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Abstract

We consider the power functions of five popular tests for AR(1) errors in a linear regression model from which relevant regressors have inadvertently been omitted. These functions are derived by numerically evaluating the finite-sample distributions of the test statistics. With this form of model mis-specification, it is found that the performances of the tests are not independent of the scale of the errors' distribution. The omission of seasonal effects or a linear trend component can have serious implications, especially if testing against positive autocorrelation, and some of the well known advantages of the "Alternative Durbin Watson test" (King (1981)) are found to still apply when the model is underspecified.

Keywords: Regression disturbances, serial independence, power, ratios of quadratic forms.

1. INTRODUCTION

When the residuals from a regression are found to be autocorrelated it is often concluded that one or more regressors may have been omitted from the fitted model. In particular, the typical time series characteristics of economic variables suggest that such an omission will induce or magnify an autoregressive error process. This argument is most convincing when both the true disturbances and the omitted variable(s) are positively autocorrelated, but it is questionable in other circumstances. More importantly, this argument takes no account of the effects of omitting variables on the probability of detecting autocorrelation. The purpose of this paper is to provide some exact finite sample numerical evidence of these effects on the power functions of several popular autocorrelation tests.

We consider five tests for first order autoregressive disturbances, all of which are well known and easily applied. These are the Durbin-Watson (DW) test of Durbin and Watson (1950), King's (1981) alternative DW (ADW) test, the Berenblut and Webb (1973) test (BW) and two versions of the point optimal test ($S(\rho_1)$) (Kadiyala (1970), King (1985)). These tests have well known power properties in correctly specified models, a thorough numerical evaluation of which is provided by King (1985).

The balance of the paper begins with an outline of the tests used in the next section. The distributions of the test statistics are discussed in Section 3, along with various computational considerations. Section 4 presents the models used in our evaluations and the results of these evaluations are given in Section 5. The final section offers some recommendations for applied workers.

2. THE TESTS

The statistic for each test studied here can be written in the form

$$r = \frac{w'Qw}{w'Mw}$$

where $w \sim N(0,\sigma_w^2 I)$ is an n×1 random vector, $M = I - X(X'X)^{-1}X'$, and Q is some non-stochastic n×n matrix.

The particular Q matrix for each test statistic is as follows. For the DW test, Q = MAM where

$$A = \begin{bmatrix} 1 & -1 & 0 & & \cdots & 0 \\ -1 & 2 & -1 & 0 & & \vdots \\ 0 & -1 & 2 & -1 & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & 0 \\ \vdots & & & \ddots & 2 & -1 \\ 0 & \cdots & & 0 & -1 & 1 \end{bmatrix} .$$

The ADW test has $Q = MA_0M$ where A_0 is A with the top left and bottom right elements replaced by 2.

For the point optimal tests

$$Q = \Sigma(\rho_1)^{-1} - \Sigma(\rho_1)^{-1}X(X'\Sigma(\rho_1)^{-1}X)^{-1}X'\Sigma(\rho_1)^{-1}$$

$$\text{where} \quad \Sigma(\rho_1) = \sigma_w^2 \begin{bmatrix} 1 & \rho_1 & \rho_1^2 & \cdots & \rho_1^{T-1} \\ \rho_1 & 1 & \rho_1 & & \vdots \\ \rho_1^2 & \rho_1 & 1 & \ddots & \\ \vdots & & \ddots & 1 & \rho_1 \\ \rho_1^{T-1} & \rho^{T-2} & \cdots & \rho_1 & 1 \end{bmatrix}$$

and ρ_1 is chosen by the researcher. Following King (1985) we choose ρ_1 = 0.5 and 0.75.

The BW test has $Q = B - BX(X'BX)^{-1}X'B$ where B is A with the top left element replaced by 2. This test can be seen as the limit of a sequence of point optimal tests as the autoregressive parameter, ρ, approaches unity.

For some purposes it is useful to write the $s(\rho_1)$ and BW test statistics in the form of a DW type test with a particular A matrix. This can be accomplished by noting, from Evans and King (1985), that as $Q = B - BX(X'BX)^{-1}X'B$, then $MQ = QM = Q$, and so $Q = MQM$.

3. THEORY

Suppose that the true data-generating process is in the form of the following multiple regression model with AR(1) errors:

$$y = X\beta + Z\gamma + u$$

$$u = \rho u_{-1} + \varepsilon$$

$$\varepsilon \sim N(0,\sigma^2 I)$$

where y and u are n×1 vectors of observations on the dependent variable and the random disturbances respectively, X and Z are non-stochastic regressor matrices of dimension n×k and n×p, and β and γ are their associated parameter vectors. Note that $u \sim N(0,\Omega(\rho))$, where

$$\Omega(\rho) = \sigma_u^2 \begin{bmatrix} 1 & \rho_1 & \rho^2 & \cdots & & \rho^{T-1} \\ \rho_1 & 1 & & & & \vdots \\ \rho_1^2 & & \ddots & & & \\ \vdots & & & & 1 & \rho \\ \rho^{T-1} & \rho^{T-2} & & \cdots & \rho & 1 \end{bmatrix}$$

If the model fitted to the data is

$$y = X\beta + v$$

then the residuals from this regression are given by

$$\hat{v} = y - X\hat{\beta}$$
$$= My$$
$$= Mw$$

where $$w = Z\gamma + u$$
$$= \phi + u.$$

and $$M = I - X(X'X)^{-1}X'.$$

We are interested in testing H_0: $\rho = 0$ vs H_a^+ : $\rho > 0$, or H_a^- : $\rho < 0$, but we are constrained to using $\hat{v}$ to do so, rather than being able to use the residuals associated with the fully specified model. This is because either it is not known that the fitted model is misspecified, or because the Z data are unavailable.

The distribution of the test statistic r is non-standard (since $E(w) = \phi \neq 0$) but we can follow the manipulations suggested by Koerts and Abrahamse (1969, pp.81-82) to compute the distribution function of r:

$$\Pr(r < r^*) = \Pr\{w'(Q-r^*M)w < 0\}.$$

Let $z = \Omega^{-1/2}w$, such that $\Omega^{-1/2}\Omega^{-1/2} = \Omega^{-1}$.

Now, $$w'(Q-r^*M)w = z'\Omega^{1/2}(Q-r^*M)^{1/2}\Omega\, z$$
$$= z'Sz$$

and $$z \sim N(\Omega^{-1/2}\phi, I_T).$$

Since S is symmetric, there exists an orthogonal matrix P

such that $$P'SP = \Lambda = \mathrm{diag}\ (\lambda_j).$$

We can now make the orthogonal transformation $P'z = q$, so that $z'Sz = q'\Lambda q$

and $q \sim N(P'\Omega^{-1/2-}\phi, I_T)$.

Then $$\Pr(r<r^*) = \Pr(q'\Lambda q < 0)$$
$$= \Pr\left[\sum_{j=1}^{T} \lambda_j\, \chi_j^{2'} < 0\right] \qquad (1)$$

where the λ_j's are the eigenvalues of S and the $\chi_j^{2\prime}$'s are independent, non-central chi-square variates with 1 degree of freedom and non-centrality parameters given by

$$\theta_j = \left[(P'\Omega^{-1/2}\phi)_j\right]^2 \quad . \tag{2}$$

When r^* is a $100\alpha\%$ critical value, the probability in (1) is the power of the test. The numerical evaluation of (1) is a fairly standard computational problem. Notice from (2) that the distribution of r depends on the scale of ε, through $\Omega^{-1/2}$. In particular $\frac{\partial\theta_j}{\partial\sigma^2} < 0$ so the effect of misspecification is reduced by more variable ε_t's.

We can gain further insight into the effect of omitting variables on the tests considered by observing the following consequences of the diagonalisation of S.

$P'P = PP' = I$ implies that $S = P\Lambda P'$, and since $\Omega^{-1/2}\Omega^{1/2} = I$ we can write

$$\Omega^{-1/2}P\Lambda P'\Omega^{-1/2} = Q - r^*M$$

$$\Lambda P'\Omega^{1/2} = P'\Omega^{1/2}(Q-r^*M)$$

$$= P'\Omega^{1/2}M(A-r^*I)M \; .$$

The last step uses the fact that, for each test, Q can be written as a quadratic form in M. We can now exploit the diagonality of Λ to show that the jth non-centrality parameter is given by

$$\left(P'\Omega^{-1/2}\phi_j\right)^2 = \left(\frac{1}{\lambda_j}\, P'\Omega^{1/2}M(A-r^*I)M\phi_j\right)^2 \quad . \tag{3}$$

If ϕ is linearly dependent on the included regressors, then $M\phi = 0$ and the non-centrality parameters are all zero. Alternatively, if ϕ is orthogonal to all of the included regressors, then all of the non-centrality parameters are positive.

For the k eigenvalues satisfying $\lambda_j = 0$, the expression in (3) is undefined. Fortunately these cases are irrelevant for the determination of the tests' powers as the corresponding χ^2 variates receive zero weight in (1).

This result provides an interesting contrast with the well known dependence of the bias of the OLS estimator on the degree of linear dependence between X and Z. In testing for autocorrelation, an orthogonal regressor is the worst type of omission possible, notwithstanding the fact that the OLS estimator is unbiased. This is intuitively appealing in the case of OLS residuals, which are orthogonal to X and, by definition, contain all possible omitted variables.

We can also use (3) to see that $\theta_j \to 0$ as $\rho \to 1$ for all j, providing the regression has an intercept. This is because as $\rho \to 1$, $\Omega \to ii'$ where $i = (1,1,\ldots 1)'$ so $\Omega^{1/2}$ is a matrix with all elements being $T^{-1/2}$. $\Omega^{1/2}$ is therefore linearly dependent on X when a constant is included in the regression, and so $\Omega^{1/2}M = 0$. This means that the mis-specified power functions must approach those for the true model as ρ approaches unity.

The distribution function of the non-central chi-square distribution with 1 degree of freedom lies to the right of that for the corresponding central distribution. Thus, drawings from a non-central distribution are likely to be smaller than their central counterparts. This does not allow <u>a priori</u> prediction of the direction of power shift however, as each chi-square variate in (1) will have a different θ_j (in general) and a weight (λ_j) which may be positive or negative. There is, therefore, a need to evaluate the power functions numerically to reveal both the direction and extent of shift in power under this form of model mis-specification.

The calculation of the power functions given by (1) and (2) is straightforward and was performed using the Davies (1980) algorithm option with the DISTRIB facility in the SHAZAM (1993) package. It is, however, worth noting that $\Omega^{-1/2}$ need not be computed unless non-centrality parameters are present, in which case it must be a symmetric matrix. All of the computations were undertaken on a VAX6340 computer under VMS 5.3.

4. MODELS

Recognising the data dependence of each test statistic's distribution, the numerical evaluations that we have undertaken involved a variety of data based on seven different design matrices, as characterised in Table 1. The numerical evaluations were conducted in two groups. For the first group, real variables were omitted which had varying degrees of linear dependence on the X matrix. To measure the degree of linear independence between the included and excluded variables, for this first group, we used the vector coefficient of alienation, defined by

$$\rho_A = \frac{\begin{vmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{vmatrix}}{|\Sigma_{11} \quad \Sigma_{22}|}$$

where Σ_{11} is the sample covariance matrix of the included variables, X, Σ_{22} = cov(Z) and Σ_{12} = cov(X,Z) and all variables measured are in deviations from their sample means. When Z has only one non-constant variable, $(1-\rho_A)$ is equivalent to the uncentred R^2 from regressing Z on X.

Table 1 Regressors used in numerical evaluations

Model	$(1-\rho_A)$	R^2(omitted/included)	Included	Omitted
1	0.79	.79	C INC	PRI
2	0.95	.95	C CPI	CPI(-1)
3	0.01	.01	C NOR	T
4	0.07	.07	C LNOR	T
5	0.00	.00	C X1	X2
6	0.17	.11	C LNOR T	S1 S2 S3
7	0.37	.25	C CPI CPIL	S1 S2 S3

In this table C is a constant, INC and PRI are the income and price variables from Durbin and Watson's (1950) "consumption of spirits" example, CPI is the quarterly Australian consumers' price index commencing 1951(1), NOR ~ N(30,4), T is a linear time-trend, LNOR ~ lognormal (2.23, 19.58), S1-S3 are quarterly seasonal dummy variables, and X1-X5 are the first five vectors from the series $(a_2+a_n)/\sqrt{2}$, $(a_3+a_{n-1})/\sqrt{2}, \ldots,$ where $a_1, \ldots, a_n$ are the eigenvectors corresponding to the eigenvalues of A arranged in ascending order. These data have been used in previous studies of this type, such as Evans (1992).

Each model was fitted with 20 observations and some limited experimentation with 60 observations was performed. All tests were conducted at a nominal 5% size and no size corrections were made. The variance of ε was set to unity although limited experiments were conducted with higher values.

The second set of power functions was constructed using a single variable, which was orthonormal to X, as the omitted regressor. This variable was constructed by normalizing the OLS residual vector from a regression of a standard normal random variable on the X matrix of the model concerned. The normal variable was generated via Brent's (1974) algorithm in the SHAZAM package.

The degree of mis-specification for both groups of power curves was controlled through γ, the values of which were chosen to reveal the degree of movement in the power curves. Notice that, from (1) and (2), the probability of rejection is independent of the sign of γ. When $\gamma = 0$ the Z matrix does not enter the true data generating process and $\theta = 0$. The reported results include the relevant values of γ.

5. RESULTS

The characteristics of all of the power functions in the correctly specified models ($\gamma = 0$) were in accord with those obtained in previous studies. With the exception of models using Watson's matrix (models 6 and 12) the power differences between the tests were relatively minor, particularly against H_a^+. Where differences were apparent they revealed lower power for the ADW test against H_a^+ and higher power for this test against H_a^-. As noted by King (1985) the $S(\rho_1)$ and BW tests can have far greater power than the DW and ADW tests when used with Watson's matrix against H_a^+, but this ranking is reversed against H_a^-.

The effect of omitting variables appears to depend mainly on the associated coefficient and the form of the omitted variable. The nature of the included regressors is generally of less importance, as is the particular test used.

The most serious power losses encountered in the first group of evaluations occurred when a seasonal shift in the intercept was not allowed for in testing against H_a^+. In this case the true sizes of all tests fell dramatically from their nominal 5% level with an associated loss of power for all $\rho > 0$. This effect occurred in model 6 and can be clearly seen in Figure 1. No single test emerged as being any more or less robust to this type of mis-specification across the relevant models.

[Figure 1 about here]

In models 3 and 4 the omission of a linear trend resulted in the true sizes of the tests against H_a^+ increasing to an average of 20.4% when the trend coefficient was 0.8. The ADW test was consistently the least distorted but still had an average size of 18.7% over the same three models. Testing against H_a^- in these models reduced the true sizes below 5% to a similar degree for all tests. These effects can be seen in Figure 2 which is based on model 3. While the size distortions in these models had predictable effects on the test powers in the neighbourhood of the null, the effects against high absolute values of ρ was less marked and the limiting powers were virtually identical. This effect was foreshadowed in the theoretical discussion above.

Models 1,2 and 7 involved the omission of real variables and produced broadly similar results, an example of which is given in Figure 3. Against H_a^+ there are power gains available by using the DW or BW tests rather than the other tests considered in these models. This conclusion is unambiguous in model 1 but is offset by size distortion, analogous to the case of an omitted linear trend discussed above, in models 2 and 7. Figure 3 also reveals a feature common to all models, though less discernable in others. This is the tendency for the power rankings of the tests to be reversed when moving from H_a^+ to H_a^-.

[Figures 2 and 3 about here]

For the second group of power functions, which examine the effect of the scale of an omitted orthogonal regressor on the powers of the tests, selected powers are presented in Tables 2 and 3. In each case considered the true sizes of tests against H_a^+ are reduced by the omission of an orthogonal regressor, the degree of reduction increasing with $|\gamma|$. The size of the DW test in model 7, for example, is reduced from 5% to 0.3% by the omission of an orthogonal variable which would have had a coefficient of ±5. It is also clear from Tables 2 and 3 that the powers of the tests are correspondingly lower as $|\gamma|$ increases.

The sizes and powers of tests against H_a^- are considerably more robust than those against H_a^+, as the tabulated values show. No general statement about the direction of size distortion is supported by the representative values in Tables 2 and 3.

[Tables 2 and 3 about here]

6. CONCLUSIONS

There are several results of interest emerging from this study. First, when relevant variables are omitted from a regression the distribution of all of the test statistics considered here is no longer independent of σ^2. It has also been shown that a failure to allow for a seasonal shift in the intercept can seriously weaken all of these tests, particularly against H_a^+. The dominance of the ADW test when testing against H_a^- is confirmed and extended to models mis-specified by the omission of variables. Furthermore the ADW test is the most robust to the effect of omitting a linear trend when testing for positive autocorrelation. The omission of an orthogonal regressor constitutes an informal bound on the degree of power function distortion. In all cases considered the true sizes and powers of tests against H_a^+ were reduced by this form of model mis-specification. The scale of the omitted variable has been shown to be important but the results are invariant to the sign of the associated coefficient.

In the light of these findings we make the following recommendations. First, the possibility of a seasonal shift in the regression intercept should be investigated before testing for AR(1) disturbances. This could be achieved by testing the significance of seasonal dummy variables prior to testing for autocorrelation. Of course, this involves a pre-test strategy, with commensurate implications for the size and power of the second test. Some evidence concerning the distortions associated with this type of strategy is provided by Giles and Lieberman (1992). Second, we recommend the use of the ADW test when negative autocorrelation is being investigated because it continues to have superior power against this alternative when relevant variables have been omitted. Finally, the rejection of H_a^+ using any of these tests should be regarded with suspicion unless one can be sure that a linear trend has not been omitted.

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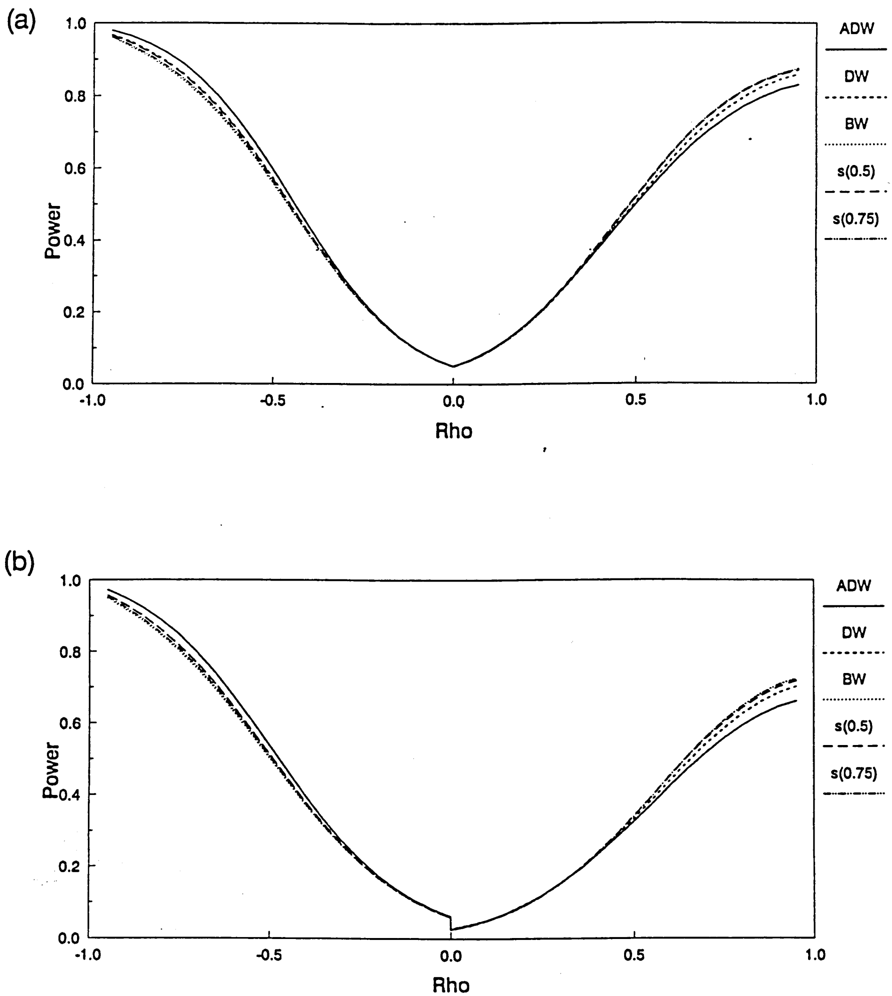


Figure 1 Power functions with lognormal data, seasonal dummies omitted. (a) $\gamma=(0,0,0)'$; (b) $\gamma=(1,1,1)'$.

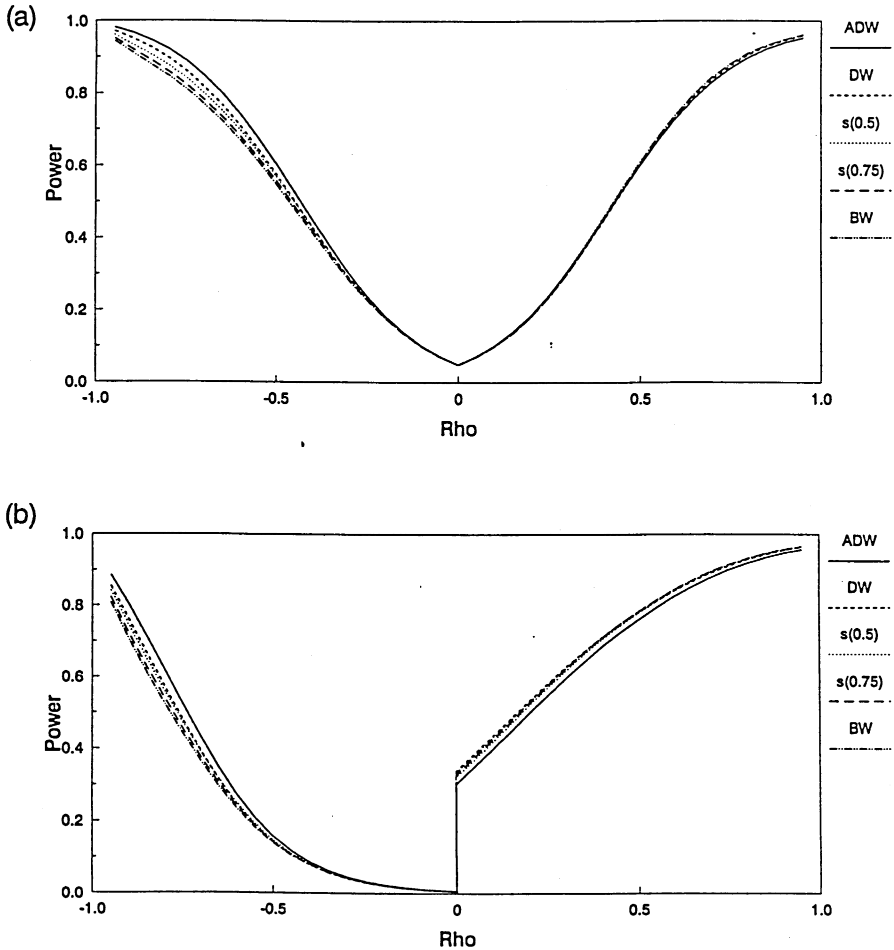
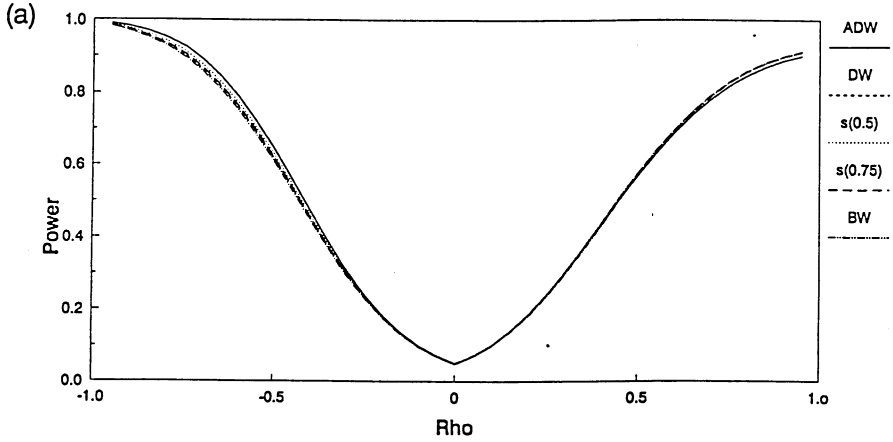


Figure 2 Power functions with normal data, linear trend omitted. (a) $\gamma=0$; (b) $\gamma=0.1$.



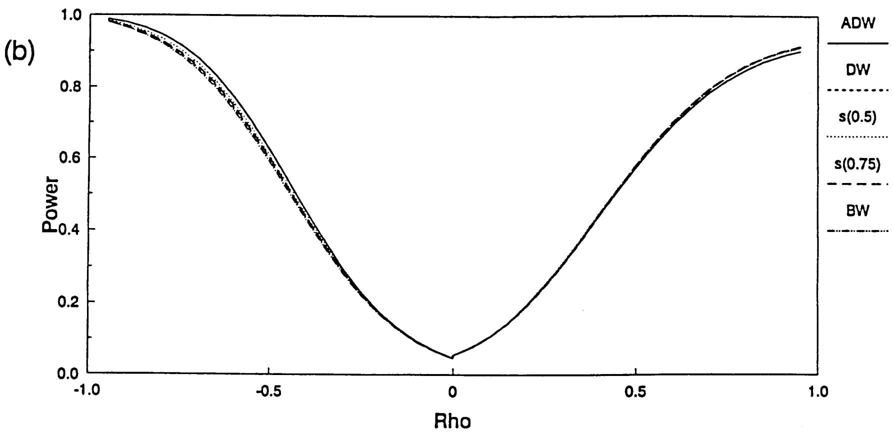


Figure 3 Power functions with spirits data, price omitted.
(a) $\gamma=0$; (b) $\gamma=5$.

Table 2 Power of AR(1) tests with Omitted Orthogonal Variables (T=20)

	DW			s(0.5)			BW		
ρ	$\gamma=0$	$\gamma=1$	$\gamma=5$	$\gamma=0$	$\gamma=1$	$\gamma=5$	$\gamma=0$	$\gamma=1$	$\gamma=5$
	Model 1								
-0.9	0.971	0.966	0.842	0.977	0.972	0.857	0.969	0.964	0.834
-0.6	0.759	0.740	0.477	0.773	0.754	0.485	0.751	0.731	0.454
-0.3	0.306	0.300	0.204	0.311	0.304	0.204	0.302	0.294	0.188
0.0	0.050	0.054	0.075	0.050	0.054	0.074	0.050	0.054	0.067
0.0	0.050	0.045	0.005	0.050	0.045	0.005	0.050	0.045	0.004
0.3	0.298	0.265	0.022	0.301	0.268	0.022	0.294	0.261	0.020
0.6	0.698	0.649	0.111	0.700	0.651	0.112	0.694	0.646	0.106
0.9	0.901	0.870	0.324	0.900	0.869	0.322	0.903	0.873	0.328
	Model 2								
-0.9	0.969	0.962	0.818	0.976	0.971	0.834	0.969	0.962	0.815
-0.6	0.753	0.732	0.436	0.770	0.748	0.439	0.753	0.731	0.424
-0.3	0.304	0.295	0.174	0.309	0.300	0.170	0.303	0.294	0.164
0.0	0.050	0.054	0.061	0.050	0.053	0.058	0.050	0.053	0.057
0.0	0.050	0.047	0.008	0.050	0.047	0.008	0.050	0.047	0.009
0.3	0.291	0.262	0.032	0.296	0.267	0.033	0.291	0.263	0.034
0.6	0.681	0.633	0.124	0.683	0.636	0.127	0.680	0.634	0.131
0.9	0.880	0.846	0.296	0.878	0.844	0.296	0.880	0.846	0.305
	Model 3								
-0.9	0.951	0.942	0.763	0.934	0.926	0.764	0.911	0.901	0.722
-0.6	0.710	0.686	0.391	0.698	0.676	0.402	0.670	0.649	0.386
-0.3	0.293	0.283	0.161	0.293	0.284	0.170	0.285	0.277	0.169
0.0	0.050	0.053	0.053	0.050	0.053	0.056	0.050	0.053	0.058
0.0	0.050	0.046	0.007	0.050	0.045	0.004	0.050	0.046	0.005
0.3	0.302	0.273	0.031	0.309	0.277	0.023	0.302	0.271	0.026
0.6	0.737	0.697	0.172	0.745	0.704	0.154	0.742	0.700	0.157
0.9	0.948	0.932	0.550	0.949	0.933	0.532	0.950	0.934	0.538
	Model 4								
-0.9	0.954	0.948	0.796	0.946	0.940	0.795	0.939	0.933	0.784
-0.6	0.732	0.709	0.384	0.719	0.696	0.369	0.709	0.686	0.364
-0.3	0.302	0.289	0.134	0.298	0.285	0.124	0.295	0.282	0.124
0.0	0.050	0.052	0.041	0.050	0.051	0.037	0.050	0.051	0.037
0.0	0.050	0.047	0.010	0.050	0.047	0.009	0.050	0.047	0.009
0.3	0.294	0.268	0.041	0.297	0.270	0.038	0.292	0.265	0.037
0.6	0.716	0.676	0.184	0.732	0.693	0.183	0.729	0.689	0.179
0.9	0.936	0.918	0.540	0.946	0.930	0.551	0.947	0.931	0.551

Table 3 Power of AR(1) tests with Omitted Orthogonal Variables (T=20)

	DW			s(0.5)			BW		
ρ	$\gamma=0$	$\gamma=1$	$\gamma=5$	$\gamma=0$	$\gamma=1$	$\gamma=5$	b3=0	b3=1	b3=5
	Model 5								
-0.9	0.669	0.658	0.484	0.488	0.482	0.367	0.448	0.441	0.330
-0.6	0.611	0.590	0.332	0.547	0.529	0.306	0.511	0.494	0.284
-0.3	0.268	0.260	0.148	0.260	0.252	0.141	0.250	0.242	0.132
0.0	0.050	0.053	0.053	0.050	0.052	0.049	0.050	0.053	0.047
0.0	0.050	0.047	0.009	0.050	0.046	0.007	0.050	0.048	0.015
0.3	0.266	0.241	0.034	0.277	0.250	0.029	0.263	0.240	0.040
0.6	0.602	0.561	0.124	0.695	0.650	0.132	0.689	0.646	0.156
0.9	0.680	0.652	0.248	0.928	0.905	0.408	0,933	0.912	0.463
	Model 6								
-0.9	0.949	0.941	0.770	0.947	0.940	0.772	0.936	0.929	0.754
-0.6	0.707	0.681	0.328	0.705	0.678	0.310	0.689	0.663	0.303
-0.3	0.282	0.268	0.103	0.282	0.266	0.092	0.277	0.262	0.091
0.0	0.050	0.051	0.032	0.050	0.050	0.027	0.050	0.051	0.028
0.0	0.050	0.048	0.014	0.050	0.048	0.014	0.050	0.048	0.014
0.3	0.262	0.239	0.043	0.268	0.246	0.046	0.263	0.241	0.045
0.6	0.626	0.581	0.135	0.644	0.602	0.147	0.642	0.600	0.146
0.9	0.846	0.809	0.293	0.859	0.826	0.312	0.862	0.830	0.319
	Model 7								
-0.9	0.965	0.961	0.852	0.963	0.958	0.834	0.950	0.943	0.788
-0.6	0.737	0.720	0.476	0.732	0.712	0.401	0.710	0.687	0.337
-0.3	0.290	0.285	0.199	0.289	0.279	0.134	0.283	0.269	0.099
0.0	0.050	0.054	0.073	0.050	0.051	0.040	0.050	0.050	0.026
0.0	0.050	0.044	0.003	0.050	0.045	0.004	0.050	0.046	0.006
0.3	0.274	0.240	0.014	0.277	0.246	0.019	0.273	0.246	0.027
0.6	0.638	0.583	0.072	0.647	0.597	0.091	0.645	0.600	0.115
0.9	0.844	0.800	0.215	0.851	0.812	0.244	0.854	0.819	0.282

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