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Abstract

This paper considers several one-sided and two-sided asymptotic tests for ARCH(q) and GARCH(p,q) regression errors, and uses Monte Carlo analysis to investigate their finite-sample sizes and powers when a regressor has been inadvertently omitted from the model. The results are compared with those obtained when the model is correctly specified. The extent to which such model mis-specification affects the tests' properties can depend on the form of the omitted regressor, but generally one-sided tests out-perform their two-sided counterparts.

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1. Introduction

Since they were first discussed by Engle (1982), autoregressive conditionally heteroscedastic (ARCH) processes have received considerable attention in the econometrics and empirical finance literatures. In particular, ARCH and generalised ARCH (GARCH) processes have been found to be useful characterisations for the disturbances in regression models relating to financial time-series data of various sorts (*e.g.*, see Bollerslev *et al.* (1992)).

There is only a limited literature on the problem of testing for white noise regression disturbances against the alternatives of ARCH or GARCH errors. Engle (1982) suggested a simple Lagrange Multiplier (LM) test for ARCH disturbances, which is shown by Lee (1991) to be also an LM test against GARCH errors. A one-sided version of this test, suitable for ARCH(1) and GARCH(1,1) processes, was considered by Engle *et al.* (1985). Recently, Lee and King (1991) have suggested a locally best score (LBS) test which also takes account of the one-sided nature of the testing problem against higher-order alternatives.

Each of these tests has (large sample) asymptotic justification, but relatively little is known about their finite-sample properties. Some evidence, based on Monte Carlo analysis, is given by Engle *et al.* (1985), Lukkonen *et al.* (1988), Bollerslev and Wooldridge (1988), Diebold and Pauly (1989), Gregory (1989) and Lee and King (1991). Even less is known about the robustness of these tests to various forms of model mis-specification. Lee and King (1991) consider the effects of conditionally leptokurtic errors, but the robustness of the tests to other departures from the underlying assumptions remain to be explored.

There is evidence (*e.g.*, Small *et al.* (1992), Giles and Saxton (1992)) that other tests based on Least Squares regression residuals lack robustness to shifts in the mean of the disturbances. In practice, this is a very common form of model mis-specification. Accordingly, in this paper we investigate the finite-sample sizes and powers of ARCH-GARCH tests when they are applied in the context of a regression model from which a relevant regressor has been unwittingly omitted.

2. The Model and Tests

Consider the linear regression model

$$y_t = b_0 + \sum_{i=1}^{k-1} b_i x_{it} + u_t \quad ; \quad t = 1,2,\ldots,T \tag{1}$$

where

$$u_t | \Phi_{t-1} \sim IN(0, \sigma_t^2) \tag{2}$$

and Φ_t is the information set available at time t. If the disturbances, u_t, follow a GARCH(p,q) process (*e.g.*, Bollerslev (1986)) then

$$\sigma_t^2 = \sigma^2 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{3}$$

where $\sigma^2 > 0$; α_i, $\beta_j \geq 0$ (all i,j) and[1] $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$. ARCH(q) errors arise as a special case of (3) when $\beta_j = 0$ (all j).

Although Ordinary Least Squares (OLS) estimation of (1) is best linear unbiased in the context of GARCH or ARCH errors, greater efficiency can be achieved by using the (non-linear) Maximum Likelihood (ML) estimator which takes account of (3). Further, ARCH and GARCH errors have unconditional distributions which are fatter-tailed than under normality. This property, and their ability to account for volatility clustering, mean that ARCH-GARCH processes are useful characterisations of prices or returns for many speculative assets. For these reasons, testing for the presence of ARCH-GARCH disturbances is of considerable importance in financial econometrics.

Specifically, we wish to test

$$H_0 : \alpha_1 = \alpha_2 = ... = \alpha_q = \beta_1 = \beta_2 = ... \beta_p = 0$$

against

$$H_A : \alpha_i \geq 0; \beta_j \geq 0 \quad \text{(with at least one strict inequality;}$$
$$i = 1,...,q; \; j = 1,...,p)$$

in the GARCH(p,q) case. In the ARCH(q) case we wish to test

$$H_0' : \alpha_1 = \alpha_2 = ... = \alpha_q = 0$$

against

$$H_A' : \alpha_i \geq 0 \quad \text{(with at least one strict inequality;}$$
$$i = 1,...,q).$$

Lee (1991) has shown that the Lagrange Multiplier (LM) test of H_0' against H_A', as proposed by Engle (1982), is also the LM test of H_0 against H_A (for the same q value). This test rejects H_0 for large values of the statistic

$$LM = \delta' W(W'W)^{-1}W'\delta/2$$

where

$$W' = (w_{q+1} \vdots \cdots\cdots \vdots w_T)$$
$$w_t' = (1, \hat{u}^2_{t-1}, \; ... \; ,\hat{u}^2_{t-q})$$
$$\delta' = \left((\hat{u}^2_{q+1}/\hat{\sigma}^2)-1, \; ... \; ,(\hat{u}^2_T/\hat{\sigma}^2)-1\right),$$

$\hat{u}_t$ is the OLS residual associated with (1), and $\hat{\sigma}^2$ is the ML estimator of σ^2 under H_0 or H_0'.

As Engle (1982) and Engle *et al.* (1985) note, an asymptotically equivalent test can be constructed using the statistic nR^2, where $n = T-q$ and R^2 is the coefficient of determination from the OLS regression of $\hat{u}^2_t$ on an intercept and q successive lags of $\hat{u}^2_t$. The LM and nR^2 statistics are each asymptotically $\chi^2_{(q)}$ under H_0 or H_0'.

One potential weakness of these two tests is that neither takes account of the one-sided nature[2] of H_A and H_A'. To compensate for this in the ARCH(1) case, Engle *et al.* (1985) suggest using a one-sided test based on the asymptotically standard normal statistic[3] $z(nR^2) = \text{sign}(\hat{\alpha}_1)(nR^2)^{1/2}$, where $\hat{\alpha}_1$ is the estimated coefficient of $\hat{u}^2_{t-1}$ in the regression used to define R^2 above. Alternatively, nR^2 can be replaced by LM, and from Lee's (1991) results, either $z(nR^2)$ or $z(LM)$ is also appropriate against a GARCH(p,1) alternative.

This approach is not helpful in the construction of a one-sided test if $q \geq 2$, which has motivated Lee and King (1991) to use the approach of King and Wu (1990) to develop a Locally Best Score (LBS) test of H_0 or H_0' against H_A or H_A'. This one-sided test is also based on an asymptotically standard normal statistic of the form

$$LBS = \frac{\sum_t \left((\hat{u}^2_t / \hat{\sigma}^2) - 1 \right) \sum_{i=1}^{q} \hat{u}^2_{t-1}}{\left\{ 2\ell' \left[\sum_t \hat{z}_t \hat{z}'_t - \sum_t \hat{z}_t \sum_t \hat{z}'_t / T \right] \ell \right\}^{1/2}}$$

where ℓ is a $(q \times 1)$ vector of ones and $\hat{z}'_t = (\hat{u}^2_{t-1}, \ldots, \hat{u}^2_{t-q})$. The LBS test of H_0 against H_A is also the LBS test of H_0' against H_A' (for the same q value), and LBS collapses to z(LM) when $q = 1$.

As noted above, there is only limited evidence concerning the finite-sample properties of these tests. This information is based on Monte Carlo simulations, and in this paper we adopt the same procedure to investigate the robustness of these tests to model mis-specification through the omission of a relevant regressor from (1). Our experimental design is described in the next section.

3. Monte Carlo Analysis

Our Monte Carlo experiment is based on a correctly specified data-generating process of the form (1) and (2) with k=3, but the researcher wrongly fits the model (and tests for ARCH or GARCH disturbances) assuming that k=2. That is, x_2 is wrongly omitted from the model. As all of the tests are based on OLS residuals they are invariant to the values of σ^2, b_0 and b_1. These are set to 0.3, 1.0 and 1.0 respectively. However, the tests' properties depend on the regressor data and the other parameters.

Our choice of regressors is similar to that of Engle *et al.* (1985) and Lee and King (1991), though the former consider only a single-regressor model through the origin, and all but one of the data sets considered by the latter involve only one regressor in addition to an intercept.[4] Specifically, we use $x_{1t} = \theta x_{1t-1} + \varepsilon_t$ ($\varepsilon_t \sim IN(0,1)$), with θ = 0, 0.8, 1.0, 1.02. Our second regressor is either $x_{2t} = 0.1x_{2t-1} + \varepsilon_t$, or $x_{2t} = t$. This allows for a wide range of time-series characteristics in our data.

The extent of model mis-specification depends on the values of both x_2 and b_2. For convenience, we measure this effect through the value of $\lambda = b_2^2/\left(2\sigma^2R(X'X)^{-1}R'\right)$, where $R = (0,0,1)$ and $X = (\underset{\sim}{1},\underset{\sim}{x}_1,\underset{\sim}{x}_2)$. The scalar λ is the non-centrality parameter associated with the usual t-test of the restriction $b_2 = 0$. Values of λ = 0, 10, 50 imply different values of b_2 which are used to generate the y_t data under various ARCH-GARCH specifications.[5] Clearly, $\lambda = 0$ implies a correctly specified model. The other λ values generate different degrees of mis-specification while maintaining plausible signal/ noise ratios.

The SHAZAM package (White *et al.* (1990)) was used on a VAX 6340 for all of our simulation analysis. ARCH(1), ARCH(2), GARCH(1,1) AND GARCH(1,2) specifications were investigated, and 5,000 replications were used throughout.[6] First, with n = 20 and 100 the true sizes of the various tests were determined for different λ values when the (asymptotic) critical values associated with nominal 1% and 10% significance levels are used. Second, for a representative selection of situations, the rejection probabilities for the tests based on these same asymptotic critical values were determined under H_A

or H'_A, as appropriate. These probabilities represent "pseudo powers", not being size-adjusted. Finally, for the same selected situations, simulation analysis was used to find the finite-sample critical values which ensure the desired significance level for each test when $\lambda = 0$. Using these critical values, genuine (size-adjusted) powers were computed, including cases where $\lambda = 10$ and 50. In keeping with the mis-specification theme of this study, the finite-sample (size-adjusted) critical values associated with $\lambda = 0$ are also pertinent if $\lambda > 0$, as the researcher would be unaware of the model's mis-specification.

4. Results

The results of the three parts of our study are discussed in the order noted above.

4.1 Actual Test Sizes

Tables 1 and 2 summarise the actual sizes of the tests against ARCH(1) or GARCH(1,1), and ARCH(2) or GARCH(1,2) errors respectively. When $\lambda = 0$ the model is correctly specified with respect to the regressor set, so these results accord with those of Lee and King (1991), in broad terms, as expected.[7] However, these results are needed as a bench-mark against which to judge the effects of model mis-specification ($\lambda > 0$).

Consistent with the findings of other studies, we observe that, when $\lambda = 0$, all of the tests have sizes which are less than the nominally assigned size when $n = 20$. Even when $n = 100$, nominal sizes of 10% over-state the true situation. Generally, there is less size-distortion with the one-sided z tests than with their two-sided counterparts in the case of an ARCH(1) alternative. While the nR^2 and LM tests generally exhibit similar sizes, the former is to be slightly favoured (especially for $n = 20$), with its one-sided variant being the least distorted of the four tests examined. In contrast, while there is little to choose between the LM, nR^2 and one-sided LBS tests in terms of size-distortion against ARCH(2) errors when n = 100, the LBS test has the greatest such distortion, and this is even more pronounced when n = 20. Finally, the two-sided tests exhibit similar results in the ARCH(1) and ARCH(2) situations, but the LBS test (which is the z(LM) test against ARCH(1)) generally shows slightly greater size-distortion against ARCH(2) disturbances than against ARCH(1) errors.

When $\lambda > 0$, the effect on the sizes of the tests depends crucially on the form of the omitted regressor. At least within sampling variation,[8] sizes increase with λ when x_2 is a time-trend, and decrease with increasing λ (for $\lambda > 0$) when x_2 is autoregressive. In the former case, with moderate sample sizes, test sizes several times greater than their nominal values are readily attained. Regardless of the form of x_2, sizes do not necessarily increase with n, when the model is mis-specified, so (ironically) the degree of size distortion can worsen with an increased sample size in such cases. While these patterns hold quite generally, the specific test sizes can be quite sensitive to the form of the included regressor (the value of θ) once a second regressor is unwittingly omitted. The extra information utilized by the one-sided tests is again generally reflected in greater sizes than for their two-sided counterparts when testing against ARCH(1) errors in a mis-specified model, as may be seen in Table 1. However, when testing against ARCH(2) errors, this is not generally true and tends to occur only when a trend variable is omitted from the regression.

Accordingly, to minimize the degree of size distortion, the $z(nR^2)$ test is typically preferred against ARCH(1) (GARCH(1,1)) errors if there is a likelihood of having omitted an AR(1) or trended regressor. An exception is that the LM test is preferred in the case of severe mis-specification through the omission of a trended regressor. On the other hand, in the event of omitting a trended (autoregressive) regressor, the LM (nR^2) test would be a conservative choice in the case of ARCH(2) (GARCH(1,2)) errors, especially in the face of possibly severe mis-specification.

4.2 Unadjusted "Pseudo Powers"

Tables 3 to 6 report simulated probabilities of rejecting H_0 or H'_0 for a range of situations. That is, pseudo "powers", which have not been size-adjusted are reported. While these probabilities represent the actual ability of each test to reject a false null, care must be taken with any inter-test comparisons.[9] These tables all relate to n = 100 and θ = 0.8, to limit the volume of representative material. For completeness, test sizes from the earlier tables are reproduced here.[10]

The following general results apply, regardless of the alternative hypothesis. First, the "power" shapes are generally orthodox, *ceteris paribus*, with increasing rejection probabilities as we depart from the null. However, exceptions can arise with GARCH(1,1) errors and a model which is severely mis-specified through the omission of a trended regressor. Secondly, these probabilities always fall, under the alternative, as the model becomes increasingly mis- specified. Generally, the form of the omitted regressor has little impact on these results, but there is a better (higher) rejection probability when a trend variable is excluded than if an AR(1) regressor is excluded under severe mis-specification.

Under either the ARCH(1) or GARCH(1,1) alternatives both the apparent and true powers of the LM test exceed those of the nR^2 test, and similarly for the one-sided variants of these tests. The raw rejection probabilities of the one-sided tests exceed those of their two-sided counterparts, though further exact power comparisons must be left until the next sub-section. These results hold whether the model is properly specified or not.

Under either the ARCH(2) or GARCH(1,2) alternatives both the apparent and true powers of the LBS test exceed those of the nR^2 test whether the model is correctly specified or not. Generally, the LBS test out-performs both two-sided tests when the model is mis-specified, though more definitive evidence on this point emerges below.[11]

Finally, it is clear that each of the tests has greater ability to reject a false null in the case of an ARCH alternative, as opposed to a GARCH alternative. This conclusion is based on the use of $\left(\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j\right)$ as an overall measure of departure from H_0.

4.3 Size-Adjusted Powers

Tables 7 to 10 illustrate both the true powers of the tests when the model is correctly specified and the corresponding rejection probabilities associated with testing unwittingly in the context of a mis-specified model when size- adjusting the tests as if the model were correctly formulated. To the extent that firm statements were possible in the last sub-section concerning power rankings, these are confirmed by Tables 7 to 10.

In particular, when testing against ARCH(1) or GARCH(1,1) disturbances, the LM test dominates the nR^2 test and the z(LM) (or LBS) test dominates the $z(nR^2)$ test. Similarly, the one-sided tests generally dominate their two-sided counterparts. These results always hold if the model is correctly specified. To the extent that exact comparisons are valid (given relative sizes) when $\lambda > 0$, these results also hold when the model is mis-specified. Further, against ARCH(2) or GARCH(1,2) errors, the LBS test always dominates the other two tests when the model is properly specified. Again, as far as valid power comparisons can be made, the same is true when $\lambda > 0$.

Again, it is clear that all of the tests are considerably more powerful against ARCH errors than against GARCH errors, whether the model is properly specified or not. Finally, at least for the cases where the relative sizes permit power comparisons, it is clear that this type of mis-specification reduces the powers of all of the tests, and that each is somewhat more robust (in terms of power) to the omission of a trend regressor than to the omission of a (stationary) AR(1) regressor.

5. Conclusions

The results of this study have some important implications for the use of several common tests for ARCH and GARCH disturbances in regression models, and hence for the modelling of financial markets. The particular form of model mis-specification that we have considered - namely the omission of a relevant regressor - occurs frequently in practice, and our results show that the tests under consideration can lack robustness to this type of specification error.

Several practical prescriptions can be drawn from our results. First, if size distortion is important, then the choice of test should reflect any prior information about the likely form of the omitted regressor. For instance, if there is a possibility of having omitted a strongly trended variable, then the LM test is a good choice. On the other hand, if the omitted variable is likely to follow a stable AR(1) process, then the $z(nR^2)$ is preferred when testing against ARCH(1) or GARCH(1,1) errors, and the nR^2 test is a good choice against ARCH(2) or GARCH(1,2) errors. In either case it must be recognised that the established result, that the exact sizes of ARCH/GARCH tests are less than their (asymptotic) nominal sizes, no longer necessarily holds if the model is mis-specified, especially through the omission of a trended regressor.

Second, if high power is desired (with or without an allowance for size distortion), then the LBS test is a good choice when testing against ARCH(2) or GARCH(1,2) errors, and the equivalent z(LM) test is preferred against ARCH(1) or GARCH(1,1) errors. In any event, the powers of the tests decline as the model becomes increasingly mis-specified, and in many cases the power performance can be very poor, especially against GARCH alternatives.

Overall, the results of this study underscore the fact that established results based on the assumption of a correctly specified model need to be reconsidered if the model is likely to be mis-specified in some way. This is true whether one is considering the absolute performance of a test, or comparing the performances of alternative tests, in terms of size distortion or power performance.

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Footnotes

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1 The last of these inequalities ensures that the unconditional variance of u_t is positive and finite.

2 One would conjecture that ignoring this information may reduce the potential powers of the tests.

3 The null hypothesis is rejected for a sufficiently large positive value of the test statistic.

4 Neither of these (or any other) studies allow for the omission of relevant regressors, as we do.

5 Note that we do not consider any mis-specifications of the error process in our analysis.

6 The simulations are straightforward to conduct. For example, note that in the ARCH(1) case the error structure may be written as $u_t = \nu_t(\sigma^2+\alpha_1 u_{t-1}^2)^{1/2}$; $\nu_t \sim IN(0,1)$, so that only a Normal random number generator is required. The SHAZAM package incorporates the generator proposed by Brent (1974).

7 Those authors report results based on 5% nominal significance levels.

8 The sampling error can be determined by noting the binomial nature of the empirical rejections. So, for example, the standard error associated with the value of 0.005 as the first entry in Table 1(a) is $\sqrt{0.005(1-0.005)/5000} = 0.000997$.

9 When the true size of one test is less than that of a second one, and the probability of rejecting a false null is greater for the first test than for the second, then the first test has the greater (true) power.

10 In these, and the following tables, the entries for $\lambda = 0$ apply to both forms of x_{2t}. Recalling that $\lambda = 0$ corresponds to $b_2 = 0$, the data-generating process is independent of x_{2t} in this case.

11 Recall footnote 9 with respect to what conclusions may be drawn in the absence of size-correction.

TABLE 1(a): ARCH(1), GARCH(1,1) TEST SIZES

Nominal Size	n	$z(nR^2)$ Test θ 0.0	0.8	1.0	1.02	nR^2 Test θ 0.0	0.8	1.0	1.02
				$\lambda = 0$					
1%	20	0.005	0.007	0.006	0.006	0.002	0.004	0.003	0.004
	100	0.012	0.013	0.011	0.011	0.009	0.010	0.008	0.008
10%	20	0.054	0.062	0.059	0.061	0.053	0.058	0.061	0.059
	100	0.080	0.080	0.082	0.082	0.078	0.079	0.077	0.076
				$\lambda = 10$					
				$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$					
1%	20	0.006	0.014	0.006	0.006	0.003	0.008	0.003	0.003
	100	0.012	0.012	0.012	0.012	0.007	0.008	0.006	0.007
10%	20	0.069	0.100	0.074	0.075	0.055	0.070	0.059	0.062
	100	0.083	0.082	0.079	0.082	0.074	0.079	0.077	0.076
				$x_{2t} = t$					
1%	20	0.016	0.033	0.014	0.009	0.007	0.020	0.006	0.004
	100	0.018	0.016	0.024	0.018	0.013	0.012	0.017	0.012
10%	20	0.111	0.181	0.109	0.075	0.084	0.122	0.079	0.073
	100	0.102	0.094	0.108	0.100	0.085	0.082	0.091	0.084
				$\lambda = 50$					
				$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$					
1%	20	0.004	0.017	0.001	0.001	0.002	0.007	0.000	0.000
	100	0.009	0.009	0.008	0.007	0.005	0.006	0.005	0.005
10%	20	0.088	0.180	0.035	0.029	0.041	0.086	0.013	0.013
	100	0.065	0.068	0.064	0.066	0.067	0.069	0.069	0.069
				$x_{2t} = t$					
1%	20	0.118	0.308	0.265	0.102	0.061	0.206	0.174	0.056
	100	0.120	0.085	0.258	0.177	0.085	0.057	0.194	0.132
10%	20	0.505	0.757	0.698	0.423	0.352	0.618	0.551	0.292
	100	0.370	0.301	0.569	0.457	0.267	0.214	0.453	0.340

TABLE 1(b): ARCH(1), GARCH(1,1) TEST SIZES

Nominal Size	n	z(LM) Test θ 0.0	0.8	1.0	1.02	LM Test θ 0.0	0.8	1.0	1.02
				$\lambda = 0$					
1%	20	0.003	0.005	0.005	0.005	0.002	0.003	0.002	0.002
	100	0.012	0.013	0.011	0.012	0.008	0.009	0.006	0.007
10%	20	0.040	0.046	0.047	0.048	0.021	0.028	0.028	0.029
	100	0.074	0.073	0.074	0.075	0.072	0.072	0.067	0.067
				$\lambda = 10$					
				$x2_t = 0.1x_{2t-1} + \varepsilon_t$					
1%	20	0.002	0.005	0.004	0.004	0.001	0.002	0.002	0.002
	100	0.014	0.012	0.011	0.012	0.008	0.009	0.008	0.008
10%	20	0.041	0.072	0.056	0.053	0.017	0.033	0.027	0.027
	100	0.074	0.075	0.074	0.074	0.067	0.071	0.066	0.065
				$x_{2t} = t$					
1%	20	0.005	0.011	0.005	0.005	0.003	0.005	0.003	0.003
	100	0.017	0.015	0.023	0.018	0.010	0.008	0.017	0.012
10%	20	0.078	0.139	0.068	0.047	0.037	0.071	0.036	0.028
	100	0.096	0.086	0.106	0.096	0.079	0.072	0.088	0.076
				$\lambda = 50$					
				$x2_t = 0.1x_{2t-1} + \varepsilon_t$					
1%	20	0.000	0.001	0.000	0.000	0.000	0.001	0.000	0.000
	100	0.009	0.009	0.007	0.007	0.005	0.005	0.005	0.005
10%	20	0.023	0.074	0.015	0.015	0.005	0.020	0.004	0.004
	100	0.059	0.064	0.057	0.060	0.058	0.060	0.059	0.059
				$x_{2t} = t$					
1%	20	0.010	0.083	0.031	0.012	0.004	0.029	0.010	0.005
	100	0.086	0.058	0.232	0.145	0.056	0.039	0.182	0.107
10%	20	0.297	0.682	0.490	0.204	0.126	0.457	0.270	0.087
	100	0.326	0.255	0.546	0.425	0.218	0.167	0.430	0.307

TABLE 2(a): ARCH(2), GARCH(1,2) TEST SIZES

Nominal Size	n	nR^2 Test θ = 0.0	0.8	1.0	1.02	LM Test θ = 0.0	0.8	1.0	1.02
		$\lambda = 0$							
1%	20	0.003	0.003	0.003	0.003	0.003	0.004	0.004	0.003
	100	0.008	0.009	0.008	0.008	0.011	0.010	0.010	0.009
10%	20	0.055	0.058	0.061	0.057	0.029	0.030	0.030	0.030
	100	0.079	0.081	0.074	0.075	0.071	0.072	0.068	0.068
		$\lambda = 10$							
		$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$							
1%	20	0.002	0.002	0.002	0.002	0.001	0.001	0.006	0.004
	100	0.010	0.010	0.009	0.011	0.011	0.010	0.011	0.012
10%	20	0.051	0.054	0.047	0.052	0.025	0.023	0.016	0.015
	100	0.076	0.076	0.079	0.075	0.069	0.065	0.066	0.069
		$x_{2t} = t$							
1%	20	0.005	0.007	0.006	0.004	0.005	0.012	0.006	0.002
	100	0.013	0.012	0.012	0.013	0.013	0.011	0.014	0.014
10%	20	0.067	0.076	0.075	0.077	0.031	0.056	0.037	0.026
	100	0.085	0.080	0.084	0.086	0.071	0.068	0.077	0.077
		$\lambda = 50$							
		$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$							
1%	20	0.001	0.001	0.000	0.000	0.006	0.002	0.000	0.000
	100	0.008	0.008	0.006	0.007	0.007	0.006	0.005	0.006
10%	20	0.042	0.046	0.022	0.020	0.014	0.006	0.002	0.002
	100	0.063	0.062	0.062	0.060	0.050	0.049	0.046	0.049
		$x_{2t} = t$							
1%	20	0.027	0.041	0.050	0.050	0.008	0.054	0.028	0.005
	100	0.086	0.060	0.176	0.162	0.053	0.036	0.161	0.132
10%	20	0.239	0.263	0.291	0.372	0.076	0.204	0.157	0.086
	100	0.289	0.225	0.447	0.412	0.210	0.152	0.413	0.370

TABLE 2(b): ARCH(2), GARCH(1,2) TEST SIZES

		LBS Test			
Nominal Size	n	θ 0.0	0.8	1.0	1.02
		$\lambda = 0$			
1%	20	0.001	0.001	0.001	0.001
	100	0.009	0.009	0.008	0.008
10%	20	0.026	0.029	0.025	0.025
	100	0.067	0.069	0.068	0.066
		$\lambda = 10$			
		$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$			
1%	20	0.004	0.000	0.000	0.000
	100	0.010	0.010	0.010	0.011
10%	20	0.013	0.013	0.008	0.009
	100	0.063	0.062	0.061	0.063
		$x_{2t} = t$			
1%	20	0.003	0.011	0.005	0.003
	100	0.010	0.008	0.013	0.013
10%	20	0.034	0.062	0.032	0.030
	100	0.070	0.062	0.093	0.087
		$\lambda = 50$			
		$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$			
1%	20	0.000	0.000	0.000	0.000
	100	0.005	0.004	0.004	0.004
10%	20	0.003	0.000	0.000	0.000
	100	0.041	0.037	0.034	0.034
		$x_{2t} = t$			
1%	20	0.010	0.066	0.009	0.006
	100	0.080	0.044	0.265	0.208
10%	20	0.131	0.263	0.087	0.131
	100	0.331	0.248	0.608	0.527

TABLE 3(a): ARCH(1) REJECTION PROBABILITIES

(Not Size-Corrected)

Nominal Size = 1%; n = 100; θ = 0.8

α_1	$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$			$x_{2t} = t$	
	λ				
	0	10	50	10	50
			$z(nR^2)$ Test		
0	0.013	0.012	0.009	0.016	0.085
0.3	0.410	0.287	0.102	0.301	0.221
0.6	0.718	0.631	0.386	0.625	0.474
0.9	0.838	0.797	0.650	0.795	0.681
			nR^2 Test		
0	0.010	0.008	0.006	0.012	0.057
0.3	0.341	0.239	0.078	0.250	0.175
0.6	0.670	0.573	0.333	0.572	0.418
0.9	0.795	0.758	0.606	0.751	0.635
			z(LM) Test		
0	0.013	0.012	0.009	0.015	0.058
0.3	0.426	0.300	0.111	0.318	0.201
0.6	0.780	0.690	0.423	0.689	0.501
0.9	0.917	0.878	0.732	0.884	0.763
			LM Test		
0	0.009	0.009	0.005	0.008	0.039
0.3	0.370	0.256	0.090	0.266	0.160
0.6	0.738	0.645	0.382	0.641	0.449
0.9	0.899	0.855	0.696	0.856	0.725

TABLE 3(b): ARCH(1) REJECTION PROBABILITIES

(Not Size-Corrected)

Nominal Size = 10%; n = 100; $\theta = 0.8$

	$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$			$x_{2t} = t$	
	λ				
α_1	0	10	50	10	50
		$z(nR^2)$ Test			
0	0.080	0.082	0.068	0.094	0.301
0.3	0.679	0.553	0.285	0.566	0.494
0.6	0.905	0.844	0.615	0.848	0.717
0.9	0.963	0.937	0.835	0.941	0.868
		nR^2 Test			
0	0.079	0.079	0.069	0.082	0.214
0.3	0.588	0.455	0.215	0.472	0.395
0.6	0.853	0.780	0.532	0.781	0.632
0.9	0.928	0.894	0.771	0.901	0.811
		z(LM) Test			
0	0.073	0.075	0.064	0.086	0.255
0.3	0.681	0.557	0.282	0.567	0.467
0.6	0.920	0.859	0.631	0.863	0.728
0.9	0.978	0.957	0.863	0.959	0.893
		LM Test			
0	0.072	0.071	0.060	0.072	0.167
0.3	0.595	0.462	0.211	0.478	0.366
0.6	0.879	0.806	0.556	0.810	0.644
0.9	0.963	0.934	0.817	0.939	0.850

TABLE 4: ARCH(2) REJECTION PROBABILITIES

(Not Size-Corrected)

$n = 100;\ \theta = 0.8$

		$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$			$x_{2t} = t$	
		λ				
α_1	α_2	0	10	50	10	50
Nominal Size = 1%						
nR^2 Test						
0.0	0.0	0.009	0.010	0.008	0.012	0.060
0.2	0.2	0.306	0.221	0.090	0.234	0.174
0.2	0.4	0.530	0.433	0.248	0.445	0.320
0.2	0.6	0.690	0.618	0.439	0.626	0.500
LM Test						
0.0	0.0	0.010	0.010	0.006	0.011	0.036
0.2	0.2	0.337	0.251	0.105	0.261	0.169
0.2	0.4	0.590	0.494	0.291	0.504	0.346
0.2	0.6	0.778	0.718	0.520	0.716	0.566
LBS Test						
0.0	0.0	0.009	0.010	0.004	0.008	0.044
0.2	0.2	0.393	0.299	0.117	0.308	0.191
0.2	0.4	0.649	0.549	0.310	0.556	0.376
0.2	0.6	0.812	0.745	0.551	0.752	0.591
Nominal Size = 10%						
nR^2 Test						
0.0	0.0	0.081	0.076	0.062	0.080	0.225
0.2	0.2	0.538	0.436	0.234	0.448	0.391
0.2	0.4	0.762	0.670	0.434	0.682	0.553
0.2	0.6	0.866	0.818	0.643	0.822	0.706
LM Test						
0.0	0.0	0.072	0.065	0.049	0.068	0.152
0.2	0.2	0.556	0.446	0.235	0.458	0.359
0.2	0.4	0.785	0.697	0.449	0.707	0.544
0.2	0.6	0.897	0.849	0.681	0.855	0.736
LBS Test						
0.0	0.0	0.069	0.062	0.037	0.062	0.248
0.2	0.2	0.673	0.552	0.294	0.563	0.456
0.2	0.4	0.859	0.783	0.529	0.793	0.641
0.2	0.6	0.939	0.901	0.741	0.903	0.802

TABLE 5(a): GARCH(1,1) REJECTION PROBABILITIES

(Not Size-Corrected)

Nominal Size = 1%; n = 100; θ = 0.8

		$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$			$x_{2t} = t$	
		λ				
α_1	β_1	0	10	50	10	50
		$z(nR^2)$ Test				
0.0	0.0	0.013	0.012	0.009	0.016	0.085
0.2	0.0	0.233	0.159	0.054	0.169	0.158
0.2	0.2	0.242	0.183	0.073	0.188	0.147
0.2	0.4	0.253	0.210	0.105	0.212	0.156
0.2	0.6	0.272	0.243	0.166	0.243	0.188
		nR^2 Test				
0.0	0.0	0.010	0.008	0.006	0.012	0.057
0.2	0.0	0.189	0.122	0.038	0.130	0.121
0.2	0.2	0.197	0.146	0.052	0.148	0.112
0.2	0.4	0.202	0.167	0.081	0.169	0.128
0.2	0.6	0.221	0.197	0.133	0.200	0.151
		z(LM) Test				
0.0	0.0	0.013	0.012	0.009	0.015	0.058
0.2	0.0	0.245	0.166	0.053	0.172	0.132
0.2	0.2	0.251	0.189	0.075	0.197	0.134
0.2	0.4	0.268	0.216	0.112	0.221	0.152
0.2	0.6	0.292	0.266	0.182	0.262	0.198
		LM Test				
0.0	0.0	0.009	0.009	0.005	0.008	0.039
0.2	0.0	0.200	0.133	0.041	0.138	0.102
0.2	0.2	0.209	0.153	0.059	0.158	0.104
0.2	0.4	0.223	0.184	0.092	0.184	0.117
0.2	0.6	0.248	0.222	0.148	0.223	0.166

TABLE 5(b): GARCH(1,1) REJECTION PROBABILITIES

(Not Size-Corrected)

Nominal Size = 10%; n = 100; θ = 0.8

		$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$			$x_{2t} = t$	
		λ				
α_1	β_1	0	10	50	10	50
		$z(nR^2)$ Test				
0.0	0.0	0.080	0.082	0.068	0.094	0.301
0.2	0.0	0.520	0.396	0.189	0.418	0.417
0.2	0.2	0.526	0.425	0.227	0.438	0.389
0.2	0.4	0.531	0.459	0.291	0.459	0.387
0.2	0.6	0.546	0.504	0.392	0.499	0.424
		nR^2 Test				
0.0	0.0	0.079	0.079	0.069	0.082	0.214
0.2	0.0	0.418	0.303	0.133	0.319	0.318
0.2	0.2	0.423	0.335	0.168	0.343	0.290
0.2	0.4	0.425	0.368	0.216	0.370	0.316
0.2	0.6	0.437	0.409	0.303	0.403	0.327
		z(LM) Test				
0.0	0.0	0.073	0.075	0.064	0.086	0.255
0.2	0.0	0.515	0.394	0.182	0.415	0.382
0.2	0.2	0.518	0.425	0.224	0.434	0.363
0.2	0.4	0.523	0.461	0.287	0.456	0.370
0.2	0.6	0.550	0.507	0.393	0.507	0.417
		LM Test				
0.0	0.0	0.072	0.071	0.060	0.072	0.167
0.2	0.0	0.415	0.295	0.127	0.318	0.277
0.2	0.2	0.419	0.327	0.160	0.343	0.264
0.2	0.4	0.428	0.364	0.211	0.368	0.272
0.2	0.6	0.450	0.418	0.305	0.411	0.324

TABLE 6: GARCH(1,2) REJECTION PROBABILITIES

(Not Size-Corrected)

n = 100; θ = 0.8

			$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$			$x_{2t} = t$	
			λ				
α_1	α_2	β_1	0	10	50	10	50
			Nominal Size = 1%				
			nR^2 Test				
0.0	0.0	0.0	0.009	0.010	0.008	0.012	0.060
0.1	0.1	0.2	0.157	0.116	0.046	0.121	0.096
0.1	0.1	0.4	0.190	0.155	0.076	0.157	0.113
0.1	0.1	0.7	0.271	0.255	0.213	0.255	0.219
			LM Test				
0.0	0.0	0.0	0.010	0.010	0.006	0.011	0.036
0.1	0.1	0.2	0.176	0.132	0.052	0.137	0.095
0.1	0.1	0.4	0.212	0.181	0.094	0.179	0.124
0.1	0.1	0.7	0.331	0.321	0.270	0.316	0.270
			LBS Test				
0.0	0.0	0.0	0.009	0.010	0.004	0.008	0.044
0.1	0.1	0.2	0.216	0.160	0.061	0.159	0.106
0.1	0.1	0.4	0.263	0.213	0.108	0.214	0.138
0.1	0.1	0.7	0.400	0.379	0.322	0.381	0.321
			Nominal Size = 10%				
			nR^2 Test				
0.0	0.0	0.0	0.081	0.076	0.062	0.080	0.225
0.1	0.1	0.2	0.359	0.284	0.162	0.292	0.274
0.1	0.1	0.4	0.400	0.340	0.224	0.347	0.286
0.1	0.1	0.7	0.495	0.475	0.421	0.476	0.420
			LM Test				
0.0	0.0	0.0	0.072	0.065	0.049	0.068	0.152
0.1	0.1	0.2	0.356	0.281	0.154	0.295	0.243
0.1	0.1	0.4	0.402	0.345	0.219	0.348	0.270
0.1	0.1	0.7	0.523	0.505	0.440	0.513	0.445
			LBS Test				
0.0	0.0	0.0	0.069	0.062	0.037	0.062	0.248
0.1	0.1	0.2	0.464	0.374	0.192	0.379	0.330
0.1	0.1	0.4	0.511	0.444	0.278	0.444	0.347
0.1	0.1	0.7	0.644	0.622	0.554	0.626	0.552

TABLE 7(a): ARCH(1) POWERS

(Size-Corrected)

Size = 1%; n = 100; θ = 0.8

	$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$			$x_{2t} = t$	
	λ				
α_1	0	10	50	10	50
		$z(nR^2)$ Test			
0	0.010	0.009	0.007	0.012	0.067
0.3	0.370	0.260	0.088	0.268	0.194
0.6	0.690	0.597	0.354	0.597	0.440
0.9	0.814	0.774	0.623	0.769	0.654
		nR^2 Test			
0	0.010	0.008	0.006	0.012	0.058
0.3	0.345	0.241	0.079	0.252	0.177
0.6	0.672	0.575	0.335	0.574	0.420
0.9	0.796	0.760	0.608	0.752	0.637
		z(LM) Test			
0	0.010	0.010	0.006	0.010	0.045
0.3	0.391	0.274	0.098	0.283	0.174
0.6	0.753	0.663	0.394	0.659	0.468
0.9	0.907	0.864	0.711	0.867	0.740
		LM Test			
0	0.010	0.009	0.006	0.011	0.044
0.3	0.389	0.272	0.098	0.281	0.172
0.6	0.751	0.660	0.394	0.656	0.465
0.9	0.905	0.862	0.708	0.865	0.737

TABLE 7(b): ARCH(1) POWERS

(Size-Corrected)

Size = 10%; n = 100; θ = 0.8

	$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$			$x_{2t} = t$	
			λ		
α_1	0	10	50	10	50
			$z(nR^2)$ Test		
0	0.100	0.102	0.083	0.113	0.339
0.3	0.712	0.589	0.320	0.603	0.534
0.6	0.924	0.866	0.649	0.869	0.746
0.9	0.973	0.948	0.855	0.949	0.886
			nR^2 Test		
0	0.100	0.104	0.092	0.110	0.241
0.3	0.622	0.491	0.247	0.504	0.427
0.6	0.873	0.802	0.561	0.806	0.660
0.9	0.941	0.912	0.794	0.916	0.830
			z(LM) Test		
0	0.100	0.102	0.083	0.116	0.314
0.3	0.725	0.605	0.328	0.614	0.526
0.6	0.935	0.883	0.672	0.888	0.762
0.9	0.984	0.965	0.883	0.965	0.910
			LM Test		
0	0.100	0.103	0.090	0.106	0.204
0.3	0.634	0.506	0.247	0.517	0.409
0.6	0.899	0.833	0.589	0.836	0.680
0.9	0.970	0.945	0.838	0.951	0.868

TABLE 8: ARCH(2) POWERS

(Size-Corrected)

$n = 100;\ \theta = 0.8$

		$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$			$x_{2t} = t$	
		λ				
α_1	α_2	0	10	50	10	50
		Size = 1%				
		nR^2 Test				
0.0	0.0	0.010	0.012	0.009	0.013	0.066
0.2	0.2	0.320	0.235	0.098	0.248	0.185
0.2	0.4	0.544	0.449	0.261	0.462	0.335
0.2	0.6	0.701	0.635	0.456	0.638	0.514
		LM Test				
0.0	0.0	0.010	0.011	0.006	0.011	0.036
0.2	0.2	0.339	0.252	0.106	0.262	0.170
0.2	0.4	0.591	0.495	0.292	0.507	0.347
0.2	0.6	0.780	0.719	0.522	0.718	0.567
		LBS Test				
0.0	0.0	0.010	0.011	0.004	0.009	0.048
0.2	0.2	0.406	0.309	0.125	0.317	0.202
0.2	0.4	0.665	0.561	0.319	0.567	0.386
0.2	0.6	0.823	0.753	0.559	0.762	0.602
		Size = 10%				
		nR^2 Test				
0.0	0.0	0.100	0.095	0.077	0.097	0.250
0.2	0.2	0.567	0.462	0.259	0.477	0.423
0.2	0.4	0.779	0.690	0.456	0.705	0.581
0.2	0.6	0.877	0.834	0.663	0.840	0.729
		LM Test				
0.0	0.0	0.100	0.098	0.077	0.095	0.197
0.2	0.2	0.595	0.486	0.270	0.494	0.401
0.2	0.4	0.809	0.728	0.487	0.739	0.584
0.2	0.6	0.911	0.869	0.705	0.875	0.764
		LBS Test				
0.0	0.0	0.100	0.091	0.062	0.092	0.329
0.2	0.2	0.734	0.621	0.356	0.635	0.535
0.2	0.4	0.892	0.829	0.590	0.836	0.710
0.2	0.6	0.958	0.925	0.786	0.927	0.844

TABLE 9(a): GARCH(1,1) POWERS

(Size-Corrected)

Size = 1%; n = 100; θ = 0.8

		$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$			$x_{2t} = t$	
		λ				
α_1	β_1	0	10	50	10	50
		$z(nR^2)$ Test				
0.0	0.0	0.010	0.009	0.007	0.012	0.067
0.2	0.0	0.205	0.137	0.045	0.148	0.136
0.2	0.2	0.213	0.160	0.060	0.161	0.128
0.2	0.4	0.224	0.185	0.092	0.186	0.130
0.2	0.6	0.240	0.217	0.145	0.217	0.165
		nR^2 Test				
0.0	0.0	0.010	0.008	0.006	0.012	0.058
0.2	0.0	0.190	0.123	0.039	0.132	0.122
0.2	0.2	0.197	0.147	0.053	0.149	0.113
0.2	0.4	0.203	0.167	0.081	0.170	0.116
0.2	0.6	0.224	0.199	0.133	0.201	0.151
		z(LM) Test				
0.0	0.0	0.010	0.010	0.006	0.010	0.045
0.2	0.0	0.215	0.144	0.044	0.152	0.113
0.2	0.2	0.226	0.168	0.063	0.173	0.113
0.2	0.4	0.239	0.198	0.098	0.198	0.126
0.2	0.6	0.265	0.239	0.161	0.235	0.178
		LM Test				
0.0	0.0	0.010	0.009	0.006	0.011	0.044
0.2	0.0	0.214	0.142	0.044	0.150	0.111
0.2	0.2	0.224	0.165	0.063	0.170	0.111
0.2	0.4	0.237	0.195	0.097	0.195	0.125
0.2	0.6	0.263	0.236	0.159	0.234	0.176

TABLE 9(b): GARCH(1,1) POWERS

(Size-Corrected)

Size = 10%; n = 100; $\theta = 0.8$

		$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$			$x_{2t} = t$	
				λ		
α_1	β_1	0	10	50	10	50
			$z(nR^2)$ Test			
0.0	0.0	0.100	0.102	0.083	0.113	0.339
0.2	0.0	0.562	0.439	0.218	0.456	0.461
0.2	0.2	0.564	0.467	0.265	0.472	0.430
0.2	0.4	0.567	0.496	0.331	0.499	0.424
0.2	0.6	0.584	0.542	0.430	0.540	0.456
			nR^2 Test			
0.0	0.0	0.100	0.104	0.092	0.110	0.241
0.2	0.0	0.456	0.336	0.160	0.356	0.354
0.2	0.2	0.460	0.364	0.197	0.376	0.330
0.2	0.4	0.463	0.402	0.246	0.399	0.326
0.2	0.6	0.476	0.446	0.335	0.438	0.361
			z(LM) Test			
0.0	0.0	0.100	0.102	0.083	0.116	0.314
0.2	0.0	0.567	0.451	0.220	0.466	0.442
0.2	0.2	0.574	0.479	0.268	0.484	0.416
0.2	0.4	0.575	0.509	0.338	0.510	0.424
0.2	0.6	0.596	0.556	0.445	0.557	0.467
			LM Test			
0.0	0.0	0.100	0.103	0.090	0.106	0.204
0.2	0.0	0.460	0.338	0.161	0.360	0.323
0.2	0.2	0.466	0.367	0.196	0.381	0.312
0.2	0.4	0.476	0.412	0.247	0.409	0.320
0.2	0.6	0.490	0.457	0.351	0.454	0.366

TABLE 10: GARCH(1,2) POWERS

(Size-Corrected)

$n = 100;\ \theta = 0.8$

			$x_{2t} = 0.1x_{2t-1} + \varepsilon_t$			$x_{2t} = t$	
					λ		
α_1	α_2	β_1	0	10	50	10	50
			Size = 1%				
			nR^2 Test				
0.0	0.0	0.0	0.010	0.012	0.009	0.013	0.066
0.1	0.1	0.2	0.168	0.123	0.050	0.128	0.106
0.1	0.1	0.4	0.200	0.165	0.083	0.166	0.122
0.1	0.1	0.7	0.287	0.270	0.226	0.269	0.231
			LM Test				
0.0	0.0	0.0	0.010	0.010	0.006	0.011	0.036
0.1	0.1	0.2	0.178	0.133	0.052	0.138	0.095
0.1	0.1	0.4	0.214	0.181	0.095	0.180	0.125
0.1	0.1	0.7	0.331	0.323	0.272	0.317	0.271
			LBS Test				
0.0	0.0	0.0	0.010	0.011	0.004	0.009	0.048
0.1	0.1	0.2	0.227	0.170	0.064	0.166	0.111
0.1	0.1	0.4	0.275	0.222	0.114	0.225	0.146
0.1	0.1	0.7	0.412	0.392	0.333	0.392	0.330
			Size = 10%				
			nR^2 Test				
0.0	0.0	0.0	0.100	0.095	0.077	0.097	0.250
0.1	0.1	0.2	0.384	0.314	0.181	0.319	0.300
0.1	0.1	0.4	0.423	0.370	0.244	0.373	0.311
0.1	0.1	0.7	0.517	0.503	0.445	0.499	0.446
			LM Test				
0.0	0.0	0.0	0.100	0.098	0.077	0.095	0.197
0.1	0.1	0.2	0.397	0.326	0.185	0.331	0.283
0.1	0.1	0.4	0.441	0.384	0.254	0.384	0.307
0.1	0.1	0.7	0.562	0.547	0.475	0.545	0.481
			LBS Test				
0.0	0.0	0.0	0.100	0.091	0.062	0.092	0.329
0.1	0.1	0.2	0.538	0.443	0.247	0.446	0.406
0.1	0.1	0.4	0.580	0.513	0.342	0.518	0.419
0.1	0.1	0.7	0.703	0.681	0.613	0.687	0.620

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