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Pre-Test Estimation in Regression

Under Absolute Error Loss*

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Abstract

We consider the risks of the Ordinary Least Squares, Restricted Least Squares and Pre-Test estimators of a regression coefficient under absolute error loss. These results are compared with their quadratic loss counterparts, and similar regions of risk dominance are found to hold, at least qualitatively.

* This note is based on some on-going collaborative research being undertaken in this general field with Offer Lieberman. I am most grateful to him for his substantial input, and to Judith Giles and Kazuhiro Ohtani for many helpful discussions.

1. Introduction

There is a large literature relating to the properties of regression estimators after some sort of preliminary hypothesis test. Estimation of the scale of the error term's distribution, and the coefficient and prediction vectors have been considered after pre-tests of linear restrictions on the coefficients or homoscedasticity of the error variance, for example. This literature is documented by Judge and Bock (1978) and Giles and Giles (1993), for instance.

Recently, Giles and Giles (1991, 1992) have explored two such estimation strategies using risk under the asymmetric LINEX loss function (*e.g.*, Varian (1975)). However, all of the other such studies to date use risk under quadratic loss as the basis for measuring an estimator's performance. This paper considers an alternative departure from quadratic loss. Symmetry of the loss function is retained, but it is taken to be of an "absolute error" form. Pre-test estimators (PTE's) have not been evaluated in such terms previously, and this paper provides some exploratory evidence (partly analytic, and partly based on Monte Carlo simulations) of the consequences of adopting such a loss structure.

In the next section we set up our model and notation; and section 3 presents some analytic results. Section 4 describes a small Monte Carlo study which focuses on the empirical risk function of the PTE itself; and some concluding comments appear in section 5.

2. The model and notation

For simplicity, and given the exploratory nature of this study, we consider the following regression model, where all data are measured as deviations about their sample means:

$$y_i = \beta_1 x_{1i} + \beta_2 x_{2i} + u_i \qquad ; \qquad u_i \sim IN(0,\sigma^2) \qquad i = 1,\ldots,n.$$

We test H_0: $\beta_2 = 0$ vs H_A: $\beta_2 \neq 0$, and reject H_0 if $|t_2| > c(\alpha/2)$, where $t_2 = \left(b_{2.1}/\text{s.e.}(b_{2.1})\right)$, $b_{2.1}$ is the Ordinary Least Squares (OLS) estimator of β_2 and s.e. denotes "standard error". The chosen significance level for the test is α, and $c(\alpha/2)$ is the critical value.

The OLS estimator of β_1 is $b_{1.2}$, which is $N(\beta_1, \sigma_1^2)$, where $\sigma_1^2 = \sigma^2/\left[\left(\sum_i x_{1i}^2\right)(1-\rho^2)\right]$ and ρ^2 is the squared sample correlation between x_1 and x_2. The Restricted Least Squares (RLS) estimator of β_1, obtained by deleting x_2 from the model, is $b_1 = b_{1.2} + \rho(\sigma_1/\sigma_2)b_{2.1}$, where $\sigma_2^2 = \sigma^2/\left[\left(\sum_i x_{2i}^2\right)(1-\rho^2)\right]$, and $b_1 \sim N(\beta_1+B, \sigma_1^{*2})$. The squared bias of b_1 is $B^2 = \beta_2^2\rho^2\left(\sum_i x_{2i}^2/\sum_i x_{1i}^2\right)$, and $\sigma_i^{*2} = \left(\sigma^2/\sum_i x_{1i}^2\right)$. The PTE of β_1 is

$$\hat{\beta}_1 = \begin{cases} b_1 & ; \ |t_2| < c(\alpha/2) \\ b_{1.2} & ; \ |t_2| \geq c(\alpha/2) \end{cases} .$$

We compare the sampling properties of $\hat{\beta}_1$ with those of its "components", $b_{1.2}$ and b_1, using risk under absolute error loss as the criterion. This is given by $R_A(\hat{\beta}_1) = E|\hat{\beta}_1 - \beta_1|$, *etc.* Comparisons are also made with the results based on quadratic risk, $R_Q(\hat{\beta}_1) = E(\hat{\beta}_1 - \beta_1)^2$, *etc.*

3. Some analytic results

The risks of $b_{1.2}$, b_1 and $\hat{\beta}_1$ under quadratic loss are well known to be

$$R_Q(b_{1.2}) = \sigma_1^2 = \sigma^2/\left[\left(\sum_i x_{2i}^2\right)(1-\rho^2)\right]$$

$$R_Q(b_1) = (\sigma_1^{*2}+B^2) = \left[\sigma^2+\beta_2^2\rho^2\sum_i x_{2i}^2\right]/\sum_i x_{1i}^2$$

$$R_Q(\hat{\beta}_1) = \left(\sigma^2\sum_i x_{2i}^2/\Delta\right) + \left(2h_\lambda(2)-h_\lambda(4)\right)\left[\beta_2\sum_i x_{1i}x_{2i}/\sum_i x_{1i}^2\right]^2$$
$$- \sigma^2 h_\lambda(2)\left(\sum_i x_{1i}x_{2i}\right)^2/\left(\Delta\sum_i x_{1i}^2\right)$$

where $\Delta = \left(\sum_i x_{1i}^2 \sum_i x_{2i}^2\right) - \left(\sum_i x_{1i}x_{2i}\right)^2$; $\lambda = \left[\beta_2^2 \sum_i x_{2i}^2 (1-\rho^2)\right]/2\sigma^2$ is the non-centrality parameter associated with the distribution of t_2; and $h_\lambda(\ell) = \Pr.\left[\left(\chi^2_{(m+\ell;\lambda)}/\chi^2_{(\nu)}\right) \le (c/\nu)\right]$, with $\nu = n - 2$. The expression for $R_Q(\hat{\beta}_1)$ follows from the first diagonal element of the risk matrix result in equation (5.3.15) of Judge and Bock (1978, p.111). $R_A(b_{1.2})$ and $R_A(b_1)$ are readily derived, as special cases of the following general result.

Proposition

Let $\tilde{\beta}$ be an estimator of the scalar β, where $\tilde{\beta} \sim N(\beta+\bar{B},\bar{s}^2)$, and let $R_A(\tilde{\beta}) = E|\tilde{\beta}-\beta|$. Then $R_A(\tilde{\beta}) = 2\bar{s}\phi(-\bar{B}/\bar{s}) + \bar{B}\left[1-2\Phi(-\bar{B}/\bar{s})\right]$, where $\Phi(q) = \int_{-\infty}^{q} \phi(z)dz$, and $\phi(z) = (e^{-z^2/2})/\sqrt{2\pi}$.

Proof

$$R_A(\tilde{\beta}) = E|\tilde{\beta}-\beta| = \int_{-\infty}^{\beta} (\beta-\tilde{\beta})p(\tilde{\beta})d\tilde{\beta} + \int_{\beta}^{\infty} (\tilde{\beta}-\beta)p(\tilde{\beta})d\tilde{\beta}$$

where $p(\tilde{\beta})$ denotes the (normal) p.d.f. of $\tilde{\beta}$.

So,

$$R_A(\tilde{\beta}) = \int_{-\infty}^{\infty} (\beta-\tilde{\beta})p(\tilde{\beta})d\tilde{\beta} + 2\int_{\beta}^{\infty} (\tilde{\beta}-\beta)p(\tilde{\beta})d\tilde{\beta}$$

$$= -\bar{B} + 2\int_{\beta}^{\infty} \tilde{\beta}p(\tilde{\beta})d\tilde{\beta} - 2\beta\left[1-\Phi(-\bar{B}/\bar{s})\right].$$

Now,

$$\int_{\beta}^{\infty} \tilde{\beta}p(\tilde{\beta})d\tilde{\beta} = \int_{-\bar{B}/\bar{s}}^{\infty} (z\bar{s}+\beta+\bar{B})\phi(z)dz,$$

where $z = (\tilde{\beta}-\beta-\bar{B})/\bar{s}$. Then,

$$\int_{\beta}^{\infty} \tilde{\beta}p(\tilde{\beta})d\tilde{\beta} = \bar{s}\phi(-\bar{B}/\bar{s})+(\beta+\bar{B})\left[1-\Phi(-\bar{B}/\bar{s})\right],$$

and so

$$R_A(\tilde{\beta}) = 2\bar{s}\phi(-\bar{B}/\bar{s})+\bar{B}\left[1-\Phi(-\bar{B}/\bar{s})\right],$$

where we have used the result $\int_a^\infty z\phi(z)dz = \phi(a)$. #

The absolute error risks of $b_{1.2}$ and b_1 then follow immediately as

$$R_A(b_{1.2}) = 2\sigma/\sqrt{2\pi(\sum_i x_{1i}^2)(1-\rho^2)}$$

$$R_A(b_1) = (2\sigma/\sqrt{\sum_i x_{1i}^2})\phi(-\rho\sqrt{2\lambda/(1-\rho^2)})$$

$$+ \rho\sigma\sqrt{2\lambda/((\sum_i x_{1i}^2)(1-\rho^2))}\left[1-2\Phi(-\rho\sqrt{2\lambda/(1-\rho^2)})\right].$$

4. Simulation results

Unfortunately, $R_A(\hat{\beta}_1)$ is not obtainable by the same analytic approach, so we have derived this function by Monte Carlo simulation with 10,000 replications, using the SHAZAM package (White *et al.* (1990)). The results we present below are data dependent, as are those for $R_Q(\hat{\beta}_1)$. However, this involves no loss of generality as it is well known that for this problem it makes no difference (qualitatively) whether we focus on coefficient estimation risk or predictive risk, and the latter is independent of the regressor values.

The simulation results are based on artificial data with the following characteristics: $n = 42$, $\sum_i x_{1i}^2 = 1496$, $\sum_i x_{2i}^2 = 1078$, $\sum_i x_{1i}x_{2i} = 1043$, so that $\rho^2 = 0.675$. We set $\alpha = 5\%$, $\sigma = \beta_1 = 1$. Then assigning values to λ, we can generate values for β_2 from the relationship $\beta_2^2 = 57.008\lambda$. This, together with normal random disturbances which are produced by Brent's (1974) algorithm, provides the information needed to generate y_i values for the simulation experiment. Exact results based on these data can be computed for $R_A(b_{1.2})$, $R_A(b_1)$ and the quadratic risks. Monte Carlo simulation results were also produced for these cases to check the accuracy of the simulation. Graphically, these results were indistinguishable from their counterparts in Figures 1 and 2.

Those figures illustrate that the well known results relating to the risks of the PTE and its component estimators under quadratic loss for this problem are unchanged qualitatively under absolute error loss. Specifically, there is always a region of the parameter space where the OLS risk is smallest of the three; a region where the RLS risk is smallest; one where the PTE has largest risk; but no region where the PTE is risk-preferred to both of its components simultaneously.

The robustness of these results may reflect the symmetry of the absolute error loss function. For example, using the asymmetric LINEX loss, Giles and Giles (1991, 1992) describe situations where PTE's of σ^2 can risk-dominate both of its component estimators. This remains a topic for further research, preferably by analytic methods.

5. Conclusions

This paper extends the literature on pre-test estimation of the regression model's coefficients by considering, for the first time, a non-quadratic loss function as the basis for estimator performance. The results are tentative, being partly analytic, and partly based on Monte Carlo simulation. However, the results to date suggest that moving from quadratic to absolute error loss does not affect the known risk-dominance results in any qualitative way.

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FIGURE 1
RISKS UNDER QUADRATIC LOSS
TWO-REGRESSOR MODEL

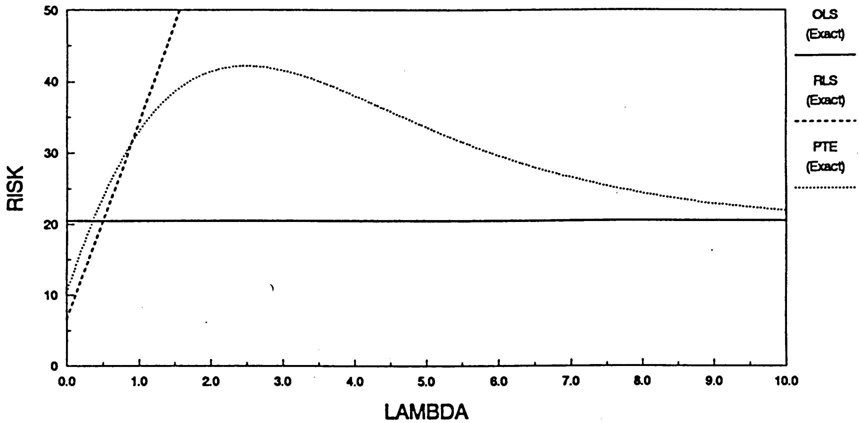
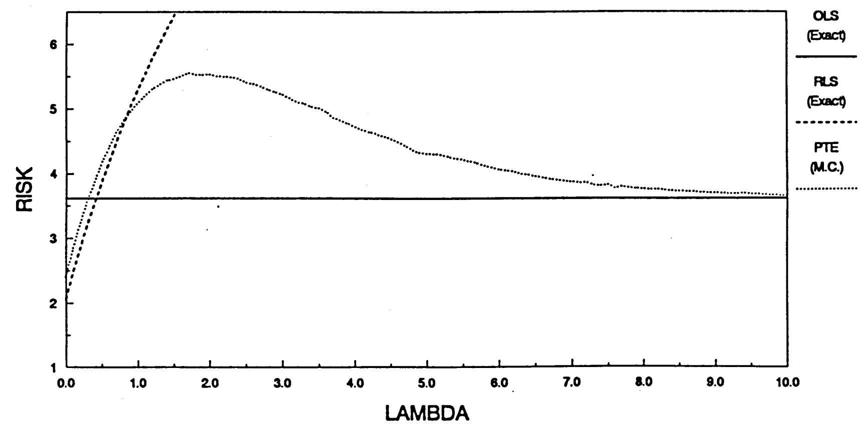


FIGURE 2
RISKS UNDER ABSOLUTE ERROR LOSS
TWO-REGRESSOR MODEL



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