,j)$$

where

$$g_d(i,j) = \frac{\Gamma(\frac{k}{2}+d+j-i)\Gamma(\frac{\nu}{2}+i)}{\Gamma(\frac{k}{2}+d)\Gamma(\frac{\nu}{2})}\left(1-I_x(\tfrac{k}{2}+d+j-i,\tfrac{\nu}{2}+i)\right) ,$$

$x = p/(p+c^*)$ and $I(.,.)$ is the incomplete beta function.

Proof

Let $\gamma = \tilde{u}'\tilde{u}/\sigma^2 \sim \chi^2_{(\nu)}$ and $\tau = \tilde{\beta}'S\tilde{\beta}/\sigma^2 \sim \chi^{2\prime}_{(k;\theta)}$. Then,

$$E\left[I_{(a/c^*,\infty)}(f^*)(\tilde{u}'\tilde{u}/\tilde{\beta}'S\tilde{\beta})^i(\tilde{\beta}'S\tilde{\beta})^j\right]$$

$$= E\left[I_{(a/c^*,\infty)}(f^*)(\gamma/\tau)^i(\tau\sigma^2)^j\right]$$

$$= E\left[I_{(p/c^*,\infty)}(\tau/\gamma)(\gamma/\tau)^i(\tau\sigma^2)^j\right]$$

$$= \sum_{d=0}^{\infty} \frac{e^{-\theta}\theta^d}{d!}(\sigma^2)^j\left[2^{(k+\nu)/2+d}\Gamma(\tfrac{k}{2}+d)\Gamma(\tfrac{\nu}{2})\right]^{-1}$$

$$\times \int\int_{(\tau/\gamma)>(p/c^*)} \gamma^{\nu/2+i-1}\tau^{j-i+k/2+d-1}\exp\left(-(\gamma+\tau)/2\right)d\gamma d\tau \ , \qquad \text{(B.4)}$$

as γ and τ are independently distributed quadratic forms.

Let $A_d = e^{-\theta}\theta^d(\sigma^2)^j\left[2^{(k+\nu)/2+d}\Gamma(\tfrac{k}{2}+d)\Gamma(\tfrac{\nu}{2})d!\right]^{-1}$, $z_1 = (\tau/\gamma)$ and $z_2 = \gamma$, so that (B.4) is

$$\sum_{d=0}^{\infty} A_d \int_{p/c^*}^{\infty}\int_0^{\infty} z_2^{\nu/2+i-1}(z_1z_2)^{k/2+d-1+j-i} \exp\left(-(z_1z_2+z_2)/2\right)(z_2)dz_2dz_1$$

$$= \sum_{d=0}^{\infty} A_d \int_{p/c^*}^{\infty}\int_0^{\infty} z_2^{(k+\nu)/2+j+d-1} z_1^{k/2+d-1+j-i}$$

$$\times \exp\left(-(1+z_1)z_2/2\right)dz_2dz_1 \ . \qquad \text{(B.5)}$$

Now, let $z_3 = (1+z_1)z_2/2$ and $z_1 = z_1$. Then (B.5) is

$$\sum_{d=0}^{\infty} A_d \int_{p/c^*}^{\infty} z_1^{k/2+d-1+j-i}(1+z_1)^{-[(k+\nu)/2+j+d]}$$

$$\times 2^{(k+\nu)/2+j+d}dz_1 \int_0^{\infty} z_3^{(k+\nu)/2+j+d-1}\exp(-z_3)dz_3$$

$$= \sum_{d=0}^{\infty} A_d\Gamma\left(\frac{\nu+k}{2}+j+d\right) \int_{p/c^*}^{\infty} z_1^{k/2+d-1+j-i}(2/(1+z_1))^{(k+\nu)/2+j+d}dz_1 \ .$$

Finally, let $z_4 = z_1/(1+z_1)$ and $x = p/(p+c^*)$, so that (B.6) is

$$\sum_{d=0}^{\infty} A_d \Gamma\left(\frac{\nu+k}{2}+j+d\right) 2^{(k+\nu)/2+j+d} \int_x^1 z_4^{k/2+d+j-i-1}(1-z_4)^{\nu/2+i-1} dz_4$$

$$= \sum_{d=0}^{\infty} A_d \Gamma\left(\frac{\nu+k}{2}+j+d\right) 2^{(k+\nu)/2+j+d} \; B\left(\frac{k}{2}+d+j-i, \frac{\nu}{2}+i\right)$$

$$\times \left(1-I_x\left(\frac{k}{2}+d+j-i, \frac{\nu}{2}+i\right)\right)$$

where B(.,.) is the beta function. The lemma then follows directly.

Applying Lemma 1 repeatedly to (B.3) we have

$$E(\hat{b}' S \hat{b}) = 2\sigma^2 \sum_{d=0}^{\infty} \frac{e^{-\theta}\theta^d}{d!} \left\{ g_d(0,1) - 2p g_d(1,1) + p^2 g_d(2,1) \right\} . \qquad \text{(B.6)}$$

To complete the expression for T_2 we need $SE(\hat{b})$.

$$S\,E(\hat{b}) = S\,E\left[I_{(a/c^*,\infty)}(f^*)\tilde{\beta} - I_{(a/c^*,\infty)}(f^*)(a/f^*)\tilde{\beta}\right]$$

$$= S \int\int_{\overline{R}} \tilde{\beta} p(\tilde{\beta}) p(\tilde{u}'\tilde{u}) d(\tilde{\beta}) d(\tilde{u}'\tilde{u})$$

$$- S\, p \int\int_{\overline{R}} \tilde{\beta}(\tilde{u}'\tilde{u}/\tilde{\beta}' S \tilde{\beta}) p(\tilde{\beta}) p(\tilde{u}'\tilde{u}) d\tilde{\beta} d(\tilde{u}'\tilde{u}) \qquad \text{(B.7)}$$

where $\overline{R}$ is the region, $\overline{R} = \left\{(\tilde{\beta}, \tilde{u}'\tilde{u}) \,|\, \tilde{\beta}' S \tilde{\beta}/\tilde{u}'\tilde{u}' > p/c^*\right\}$ and $p(\tilde{\beta})$ and $p(\tilde{u}'\tilde{u})$ are the probability density functions of $\tilde{\beta}$ and $\tilde{u}'\tilde{u}$ respectively. We require the following Lemma to proceed further.

Lemma 2

Let Q(i) be defined by

$$Q(i) = \int\int_{\overline{R}} \tilde{\beta}(\tilde{u}'\tilde{u}/\tilde{\beta}' S \tilde{\beta})^i p(\tilde{\beta}) p(\tilde{u}'\tilde{u}) d\tilde{\beta} d(\tilde{u}'\tilde{u}), \; i = 0,1,2,\ldots .$$

Then

$$S\,Q(i) = S\beta \sum_{d=0}^{\infty} \frac{e^{-\theta}\theta^{d}}{d!}\, g_{d+1}(i,0)\ .$$

Proof

Let

$$G(i) = E\left[I_{(a/c^*,\infty)}(f^*)(\tilde{u}'\tilde{u}/\tilde{\beta}'S\tilde{\beta})^{i}\right]$$

$$= \int_{\bar{R}}\int (\tilde{u}'\tilde{u}/\tilde{\beta}'S\tilde{\beta})^{i}\, p(\tilde{\beta})p(\tilde{u}'\tilde{u})\,d\tilde{\beta}\,d(\tilde{u}'\tilde{u})$$

$$= \int_{\bar{R}}\int (\tilde{u}'\tilde{u}/\tilde{\beta}'S\tilde{\beta})^{i}\left[(2\pi)^{k/2}|\sigma^2S^{-1}|^{\frac{1}{2}}\right]^{-1}$$

$$\times \exp\left[-(\tilde{\beta}-\beta)'S(\tilde{\beta}-\beta)/(2\sigma^2)\right]p(\tilde{u}'\tilde{u})\,d\tilde{\beta}\,d(\tilde{u}'\tilde{u})$$

as $\tilde{\beta} \sim N(\beta,\sigma^2S^{-1})$. Then,

$$\frac{\partial G(i)}{\partial\beta} = \int_{\bar{R}}\int (\tilde{u}'\tilde{u}/\tilde{\beta}'S\tilde{\beta})^{i}\left[(2\pi)^{k/2}|\sigma^2S^{-1}|^{\frac{1}{2}}\right]^{-1}$$

$$\times \left\{(S\tilde{\beta}/\sigma^2)\exp\left(-\frac{1}{2\sigma^2}(\tilde{\beta}-\beta)'S(\tilde{\beta}-\beta)\right)\right.$$

$$\left. - (S\beta/\sigma^2)\exp\left(-\frac{1}{2\sigma^2}(\tilde{\beta}-\beta)'S(\tilde{\beta}-\beta)\right)\right\}p(\tilde{u}'\tilde{u})\,d\tilde{\beta}\,d(\tilde{u}'\tilde{u})$$

$$= (S/\sigma^2)\int_{\bar{R}}\int \tilde{\beta}(\tilde{u}'\tilde{u}/\tilde{\beta}'S\tilde{\beta})^{i}p(\tilde{\beta})p(\tilde{u}'\tilde{u})\,d\tilde{\beta}\,d(\tilde{u}'\tilde{u}) - (S\beta/\sigma^2)G(i)$$

$$= (S/\sigma^2)Q(i) - (S\beta/\sigma^2)G(i)\ . \qquad \text{(B.8)}$$

Further,

$$G(i) = \sum_{d=0}^{\infty} \frac{e^{-\theta}\theta^{d}}{d!}\, g_{d}(i,0) \qquad \text{(B.9)}$$

and $\partial\theta/\partial\beta = S\beta/\sigma^2$ so that using (B.9) we have

$$\frac{\partial G(i)}{\partial \beta} = -(S\beta/\sigma^2)\sum_{d=0}^{\infty} \frac{e^{-\theta}\theta^d}{d!} g_d(i,0)$$

$$+ (S\beta/\sigma^2)\sum_{d=0}^{\infty} \frac{e^{-\theta}\theta^d}{d!} g_{d+1}(i,0)$$

$$= -(S\beta/\sigma^2)G(i) + (S\beta/\sigma^2)\sum_{d=0}^{\infty} \frac{e^{-\theta}\theta^d}{d!} g_{d+1}(i,0) \ . \qquad \text{(B.10)}$$

Equating (B.8) and (B.10) gives the required result.

Using Lemma 2 with $i = 0$ and $i = 1$ in (B.7) we have

$$SE(\hat{b}) = S\beta \sum_{d=0}^{\infty} \frac{e^{-\theta}\theta^d}{d!} g_{d+1}(0,0) - pS\beta \sum_{d=0}^{\infty} \frac{e^{-\theta}\theta^d}{d!} g_{d+1}(1,0)$$

so that

$$T_2 = 2\sigma^2 \sum_{d=0}^{\infty} \frac{e^{-\theta}\theta^d}{d!}\left\{g_d(0,1)-2pg_d(1,1)+p^2g_d(2,1)\right\}$$

$$- 2\beta' S\beta \sum_{d=0}^{\infty} \frac{e^{-\theta}\theta^d}{d!}\left\{g_{d+1}(0,0)-pg_{d+1}(1,0)\right\} + \beta' S\beta$$

$$= 2\sigma^2\left\{\theta + \sum_{d=0}^{\infty} \frac{e^{-\theta}\theta^d}{d!}\left[g_d(0,1)-2pg_d(1,1)+p^2g_d(2,1)\right.\right.$$

$$\left.\left. - 2\theta g_{d+1}(0,0)+ 2\theta p g_{d+1}(1,0)\right]\right\} \ . \qquad \text{(B.11)}$$

Finally, we require

$$T_1 = E(y-X\hat{b})'(y-X\hat{b})$$

$$= E(\tilde{u}'\tilde{u}) + E(\tilde{\beta}'S\tilde{\beta}) - E\left(\tilde{\beta}'S\tilde{\beta}I_{(a/c^*,\infty)}(f^*)\right)$$

$$+ E\left(\tilde{\beta}'S\tilde{\beta}I_{(a/c^*,\infty)}(f^*)(a^2/f^{*2})\right) \ . \qquad \text{(B.12)}$$

As $(\tilde{u}'\tilde{u}/\sigma^2) \sim \chi^2_{(\nu)}$ and $(\tilde{\beta}'S\tilde{\beta}/\sigma^2) \sim \chi^{2\prime}_{(k;\theta)}$, it follows that $E(\tilde{u}'\tilde{u}) = \sigma^2\nu$ and $E(\tilde{\beta}'S\tilde{\beta}) = \sigma^2(k+2\theta)$.

Further, using Lemma 1

$$E\left(\tilde{\beta}' S\tilde{\beta} I_{(a/c^*,\infty)}(f^*)\right) = 2\sigma^2 \sum_{d=0}^{\infty} \frac{e^{-\theta}\theta^d}{d!} g_d(0,1)$$

and

$$E\left(\tilde{\beta}' S\tilde{\beta} I_{(a/c^*,\infty)}(f^*)(a^2/f^{*2})\right) = 2\sigma^2 p^2 \sum_{d=0}^{\infty} \frac{e^{-\theta}\theta^d}{d!} g_d(2,1)$$

so that (B.12) becomes

$$T_1 = \sigma^2\left[\nu+k+2\theta-2\sum_{d=0}^{\infty} \frac{e^{-\theta}\theta^d}{d!}\left(g_d(0,1)-p^2 g_d(2,1)\right)\right] . \qquad \text{(B.13)}$$

Substituting (B.11) and (B.13) into (B.1) and rearranging terms gives the required result.

REFERENCES

Bancroft, T.A. (1944), "On Biases in Estimation Due to the Use of Preliminary Tests of Significance", *Annals of Mathematical Statistics*, 15, 190-204.

Baranchik, A.J. (1964), "Multiple Regression and Estimation of the Mean of a Multivariate Normal Distribution", Technical Report No. 51, Department of Statistics, Stanford University.

Brook, R.J. (1976), "On the Use of a Regret Function to Set Significance Points in Prior Tests of Estimation", *Journal of the American Statistical Association*, 71, 126-131.

Clarke, J.A., Giles, D.E.A. and Wallace, T.D. (1987), "Estimating the Error Variance in Regression After a Preliminary Test of Restrictions on the Coefficients", *Journal of Econometrics*, 34, 293-304.

Giles, D.E.A. (1993), "Pre-Test Estimation in Regression Under Absolute Error Loss", *Economics Letters*, in press.

Giles, J.A. and Giles, D.E.A. (1993a), "Preliminary-Test Estimation of the Regression Scale Parameter When the Loss Function is Asymmetric", *Communications in Statistics, A*, in press.

Giles, J.A. and Giles, D.E.A. (1993b), "Risk of a Homoscedasticity Pre-Test Estimator of the Regression Scale Under LINEX Loss", *Journal of Statistical Planning and Inference*, in press.

Giles, J.A. and Giles, D.E.A. (1993c), "Pre-Test Estimation and Testing in Econometrics: Recent Developments", *Journal of Economic Surveys*, in press.

James, W. and Stein, C. (1961), "Estimation With Quadratic Loss", *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability*, Berkeley: University of California Press, 361-379.

Judge, G.G. and Bock, M.E. (1978), *The Statistical Implications of Pre-Test and Stein-Rule Estimators in Econometrics*, Amsterdam: North-Holland.

Ohtani, K. (1993), "A Comparison of the Stein-Rule and Positive Part Stein-Rule Estimators in a Misspecified Linear Regression Model", *Econometric Theory*, in press.

Ohtani, K., Giles, D.E.A. and Giles, J.A. (1993), "The Risk Behavior of a Pre-Test Estimator in a Regression Model With Possible Heteroscedasticity Under the LINEX Loss Function", mimeo., Faculty of Economics, Kobe University.

Press, W.H., Flannery, B.P., Teukolsky, S.A. and Vetterling, W.T. (1986), *Numerical Recipes: The Art of Scientific Computing*, Cambridge: Cambridge University Press.

Rodrigues, J. and Zellner, A. (1992), "Weighted Balanced Loss Function for the Exponential Mean Time to Failure", mimeo., Graduate School of Business, University of Chicago.

Stein, C. (1956), "Inadmissibility of the Usual Estimator for the Mean of a Multivariate Normal Distribution", *Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability, Vol. 1*, Berkeley: University of California Press, 197-206.

Stein, C. (1966), "An Approach to the Recovery of Inter-Block Information on Balanced Incomplete Block Designs", in F.N. David (ed.), *Research Papers in Statistics*, New York: Wiley, 351-366.

Toro-Vizcarrondo, C. and Wallace, T.D. (1968), "A Test of the Mean Square Error Criterion for Restrictions in Linear Regression", *Journal of the American Statistical Association*, 63, 558-572.

Varian, H. (1974), "A Bayesian Approach to Real Estate Assessment", in S.E. Fienberg and A. Zellner (eds.), *Studies in Bayesian Econometrics and Statistics in Honor of Leonard J. Savage Vol. 1*, Amsterdam: North-Holland, 195-208.

Zellner, A. (1991), "Bayesian and Non-Bayesian Estimation Using Balanced Loss Functions", Working Paper 91-109, Graduate School of Business, University of Chicago.

FOOTNOTES

* Judith A. Giles is Senior Lecturer and David E.A. Giles is Professor, Department of Economics, University of Canterbury, Private Bag 4800, Christchurch, New Zealand. Kazuhiro Ohtani is Professor, Faculty of Economics, Kobe University, Rokkodai, Nada-ku, Kobe 657, Japan. This work was supported in part by University of Canterbury Research Grants # 1770901, # 1787148, and # 1894183. We are grateful to Matthew Cunneen for his assistance.

1. The evaluations for the figures in this paper were undertaken with FORTRAN code written by the authors, incorporating routines from Press *et al.* (1986), and executed on a VAXstation 4000 under VMS 5.5. In all of the figures $\sigma^2 = 1$, which results in no loss of generality.

2. In Figures 1-4, two choices of c are considered: $c = 4.35$ ($\alpha = 5\%$), and $c = 1.895$ ($\alpha = 18.5\%$). The latter choice is Brook's (1976) "optimal" c in the case of quadratic loss.

3. That is, regardless of the value of m or ν, the "optimal" value of c will lie between zero and two in value, approximately.

FIGURE 1

RISKS UNDER BALANCED LOSS FUNCTION
(w = 0.0, m = 1, v = 20, k = 3)

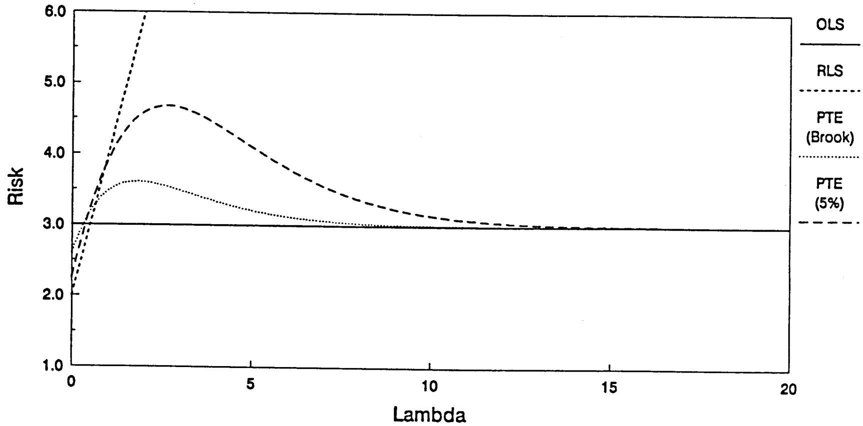


FIGURE 2

RISKS UNDER BALANCED LOSS FUNCTION
(w = 0.5, m = 1, v = 20, k = 3)

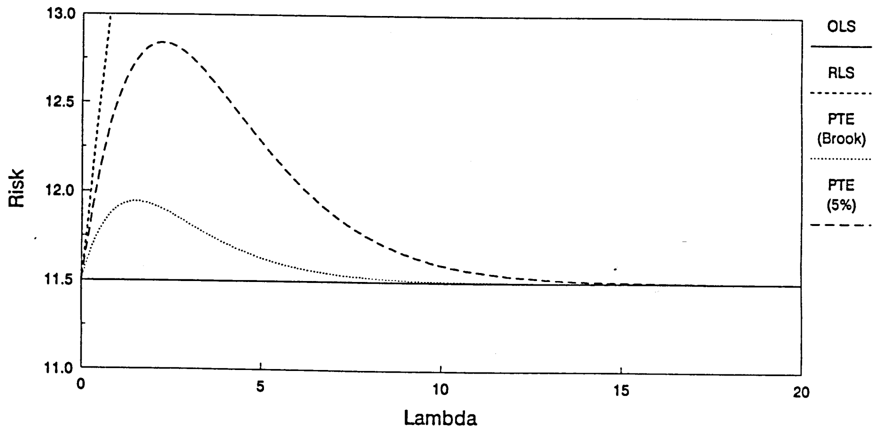


FIGURE 3

RISKS UNDER BALANCED LOSS FUNCTION
(w = 0.75, m = 1, v = 20, k = 3)

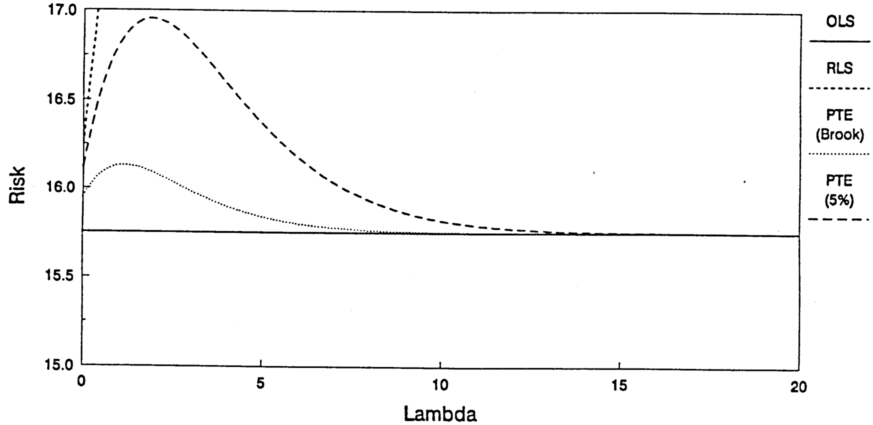


FIGURE 4

RISKS UNDER BALANCED LOSS FUNCTION
(w = 1.0, m = 1, v = 20, k = 3)

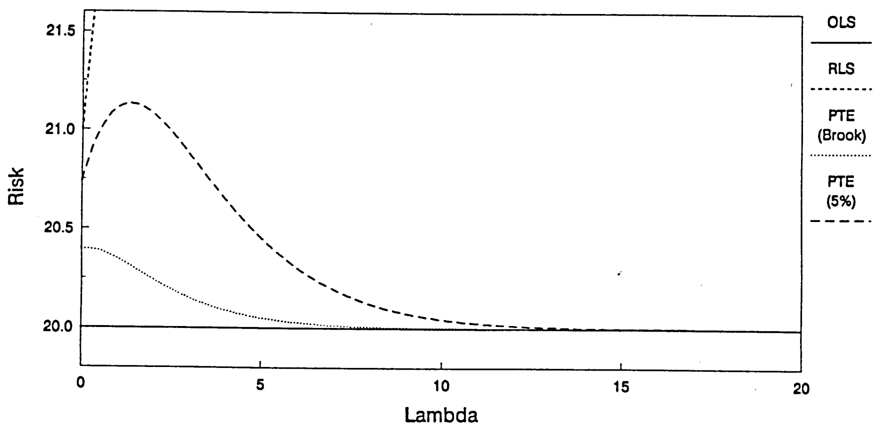


FIGURE 5

RISKS UNDER BALANCED LOSS FUNCTION

(w = 0.0, k = 3, v = 20, p = 0.045045)

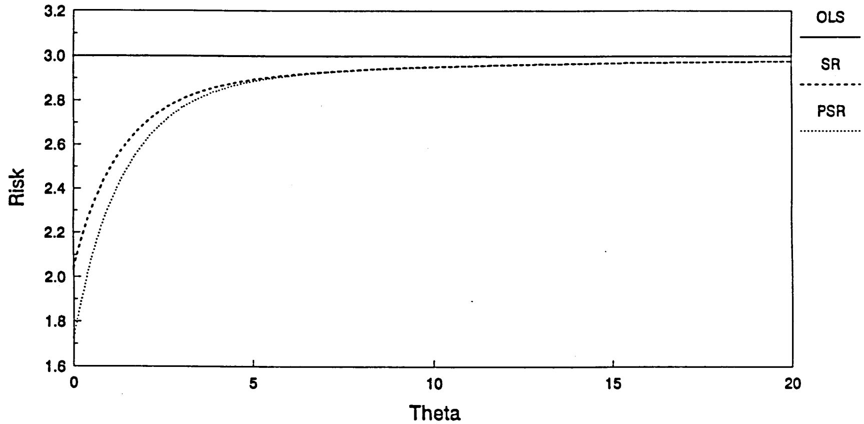


FIGURE 6

RISKS UNDER BALANCED LOSS FUNCTION

(w = 0.75, k = 3, v = 20, p = 0.01136363)

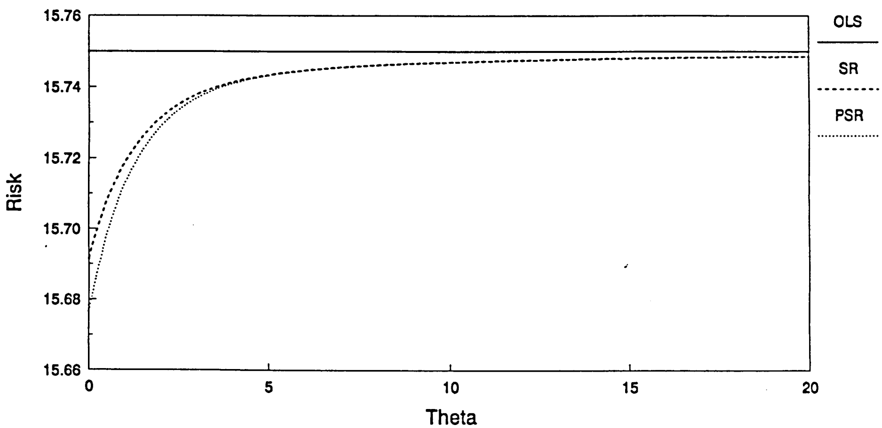


FIGURE 7
RISKS UNDER BALANCED LOSS FUNCTION
(w = 0.75, k = 3, v = 20, p = 0.045045)

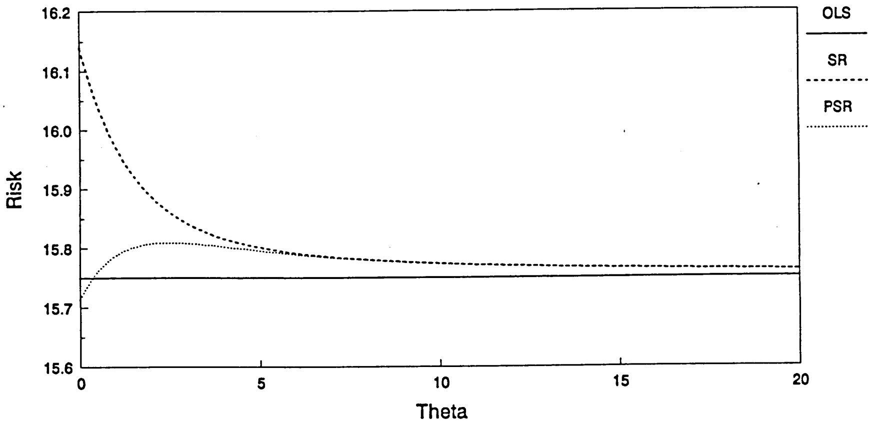
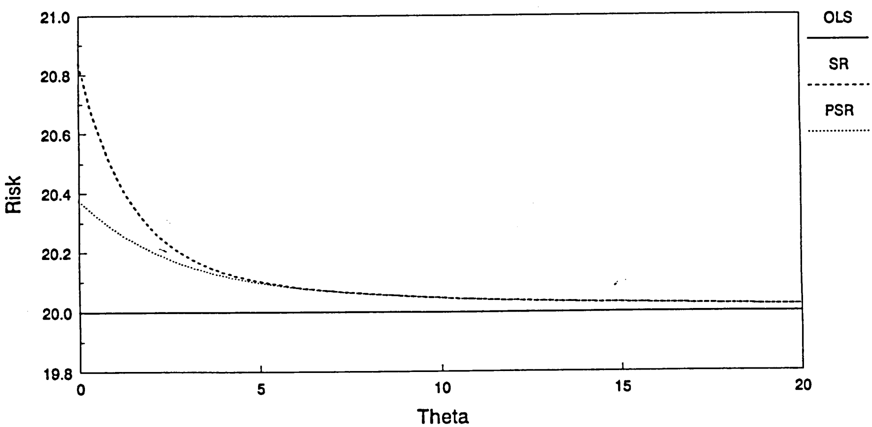


FIGURE 8
RISKS UNDER BALANCED LOSS FUNCTION
(w = 1.0, k = 3, v = 20, p = 0.045045)



LIST OF DISCUSSION PAPERS*

No. 8901 Testing for Financial Buffer Stocks in Sectoral Portfolio Models, by P. Dorian Owen.

No. 8902 Provisional Data and Unbiased Prediction of Economic Time Series by Karen Browning and David Giles.

No. 8903 Coefficient Sign Changes When Restricting Regression Models Under Instrumental Variables Estimation, by David E. A. Giles.

No. 8904 Economies of Scale in the New Zealand Electricity Distribution Industry, by David E. A. Giles and Nicolas S. Wyatt.

No. 8905 Some Recent Developments in Econometrics: Lessons for Applied Economists, by David E. A. Giles.

No. 8906 Asymptotic Properties of the Ordinary Least Squares Estimator in Simultaneous Equations Models, by V. K. Srivastava and D. E. A. Giles.

No. 8907 Unbiased Estimation of the Mean Squared Error of the Feasible Generalised Ridge Regression Estimator, by V. K. Srivasatva and D. E. A. Giles.

No. 8908 An Unbiased Estimator of the Covariance Matrix of the Mixed Regression Estimator, by D. E. A. Giles and V. K. Srivastava.

No. 8909 Pre-testing for Linear Restrictions in a Regression Model with Spherically Symmetric Disturbances, by Judith A. Giles.

No. 9001 The Durbin-Watson Test for Autocorrelation in Nonlinear Models, by Kenneth J. White.

No. 9002 Determinants of Aggregate Demand for Cigarettes in New Zealand, by Robin Harrison and Jane Chetwyd.

No. 9003 Unemployment Duration and the Measurement of Unemployment, by Manimay Sengupta.

No. 9004 Estimation of the Error Variance After a Preliminary-Test of Homogeneity in a Regression Model with Spherically Symmetric Disturbances, by Judith A. Giles.

No. 9005 An Expository Note on the Composite Commodity Theorem, by Michael Carter.

No. 9006 The Optimal Size of a Preliminary Test of Linear Restrictions in a Mis-specified Regression Model, by David E. A. Giles, Offer Lieberman, and Judith A. Giles.

No. 9007 Inflation, Unemployment and Macroeconomic Policy in New Zealand: A Public Choice Analysis, by David J. Smyth and Alan E. Woodfield.

No. 9008 Inflation — Unemployment Choices in New Zealand and the Median Voter Theorem, by David J. Smyth and Alan E. Woodfield.

No. 9009 The Power of the Durbin-Watson Test when the Errors are Heteroscedastic, by David E. A. Giles and John P. Small.

No. 9010 The Exact Distribution of a Least Squares Regression Coefficient Estimator After a Preliminary t-Test, by David E. A. Giles and Virendra K. Srivastava.

No. 9011 Testing Linear Restrictions on Coefficients in a Linear Regression Model with Proxy variables and Spherically Symmetric Disturbances, by Kazuhiro Ohtani and Judith A. Giles.

No. 9012 Some Consequences of Applying the Goldfeld-Quandt Test to Mis-Specified Regression Models, by David E. A. Giles and Guy N. Saxton.

No. 9013 Pre-testing in a Mis-specified Regression Model, by Judith A. Giles.

No. 9014 Two Results in Balanced-Growth Educational Policy, by Alan E. Woodfield.

No. 9101 Bounds on the Effect of Heteroscedasticity on the Chow Test for Structural Change, by David Giles and Offer Lieberman.

No. 9102 The Optimal Size of a Preliminary Test for Linear Restrictions when Estimating the Regression Scale Parameter, by Judith A. Giles and Offer Lieberman.

No. 9103 Some Properties of the Durbin-Watson Test After a Preliminary t-Test, by David Giles and Offer Lieberman.

No. 9104 Preliminary-Test Estimation of the Regression Scale Parameter when the Loss Function is Asymmetric, by Judith A. Giles and David E. A. Giles.

No. 9105 On an Index of Poverty, by Manimay Sengupta and Prasanta K. Pattanaik.

No. 9106 Cartels May Be Good For You, by Michael Carter and Julian Wright.

No. 9107 L_p-Norm Consistencies of Nonparametric Estimates of Regression, Heteroskedasticity and Variance of Regression Estimate when Distribution of Regression is Known, by Radhey S. Singh.

(Continued on next page)

No. 9108 Optimal Telecommunications Tariffs and the CCITT, by Michael Carter and Julian Wright.

No. 9109 Price Indices : Systems Estimation and Tests, by David Giles and Ewen McCann.

No. 9110 The Limiting Power of Point Optimal Autocorrelation Tests, by John P. Small.

No. 9111 The Exact Power of Some Autocorrelation Tests When the Disturbances are Heteroscedastic, by John P. Small.

No. 9112 Some Consequences of Using the Chow Test in the Context of Autocorrelated Disturbances, by David Giles and Murray Scott.

No. 9113 The Exact Distribution of R^2 when the Disturbances are Autocorrelated, by Mark L. Carrodus and David E. A. Giles.

No. 9114 Optimal Critical Values of a Preliminary Test for Linear Restrictions in a Regression Model with Multivariate Student-t Disturbances, by Jason K. Wong and Judith A. Giles.

No. 9115 Pre-Test Estimation in a Regression Model with a Misspecified Error Covariance Matrix, by K. V. Albertson.

No. 9116 Estimation of the Scale Parameter After a Pre-test for Homogeneity in a Mis-specified Regression Model, by Judith A. Giles.

No. 9201 Testing for Arch-Garch Errors in a Mis-specified Regression, by David E. A. Giles, Judith A. Giles, and Jason K. Wong.

No. 9202 Quasi Rational Consumer Demand — Some Positive and Normative Surprises, by John Fountain.

No. 9203 Pre-test Estimation and Testing in Econometrics: Recent Developments, by Judith A. Giles and David E. A. Giles.

No. 9204 Optimal Immigration in a Model of Education and Growth, by K-L. Shea and A. E. Woodfield.

No. 9205 Optimal Capital Requirements for Admission of Business Immigrants in the Long Run, by K-L. Shea and A. E. Woodfield.

No. 9206 Causality, Unit Roots and Export-Led Growth: The New Zealand Experience, by David E. A. Giles, Judith A. Giles and Ewen McCann.

No. 9207 The Sampling Performance of Inequality Restricted and Pre-Test Estimators in a Mis-specified Linear Model, by Alan T. K. Wan.

No. 9208 Testing and Estimation with Seasonal Autoregressive Mis-specification, by John P. Small.

No. 9209 A Bargaining Experiment, by Michael Carter and Mark Sunderland.

No. 9210 Pre-Test Estimation in Regression Under Absolute Error Loss, by David E. A. Giles.

No. 9211 Estimation of the Regression Scale After a Pre-Test for Homoscedasticity Under Linex Loss, by Judith A. Giles and David E. A. Giles.

No. 9301 Assessing Starmer's Evidence for New Theories of Choice: A Subjectivist's Comment, by John Fountain.

No. 9302 Preliminary-Test Estimation in a Dynamnic Linear Model, by David E. A. Giles and Matthew C. Cunneen.

No. 9303 Fans, Frames and Risk Aversion: How Robust is the Common Consequence Effect? by John Fountain and Michael McCosker.

No. 9304 Pre-test Estimation of the Regression Scale Parameter with Multivariate Student-t Errors and Independent Sub-Samples, by Juston Z. Anderson and Judith A. Giles

No. 9305 The Exact Powers of Some Autocorrelation Tests When Relevant Regressors are Omitted, by J. P. Small, D. E. Giles and K. J. White.

No. 9306 The Exact Risks of Some Pre-Test and Stein-Type Regression Estimators Under Balanced Loss, by J. A. Giles, D. E. A. Giles, and K. Ohtani.

* Copies of these Discussion Papers may be obtained for $4 (including postage, price changes occasionally) each by writing to the Secretary, Department of Economics, University of Canterbury, Christchurch, New Zealand. A list of the Discussion Papers prior to 1989 is available on request.