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Preliminary-Test Estimation in a Dynamic Linear Model*

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Abstract

This paper considers the estimation of a dynamic linear regression model after a pre-test of exact linear restrictions on the coefficient vector. Monte Carlo evidence illustrates that pre-testing can be risk-superior to both ordinary and restricted least squares.

J.E.L. Classification: C22

1. Introduction

The implications of testing the validity of exact linear restrictions on the coefficients of a linear regression model, prior to estimating those coefficients, are well known when the model satisfies the usual "ideal" assumptions. Judge and Bock (1978) and Giles and Giles (1993), among others, document the risk properties of this and related "pre-test" estimators.

The latter authors also discuss recent work which focuses on the sampling performance of certain common pre-test regression coefficient estimators when some of the assumptions underlying the standard regression framework are violated. For example, the effects of wrongly omitted or included regressors (Ohtani (1983), Mittelhammer (1984), Giles (1986)); non-Normal errors (Giles (1991a,b)); or a non-spherical error covariance matrix (Albertson (1993)) have all been considered.

However, to date no account has been taken of the effects of stochastic regressors[1] on the risk properties of the regression coefficient pre-test estimator. An important practical example of this arises with a model which is "dynamic", or autoregressive, in the sense of including lagged values of the dependent variable as regressors.

In this paper we consider this problem, and use Monte Carlo simulation to compute the risks of the above pre-test estimator and its "component" estimators in models with one or two lagged dependent variables. We show, in particular, that the presence of model dynamics can alter some of the established risk-dominance results.

2. The model framework

Consider the model

$$y = X\beta + u \;\; ; \;\; u \sim N(0, \sigma^2 I)$$

where y is $(n \times 1)$, β is $(k \times 1)$, and X is $(n \times k)$ and of rank k. We allow for the possibility that at least one of the columns of X is stochastic. Then, suppose that we test H_0: $R\beta = r$ (where R is $(m \times k)$ and of rank m, and r is $(m \times 1)$, both non-stochastic) prior to estimating β. The pre-test estimator of β, say $\hat{\beta}$, is *either* $\tilde{\beta} = (X'X)^{-1}X'y$ (the Ordinary

Least Squares, or OLS, estimator) if H_0 is rejected; *or* $\beta^* = \tilde{\beta} + (X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}(r - R\tilde{\beta})$ (the Restricted Least Squares, or RLS, estimator) if H_0 is *not* rejected.

Typically, one would ignore the randomness of X and test H_0 using[2] the usual "F-statistic", $f = [(u^{*\prime}u^* - \tilde{u}'\tilde{u}) / \tilde{u}'\tilde{u}]\,[(n - k)/m]$, where $u^* = y - X\beta^*$ and $\tilde{u} = y - X\tilde{\beta}$. Then, we can write $\hat{\beta} = \beta^* I_{[0,c)}(f) + \tilde{\beta}\, I_{[c,\infty)}(f)$, where $I_{[a,b)}(f) = 1$ if $f \in [a,b)$, zero otherwise; and c is the usual $F_{m,n-k}$ critical value for a nominal $\alpha\%$ significance level.

We consider the case where X is random because some of its columns are lagged values of y. Then, it is well known that $\tilde{\beta}$ and β^* are biased (even if H_0 is true), and their covariance matrices differ (in a manner depending on the randomness of the data) from what they would be with fixed regressors. Also, f is *not* F-distributed in this case (*e.g.*, Evans and Savin (1982)). We determine the risk of $\hat{\beta}$ under quadratic loss. That is, we use $\rho(\beta) = E(\hat{\beta} - \beta)'(\hat{\beta} - \beta)$ as the measure of estimator performance. If X is *non-stochastic* this risk (the trace of the matrix mean squared error) is readily computed analytically (*e.g.*, Judge and Bock (1978)). However, if X is random the situation is complicated to the extent that Monte Carlo simulation must be used. We compare $\rho(\hat{\beta})$ with $\rho(\tilde{\beta})$ and $\rho(\beta^*)$.

3. Monte Carlo experiment

We consider two situations

<u>Model I</u>: $\quad y_t = \beta_1 + \beta_2 y_{t-1} + u_t\ ;\ \ u_t \sim N(0, \sigma^2)\ ;\ \ t = 1,2,\ldots,n$

with[3]

$$H_0 : \beta_1 = 0 \ \ vs.\ H_a : \beta_1 > 0 .$$

<u>Model II</u>: $\quad y_t = \beta_1 + \beta_2 y_{t-1} + \beta_3 y_{t-2} + u_t\ ;\ \ u_t \sim N(0, \sigma^2)\ ;\ \ t = 1,2,\ldots,n$

with

$$H_0 : \beta_3 = 0 \ \ vs.\ H_a : \beta_3 > 0 .$$

Our Monte Carlo simulations involve 20,000 replications of each of several experimental designs, all of the computations being undertaken with the SHAZAM package

(White *et al.* (1990)) on a VAX 6340 and a VAXstation 4000 under VMS 5.5. SHAZAM uses the random number generator proposed by Brent (1974). We set $y_0 = 0$ and $\sigma^2 = 1$, and consider n = 10, 50.

With Model I we set c = 5.32 (n = 10) and 4.04 (n = 50). These values correspond to a (nominal) significance level of 5%. We also used Brook and Fletcher's (1981) "optimal" critical value (for non-orthonormal regressors) under a mini-max regret criterion in each case[4]. These values are functions of the y-data, so they differ across replications of the Monte Carlo experiment, *ceteris paribus*. By way of information, the average such critical values for n = 10 (50) were 2.38 (2.61), corresponding to nominal significance levels of 16.2% (11.3%). With Model II the (nominal) 5% critical values are c = 5.59 (n = 10) and 4.05 (n = 50). Brook and Fletcher's "optimal" values were also used. They are again data-dependent (though independent of the parameter values), and their average values over the 20,000 replications were 1.13 (1.02) for n = 10 (50), corresponding to nominal significance levels for the "F-test" of 31.8% (31.7%). With Model I we evaluated the risks of $\hat{\beta}$, $\tilde{\beta}$ and β^* when $\beta_2 = 0.1\ (0.1)\ 1.0$, in each case varying β_1 so as to compute the risks as functions of $\lambda = (\sqrt{\beta_1})/(2n)$. The latter is the non-centrality parameter associated with the[5] "F-test" of H_0 if $\bar{y} = 0$. So, $\lambda = 0$ corresponds to H_0 being true, and λ increases monotonically as we depart from H_0. This makes the presentation of our results comparable with those in the established pre-test literature. With Model II we set $\beta_1 = 1$, considered $\beta_2 = 0.1, 0.5, 0.9, 1.0$, and computed the risks in each case as a[6] function of $\beta_3 \in [0,1]$. These choices of parameters allow for the important unit root situation and result in 56 design matrices.

4. Results

The precise shapes of the various risk functions for the full regression coefficient vector depend to some extent, for both Models I and II, on the parameter values, sample size, and choice of significance level for the pre-test. However, the general *qualitative* features of these results are quite systematic and these are illustrated in Figures 1 and 2 for Models I and II respectively.

When the model includes a single lagged value of the dependent variable, regions of risk-dominance always arise which *qualitatively* match those which are well-known (*e.g.*, Judge and Bock (1978)) for the fixed-regressor model when testing for exact linear

Figure 1(a)

Risks of OLS, RLS & Pre-Test Estimators
Model I (n=10, Beta2=0.1, 5% critical value)

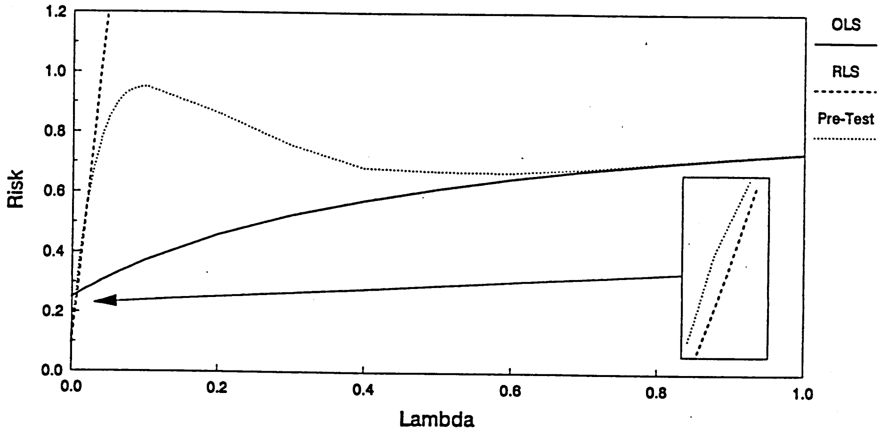


Figure 1(b)

Risks of OLS, RLS & Pre-Test Estimators
Model I (n=10, Beta2=1.0, 5% critical value)

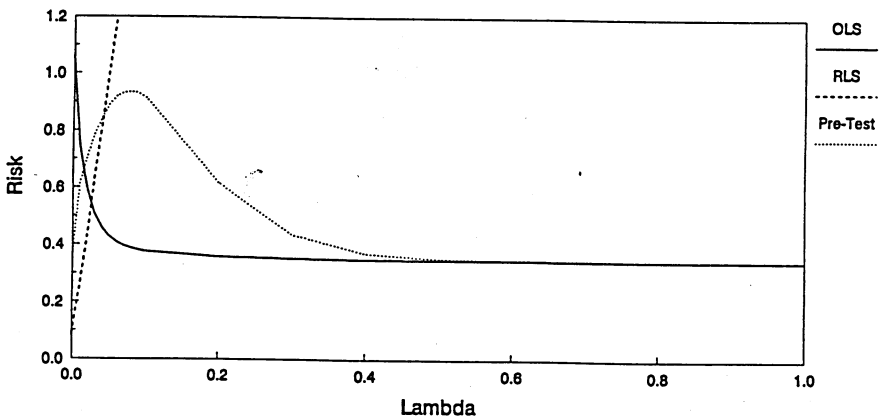


Figure 2(a)

Risks of OLS, RLS & Pre-Test Estimators
Model II (n=50, Beta2=0.1, Brook-Fletcher critical value)

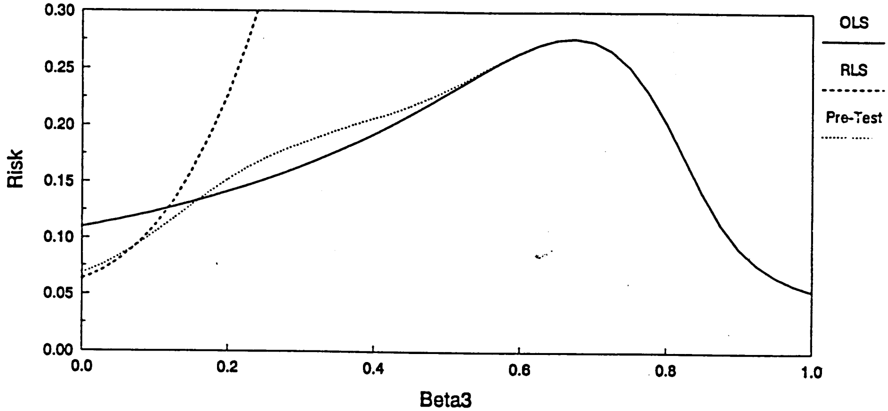
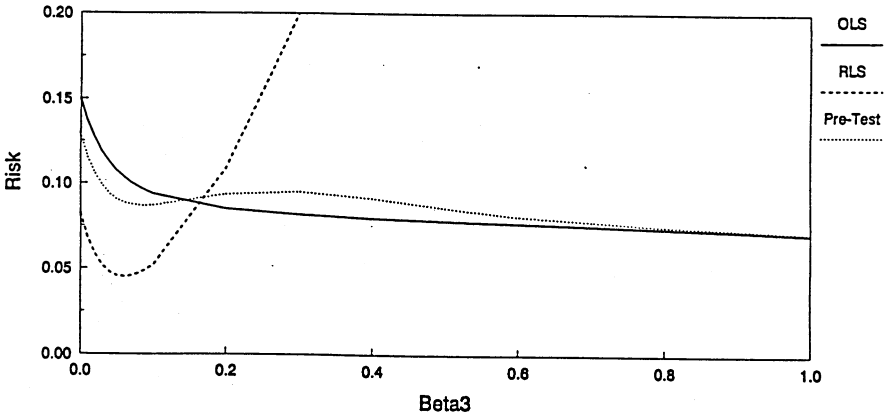


Figure 2(b)

Risks of OLS, RLS & Pre-Test Estimators
Model II (n=50, Beta2=1.0, Brook-Fletcher critical value)



restrictions on the coefficients. That is, there is always a region of the parameter space (*i.e.*, a range of λ values) for which the OLS estimator is preferred among the three estimators under consideration; there is always a region where the RLS estimator is preferred; there is always a region where the pre-test estimator is the *worst* of the three; but nowhere is the latter estimator "best", in the sense of having quadratic risk which is simultaneously less than that of the other two estimators. This is illustrated in Figures 1(a) and 1(b).

On the other hand, when the model is second-order autoregressive (with a drift term), it *is* possible to generate situations where the pre-test estimator simultaneously dominates *both* the OLS and RLS estimators. Regardless of the sample size, this arises for relatively small values of the coefficient (β_2) on the first lagged regressor. This effect is less pronounced when a conventional (nominal) significance level is used for the pre-test itself, than when the relatively large significance levels associated with Brook and Fletcher's "optimal" critical "F-values" are used. In these same cases, there is never a region of the parameter space where pre-testing is the "worst" of the three estimation strategies under consideration. This is illustrated in Figure 2(a). Considering larger values of β_2 in these cases (as in Figure 2(b)), we see that the more familiar dominance regions re-emerge, though the region over which pre-testing is the least desirable strategy is always very small.

5. Conclusions

Recent investigations of a range of preliminary-test estimation problems have illustrated that if the model is mis-specified in various ways, then a number of the standard results in the pre-test literature may be violated. In particular, in such cases it is possible for certain pre-test estimators to risk-dominate *both* of the associated component estimators simultaneously, at least in parts of the parameter space. Examples of these results are discussed, for instance, by Giles and Giles (1993), and they undermine the common view that pre-testing is inherently "bad".

We provide another important illustration of the non-robustness of such pre-test results to model mis-specification, here through the presence of particular stochastic regressors. The results presented here are preliminary, but they suggest that a more detailed analysis of this problem is warranted. Such further work might focus on the effects of the presence of other non-stochastic regressors, and the sensitivity of the results to the choice of loss function.

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Footnotes

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1. However, for some properties of autocorrelation pre-test estimators in a dynamic linear model, see Giles and Beattie (1987).

2. Treating (mf) as asymptotically χ^2 would not effectively alter the following results - it would be merely equivalent to using a different critical value for the tests.

3. The use of one-sided rather than two-sided alternative hypotheses simply affects the implicit significance levels being used.

4. Note that these critical values relate to a nominal "F-test", rather than "t-test" of H_0 in each case.

5. As the model is autoregressive, we cannot control the Monte Carlo results with respect to the non-centrality parameter without some constraint on the data.

6. With Model II it is simpler to use the value of β_3 as a direct measure of the departure from H_0, rather than re-expressing this measure as a function of the non-centrality parameter of the (nominal) distribution of f.

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