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QUASI RATIONAL CONSUMER DEMAND —  
SOME POSITIVE AND NORMATIVE SURPRISES

John Fountain

*Discussion Paper*

No. 9202

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***Discussion Paper No. 9202***

February 1992

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SOME POSITIVE AND NORMATIVE SURPRISES**

**John Fountain**

# **Quasi Rational Consumer Demand - Some Positive and Normative Surprises**

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**February , 1992**

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## Abstract

This paper develops a theory of consumer demand based on a notion of quasi rational decision making behaviour. Two ideas drawn from empirical studies of preferences turn out to be useful: the idea of 'framing' used in explanations of empirical inconsistencies in preference and the idea that choice is, even for an individual, a compromise among a divided self. A simple model of consumer choice is developed using a combination of these two ideas in conjunction with the conventional notion of constrained optimization. The model predicts both demand curves and specific patterns of inconsistency in binary choices. A specification of the model in terms of quadratic utility predicts the simplest of economic relationships: a linear relationship between price and quantity demanded. The demand functions of a quasi rational consumer have the standard property that the substitution effect of a price change is negative. However, the total effect of the price change is decomposable into three, rather than two, parts: a substitution effect, an income effect, and an additional effect called the inconsistency effect. The expenditure minimisation problem for a quasi rational consumer turns out to be well defined, but duality does not hold. A quasi rational consumer is not an expenditure minimiser. This implies that competitive market price does not measure marginal value to a quasi rational consumer. An example is used to show that *measurement* errors and *interpretation* errors are likely if ordinary demand curves are used to calculate consumer surplus type gains and losses for quasi rational individual consumers, even in the absence of income effects. We also have the somewhat surprising implication that an income tax may be inferior to a sales tax that raises the same revenue from a quasi rational consumer.

## **I. Introduction**

This paper develops a theory of consumer demand based on a notion of quasi rational decision making behaviour. Two ideas drawn from empirical studies of preferences turn out to be important in this theory : the idea of 'framing' used in explanations of empirical inconsistencies in preference and the idea that choice is, even for an individual, a compromise among a divided self. A simple model of consumer choice is developed using a combination of these two ideas in conjunction with the conventional notion of constrained optimization. The model predicts both demand curves and specific patterns of inconsistency in binary choices. A specification of the model in terms of quadratic utility predicts the simplest of economic relationships: a linear relationship between price and quantity demanded

The demand functions of a quasi rational consumer have the standard property that the substitution effect of a price change is negative. However, the total effect of the price change is decomposable into three parts: a substitution effect, an income effect, and an additional effect called the inconsistency effect. The expenditure minimisation problem for a quasi rational consumer turns out to be well defined, but duality does not hold. A quasi rational consumer is not an expenditure minimiser. This implies that competitive market price does not measure marginal value to a quasi rational consumer.

Some implications of quasi rational consumer demand for the measurement of welfare effects of price changes in competitive markets are discussed. An example is used to show that measurement errors and interpretation errors will abound if ordinary demand curves are used to calculate consumer surplus type gains and losses for quasi rational individual consumers, even in the absence of income effects. On a practical level we also have the somewhat surprising implication that an income tax may be inferior to a sales tax that raises the same revenue from a quasi rational consumer .

The paper begins in Section II by explaining the idea of framing and the basic hypotheses about individual preferences that will be used to circumscribe the notion of quasi rationality. Section III provides a simple model of how a consumer with divided interests can reach an equilibrium or compromise between his divided selves within the context of a frame. Demand functions are derived from this equilibrium. Predictable intransitivities are derived in section IV. Section V poses the expenditure minimisation problem for the quasi rational consumer and derives comparative static properties of the equilibrium demand functions. Section VI analyses the problem of measuring and interpreting consumer surplus for quasi rational consumers. Section VII offers a brief conclusion and directions for future research.

## II. Framing Effects and Inconsistent Choices

There is a growing body of research on individual preferences that offers a challenge to the positive economic theory of the consumer and the normative applications of that theory in welfare economics and public choice<sup>1</sup>. Although much of this research investigates preferences over uncertain prospects, it also offers the following suggestive hypotheses relevant to the economist's static theory of consumer choice:

- S1• Individual people do not have well thought out rational preferences, but may be viewed as having divided minds with different aspirations.
- S2• The process of choice is an act of compromise among the different selves.
- S3• Complex choice problems are decomposed into component parts or 'frames' to simplify the process of decision making.
- S4• Individual people do not readily use available but implicit information about their decision problems to produce alternative frames; in particular they do not readily produce alternative frames that facilitate the evaluation of interactive or aggregated effects of the simplified component decisions.
- S5• Choices from the initial frames individual people work with can violate fundamental axioms of rational choice, in particular dominance, transitivity, and invariance, unless the frames in which the decision problem are analysed make such 'inconsistencies' transparent
- S6• When transparent, normative criteria such as transitivity and dominance are valued by the individual, but the resolution of conflicting values into a coherent rational preference satisfying transitivity and dominance is a tentative and exploratory process that many people find difficult, even with assistance, and some find impossible<sup>2</sup>

The rest of this section offers a very brief explanation of and justification for these hypotheses. For an in depth treatment the symposia edited by Bell Raiffa and Tversky (1988) should be consulted.

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<sup>1</sup> See the symposia edited by Bell, Raiffa and Tversky (1988)

<sup>2</sup> Einhorn and Hogarth p.147 ff. Tversky and Kahneman (1981) comment "Individuals who face a decision problem and have a definite preference (i) might have a different preference in a different framing of the same problem, (ii) are normally unaware of alternative frames and their potential effects on the relative attractiveness of options, (3) would wish their preferences to be independent of the frame, but (4) are often uncertain how to resolve detected inconsistencies."(p 457)



Empirical studies of preferences in decision making under uncertainty show that the following four fundamental axioms of rational choice are frequently and systematically violated. Violations of these axioms of choice are often called, indiscriminately, 'inconsistencies'.

- Transitivity: *a* preferred to *b* and *b* preferred to *c* implies *a* preferred to *c*
- Dominance: if one option is better in one state and at least as good in all other states when compared to a second option, the dominant option should be chosen
- Cancellation (sure thing principle, independence): the choice between risky options should only depend on states in which they yield different outcomes
- Invariance: different representations of the same choice problem should yield the same preference

The idea of 'framing' of alternatives is widely used in systematic explanations of these empirical inconsistencies. Essentially framing describes the way individuals perceive and process information they deem relevant to a decision situation prior to evaluation and selection. The following example of a violation of invariance and dominance, from Kahneman and Tversky (1988), illustrates the role that framing effects can play in generating inconsistencies of several types.

150 people in an experiment were presented with the decision problem in Table 1. They were instructed that two concurrent decisions were to be made. The numbers in parentheses indicate the percentage of respondents who chose that respective option.

**TABLE 1**  
**Disaggregated frame**

<b>Decision 1</b>		
Choose between		
A	a sure gain of \$240	[84%]
B	25% chance to gain \$1000 and a 75% chance to gain nothing	[16%]
 <b>Decision 2</b>		
Choose between		
C	a sure loss of \$750	[13%]
D	75% chance to lose \$1000 and a 25% chance to gain nothing	[87%]

Because the subjects considered two decisions simultaneously they were implicitly choosing between two portfolios, the probabilities and outcomes of which are shown in Table 2.. The vast majority effectively preferred portfolio A & D over portfolio B & C. Yet portfolio A & D is actually dominated by portfolio B & C.

**TABLE 2**  
**Aggregated frame**

Portfolio A & D	25% chance to win \$240 and a 75% chance to lose \$760
Portfolio B & C	25% chance to win \$250 and a 75% chance to lose \$750

When subjects were presented with the options in the aggregated form, as portfolios, the dominated options were almost always rejected. Yet when presented in the disaggregated form, as two separate but concurrent decisions, almost 3/4 chose the dominated portfolio A&D. The principle of invariance is also violated in this example in that the two forms or 'frames' of the problem are mathematically equivalent, yet individual choices are sensitive to the form in which the choice is presented.

Theorists analysing these and similar experiments make the following relevant comments. Slovic et al (1988, p 153) suggest that what we are observing is a form of concrete perceiving and thinking, where individuals only use the information that is displayed explicitly in the formulation of the problem. Information that has to be *logically deduced* from the initial display frame or *created* and *processed* by some kind of conceptual transformations tends to be ignored. Kahneman and Tversky (1988) point out that people do not spontaneously aggregate concurrent prospects or transform all outcomes into a common frame that facilitates comparison of their combined effects<sup>3</sup>. In the above illustrative experiment, the situation as presented in Table 1 simplifies the decision problem by focusing attention on making a choice *first* between A and B and *then* between C and D. Yet this decomposition obscures the relationship of dominance between the combined alternatives, a relationship that is transparent in the second, 'portfolio' frame of Table 2. Subjects systematically failed to make the transformation of the two separate decisions in Table 1 into a 'standard' form, such as a cumulative probability distribution, that focuses attention on the combined interactions of the two decisions and facilitates the detection of dominated alternatives. Indeed, subjects making the 'inconsistent' choices were very surprised to learn that the combination of two apparently reasonable choices they made led them to select a dominated option. In related experiments, as well as in consultancy situations<sup>4</sup>, when subjects are presented with their inconsistencies, they experience considerable difficulty in resolving them, even with the help of attentive and skilled advice.

<sup>3</sup>Edwards et al, studying the portfolio choice problems of the Construction Engineering Laboratory (CERL) of the US Army Corps of Engineers, make the point that virtually everyone begins by treating project proposals independently, ignoring their possible, but difficult and potentially complex, interactions with one another. The experimental work by Kahneman and Tversky and others suggests that in the absence of informed consultants or other devices for producing alternative frames, evaluative assessment will also tend to end at this point too.

<sup>4</sup>Edwards et al (), Bell Raiffa, Tversky()

Empirical examples like the one we have used as an illustration and the interpretative analysis of these experiments form the basis for hypotheses S3 to S6 above<sup>5</sup>. Hypotheses S1 and S2 have a slightly different origin, in methodological reflection.

Bell Raiffa and Tversky's (1988a) argue strongly for a new 'prescriptive' methodology in the social and behavioural sciences. They add to the experimental evidence their own experience as consultants. In their view, the conventional theory of how idealized rational super intelligent people should think and act simply does not come to grips with the

- internal turmoils,
- shifting values,
- anxieties,
- post decision disappointments and regrets,
- memory and attention span limitations, and
- calculating (in)abilities

that they meet with in their clients. 'How can we help such people make better decisions?' they ask. In their view, this question cannot fruitfully be answered by assuming that deep down every individual has a complete rational preference ordering for whatever alternative choices are before him. In a complex, emotional choice situation, a decision maker is often very confused about what he should be doing in his own best interests. It is a "Platonic myth" (p 21) that hidden subjective probabilities and utilities exist only to be 'discovered' by the discerning consultant. As an alternative, they suggest the following methodological premise:

"not that people have well thought out preferences, but that they may be viewed as having divided minds with different aspirations, that decision making, even for the individual, is an act of compromise among the different selves" p9

This premise is the basis for suggestive hypotheses S1 and S2.

There are other, broader issues, than those raised in hypotheses S1 through S6. Why would someone sensibly and intelligently accept a set of imperfectly reconciled values in the long run? As a first consideration, human abilities to remove ambiguities and inconsistencies through processing of information are limited and costly. If it were possible to be consistent at reasonable cost we probably would want to be. This is obviously a 'bounded rationality' type consideration and the types of theories designed to explain inconsistent choices can be viewed as articulating theories of the limits of human perception and judgement. But even if human computational abilities were more powerful, or more accessible and cheaper, it is doubtful that all ambiguities in and conflicts between diverse goals within ourselves would (not just could) be eliminated. As noted above, even after extensive

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<sup>5</sup>Grether & Plott's (1977) early work on preference reversals should be read by all skeptical economists. Their experiments were designed specifically to take account of a series of 'standard' objections economists might make to these experiments on preference reversals. Their own findings reinforced what experimental decision theorists and psychologists had already established. .

discussion some subjects in controlled experiments and some clients in consultancy situations who are confronted with their inconsistent choices are unwilling to change their choices simply to maintain consistency. As Emerson put it so poetically

“A foolish consistency is the hobgoblin of little minds, adored by little statesman and philosophers and divines”.

Consistency in values is not an end in itself for some (most?) people.

Taking a lifetime perspective, tastes and goals are not ‘given’ but cultivated and developed through time. The process of cultivation and development of tastes appears to require as an input constant confrontation and dialogue, both private and social, between preferences and actions inconsistent with them, and between conflicting preferences. A fully integrated set of tastes lacking inconsistencies and ambiguities foregoes this opportunity for new growth and change. Acts of deliberation and intellectual justification of choice, to ones self as well as to others, appear to play a critical, and creative role, in assessing and managing these internal preference conflicts and confrontations in wise ways. As well, these same processes can be used in the hands of skillful prescriptionists using the veil of consistency to harness or suppress powerful desires or wants that are regarded, in another context and frame, as simply wrong, immoral or inappropriate. Settling for some inconsistency or incoherence may be a safeguard against manipulative exploitation by other persuasive people.<sup>6</sup>

These types of considerations, empirical, methodological, and dynamic, don’t fit well into consumption theories in economics that rely on the assumption of rational individual choice. In the following section I develop a simple theory of consumer choice in a static setting that incorporates hypothesis S1 through S6.

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<sup>6</sup>For example, Miller’s work *Social Justice* argues that the great diversity of opinions on evaluative criteria for fairness can be reduced, after considerable philosophical argument and deliberation, to three fundamental, but often conflicting, principles. Any further reduction, in the name of coherence, will tend to suppress one valued principle of justice at the expense of another. Whether or not Miller’s substantive argument about justice is correct, the idea that incoherence needs to be tolerated in order to maintain the integrity of diverse values is perfectly respectable in academic circles. Why not, then, for ordinary people as consumers?

### III. Demand as a Compromise Between Divided Selves

This section derives the simplest of all economic relationships, demand functions, from a model of consumer behaviour consistent with hypotheses S1 - S6 above. As in prospect theory choice is viewed as a two stage process. The first phase consists of an editing and framing phase during which a preliminary analysis is made, simplifying the decision situation and outlining effective strategies for coping with the problem. The second phase consists of evaluation and assessment involving processes of constrained optimization and of compromise between conflicting interests<sup>7</sup>, all within the limitations imposed by the initial frame.

Imagine a consumer with multiple interests, two to be precise, labelled A and B. Each interest is a way of looking at activities in a simplified, yet focussed, way. That is, by narrowing down his field of view to a few things that matter and ignoring a host of complex interactions, each type of focussed interest can aid the consumer in evaluating alternative courses of action (hypothesis S3). These interacting, multiple interests all matter to the consumer. Of course, life is seldom so neatly compartmentalized. Choices guided by one focussed interest will often influence evaluations and choices made under the guidance of the another. Looking at activities in narrow, focussed ways may generate possible value conflicts and inconsistencies from some theoretical overall point of view (hypothesis S5). But it has the advantage of simplicity and integrity - the values that matter to the consumer are taken account of, even if all the potential value conflicts are not 'worked out' (hypothesis S4). Trying to integrate potentially conflicting values into one single overriding value is a complex and difficult task (hypotheses S1,S6). There may be simpler<sup>8</sup>, alternative strategies to make choices with in the presence of conflicting values (hypotheses S2,S3) than to try to integrate a set of conflicting values into an overall single criterion.

To be concrete, imagine our consumer has social interests, labelled A, which involve drinking alcohol,  $x_a$ , and eating food in combination with drinking,  $z_a$ . He also has sporting interests B which involve some activity,  $y_b$ , and food,  $z_b$ . The consumer frames the decision problem in the following way. Ignoring the social interests and associated activities for simplicity, our consumer focuses on activities related to his sporting interests ( $y_b, z_b$ ) and ranks alternative combinations of ( $y_b, z_b$ ) in a fully rational manner according to a utility function  $UB(\bullet)$  that measures the degree of achievement of sporting interests B. Assume as well that our consumer is aware that some activities associated with drinking,  $x_a$ , also influence the utility derived from B interests. Alternatively, using a less cardinal interpretation of the utility function  $UB$ , we assume that the tradeoffs between  $y_b$  and  $z_b$  the consumer

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<sup>7</sup>The second phase in prospect theory's analysis of choice under uncertainty involves processes of cancellation of common components and elimination of transparent dominated alternatives. Tversky and Kahneman (1988) p 172  
<sup>8</sup>See Edwards et al (1988) for a discussion of the premium ordinary decision makers place on simplicity in assessment techniques.

is willing to make under the guidance of focussed sporting interests UB will vary with changes in the levels of his drinking activities  $x_a$ . Thus

$$1 \quad UB = UB(y_b, z_b; x_a)$$

Symmetrically we write

$$2 \quad UA = UA(x_a, z_a; y_b)$$

for the rational preferences describing focussed interest A.

In this formulation, activities  $x_a$  and  $y_b$  are two distinct types while  $z_a$  and  $z_b$  are two different amounts of the same type, so aggregation  $z_a + z_b$  makes sense.

The consumer is a price taker with an exogenous source of income. For convenience we normalize with  $z$  as numeraire so  $Pz=1$ . The theory hypothesizes that purchasing decisions are made in the following way.

Income  $I$  is divided into two amounts ( $I_a, I_b$ ),  $I_a + I_b = I$ , dedicated to spending on A interests and on B interests respectively. The following budget constraints with parameters  $(P, I) = (p_x, p_y, Pz, I_a, I_b)$  then hold<sup>9</sup>:

$$3a \quad p_x \cdot x_a + z_a = I_a$$

$$3b \quad p_y \cdot y_b + z_b = I_b$$

Using income  $I_a$  and the focussed interest UA the consumer maximizes his A type interests over the narrow sphere of activities  $(x_a, z_a)$ . This optimization process yields, under suitable regularity conditions, demand functions for  $x_a$  and  $z_a$  conditional on the level of activity  $y_b$ . Similarly, using income  $I_b$  and the focussed interest UB the consumer maximizes his B type interests over the narrow sphere of activities  $(y_b, z_b)$ . This optimization process yields demand functions for  $y_b$  and  $z_b$  conditional on the level of activity  $x_a$ .

Let

$$4a \quad x_a^{\text{cond}} = x_a^*(y_b, P, I) \text{ and}$$

$$4b \quad y_b^{\text{cond}} = y_b^*(x_a, P, I)$$

be the respective conditional demand functions arising from the solutions to the first order conditions (6) and budget constraints (3) for the following optimization problems (5)

<sup>9</sup>Note that we are using the same symbol  $I$  for a scalar, exogenous income, and a vector of expenditures  $(I_a, I_b)$ .

$$5a \quad VA(y_b) = \max_{x_a} UA(x_a, I_a - P_x \cdot x_a, y_b)$$

$$5b \quad VB(x_a) = \max_{y_b} UB(y_b, I_b - P_y \cdot y_b, x_a)$$

$$6a \quad \frac{\partial UA}{\partial x_a} / \frac{\partial UA}{\partial z_a} = P_x$$

$$6b \quad \frac{\partial UB}{\partial y_b} / \frac{\partial UB}{\partial z_b} = P_y$$

The conditional demand functions may - if a unique solution to (5) exists - be solved simultaneously to obtain equilibrium demands  $x_a^e = x_a^*(P, I)$  and  $y_b^e = y_b^*(P, I)$  satisfying

$$7a \quad x_a^e(P, I) = x_a^*(y_b^e(P, I), P, I)$$

$$7b \quad y_b^e(P, I) = y_b^*(x_a^e(P, I), P, I)$$

The budget constraints yield the equilibrium demands for  $z_a$  and  $z_b$ . The superscript "e" symbolizes equilibrium. The equilibrium described in these equations is the compromise the consumer strikes between his divided selves (hypothesis S2).

Consider an example. The following quadratic utility functions describe interests UA and UB. Parameters  $\bar{b}$ ,  $b$ ,  $s$ , and  $t$  are assumed to be negative and  $a$  sufficiently large to ensure positive marginal utilities for  $x_a$  and  $y_b$ , in UA and UB respectively<sup>10</sup>

$$7a \quad UA = a \cdot x_a + z_a + [x_a \ y_b] \begin{bmatrix} b & s \\ s & 0 \end{bmatrix} \begin{bmatrix} x_a \\ y_b \end{bmatrix} \\ = a \cdot x_a + z_a + b \cdot x_a^2 + 2 \cdot s \cdot x_a \cdot y_b$$

$$7b \quad UB = a \cdot y_b + z_b + [x_a \ y_b] \begin{bmatrix} 0 & t \\ t & \bar{b} \end{bmatrix} \begin{bmatrix} x_a \\ y_b \end{bmatrix} \\ = a \cdot y_b + z_b + \bar{b} \cdot y_b^2 + 2 \cdot t \cdot x_a \cdot y_b$$

<sup>10</sup>We also assume that  $\bar{b} \cdot b - (s + t)^2 > 0$  so that the expenditure minimisation problem (10) has a unique solution.

The conditional demand functions (5) arising from maximizing each interest separately are:

$$8a \quad x_a^{\text{cond}} = \frac{(-a + P_x)}{2 \cdot b} - \frac{s}{b} y_b$$

$$8b \quad y_b^{\text{cond}} = \frac{(-a + P_y)}{2 \cdot \bar{b}} - \frac{t}{\bar{b}} x_a$$

The resulting equilibrium demand functions for  $x_a$  and for  $y_b$  are:

$$9a \quad x_a^e = \left[ \frac{(a - P_x) + (a - P_y) \cdot s}{2 b + 2 b \cdot \bar{b}} \right] \frac{b \cdot \bar{b}}{b \cdot \bar{b} - s \cdot t}$$

$$= \frac{a \cdot (s - \bar{b})}{2 \cdot (b \cdot \bar{b} - s \cdot t)} + \frac{\bar{b}}{2 \cdot (b \cdot \bar{b} - s \cdot t)} \cdot P_x - \frac{s}{2 \cdot (b \cdot \bar{b} - s \cdot t)} \cdot P_y$$

$$9b \quad y_b^e = \left[ \frac{(a - P_y) + (a - P_x) \cdot t}{2 \bar{b} + 2 b \cdot \bar{b}} \right] \frac{b \cdot \bar{b}}{b \cdot \bar{b} - s \cdot t}$$

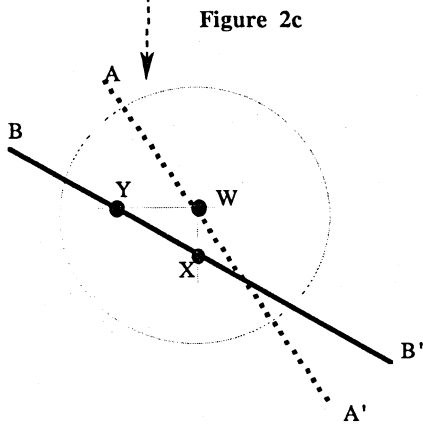
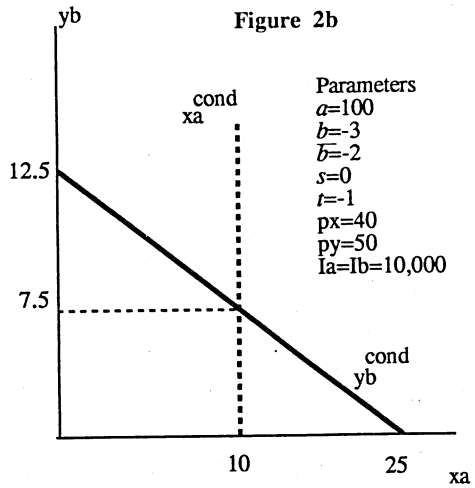
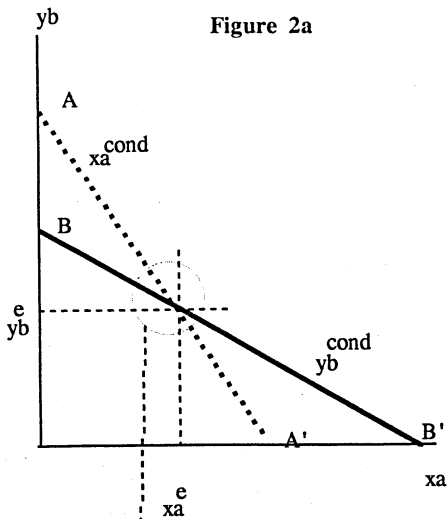
$$= \frac{a \cdot (t - b)}{2 \cdot (b \cdot \bar{b} - s \cdot t)} + \frac{b}{2 \cdot (b \cdot \bar{b} - s \cdot t)} \cdot P_y - \frac{t}{2 \cdot (b \cdot \bar{b} - s \cdot t)} \cdot P_x$$

The equations (9) are simple linear relationships between quantity demanded and prices.

For given budget parameters, the conditional demand functions are plotted in Figure 2 in general (Fig 2a) and for the special case where  $y_b$  has no detrimental influence on interest A but  $x_a$  does have a detrimental influence on interest B<sup>11</sup>. (Fig 2b). The example has been chosen to make the existence and uniqueness of the demand functions obvious. In general this will not be the case. There may or may not be a solution to the system of equations (6) describing the conditional demands and, if a solution exists, it may not be unique.

<sup>11</sup>Specifically :  $a=100, b=-3, \bar{b}=-2, s=0, t=-1, p_x=40, p_y=50, I_a=I_b=10,000$





#### IV. Predictable Intransitivities

As well as generating demand functions the theory can also be used to predict certain kinds of intransitive binary choices. First we develop a model based on the idea of framing. Second we show how to exploit the demand functions to produce intransitivities in revealed preferences.

The framing of the decision problem in our theory focuses attention on solving two separate, but interrelated, decision problems. In this process explicit information about UA and UB values is generated over the decision space  $(x_a, y_b)$ . When interest UB is maximized, information is generated on the UB-best choices of  $y_b$  and  $z_b$  for various levels of  $x_a$ . In Figure 2a this information can be simply framed or represented by focussing attention and interpretation on vertical lines and the line segment BB'. Along any such vertical lines the UB interest is maximized along BB'. Binary comparisons between points on a vertical line are facilitated since being on BB' is shown within the frame to be UB-better than being vertically above or below BB'. Also, since the direction of increase in UB interests is from B' towards B, any two points along BB' are readily comparable in terms of UB interests. Notice that this simplified frame does not explicitly generate and display information about how UA changes along vertical lines or along BB'. That is, the frame masks the changing UA interests along vertical line segments and along BB'. Symmetrically, by focusing attention on horizontal lines, the curve AA' can be used to compare points in terms of UA interests. Along a horizontal line points off AA' are UA inferior to the corresponding point on AA'. Similarly, points along AA' are easily comparable in terms of UA interests since the direction of increasing UA is from A to A'. The frame masks the changing UB interests along horizontal line segments and along AA'.

Now consider in Figure 2c a comparison between three alternative bundles W, X and Y in terms of information readily available from the frame. W is UA-better than Y, looking along a horizontal line between W and Y. W is not readily UB-comparable to Y in terms of the frame since the two points are not along a vertical line. If a binary choice has to be made and no alternative frame is generated, hypothesis S4 leads us to predict that W would be chosen over Y. Similarly, Y is UB-better than X since both lie along BB', but Y and X are not readily UA comparable in terms of the frame. We predict Y would be chosen over X in a binary choice. Finally, since X is UB-better than W being vertically below W and on BB' but not readily UA comparable with W, we predict X will be chosen over Y in a binary choice - an intransitive pattern of binary choices.

This procedure for predicting intransitivities in binary choices is crude, even within the context of the frame.<sup>12</sup> The frame recognizes that  $x_a$  affects UB interests negatively and vice versa. Thus, 'large' differences between alternatives in the decision space  $(x_a, y_b)$  are likely to arouse some suspicion in the decision maker's mind that selectively focussing on one dimension of value at a time will create some sort of difficulties. Agreed. We can even take such cognitive dissonance as a prediction of the

<sup>12</sup>It is similar in spirit to Tversky's (1969) use of lexicographic semi orders to explain intransitivities.

theory. But the point is that without changing the frame the decision maker is not likely to be able to avoid intransitivities and hypothesis S4 suggests people do not spontaneously produce alternative frames that facilitate the evaluation of combined effects in interactive decisions.

At a more sophisticated level our theory predicts cycles in the binary revealed preference relation  $\mathcal{R}$  between commodity bundles  $a$  and  $b$

$a \mathcal{R} b$  if  $a \neq b$  and  $b$  is affordable in the budget situation under which  $a$  is demanded

Consider the system of equations describing equilibrium demands (9) for a particular set of parameters in the quadratic utility specification.<sup>13</sup>

$$\begin{aligned}
 x_a^e &= -\frac{1}{6} p_x + \frac{50}{3} \\
 y_b^e &= p_x - \frac{1}{4} p_y - 75 \\
 z_a^e &= \frac{1}{6} p_x^2 - \frac{50}{3} p_x + I_a \\
 z_b^e &= \frac{1}{4} p_y^2 - p_x \cdot p_y + 75 p_y + I_b
 \end{aligned}
 \tag{10}$$

There are two notable features of this system of equations. First, the demands for  $x_a$  and  $y_b$  exhibit zero income effects, a peculiarity due to the quasi linear nature of the utility functions used in our example. Second, the cross price effects between products  $x_a$  and  $y_b$ , even after subtracting out (zero) income effects, are asymmetric. This second feature turns out to be a general prediction of our theory of quasi rational consumer demand (see section V below) It is due essentially to the 'inconsistency effect' of a price change, an additional wedge between the (symmetric) substitution effect of a price change and the total effect of a price change. Generally speaking, the inconsistency effect of a price change implies that the demand functions of this model will not satisfy the integrability conditions, an obvious point in the case of the system of equations (10). Failure to satisfy the integrability conditions can be exploited to derive predictable cycles in revealed preferences. The following example using the equilibrium demands (10) illustrates the method.

Imagine a sequence of discrete price changes from case A through case E as outlined in the columns of Table 3 below. The first two rows indicate the prices of the goods that are changing. The next two rows indicate the quantities of  $x_a$  and  $y_b$  demanded at each price. The last row is the cost difference between one budget situation and the immediately preceding one. By cost difference we mean the extra income ( $\pm$ ) just sufficient to keep the previous commodity bundle affordable. For example, the

<sup>13</sup>Specifically :  $a=100, b=-3, \bar{b}=-2, s=0, t=12$ ,

change from A to B is a price increase of 2. The cost difference of 4 in the last row of budget situation B is the extra income required to make budget situation A affordable<sup>14</sup>. Figure 3 provides a visual aid.

Table 3

Budget situation

Py

Px

Equilibrium demands

Yb

Xa

	A	B	C	D	E
Py	40	40	39	39	40
Px	88	90	90	88	88

Yb	3.00	5.00	5.25	3.25	3.00
Xa	2.00	1.67	1.67	2.00	2.00

Cost difference from immediately preceding budget situation

		4.00	-5.00	-3.33	3.25
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Figure 3a

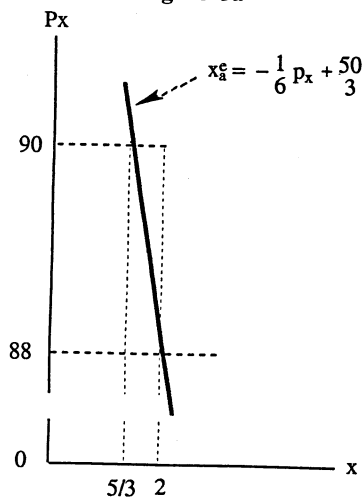
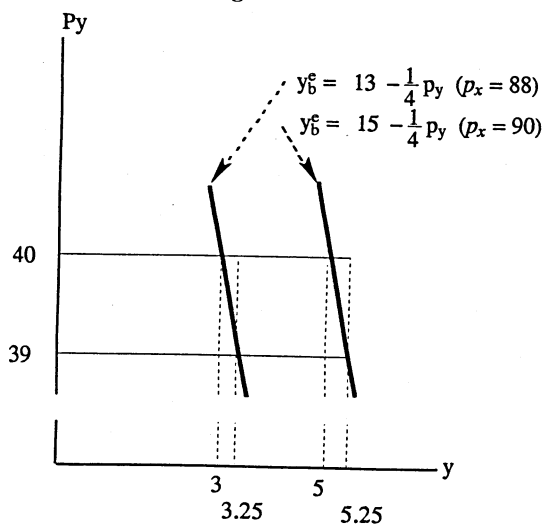


Figure 3b



<sup>14</sup>The demands for  $z_a$  and  $z_b$  can be inferred from the budget constraints and are suppressed in Table 3 for simplicity since they are not required in order to calculate cost differences.

The notation  $\mathcal{D}(A)$  will be used for the vector of equilibrium demands  $(x_a, z_a, y_b, z_b)$  in budget situation A, etc. As we move from one budget situation to next across the top row of Table 3 suppose the consumer's income is changed by the cost difference. That is, in situation B the consumer's income is increased relative to what it was in A by 4. The bundle chosen in A's budget situation therefore becomes affordable at B's prices. In situation C income is decreased relative to situation B by 5, so that bundle chosen in B's budget situation becomes affordable at C's prices. Proceeding across the top row of the table leads us to the following sequence of revealed preference relations:

$$11 \quad \mathcal{D}(B) \mathcal{R} \mathcal{D}(A), \mathcal{D}(C) \mathcal{R} \mathcal{D}(B), \mathcal{D}(D) \mathcal{R} \mathcal{D}(C), \mathcal{D}(E) \mathcal{R} \mathcal{D}(D)$$

However, budget situation E now has the initial income plus the cumulative effects of the sequential changes from A through E, 1.08 less than initially. Since A and E have the same prices and E has less income,  $\mathcal{D}(A) \mathcal{R} \mathcal{D}(E)$ . Along with (11) this is a cycle in the revealed preference relation  $\mathcal{R}$ .

Effectively we have chosen to evaluate the line integral (12) along the specific closed path of prices  $\alpha(t)$  shown in Figure 4..

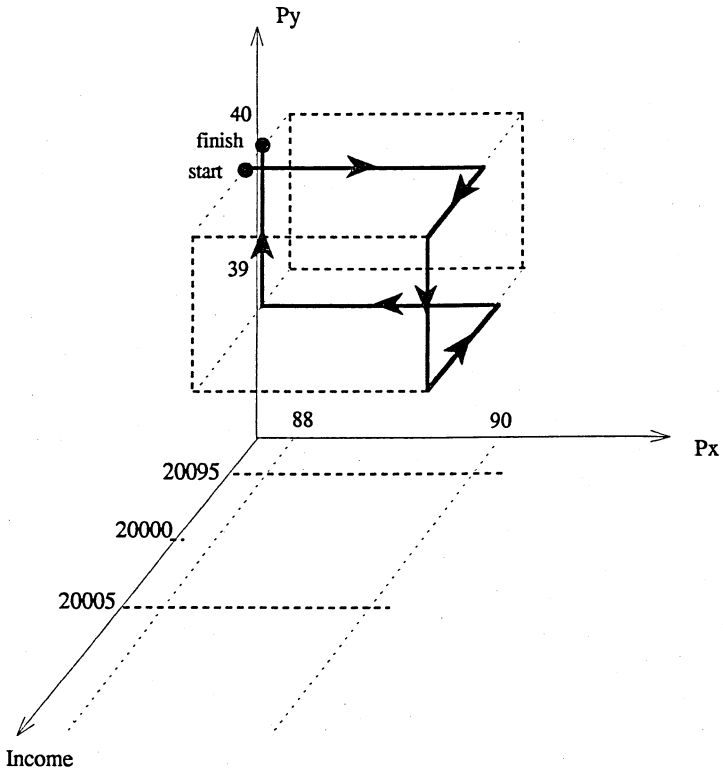
$$12 \quad \int_0^{1/4} x_a^c(\alpha(t)) d\alpha_1 + \int_{1/4}^{1/2} y_b^c(\alpha(t)) d\alpha_2 + \int_{1/2}^{3/4} x_a^c(\alpha(t)) d\alpha_1 + \int_{3/4}^1 y_b^c(\alpha(t)) d\alpha_2$$

The line integral (12) is path dependent because of the failure of the integrability conditions to hold: the cross price derivatives of equilibrium demands (10) are not equal, even *after subtracting out the income effect of the price changes*. Each component of the line integral can be viewed as the limit of partial sums of cost differences of the sort computed in Table 3. The terms of each partial sum for a component integral define a finite sequence of budget situations and revealed preference relationships. The path dependency of the line integral implies that through a suitable choice of path we can make the line integral along a closed path equal to any positive or negative magnitude we choose<sup>15</sup>. Therefore there will always exist a finite sequence of budget situations leading to a cycle of revealed preference relations of the sort we have proved in our numerical example<sup>16</sup>.

<sup>15</sup>The cycle in revealed preference we have exhibited violates the Strong Axiom of Revealed Preference, SARP. SARP is necessary and sufficient for a finite set of data to be consistent with utility maximization in the case of single valued demand functions (Varian, p 143). Therefore the demand functions (10) cannot be consistent with utility maximization.

<sup>16</sup>Actually finding a path with a small number of terms in a specific instance was no simple matter! In the numerical calculations I carried out shorter paths were found when the asymmetry in cross price effects was larger.

Figure 4



$\alpha(t)$  is a vector valued step function with graph shown above

$t=0$	$\alpha(t) = (p_x(t), p_y(t), I(t))$	$= (88, 40, 20000)$
$t \in (0, 1/4]$	$\alpha(t) = (p_x(t), p_y(t), I(t))$	$= (90, 40, 20004)$
$t \in (1/4, 1/2]$	$\alpha(t) = (p_x(t), p_y(t), I(t))$	$= (90, 39, 19999)$
$t \in (1/2, 3/4]$	$\alpha(t) = (p_x(t), p_y(t), I(t))$	$= (88, 39, 19995.7)$
$t \in (3/4, 1]$	$\alpha(t) = (p_x(t), p_y(t), I(t))$	$= (88, 40, 19998.9)$

## V. Expenditure minimisation and a decomposition of the total effect of a price change

The following problem

$$12 \quad \underset{x_a, y_b, z_a, z_b}{\text{minimize}} \quad P_x \cdot x_a + P_y \cdot y_b + P_z \cdot (z_a + z_b) \quad \text{subject to} \quad UA(\cdot) \geq u_a \quad \text{and} \quad UB(\cdot) \geq u_b$$

has a unique solution (13) if the utility functions are strictly quasi-concave.

$$13a \quad x_a^{cXP} = x_a^q(P, u_a, u_b)$$

$$13b \quad y_b^{cXP} = y_b^q(P, u_a, u_b)$$

$$13c \quad z_a^{cXP} = z_a^q(P, u_a, u_b)$$

$$13d \quad z_b^{cXP} = z_b^q(P, u_a, u_b)$$

The first order conditions for this problem are

$$14a \quad \frac{\frac{\partial UA}{\partial x_a}}{\frac{\partial UA}{\partial z_a}} + \frac{\frac{\partial UB}{\partial x_a}}{\frac{\partial UB}{\partial z_b}} = P_x$$

$$14b \quad \frac{\frac{\partial UB}{\partial y_b}}{\frac{\partial UB}{\partial z_b}} + \frac{\frac{\partial UA}{\partial y_b}}{\frac{\partial UA}{\partial z_a}} = P_y$$

The expenditure function

$$15 \quad e(P, u_a, u_b) = P_x \cdot x_a^{cXP} + P_y \cdot y_b^{cXP} + P_z \cdot (z_a^{cXP} + z_b^{cXP})$$

is well defined and has the standard properties (eg Varian, p.123) of an expenditure function in the theory of rational consumer choice (assuming both utility constraints are active and the non linear programming constraint qualification holds)

- $e(P, u_a, u_b)$  is *nondecreasing* in  $P$
- $e(P, u_a, u_b)$  is *homogeneous of degree zero* in  $P$
- $e(P, u_a, u_b)$  is *concave* in  $P$
- $e(P, u_a, u_b)$  satisfies the *derivative property* whenever the derivatives are well defined. That is, the derivative of  $e(P, u_a, u_b)$  with respect to the price of a specific commodity is equal to the expenditure minimizing demand for that commodity. e.g.

$$16 \quad \frac{\partial e(P, u_a, u_b)}{\partial p_a} = x_a^q(P, u_a, u_b)$$

For a rational consumer duality between expenditure minimisation and utility maximization holds. Facing a given price budget situation (P,I) a rational consumer will purchase a utility maximizing commodity basket, deriving utility  $u$ . This same commodity basket is an expenditure minimizing commodity basket subject to being as well off as at  $u$ . That is, given the minimum amount of income required to reach utility level  $u$  the consumer will spend it on the expenditure minimizing commodity basket.

However, duality will not generally hold for the quasi rational consumer. This is most obvious by comparing the equilibrium demand conditions (6) with the first order conditions (14) for the expenditure minimisation problem. A solution to one problem will not generally be a solution to the other if there are interactive effects between different spheres of interest.

In the case of our quadratic utility example, optimal demand for  $x_a$  is

$$17 \quad x_a^{exp} = \frac{1}{2} \frac{(-a - p_x) + ((a - p_y) \cdot (s + t))}{b \bar{b} - (s + t)^2}$$

yet from (9a)

$$x_a^e = \left[ \frac{(a - p_x)}{2b} + \frac{(a - p_y) \cdot s}{2b \cdot \bar{b}} \right] \frac{b \cdot \bar{b}}{b \cdot \bar{b} - s \cdot t}$$

For the specific parameters underlying Figure 2b the equilibrium demands are  $(x_a, y_b) = (10, 7.5)$  while the optimum demands are  $(x_a, y_b) = (7, 9)$ . Essentially, by ignoring the negative interactive effects between the two spheres of interest the quasi rational consumer consumes 'too much'  $x$  and 'too little'  $y$ , relative to what is needed to be consumed in order to keep expenditure to a minimum. This establishes the following two propositions

**Proposition 1** If a quasi rational consumer has income just sufficient to reach given levels of utility  $u_a, u_b$  that income will not be spent on the expenditure minimizing levels of commodities. e.g.

$$x_a^g(P, u_a, u_b) \neq x_a^{**}(P, e(P, u_a, u_b))$$

**Proposition 2** A quasi rational consumer in a competitive market does not minimize the cost of reaching the utility levels he chooses.

Looked at in another way, Proposition 2 implies a failure to reach the  $(u_a, u_b)$  utility possibility frontier consistent with his aspirations and limited budget. That is, the compromise choice the quasi rational consumer makes leads him to select dominated consumption bundles (hypothesis S5). The frame in which the consumer is assumed to analyse his decision situation is essentially  $(x_a, y_b)$  space. It is logically possible to form an analysis of the problem in  $(u_a, u_b)$  space that makes it transparent that



the equilibrium solution will be dominated by some other choice. However, by hypothesis S4, consumers do not spontaneously make such changes of frame.

The comparison between equilibrium and optimal first order conditions is illuminating. To reach an equilibrium level of consumption (6) states that the quasi rational consumer equates the marginal rate of substitution, MRS, for each commodity to its price *ignoring the integrated effects of purchases in other spheres of interest*. To reach an optimum<sup>17</sup> in the sense of minimising expenditures to reach a certain standard of living ( $u_a, u_b$ ) the sum of the MRS's across relevant spheres of interest needs to be equated with price. Therefore, if we interpret a commodity's MRS's from each separate interest as indicating marginal valuations of a commodity to a consumer, the maximum amount of numeraire commodity that particular interest would be prepared to sacrifice for an extra unit of that commodity, we have

**Proposition 3** Competitive market price does not measure marginal value for a quasi rational consumer in equilibrium

The implications of this proposition for policy applications of economic theory are potentially serious, as suggested below. Before exploring some of the policy implications of this statement, however, it will be helpful to establish the following comparative statics proposition.

**Proposition 4** The total effect of a price change can be decomposed into a substitution effect, an income effect, and an inconsistency effect

$$18 \quad \frac{\partial x_a^e(P, I)}{\partial p_x} = \underbrace{\frac{\partial x_a^*(y_b^e, P, I)}{\partial y_b^e} \cdot \frac{\partial y_b^e(x_a^e, P, I)}{\partial p_x}}_{\text{inconsistency effect}} + \underbrace{\left[ \frac{\partial x_a^*(y_b^e, P, I)}{\partial p_x} \right]_{\text{UA constant}}}_{\text{substitution effect}} - \underbrace{\frac{\partial x_a^*(y_b, P, I)}{\partial I} \cdot x_a^*(y_b, P, I)}_{\text{income effect}}$$

<sup>17</sup>A similar condition to (13) holds if we express the problem as ensuring the consumer reach an undominated alternative on the utility possibility frontier for the consumer.

We will prove the proposition for an 'own price' price change. For any given value of  $y_b$  the conditional demand function 4a

$$x_a^{\text{cond}} = x_a^*(y_b, P, I)$$

derived from maximizing utility  $U_A$  subject to budget constraint 3a satisfies the conventional decomposition of a price change into income and substitution effects using the Slutsky equation:

$$19 \quad \frac{\partial x_a^*(y_b, P, I)}{\partial p_x} = \left[ \frac{\partial x_a^*(y_b, P, I)}{\partial p_x} \right]_{U_A \text{ constant}} - \frac{\partial x_a^*(y_b, P, I)}{\partial I} \cdot x_a^*(y_b, P, I)$$

substitution effect
income effect

Differentiating 7a with respect to  $p_x$  and substituting in (19) yields (18)

For a quasi rational consumer equation (18) shows a price change has three, not two, effects. The intuition behind equation (18) is as follows. The substitution effect induces the consumer to economize on income in achieving  $U_A$  utility by substituting away from the good whose price has risen, holding  $U_A$  and  $y_b$  constant. Utility  $U_A$  is not constant however, and a price change is like reducing income devoted to interest  $U_A$  - the income effect, still holding  $y_b$  constant. A price change will also induce a change in the equilibrium level of  $y_b$  as the consumer attempts to strike his compromise between  $x_a$  and  $y_b$ . Thus  $y_b$  will change and therefore the equilibrium level of  $x_a$ . This latter effect is appropriately called the inconsistency effect of the price change.<sup>18</sup> since, for a rational consumer maximizing a single interest subject to a budget constraint, equilibrium and optimal demands are identical, the first term of (19) vanishes, and (19) reduces to the usual Slutsky equation.

What about the sign of the inconsistency effect? If  $\frac{\partial x_a^*(y_b, P, I)}{\partial y_b^c} < 0$ , as in our quadratic utility example, higher  $y_b$  leads to lower marginal  $U_A$  utility for  $x_a$  and a reduction in it's conditional demand. If equilibrium demands for  $x_a$  and  $y_b$  show them to be gross substitutes, cross price effects  $\frac{\partial y_b^c(x_a, P, I)}{\partial p_a}$  will be positive. In this case the inconsistency effect is negative and will reinforce the substitution effect. Other cases may be constructed so that the inconsistency effect works in the opposite direction to the substitution effect.

<sup>18</sup>This additional effect is similar in form, but not substance, to the inconsistency effect for nontransitive consumers Fountain(1981). There, the consumer was assumed to have a single intransitive preference ordering, not multiple transitive ones.

We have derived a decomposition of the total effect of an *own* price change into three components. A similar derivation can be used to show the following decomposition for *cross* price effects.

$$\begin{aligned}
 20 \quad \frac{\partial x_a^c(P, I)}{\partial p_y} &= \underbrace{\frac{\partial x_a^*(y_b^c, P, I)}{\partial y_b^c}}_{\text{inconsistency effect}} \cdot \frac{\partial y_b^c(x_a^c, P, I)}{\partial p_y} + \underbrace{\left[ \frac{\partial x_a^*(y_b^c, P, I)}{\partial p_y} \right]_{\text{UA constant}}}_{\text{substitution effect}} - \underbrace{\frac{\partial x_a^*(y_b, P, I)}{\partial I} \cdot y_b^*(x_a, P, I)}_{\text{income effect}}
 \end{aligned}$$

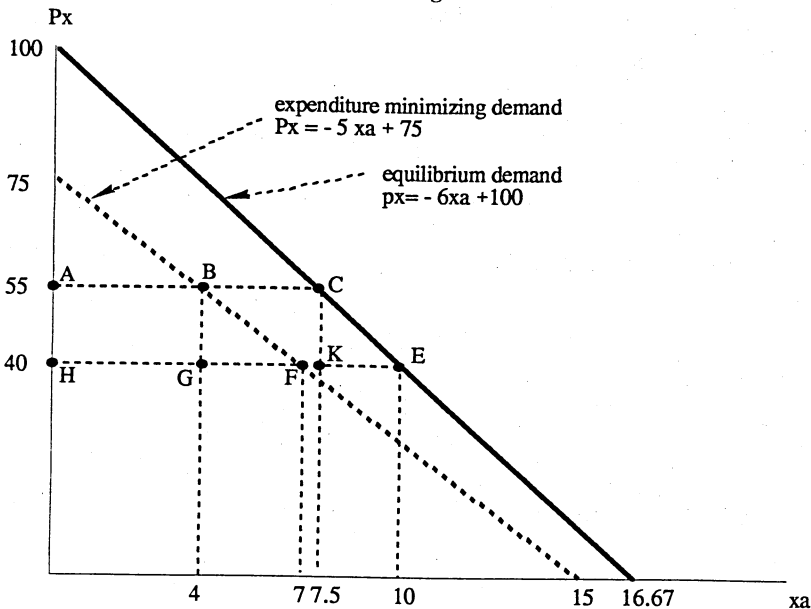
The only symmetric term that theory can predict in the decomposition (20) is the substitution effect. Generally neither the income effect nor the inconsistency effect of a price change can be expected to be symmetric. Thus, the integrability conditions (Varian p 137) will generally not hold for demand systems derived for quasi rational consumers. Moreover, because of the inconsistency effect the substitution effect cannot be recovered from the total effect of a price change by subtracting out the empirically observable income effect.

## VI. Implications for Welfare Economics

Propositions 1, 2 and 3 have serious implications for applied welfare economics. Consider the use of consumer surplus measures in a partial equilibrium analysis of the welfare effects of a price increase due to a sales tax.

Using our quadratic utility function example, the inverse demand curves for commodity  $x$  are plotted in Figure 5. If the ordinary demand function is used to calculate a compensating variation for a price rise from, say, 40 to 55, the required compensating payment, area ACEH, is estimated as 131.25. Since only  $15 \times 7.5 = 112.5$  is raised in revenue the welfare loss associated with this price increase is *measured* as 18.75 and *interpreted* as the extra amount a rational consumer would pay out of his actual income to avoid the sales tax in favour of a lump sum income tax raising the same revenue. However, both this measurement and its interpretation are subject to several sorts of errors.

Figure 5



First, it is obvious from Figure 5 in particular, and Proposition 1 in general, that the equilibrium demand curve and the expenditure minimizing demand curve do not coincide for quasi rational consumers. It is the area to the left of the latter demand curve that is the correct one to use from a theoretical viewpoint of measuring the changes in the minimum income necessary to keep a consumer

at fixed utility levels. It is worth pointing out that there are no approximation errors due to income changes in this example (as analyzed by Willig(1976)). The quasi linear nature of the utility functions implies zero income effects for commodities x and y, as the equilibrium demand functions (9) show. Figure 5 (or equation 19 with the income effect zero) shows that there is a discrepancy between equilibrium and expenditure minimizing demand curves *even in the absence of income effects*. This difference can be attributed to the quasi rational nature of demand and in some sense is a measure of the 'degree of inconsistency' of the consumer.

Second, the interpretation of this surplus measure as the minimum income change required to compensate the consumer for a price change is erroneous for quasi rational consumers. Consider our quadratic utility example. Some quick calculations will show that the minimum income required to achieve the utility levels for the  $p_x=40$  budget set,  $(u_a, u_b)=(10300, 10112)$ , is 19977 when  $p_x=40$  and 20059.5 when  $p_x=55$ <sup>19</sup>. The 82.5 difference is the surplus measured as area ABFH in Figure 5<sup>20</sup>. Does 82.5 indicate anything about an amount that could be added to the quasi rational consumer's actual income to compensate him for the price increase? No. It only tells us how much to add to the theoretical minimum income, not the consumer's actual income, to make it financially possible for the consumer to reach initial utility levels  $(u_a, u_b)=(10300, 10112)$ . A quasi rational consumer will not usually buy the expenditure minimizing commodity bundle. In this case, at  $p_x=55$  prices and with either the theoretical minimum income of 20059.5 or his actual income of 20,000 the quasi rational consumer will purchase  $(x_a, y_b)=(7.5, 8.75)$ , not  $(x_a, y_b)=(4, 10.5)$  as required for expenditure minimisation.

In fact, in this example we can add 90 to the consumer's actual income and leave him with enough income to just afford to achieve the initial utility level  $(u_a, u_b)=(10300, 10112)$ <sup>21</sup>. The reason the figure of 90 differs from the surplus measure of 82.5 is due to the discrepancy between actual income and expenditure minimizing income for a quasi rational consumer aiming to achieve a fixed utility target. Quasi rational consumer's don't cost minimize!

<sup>19</sup>Optimal consumption under the initial  $p_x=40$  budget parameters is  $(x_a, z_a, y_b, z_b)=(7.9747, 9.9500)$  yielding utility  $(u_a, u_b)=(10300, 10112)$ . Pricing out this commodity basket yields 19977. Similarly under the  $p_x=55$  budget parameters optimal  $(x_a, z_a, y_b, z_b)=(4.9948, 10.5, 9.366.5)$ , which prices out at 20059.5

<sup>20</sup>The derivative property (16) assures us that changes in value of the expenditure function can be measured by areas to the left of appropriate expenditure minimizing demand curves.

<sup>21</sup> At the new higher price of  $x_a$ , less  $x_a$  is bought: the equilibrium  $(x_a, y_b)$  changes from  $(10, 7.5)$  to  $(7.5, 8.75)$ . The utility derived from the new level of  $(x_a, y_b)$  can be calculated as  $(u_a, u_b) = (581.25, 590.62)$ .  $(z_a, z_b) = (9718.8, 9521.4)$  will bring the overall level of utility up to  $(u_a, u_b)=(10300, 10112)$ . Pricing out this commodity basket yields an income of 20090. (If the consumer divides his income into  $(I_a, I_b)=(10131, 9958.9)$  he actually will achieve, in equilibrium, the initial utility levels.)

Let us recapitulate and generalize these observations about the correct way to calculate compensating changes for quasi rational consumers. Initial income  $I$  and initial prices  $P$  leads to utility levels  $(u_a, u_b)$ .

- 1•  $[e(P, u_a, u_b) - I]$  measures the extent to which actual income exceeds minimum required income at initial prices  $P$  for a quasi rational consumer.
- 2• The surplus measured to the left of the expenditure minimizing demand curve  $[e(P', u_a, u_b) - e(P, u_a, u_b)]$  indicates the theoretical minimum compensating income payment for the price change from  $P$  to  $P'$  for a cost minimising consumer.
- 3• The actual income  $I'$  required to reach initial utility levels at new prices  $P'$  through equilibrium choices exceeds the theoretical minimum by an amount  $[I' - e(P', u_a, u_b)]$ .

The following table summarizes this discussion.

General concept	Numerical e.g.	verbal interpretation
$e(P, u_a, u_b) - I$	19977 - 20000 = - 23	discrepancy between actual initial income and expenditure minimizing income
$e(P', u_a, u_b) - e(P, u_a, u_b)$	20059.5 - 19977 = 82.5	theoretically minimum compensating variation in income
$I' - e(P', u_a, u_b)$	20090 - 20059.5 = 30.5	discrepancy between actual initial income and expenditure minimizing income
$I' - I$	90	change in actual income required to compensate the consumer for the price rise

The previous observations on measuring consumer benefits have important policy implications. Interestingly enough, in our numerical example the income required to compensate the consumer for the price rise is actually *less than the tax revenue generated from the price increase* (interpreted as a sales tax). The tax revenue is  $15 \cdot 7.5 = 112.5$ . We have the surprising result that there is no efficiency loss from the sales tax in spite of the fact that conventional partial equilibrium methods would impute an efficiency loss of 18.75, equal to area CEK in Figure 5 for this tax. A direct comparison of the revenue from the tax, 112.5, and the loss as measured by the true compensating variation in income, 90, show, in fact, that there is an efficiency gain of 22.5 from this sales tax.

The reason for this efficiency gain in this example is not hard to discern. The sales tax on  $x$  acts as a corrective for the negative externality between interest  $U_A$  and interest  $U_B$ , an externality that the quasi rational consumer has not fully internalized. If the tax revenue were returned to the consumer

the consumer could be strictly better off than without the tax. Alternatively, an income tax that raised the same revenue as the sales tax leaves unchanged the extent to which the consumer internalizes the externality between the two interests UA and UB since the demand for commodities x and y is unaffected by income in our example. An income tax that raised the same revenue as the sales tax would not leave the consumer able to achieve his initial utility levels. Thus, when dealing with quasi rational consumers the applied welfare economists cannot rely on the general folk theorem that a lump sum income tax is superior to a sales tax yielding the same revenue.

## ***VII. Conclusion***

Recently, Akerlof and Yellen (1985) and Russel and Thaler (1985), have explored the implications of 'quasi rational' type behaviour for market equilibria. Our analysis has focussed on the individual agent rather than on market equilibria. We have shown that quasi rational behaviour has significant implications for the theory of consumer demand and applied welfare economics. The present paper has only begun to scratch the surface. Questions about the generality of the analysis, and about the framing of income budgeting decisions, have yet to be addressed.

For the past century the economic theory has used a powerful paradigm for generating testable theories about individual choice and applying those theories to important questions of policy. The paradigm is constrained maximization of a single utility function. The theory of quasi rational consumer behaviour is only in its infancy compared to these developments. But the evidence from empirical psychology and the decision sciences suggests a paradigm shift is required. No doubt there will be many skeptics, especially those wedded to the intellectual power and beauty of the rational consumer model.<sup>22</sup> Arrow's (1988) comments on framing are a propos.

"Economists would tend to argue that the choices made in the market, where the stakes to the individual are high, reflect the correct choice of frame. But this is probably too complacent a view. It may well be true that the individual makes different tradeoffs in contexts which, to the analyst, appear to be identical. But this is a deep topic for further study."

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<sup>22</sup>Stiglitz and Becker (1977)

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