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TESTING AND ESTIMATION WITH SEASONAL
AUTOREGRESSIVE MIS-SPECIFICATION

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Abstract

The problem of testing for AR(1) disturbances is considered using a model
in which fourth order autocorrelation is also present. The effect of this
mis-specification of the model on the power of some popular AR(1) tests is
shown. Effects at the unit root boundaries of the parameter space are
incorporated into the analysis. The efficiency of OLS estimation in this
model is considered using spectral analysis of the disturbances.

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1. Introduction

Several authors have suggested that time series regressions using quarterly data could produce residual autocorrelation which has both first and fourth order components (see Harvey (1990, p.205), for example). This is entirely consistent with the standard rationale for the existence of a random disturbance term in a regression model. The possibility of simple AR(4) disturbances has been considered as a separate issue by Wallis (1972) and Vinod (1973) who proposed a fourth order generalisation of the Durbin-Watson (1950,1951) test, and by King (1984) who constructed the associated point optimal invariant test. In addition, King (1989) presented a test designed to detect a simple AR(4) process when it is already known that AR(1) errors exist.

The aim of this paper is to take a step back from the analysis of King (1989) and seek the answers to two questions. First, how does the joint presence of AR(1) and simple AR(4) error processes affect the probability of detecting the AR(1) component? This will be answered be evaluating the power functions of several popular AR(1) tests under this form of mis-specification. The second question concerns the estimation efficiency of OLS relative to a feasible GLS estimator which might be used for final estimation, depending on the outcome of the AR(1) test. This issue could be addressed as a pre-testing problem by considering the risk, under some loss function, of the pre-test estimator and its components. The approach taken here, however, will focus on the spectral density of the error process.

The paper is organised in the following way. The next section introduces the AR(1) tests and discusses some issues associated with computing their powers. Section 3 presents the results of the numerical

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evaluations. This motivates the analysis, in Section 4, of the efficiency of OLS for the model used. Section 5 offers some concluding comments.

2. Test Power

Consider the standard linear regression model

$$y = X\beta + u \tag{1}$$

where y is $T\times1$, X is $T\times k$, independent of u and of rank $k < T$, β is a $k\times1$ parameter vector and u is a $T\times1$ vector of disturbances. Assuming that the data are observed quarterly, the following model is considered for u:

$$(1-\phi_1 L)(1-\phi_4 L^4)u_t = \varepsilon_t \qquad\qquad t = 1,2,\ldots,T \tag{2}$$

where $\varepsilon_t \sim N(0,\sigma_\varepsilon^2)$ and L is the usual lag operator, such that $u_t(1-\phi_1 L) = u_t - \phi_1 u_{t-1}$. Stationarity of (2) requires that $|\phi_1|$, $|\phi_4| < 1$ and these conditions will generally be imposed. This process can be seen as a restricted AR(5) scheme by writing (2) as

$$u_t = \phi_1 u_{t-1} + \phi_4 u_{t-4} - \phi_1\phi_4 u_{t-5} + \varepsilon_t \quad . \tag{3}$$

To study the effect of seasonal autoregressive mis-specification, the power of five tests of H_0: $\phi_1 = 0$ vs H_a: $\phi_1 > 0$ will be considered, ignoring the possibility that ϕ_4 is non-zero. The tests used are the Durbin-Watson (DW) test, King's (1981) alternative DW test (ADW), the Berenblut and Webb (1973) test (BW) and two versions of King's (1985) point optimal test, which will be denoted S(0.5) and S(0.75) to indicate the value of ϕ_1 at which each is most powerful invariant. Both the BW and the point optimal test are special cases of a more general test due to Kadiyala (1970). Each of these tests has optimality properties in particular regions of the ϕ_1 space which are well established for correctly specified models[1]

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The statistic for each test can be written as a ratio of quadratic forms in u, the general form of which is

$$r = \frac{u'Qu}{u'Mu} \qquad (4)$$

where $M = I_T - X(X'X)^{-1}X'$ and Q is some non-stochastic T×T matrix defining the individual test statistic.

The exact versions of the tests reject H_0 if $r < r^*$ where r^* is the exact critical value for some $100\alpha\%$ size ($\alpha = 0.05$ throughout this study). To compute the exact power of each test the manipulations of Koerts and Abrahamse (1969) are used to write

$$\text{pr}(r<r^*|V) = \text{pr}\left\{ \sum_{j=1}^{T} \lambda_j Z_j^2 < 0 \right\} \qquad (5)$$

where V is the true covariance matrix of u (up to a scalar multiple), the Z_j^2 are $\chi_{(1)}^2$ and independent, and the λ_j are the eigenvalues of $(Q-r^*M)V$. Several algorithms are available for computing the probabilities in (5) such as the procedures of Imhof (1961) and Shively, Ansley and Kohn (1990).[2] In this study the probabilities were evaluated using the FORTRAN version of Davies (1980) algorithm contained in the SHAZAM (White et.al (1990)) computer package running on a Vax 6340 under VMS 5.5.

To implement the procedure outlined above, the form of V is required. The covariance matrix used by King (1989) does not truly reflect (2) but the correct form can be derived from the Yule-Walker equations for this process.[3] Denoting the autocovariance function by $\gamma_k = \gamma_{-k} = \text{cov}(u_t u_{t-k})$ gives

$$\gamma_0 = \phi_1 \gamma_1 + \phi_4 \gamma_4 - \phi_1 \phi_4 \gamma_5 + \sigma_\varepsilon^2$$

$$\gamma_1 = \phi_1 \gamma_0 + \phi_4 \gamma_3 - \phi_1 \phi_4 \gamma_4$$

$$\gamma_2 = \phi_1 \gamma_1 + \phi_4 \gamma_2 - \phi_1 \phi_4 \gamma_3$$

$$\gamma_3 = \phi_1 \gamma_2 + \phi_4 \gamma_1 - \phi_1 \phi_4 \gamma_2$$

$$\gamma_4 = \phi_1 \gamma_3 + \phi_4 \gamma_0 - \phi_1 \phi_4 \gamma_1$$

and

$$\gamma_k = \phi_1 \gamma_{k-1} + \phi_4 \gamma_{k-4} - \phi_1 \phi_4 \gamma_{k-5} \qquad \text{for all } k > 4.$$

The simultaneous solution of these equations provides the autocovariance function and subsequent division by γ_0 gives the following autocorrelation function, where ρ_k represents the correlation between u_t and u_{t-k}:

$$\rho_0 = 1$$
$$\rho_1 = \phi_1(1+\phi_1^2\phi_4)/(1+\phi_1^4\phi_4)$$
$$\rho_2 = \phi_1^2(1+\phi_4)/(1+\phi_1^4\phi_4)$$
$$\rho_3 = \phi_1(\phi_1^2+\phi_4)/(1+\phi_1^4\phi_4)$$
$$\rho_4 = (\phi_1^4+\phi_4)/(1+\phi_1^4\phi_4)$$
$$\rho_k = \phi_1\rho_{k-1} + \phi_4\rho_{k-4} - \phi_1\phi_4\rho_{k-5} , \qquad \text{for } k > 4 .$$

The scale factor was found by this method to be

$$\gamma_0 = \sigma_u^2 = \frac{\sigma_\varepsilon^2(1+\phi_1^4\phi_4)}{(1-\phi_1^2)(1+\phi_1^4\phi_4^3-\phi_1^4\phi_4-\phi_4^2)} .$$

It is immediately apparent that these expressions collapse to those for the well known AR(1) case when $\phi_4 = 0$.

By routinely testing data for unit roots, econometricians explicitly acknowledge the fact that many economic time series are non-stationary. Also widely accepted, is the virtual inevitability that relevant variables are omitted from many regression models. The clear implication of these two facts is that we may often encounter non-stationary residuals. Consequently, it was considered desirable to explore the power properties of these tests along the unit root boundaries of the stationary parameter space. For this problem these boundaries are the closed curve defined by $\phi_1,\phi_4 = \pm 1$. The power of each test was computed numerically along these line segments using a modification of the techniques suggested by Krämer

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and Zeisel (1989). When $\phi_1 = 1$, for example, $V = \iota\iota'$ where $\iota = (1,1,\ldots,1)'$ and $MV = 0$ for regressions with an intercept. Thus all the λ_j of (5) are zero and the power of the test is undefined. The limiting power as $\phi_1 \to 1$ can, however, be computated by replacing V with a transformation matrix W such that

$$W = \lim_{\phi_1 \to 1} (1-\phi_1)^{-1}(V-\iota\iota') .$$

This matrix W can be shown to be a Toeplitz matrix with first column equal to

$$W_1 = \frac{\phi_4-1}{\phi_4+1} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 + 2\phi_4 \\ 6 + 4\phi_4 \\ 7 + 6\phi_4 \\ 8 + 8\phi_4 \\ 9 + 10\phi_4 + 2\phi_4^2 \\ \vdots \end{bmatrix}$$

It can also be easily seen by inspection of the autocorrelation function that $V(\pm 1, -1) = I_T$, where the arguments of V are the values of ϕ_1, ϕ_4. This means that the power of each test at these points is equal to the true size of the test. It can further be shown that at all points on the $\phi_4 = -1$ boundary, except the endpoints, the power of each test is either zero or one. This result draws on the findings of Krämer (1985) and Small (1991) and the proof is omitted here in the interests of brevity.

To conclude this section we consider the seasonal unit root case defined by $\phi_4 = 1$. From the general form of the autocorrelation function it can be seen that setting $\phi_4 = 1$ gives:

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$$\rho_0 = 1$$
$$\rho_1 = \phi_1(1+\phi_1^2)/(1+\phi_1^4)$$
$$\rho_2 = 2\phi_1^2/(1+\phi_1^4)$$
$$\rho_3 = \rho_1$$
$$\rho_4 = 1.$$

This pattern repeats indefinitely so that the individual autocorrelations must take one of only three values. The rank of V, and the number of non-zero eigenvalues in (5), is therefore three.

The power of each test was computed under these conditions for the entire range of data outlined in Section 3 below. ϕ_1 took values ranging from zero to 0.9. In every case, each of the three non-zero eigenvalues of (5) were found to be positive so that test power was always zero.

3. Numerical Results

The well known dependence of the powers of these tests on the regressor data was allowed for by examining a range of data conditions. The design matrices used were:

X1   The income and price series from Durbin and Watson's (1951) spirits example.

X2   The quarterly Australian consumers price index commencing 1959(1) and the same series lagged one period.

X3   A linear time trend and observations drawn from the Normal (30,4) distribution.

X4   A linear time trend and a Uniform [0,10] series.

X5   A linear time trend and a lognormal (2.23, 19.58) series.

X6 $(a_2+a_T)/\sqrt{2}$ and $(a_3+a_{T-1})/\sqrt{2}$ where $a_1,\ldots,a_T$ are the eigenvectors corresponding to the eigenvalues of the DW first differencing matrix, A, arranged in increasing order.[4]

X7   A linear time trend and the logarithm of quarterly registered
unemployed in New Zealand, commencing 1952(2).

Each design matrix also included an intercept.   The first six data
sets have been used in several related studies and are discussed by Evans
(1992).   X6 is often referred to as Watson's X-matrix, and was shown by
Watson (1955) to produce the most inefficient OLS estimates within the
class of orthogonal matrices.   The X7 matrix was chosen for its strong
seasonality, which is an important data characteristic in this study.

Using a sample size of 20, a thorough investigation was conducted
across all tests and design matrices along 20 lines in the parameter space.
A selection of the resulting power curves is presented in the Figures 1 to
3 to support the general conclusions outlined below, while Table 1 shows
the lowest and highest power obtained across the tests for a variety of
cases.   A further more limited, study used a sample size of 60.   This
latter work confirmed the findings of previous studies (e.g., King (1985))
that a larger sample increases the power of each test and reduces the
power, differences between the tests.

The following features were observed with all seven data sets and each
test and are stated relative to power against pure AR(1) disturbances.
First, the true sizes of the tests are decreased (increased) by the
introduction of a positive (negative) fourth order component.   The only
exceptions to this were for S(0.75) and BW when using X6, where slight size
increases were registered as $\phi_4 \rightarrow 1$.   On average, sizes were 29.5% as
$\phi_4 \rightarrow -1$ and 0.87% as $\phi_4 \rightarrow 1$.

Second, serious losses of power were found when ϕ_4 fell in the
interval (0.4,1.0) for all $\phi_1 > 0$.   This is not unexpected in view of the
size effect noted above when $\phi_4 > 0$.   No size corrections were made to the

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power functions, since $\phi_4 \neq 0$ is assumed to be a mis-specification. Table 1 provides power values which show that when $\phi_1 = 0.4$, the introduction of a fourth order component with $\phi_4 = 0.4$ reduces power from around 40% to 25%. Increasing ϕ_4 to 0.6 further reduces power to around 15% while when $\phi_4 = 0.8$ power was generally about 7%. It is apparent from Table 1 that some data cause large spreads in power across tests. This feature is well known for X6 (see King (1985), for example).

The third feature of the numerical results is that the power of all tests is reduced when ϕ_4 falls in the interval $(-1,-0.4)$ for all $\phi_1 > 0$. In this region the power reduction is somewhat less serious, being offset by increased size.

4. Estimation Efficiency

The power effects summarised above suggest that an applied researcher has a greatly reduced chance of detecting an AR(1) process in the regression residuals if the true process is given by (2) and ϕ_4 is moderately large. Under these circumstances it would be useful to know something of the likely effect of failing to detect autocorrelation on the efficiency of OLS estimation.

Grenander (1954) and Grenander and Rosenblatt (1957) showed that if the spectral density function of the true disturbances is flat at all frequencies where the exogenous variables have spectral weight, then OLS is asymptotically fully efficient. Combining this with the finding of Granger (1966) that economic variables typically have their spectral weight at low frequencies, we follow Engle (1974) in concluding that OLS will be efficient if the disturbance spectrum is flat at low frequencies.

The spectrum of the covariance stationary process (2) is given by

$$f(\lambda) = \sigma_\varepsilon^2 \left\{ 2\pi |e^{5i\lambda} + \phi_1 e^{4i\lambda} + \phi_4 e^{i\lambda} - \phi_1\phi_4|^2 \right\}^{-1}$$

$$= \sigma_\varepsilon^2 \left\{ 2\pi \left[(1+\phi_1^2)(1+\phi_4^2) + 2\phi_1\phi_4(\cos 5\lambda + \cos 3\lambda) \right. \right.$$

$$\left. \left. - 2\phi_4 \cos 4\lambda (1+\phi_1^2) - 2\phi_1 \cos\lambda(1+\phi_4^2) \right] \right\}^{-1}.$$

Using a grid of frequencies in the range $-2\pi \leq \lambda \leq 2\pi$, $f(\lambda)$ was evaluated at 42 settings of ϕ_1, $\phi_4 \geq 0$ with σ_ε^2 arbitrarily set to 2π. A selection of the resulting spectra is presented in Figure 4. It is immediately apparent from these graphs that the spectra corresponding to the individual components of u reinforce each other at low frequencies. This suggests that, in general, the relative efficiency of OLS to GLS, which is known to decline with ϕ_1 is also decreasing in ϕ_4.[5] We can also see, however, that provided $\phi_4 \neq 0$, values of λ: $0 \leq \lambda \leq \pi$ exist for which the spectrum of u is flat. For design matrices whose spectral weight is concentrated on these frequencies we can conclude that OLS is (asymptotically) fully efficient.

The cause of these flat regions in $f(\lambda)$ can be seen by considering the log spectrum of u which has the same turning points as $f(\lambda)$. Apart from a constant this is given by

$$\ln f(\lambda) = \ln \frac{1}{|1-\phi_1 e^{i\lambda}|^2} + \ln \frac{1}{|1-\phi_4 e^{4i\lambda}|^2} \quad . \tag{6}$$

The spectrum of u is therefore the sum of the spectra of simple AR(1) and simple AR(4) processes.[6] The first term in (6) has a unique maximum (over the $[0,\pi]$ interval) at $\lambda = 0$ while the second term has maxima at 0, $\pi/2$ and π. Comparing the spectrum of u with that of a simple AR(1) process, $f^1(\lambda)$ we can therefore conclude that $f(\lambda) > f^1(\lambda)$ when $\lambda \in (0, \pi/2, \pi)$ and the

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integration constraint then requires that the inequality be reversed for some other λ between 0 and π.

5. Conclusion

This paper has considered the problem of detecting first order serial correlation when a fourth order component is also present. It has been shown that the power of several popular tests for AR(1) errors is considerably reduced by positive fourth order autocorrelation. It is suggested that this also reduces the chances of an applied researcher either adopting a suitable alternative estimator to OLS or investigating the residuals further to discover the true autocorrelation process. The possible consequences of this (lack of) action were revealed by a study of the disturbance spectrum, which showed that the relative efficiency of OLS is likely to be lower when u follows a joint first and fourth order autoregressive scheme rather than either of these as a simple process.

Footnotes

*   Helpful comments from David Giles, Judith Giles, Howard Doran and Philip Franses are gratefully acknowledged. The author is solely responsible for any errors.

(1)   For a good discussion of these, and related, issues see King (1987).

(2)   A more general algorithm due to Lieberman (1992) uses a saddlepoint expansion to evaluate the p.d.f. of (4).

(3)   The following autocorrelation function was also derived independently by Wu (1991).

(4)   A is a tri-diagonal (TxT) matrix with (1,1) and (T,T) elements as unity, 2 elswhere on the leading diagonal and -1 for the leading off-diagonal elements.

(5)   An obvious exception is when the columns of X are linear combinations of the eigenvectors of V (Anderson (1948)).

(6)   I am grateful to Howard Doran for pointing this out.

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Table 1

Power Range Across Tests

Selected Values

Data	ϕ_4	$\phi_1 = 0.4$		$\phi_1 = 0.6$		$\phi_1 = 0.8$	
X1	0	.40	.41	.65	.66	.81	.82
	.4	.25	.27	.50	.52	.72	.75
	.6	.15	.18	.37	.41	.62	.66
	.8	.06	.09	.19	.24	.43	.49
X2	0	.40	.40	.64	.65	.80	.81
	.4	.23	.23	.46	.47	.67	.69
	.6	.14	.14	.33	.34	.56	.58
	.8	.05	.06	.16	.17	.38	.39
X3	0	.40	.40	.65	.66	.81	.83
	.4	.24	.25	.49	.50	.71	.73
	.6	.14	.15	.35	.36	.60	.62
	.8	.05	.06	.17	.18	.39	.41
X4	0	.40	.40	.64	.66	.81	.83
	.4	.24	.25	.49	.50	.71	.73
	.6	.15	.16	.36	.37	.60	.62
	.8	.06	.06	.18	.19	.40	.42
X5	0	.38	.39	.61	.64	.77	.82
	.4	.21	.22	.43	.46	.64	.69
	.6	.12	.13	.30	.32	.53	.58
	.8	.04	.05	.14	.16	.33	.37
X6	0	.30	.34	.42	.60	.44	.83
	.4	.13	.26	.22	.52	.24	.79
	.6	.06	.22	.12	.46	.14	.75
	.8	.02	.15	.04	.32	.06	.63
X7	0	.38	.38	.60	.62	.75	.79
	.4	.21	.24	.42	.46	.62	.68
	.6	.13	.17	.29	.35	.50	.58
	.8	.05	.12	.15	.23	.32	.42

Figure 1a

Spirits Data; T = 20
Phi4 = -0.8

Power

ADW

DW

S(0.5)

S(0.75)

BW

DW0

Phi 1

DW0 Is DW Test When Phi4 = 0
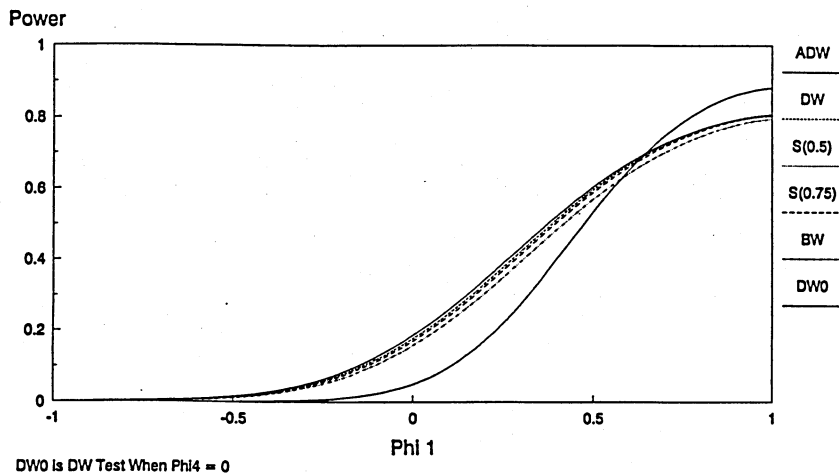
Figure 1b

Spirits Data; T = 20
Phi4 = 0.8

Power

ADW

DW

S(0.5)

S(0.75)

BW

DW0

Phi 1

DW0 Is DW Test When Phi4 = 0
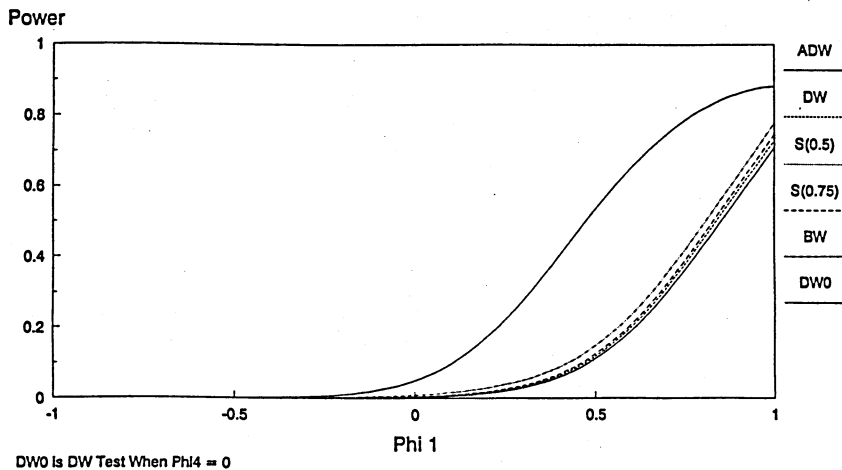
Figure 2a
Unemployed Data ; T=20
Phi4 = 0.6

Power

ADW

DW

S(0.5)

S(0.75)

BW
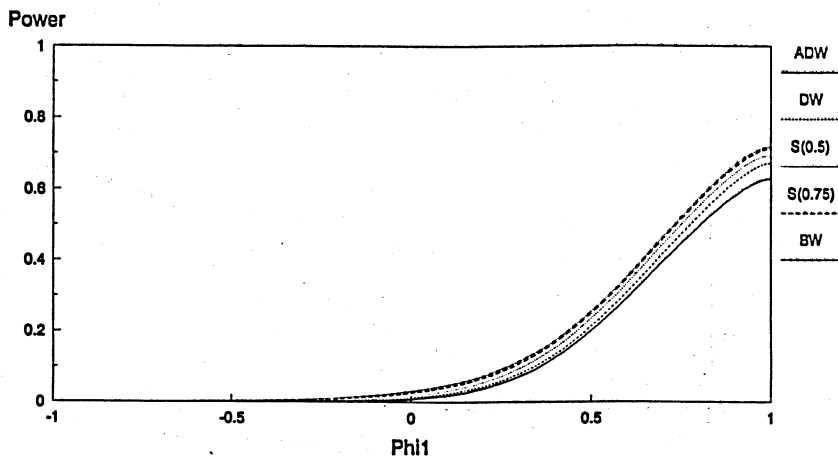
Phi1

Figure 2b
Unemployed Data ; T=20
Phi4 = 0.8

Power

ADW

DW

S(0.5)

S(0.75)

BW
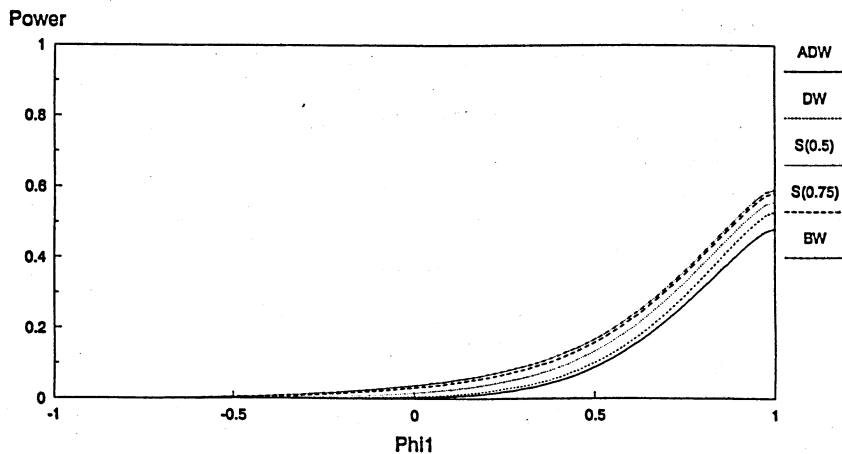
Phi1

Figure 3a
Normal Data ; T=20
Phi4 = -1

Power

ADW

DW

S(0.5)

S(0.75)

BW
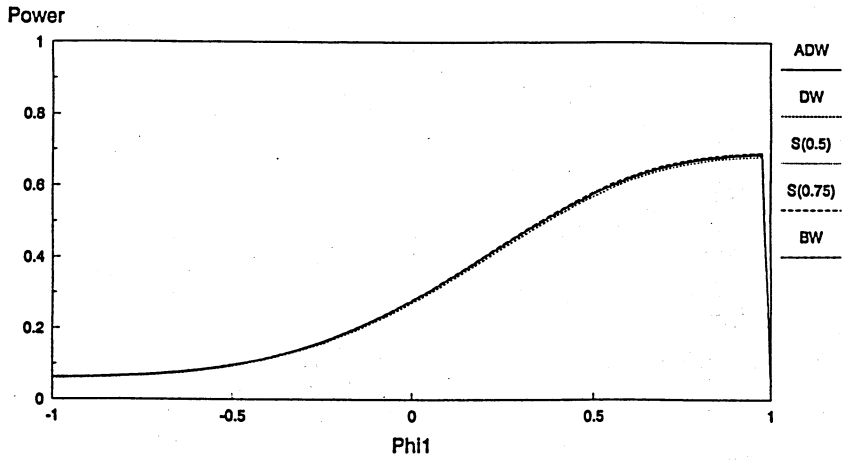
Phi1

Figure 3b
Spirits Data ; T=20
Phi1 = 0.8

Power

ADW

DW

S(0.5)

S(0.75)

BW

Phi4
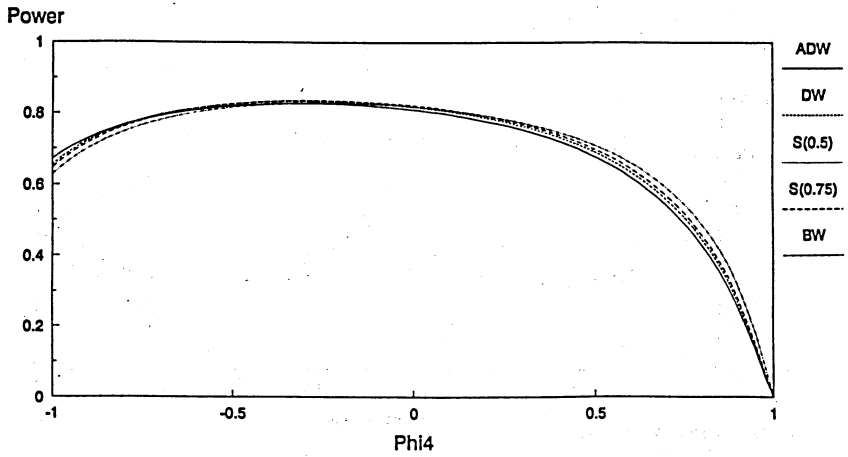
Figure 4a
Spectral Density of u
Phi1=0.6
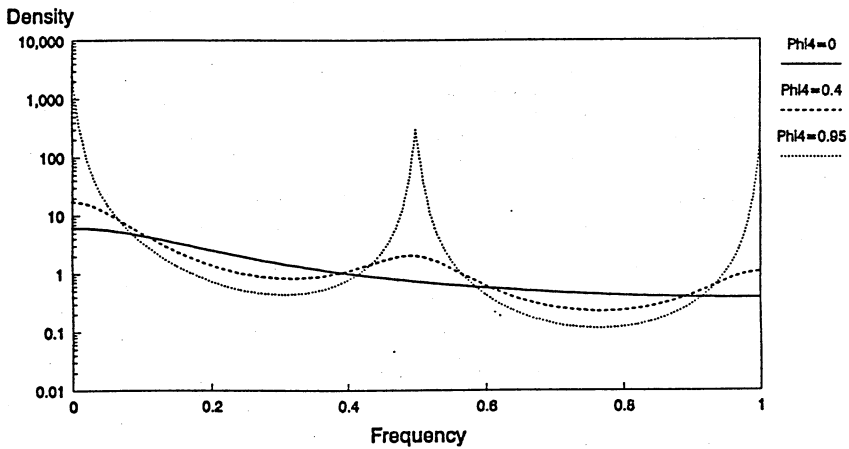
Note Logarithmic Density Scale

Figure 4b
Spectral Density of u
Phi4=0.6
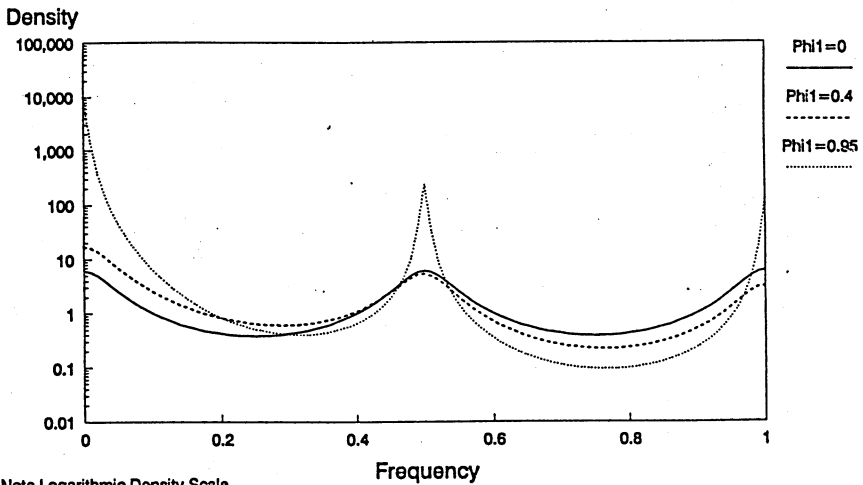
Note Logarithmic Density Scale

LIST OF DISCUSSION PAPERS*