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FACTOR DEMAND FUNCTIONS FOR
CONSTANT RETURNS TO SCALE TECHNOLOGIES

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A B S T R A C T

If the production function is homogeneous of degree one, factor demand functions cannot be obtained from the solution of a profit maximization problem. Application of Shepard's Lemma to a cost function yields factor demand functions, but these will not capture output effects.

This paper presents a simple way to construct factor demand functions that capture output and substitution effects for the case of constant returns to scale. The properties of these functions are investigated, and the results are contrasted with those obtained using the constant-output type functions.

Specific functional forms for the new functions are derived from assumed functional forms of the cost function. The obtained forms can then be estimated directly using the appropriate econometric techniques.

"FACTOR DEMAND FUNCTIONS FOR CONSTANT RETURNS TO SCALE TECHNOLOGIES"

I. Introduction

Production functions which exhibit constant returns to scale play an important role in the pure theory of international trade, the theory of growth, and general equilibrium theory. Nevertheless, when constant returns to scale prevail, it is not possible to derive factor demand functions through the usual analysis of profit maximization. Instead, when the production function is homogeneous of degree one, factor demand functions are derived from a cost minimization problem via the applications of Shepard's Lemma. The functions thus obtained, however, can only capture substitution effects triggered by changes in factor prices since the level of output is, by construction, held constant.

It can be argued, therefore, that these functions are incomplete as, in general, when factor prices change the same will happen to product price and there will be a corresponding output adjustment that will, in turn, affect the demand for factors.

In this paper we develop an alternative methodology which allows one to construct factor demand functions that capture both 'output' as well as 'substitution' effects. The functions thus obtained can then be used to derive the complete (i.e., inclusive of output adjutment) comparative statics generated by changes in factor prices.

The conclusions obtained from the 'complete' factor demand functions can, under certain circumstances, be qualitative different from those derived using the constant output type functions. This, in turn, is important for policy analysis which try to predict the effects of the change in the price of the i^{th} factor on the use of the j^{th} factor.

The paper is organized as follows: In section II we state the problem, show why factor demand functions cannot be derived from a profit maximiztion problem, and discuss briefly the solutions that have been offered. Section III shows how complete factor demand functions can be constructed under certain assumptions. Section IV discusses how our anlysis is related to results presented by other authors. Section V develops functional forms for the complete factor demand functions, given specific functional forms for the cost function and the demand function for the final product.

Section VI contains a summary of our findings. Finally, a brief appendix shows how our work is related to results previously obtained by Hicks (Hicks, 6, ch. 9, p. 244).

II. Statement of the Problem

Let q denote output level, $x = (x_1, \ldots, x_n)$ a non-negative vector of services derived from the use of factors, $w = (w_1, \ldots, w_n)$ a strictly positive vector of factor prices, π the level of profits and p the price of product q. Finally, let f(x) be the firm's production function.[1] Under perfect competition the problem is to:

$$\max_{x} \pi(w,p) = p \cdot f(x) - wx \tag{1}$$

First order conditions are:

[1] It will be assumed that f is concave and twice differentiable.

$$p \cdot \nabla_x \cdot f(x) = w \qquad (2)$$

where $\nabla_x \cdot f(x)$ is the gradient vector of first partial derivatives of of f.

System (2) consists of n equations in n unknowns. If certain conditions are met, one can use the implicit function theorem so as to obtain (Intriligator, 1, pp. 86-67):

$$x = x\ (w,p) \qquad (3)$$

where (3) gives us the n factor demand functions consistent with the profit maximization hypothesis.

The existence of the n functions postulated in (3) requires that the endogenous variable jacobian determinant of (2) be different from zero. That is, it is required that:

$$\det p \cdot \frac{\partial^2 f}{\partial x^2} = p \cdot |H| \neq 0$$

where H is the Hessian matrix of the function f(x).

If, however, f(x) is homogeneous of the first degree, then it is well known that its Hessian matrix will not be of full rank, and condition (4) will not be fulfilled (Samuelson, 2, Ch. 4, p. 78). We conclude, therefore, that under constant returns to scale problem (1) cannot be solved, and hence it will not be possible to obtain factor demand functions (3).

The economic rationale behind this result is stated clearly by Samuelson: "Unit costs being constant, and demand being horizontal, there are only three possibilities: price being everywhere greater than marginal costs, it will pay the firm to expand indefinitely, i.e., until competition ceases to be pure; or price is less than marginal cost, no output will be produced; or, finally, if price is identically equal to marginal cost, the exact output of the firm will be a matter of indifference." (Samuelson, 2, Ch. 4, p. 78).

There have been various alternative approaches to solve this problem. One is to maximize a restricted profit function:[1]

$$\text{Max } \pi(w,p,q_o) = p.f(x)-wx \qquad (5)$$
$$\text{s.t. } f(x) = q_o$$

where q_o is an exogenously specified level of output.

Another approach, exactly equivalent to (5), is to minimize the cost of a given level of output:

$$\text{Min } C(w,q_o) = wx \qquad (6)$$
$$\text{s.t. } f(x) = q_o$$

In either case, the added constraint has the effect of making definite the level of output, and hence eliminating the indeterminacy that was mentioned before. Mathematically, the Hessian matrix of the

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[1] Sometimes the restricted profit function takes the form: $\pi(w,p;\bar{X}_1) = p.f(X_2,\ldots,X_n|\bar{X}_1) - \sum_{j=2}^{n} w_j x_j - w_1\bar{X}_1$ i.e., the quantity of a factor is exogenously fixed. Strictly, this should be called a "short-run" profit function. In this case the degree of homogeneity of f(x) is irrelevant.

Lagrangean obtained from solving problems (5) or (6) will be of full rank, and it will be possible to use the implicit function theorem to obtain the factor demand functions of the form:

$$x = x\ (w,q_o) \tag{7}$$

A more recent approach, based on the results of duality theory, consists in constructing the cost function by solving (6) to obtain:

$$C^* = C^*(w,q_o) = w.x^* \tag{8}$$

where x^* is the solution vector to problem (6). Once the cost function is constructed, Shepard's Lemma can be used to obtain:[1]

$$\nabla wC^*(w,q_o) = x(w,q_o) \tag{9}$$

where $\nabla wC^*(w,q_o)$ is the gradient vector of first partial derivates of C^* with respect to w.

Whichever approach is followed, either through the constrained maximization or minimization problems or via Shepard's Lemma, one always obtains factor demand functions where output is held constant. The problem with these approaches, however, is that the assumption of constant output is not consistent with profit maximization as, in general, output level will not be exogenous but will be chosen by firms so as to maximize profits. As a consequence, price elasticities derived from (7) and/or

[1] One need not solve problem (6) to obtain x^* and construct C^*. It is also possible to specify a specific functional form for C^* and, as long as it satisfies certain regularity conditions, one is assured that the obtained functions $x(w,q_o)$ do come from some underlying production function. (Diewert, 3, p. 546).

(9) will not be the correct measure of response of a profit maximizing firm to a change in factor prices.

III. Constructing Complete Factor Demand Functions

As other authors have proved (Diewert, 3, p. 551) if f(x) is homogeneous of degree one then the associated cost function, defined as the solution to problem (6), will take the form:

$$C = C(w,q_o) = q.A(w) \tag{10}$$

For (10) to classify as a cost function, however, certain conditions must be satisfied. In particular, the function A(w) must be concave as well as homogeneous of degree one in w.

Although costs functions have been extensively analyzed (Diewert, 3, pp. 538-539), it is useful, given our purposes, to recall two important properties.

First, Uzawa (4) has proven that the partial (Allen) elasticity of substitution between two inputs can be written:

$$v_{ij} = C.C_{ij}/C_i.C_j \tag{11}$$

where subscripts indicate partial differentiation with respecto to the i^{th} factor price.

Second, and again assuming that C is differentiable, Shepard's Lemma implies that the cost minimizing constant output factor demand functions are given by:

$$C_i = x_i(w,q_o) \tag{12}$$

Since $C(w,q_o)$ takes the special form (10) under constant returns to scale, one can re-write the elasticity of substitution as:

$$v_{ij} = \frac{q.A(w).q.A_{ij}}{q.A_i.q.A_j} = \frac{A(w)A_{ij}}{A_iA_j} \tag{13}$$

Given that $A(w)$ is concave, A_i and A_j are both positive, while A_{ii} is negative (evidently, $A(w)$ is positive). However, the sign of A_{ij} $(i \neq j)$ cannot be determined a priori. Note that as:

$$A_{ij} \gtrless 0 \rightarrow v_{ij} \gtrless 0 \text{ and factors i and j are } \left\{\begin{matrix}\text{substitutes}\\ \text{complements}\end{matrix}\right\} \tag{14}$$

On the other hand, constant output factor demand functions take the simple form:

$$x_i(w,q_o) = q.A_i \tag{15}$$

Lastly, it is useful to note that the price elasticity of function (15) is given by:

$$\varepsilon_{ij} = \frac{\partial x_i}{\partial w_j} \cdot \frac{w_j}{x_i} = \frac{q.A_{ij}.w_j}{q.A_i} = A_{ij}w_j/A_i \tag{16}$$

These elasticities can be expressed in terms of the Allen elasticities of substitution if we combine (16) and (13):

$$\varepsilon_{ij} = \frac{A_{ij}w_j}{A_i} \cdot \frac{A(w)A_{ij}}{A_iA_j} \cdot \frac{A_iA_j}{A(w)A_{ij}} = v_{ij}\,\frac{w_jA_j}{A(w)} \tag{17}$$

Since, however, under constant returns to scale price will equal average cost:

$$\frac{C}{q} = p = \frac{q.A(w)}{q} = A(w) \tag{18}$$

Substituting (18) into (17) and noting that $A_j = x_j/q$ (from (15)), we obtain:

$$\varepsilon_{ij} = v_{ij}\, w_j\, x_j/p.q = v_{ij}\, s_j \tag{19}$$

where s_j is the share of factor j in total cost.

Result (19) is well known (Allen, 5, pp. 503-509). It is important to emphasize, nevertheless, that these ε_{ij}'s do not capture the full response of a firm to a change in factor prices, as output adjustment is excluded.

We define the price elasticity of demand for the final product as:

$$\eta = \frac{dq}{dp} \cdot \frac{p}{q} \leq 0 \tag{20}$$

Assuming η is a constant, (20) implies that the demand function for the final product takes the simple form:

$$q = g(p) = \gamma p^{\eta} \quad ; \quad \gamma > 0 \tag{21}$$

Using (18) it is possible to express the demand for the final product as a function of factor prices only, that is:

$$q = \gamma\{A(w)\}^{\eta} \tag{22}$$

Under constant returns to scale, the size of each firm within an industry is undefined (Samuelson, 2, Ch. 4, p. 79). It is possible, therefore, to model our problem 'as if' there was only one firm in that industry although, given constant returns to scale and free entry, this firm cannot exercise any monopoly power. Under these conditions, however, the level of output cannot be arbitrarily set, but must necessarily coincide with the level of output that, at the given factor prices, the market will demand.

The construction of factor demand functions which include output and substitution effects is built upon this simple idea.[1] Equation (15) expresses the demand for a factor as a function of factor prices and an arbitrarily given level of output. We can, as a consequence, make this arbitrarily given level of output equal to the equilibrium level of output, as expressed in (22). By doing so we obtain factor demands that depend only on factor prices. Substituting (22) in (15) we get:

$$\hat{x}_i = \hat{x}_i(w) = \gamma\{A(w)\}^{\eta} A_i \tag{23}$$

We will call $\hat{x}_i$ a 'complete' factor demand function. It is clear that the comparative statics derived from (23), as opposed to those obtained from (15), will have already endogenized the response of changes in output to changes in factor prices.

More concretely, functions $\hat{x}_i$ give us factor demands which insure that the level of output produced will be equal to output demanded, at the given factor prices. Alternatively, functions $\hat{x}_i$ - as opposed, once again, to x_i in (15) - take into account information not only about technology and factor prices, but also about demand for the final product. Thus, the results derived using functions $\hat{x}_i$ will be more consistent with the behavior of the profit maximizing firm, than those obtained using

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[1] No claims to originality are made. We simply want to explore the consequences of carrying out this approach. In particular, functions similar to (23) - see below - are briefly mentioned by Samuelson, although they are not anlayzed in detail (Samuelson, 2, Ch. 4, pp. 76-77).

functions x_i.

We turn to analyze the properties of (23). First, note that since $A(w)$ is homogeneous of degree one in w, $A_i(w)$ will be homogeneous of degree zero, such that:

$$\lambda^{\eta}.\hat{x}_i = \gamma\{A(\lambda w)\}^{\eta} A_i(\lambda w), \text{ for } \lambda > 0 \tag{24}$$

that is, functions $\hat{x}_i$ are homogeneous of degree η.

This result is consistent with economic reasoning. If all factor prices increase by λ, the same will happen to product price. A λ increase in product price, however, causes a λ^{η} decrease in output demanded, which in turn decreases demand for each factor by the same proportion.

Second, note that:

$$\frac{\partial \hat{x}_i}{\partial w_i} = \gamma\eta\{A(w)\}^{\eta-1} A_i^{\;2} + A_{ii}\gamma\{A(w)\}^{\eta} \tag{25}$$

which is unambiguously negative, as $\eta \leq 0$ and $A_{ii} < 0$ by the concavity of $A(w)$.

Third, the price elasticity of $\hat{x}_i$ is given by:

$$\hat{\varepsilon}ij = \frac{\partial \hat{x}_i}{\partial w_j} \frac{w_j}{\hat{x}_i} = \left(\gamma\, \eta\, A(w)\right\}^{\eta-1} A_j A_i + \gamma A_{ij}\{A(w)\}^{\eta}\Big\} w_j / \gamma\, \{\, A(w)\}^{\eta}\; A_i$$

$$= \eta.w_j A_j / A(w) \; + w_j A_{ij} / A_i$$

Making use of (16), however:

$$\hat{\varepsilon}_{ij} = \eta.w_j A_j / A(w) + \varepsilon_{ij} \tag{26}$$

Therefore, the price elasticity of the 'complete' factor demand function is equal to the price elasticity of the constant output factor demand function plus another term (which is always non-positive) depending on the price elasticity of demand for the final product. It is clear that:[1]

$$|\hat{\varepsilon}_{ii}| > |\varepsilon_{ii}| \quad \text{if} \quad |\eta| > 0$$

$$\hat{\varepsilon}_{ij} = \varepsilon_{ij} \quad \text{only if} \quad \eta = 0 \tag{27}$$

which is what one would expect.

If we note that (see (15) and (18)) $\eta w_j A_j/A(w) = \eta . w_j x_j/p.q = \eta s_j$ and recall that $\varepsilon_{ij} = v_{ij}s_j$, (by (19)), then expression (26) can be re-written as:

$$\hat{\varepsilon}_{ij} = \eta s_j + s_j v_{ij} \tag{28}$$

We can call the first term on the R.H.S. of (28) the 'output' effect, and the second term the 'substitution' effect.[2]

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[1] If one is willing to call $\hat{x}_i$ and x_i the 'long run' and 'short run' factor demand functions, respectively, the results (27) are simply a consequence of the well known 'Le Chatelier' effects.

[2] The decomposition of the price elasticity of demand for a factor into an 'output' and a 'substitution' effect can be found in Hicks (6). A brief discussion of this is presented as an appendix.

Formula (28) is of some importance. Studies of demand for factors that are based on Shepard's Lemma (e.g. Berndt and Wood (7)) can serve only to calculate the price elasticities defined in (19), or what we have called the 'substitution' effect. These elasticities, however, can give incorrect results as to the effects of a change in the price of the i^{th} factor on the use of the j^{th} factor. In particular, if factors i and j are substitutes, then we know from (14) that $v_{ij} > 0$ such that $\varepsilon_{ij} > 0$. However, as (28) indicates, this does not imply that $\hat{\varepsilon}_{ij} > 0$.

In fact, directly from (28) it can be stated that factors i and j are substitutes, then:

$$\hat{\varepsilon}_{ij} \lesseqgtr 0 \quad \text{as} \quad |\eta| \gtreqless |v_{ij}| \tag{29}$$

Of course, if factors i and j are complementary (or if i=j) then ε_{ij} and $\hat{\varepsilon}_{ij}$ will have the same sign but ε_{ij} will underestimate the value of the price elasticity (as long as $\eta < 0$).

IV. An Important Clarification

The inclusion of shifts in isoquants as one of the determinants of price elasticities of factors has been mentioned by Halvorsen (8, p. 387) and formally modelled by Berndt and Wood (B-W, 9, pp. 343-346). These analyses, however, are fundamentally different from the one presented in section III, as the output effects discussed by these authors are in the context of a partition of the input set and refer to shifts of the isoquants of the respective "subfunctions" that produce each aggregate input.

Following B-W(9, p. 343), partition the set of n inputs into r mutually exclusive and exhaustive subsets, $N_1, \ldots\ldots, N_r$, a partition denoted R. Then, if the production function $q = f(x_1, \ldots\ldots, x_n)$ is weakly separable with respect to the partition R it can be re-written as $q = f^*(X^1, \ldots\ldots, X^r)$, where X^m is a positive strictly quasi-concave homothetic production subfunction of only the elements within N_m, i.e., $X^m = f_m(x_i)$ $i \varepsilon N_m$, $m = 1, 2, \ldots\ldots, r$. Function f* is now called the "master" production function, while the r f_m functions are the production "sub-functions." Thus, production is now envisioned as a two-step process. First, producers chose optimal quantities of inputs wihin each subfunction and obtain an aggregated input X^m. Second, the r aggregated inputs are combined to produce the final output q.

Within this context, B-W define the gross price elasticity of demand for a factor, ε_{ij}^*, as $\partial \ln x_i / \partial \ln p_j$ $(i, j \varepsilon N_m)$, where the output of the production sub-function, X^m, is held constant. On the other hand, the net price elasticity of demand, ε_{ij}, is defined as $\partial \ln x_i / \partial \ln p_j$ $(i, j \varepsilon N_m)$ where the "master" output level is held constant (say at $q = \bar{q}$), while the level of output of the production sub-function X^m is allowed to vary. Assuming the f_m functions to be linearly homogeneous, these two elasticities are related by:

$$\varepsilon_{ij} = \varepsilon_{ij}^* + s_{jm}\varepsilon_{mm} \qquad (i, j \varepsilon N_m) \tag{30}$$

where s_{jm} is the cost share of the j^{th} input in total cost of producing X^m, and ε_{mm} is the own price elasticity of demand for X^m along a q ("master") isoquant. B-W call the term $s_{jm}\ \varepsilon_{mm}$ the "expansion elasticity."

It is clear, nevertheless, that these "expansion elasticities" refer to shifts of the isoquants of the sub-functions that produce X^m and not to shifts of the isoquant of the "master" production function. Therefore, the term ε_{ij} in equation (30) captures output effects only within the aggregated inputs but not with respect to final output as, by construction, this term is defined along a fixed "master" isoquant. Shifts of the master isoquant must be related, as we argued before, to the price elasticity of demand for the final product, η, an argument not included in (30).

Thus, the net price elasticity of demand, ε_{ij}, corresponds to what we have called the "substitution" effect measured by the second term of (28). The analysis of Halvorsen and B-W consists of further decomposing this term into a gross price elasticity and an expansion elasticity within a given partition of the input set.[1] It follows, therefore, that if one desires to introduce the said partition, it is possible to combine their results with ours. Substituting (30) into (28) and noting that (by (19)) $\varepsilon_{ij} = s_j\ v_{ij}$ we obtain:

$$\hat{\varepsilon}_{ij} = \eta s_j + \varepsilon_{ij}^* + s_{jm}\ \varepsilon_{mm} \qquad (i, j \in N_m) \tag{31}$$

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[1]From the B-W discussion it appears that the distinction between net and gross price elasticities only applies when i and j belong to the same input subset. On the other hand, the term ε_{ij} in (16) is defined regardless of any partition of the input set.

Following our previous discussion, the first term of (31) is the output effect (which is always non-positive), while the last two terms measure the substitution effect which can be positive or negative depending on whether inputs inside each production subfunction f_m are substitutes or complements ($\varepsilon_{ij}^* \gtrless 0$) and on the size of the expansion elasticity ($s_{jm}\ \varepsilon_{mm}$).[1] It nevertheless remains true that factors i and j can be net and gross substitutes ($\varepsilon_{ij} > 0$, $\varepsilon_{ij}^* > 0$) but still complements as measured by $\hat{\varepsilon}_{ij}$.[2] It is this last elasticity, however, that measures the total effect of a change in a factor price and is thus the once that should be used for policy analysis.

Of course, if one does not wish to introduce any partitions within the input set, the decomposition of the substitution effect presented in (30) is not necessary, and one is left with expression (28) as the measure of the complete price elasticity of demand for a factor. This is the approach that will be followed in the remainder of this paper.

V. Functional Forms for Econometric Estimation

As is well established now from the results of duality theory, one need not estimate a production function to know the parameters that describe the technology. Instead, one can postulate a specific functional form for a cost function, and as long as it satifies certain

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[1]As ε_{mm} is negative by the strict quasi-concavity of the f_m function, and s_{jm} is evidently positive, this expansion elasticity will always be negative.

[2]Of course, other possibilites are possible, depending on the size and signs of the various elasticities.

regularity conditions, obtain the desired information by estimating directly the given cost function.

Following the discussion of section III, this approach can be extended to obtain functional forms for the complete factor demand functions $\hat{x}_i$, and thus circumvent the indeterminacy problem mentioned in section II. Again, one needs to specify a specific functional form for the cost function. In this case, however, it is also required to specify a specific form for the final product demand function.

We will present some specific examples of this procedure, assuming the demand function for the final product takes the simple form stated in (21).[1]

Example # 1: The Cobb-Douglas Cost Function.

$$C = q.A(w) = q.\prod_{i=1}^{n} w_i^{\alpha_i}; \qquad \alpha_i > 0 \quad \forall_i; \qquad \sum_{i=1}^{n} \alpha_i = 1$$

then

$$\hat{x}_i = \gamma\{\prod_{i=1}^{n} w_i^{\alpha_i}\}^{\eta} . \alpha_i w_i^{\alpha_i - 1} \prod_{\substack{j=1 \\ j\neq i}}^{n-1} w_j^{\alpha_j} \tag{32}$$

Example # 2: The C.E. S. Cost Function.

$$C = q.A(w) = q.\{(\sum_{i=1}^{n} \delta_i w_i^{e})^{\frac{1}{1+e}}\}^{\frac{1+e}{e}}; \; e > -1; \; \delta_i > 0, \; \forall_i; \; \sum_{i=1}^{n} \delta_i = 1$$

then

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[1] This assumption is not required. In general we can write, following (21), $\hat{x}_i = g\{A(w)\}.A_i$. Of course, one needs then to postulate an alternative form for g.

$$\hat{x}_i = \gamma\left(\{(\sum_{i=1}^{n} \delta_i w_i^e)^{\frac{1}{1+e}}\}^{\frac{1+e}{e}}\right)^{\eta}.\{(\sum_{i=1}^{n} \delta_i w_i^e)^{\frac{1}{1+e}}\}^{\frac{e}{1+e}-1} \delta_i^{\frac{1}{1+e}}$$

$$w_i^{\frac{e}{1+e}-1} \tag{33}$$

Example # 3: The Generalized Leontief Cost Function.

$$C = q.A(w) = q.\sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} w_i^{1/2} w_j^{1/2} \quad ; b_{ij} \geq 0 \qquad \forall_{i,j}; \quad b_{ji} = b_{ij}$$

then

$$\hat{x}_i = \gamma\{\sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} w_i^{1/2} w_j^{1/2}\}^{\eta}. \sum_{j=1}^{n} b_{ij} (w_j/w_i)^{1/2} \tag{34}$$

Functions (32), (33), or (34) can be estimated with the appropriate econometric techniques. Estimating (32) is, of course, much simpler than estimating (33) or (34) as it is a log-linear form, whereas (33) and (34) require non-linear estimating methods. On the other hand, (34) has more 'desirable' properties, as it does not restrict a priori the values of the Allen elasticities of substitution.[1]

VI. Summary and Conclusions

The main result of this paper can be stated as follows: Application of Shepard's Lemma to a cost function yields constant output factor demand functions. However, by making the exogenously specified level of output coincide with the equilibrium level of output, one can extend the

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[1]Whereas $v_{ij}=1 \ \forall_{i,j}$ in the case of (32) and $v_{ij}= \frac{1}{1+e} \ \forall_{i,j}$ in the case of (33).

factor demand functions to include output effects. Given the demand function for the final product one can make this equilibrium quantity a function of product price. Furthermore, since under constant returns to scale product price is a function of factor prices only, it turns out that the only parameters that enter the complete factor demand functions are factor prices.

On the other hand, by assuming functional forms for the cost function, one can construct functional forms for the complete factor demand equations. These equations can then be estimated with the appropriate techniques to yield complete values for all price elasticities.

It remains to be seen whether the numerical values of the various elasticities obtained from estimating the complete functions differ from those obtained estimating the constant output type functions. With that information it will be possible to assess whether the inclusion of 'output' effects yields statistically different results that, in turn, will lead to a change in policy recommendations.

A P P E N D I X

The elasticity formula (28) was, in a different context, previously developed by Hicks. In the Appendix to Chapter 11 of his "Theory of Wages" he derives - for the two factor case - a formula for the (direct) price elasticity of demand for a factor (Hicks, 6, pp. 244-245). In our notation:

$$\hat{\varepsilon}_{11}\Big|_{\text{Hicks}} = \frac{v_{12}(\eta+e) + es_1(\eta - v_{12})}{v+e - s_1(\eta - v_{12})} \qquad \text{(A.1)}$$

where e is the price elasticity of supply of factor 2. Note, however, that Hicks defined both $\hat{\varepsilon}_{11}$ and η to be positive.

The conditions stated in the text imply that $e = \infty$, hence

$$\lim_{e \to \infty} \hat{\varepsilon}_{11}\Big|_{\text{Hicks}} = \eta s_1 + (1-s_1)v_{12} \qquad ; \qquad \hat{\varepsilon}_{11} > 0, \quad \eta \geq 0$$

To make the Hicksian definition coincide with ours, we multiply this expression by (-1) to obtain:

$$\hat{\varepsilon}_{11}\Big|_{\text{Hicks}} = \eta s_1 - (1-s_1)v_{12} \qquad \text{(A.2)}$$

with $\eta \leq 0$ and $\hat{\varepsilon}_{11} < 0$.

On the other hand, (28) implies that, for the case of i=j:

$$\hat{\varepsilon}_{11} = \eta s_1 + s_1 v_{11} \qquad \text{(A.3)}$$

All we need to show, therefore, is that $s_1 v_{11} = -(1-s_1)v_{12}$.

First, note that since (15) is homogeneous of degree zero in w we can use Euler's Theorem to obtain

$$0 . x_1 = \frac{\partial x_1}{\partial w_1} \cdot w_1 + \frac{\partial x_1}{\partial w_2} w_2$$

from which $\varepsilon_{11} + \varepsilon_{12} = 0$

Second, note from (19) that $\varepsilon_{ij} = s_j v_{ij}$, such that $\varepsilon_{11} = s_1 v_{11}$ and $\varepsilon_{12} = s_2 v_{12} = (1-s_1)v_{12}$.

Since $\varepsilon_{11} = - \varepsilon_{12}$, a simple substitution shows that $s_1 v_{11} = -(1-s_1) v_{12}$ such that (A.2) = (A.3).

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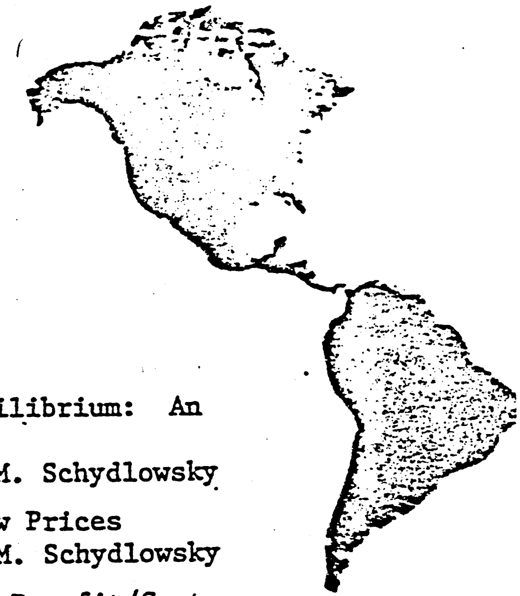

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