Will there be a Concentration of Alikes?
The Impact of Labor Market Structure on Industry Mix in the Presence of Product Market Shocks

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Will there be a Concentration of Alikes?  
The Impact of Labor Market Structure on Industry Mix in the Presence of Product Market Shocks

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Abstract

We analyze determinants of regional industry mix and focus especially on the influence of labor market characteristics. By combining a labor market pooling argument with an argument involving the cost of switching a worker from one firm to another, we show that in the presence of product market shocks there exists an interesting trade-off for the concentration of firms of the same industry in one region. Firms belonging to different industries are hedged against industry-specific shocks if they settle in the same region, but face higher switching costs (retraining costs for workers moving from one firm to another). In addition, with a given supply of labor there is an additional rationing effect affecting the location decisions of firms. Against the background of these trade-offs we analyze the resulting location decisions of firms in a two-regions-two-industry-four-firm framework. We analyze the impact of different parameters on the location choice of firms.

JEL Classification: J4, L1, R3
Keywords: local labor market, product market shocks, industry structure, imperfect competition
1 Introduction

Industries as well as industry-related employment are concentrated in space, and differences in industrial structure among local labor markets are substantial. These features are amply demonstrated in the literature, e.g. by Topel (1986) or Neumann and Topel (1991). These authors argue forcefully that the structure of local labor markets is an important determinant of regional economic activity, and vice versa.

The purpose of this paper is to analyze this relationship in a world of demand uncertainty, and their consequences on factor demand. More specifically, we ask for the circumstances under which a firm prefers to locate close to firms of the same industry, vs. firms of different industries. Firms face stochastic product demand, leading to uncertain labor demand. The firms’ location decisions serve as a mechanism to deal with the problems (and potential advantages) of uncertainty in a profit-maximizing manner.

In such an environment firms face a principle trade-off when deciding on their location. If partaking in the same labor market with firms of the same industry, the firms are hedged against firm-specific shocks, as long as these shocks are perfectly negatively correlated. Yet problems of wage increases or labor supply rationing may arise if these firms are subject to the same positive industrywide shock.

Conversely, if partaking in the same labor market with firms from other industries, the firms are hedged additionally against industry-specific shocks, as long as these shocks are perfectly negatively correlated. Yet retraining costs for workers employed in another industry are larger than for workers of the same industry. Thus they face higher costs of attracting workers from other firms in the same region. Against this background, we analyze the determinants of the regional industry mix and the impact of local labor market characteristics.

The hedging argument is not new. Our approach builds upon and extends the labor market pooling argument due to Marshall (1920). It was used again by David and Rosenbloom (1990) and especially Krugman (1991) to explain the agglomeration of
workers and firms in a single region. According to this argument, agglomerations depict an efficient manner to cope with fluctuating labor demand since it saves migration costs. Whereas the David and Rosenbloom (1990) analysis lacks a microeconomic underpinning, Krugman’s (1991) work popularized the labor market pooling argument by presenting a model with a rather simple microeconomic structure. He argues that firm-specific shocks provide a strong reason for (complete) regional concentration of economic activity.

We use this basic argument and extend it in various dimensions. We distinguish between firm-, and industry-specific shocks. This allows us to analyze the hedging effect in various combinations of firms from the same, and from different industries. In contrast to Krugman (1991), we allow for the possibility that attracting workers from other firms may not be costless.\(^1\) Switching, or retraining costs may arise. They increase in the skill differences demanded by the different firms.

In fact, the level of these retraining costs, together with the relative importance of firm-to-industry-specific demand shocks doubly indicates the degree of differentiation between firms in our local economies. We argue that the more important, in relative terms, firm-specific shocks, the more firms differ in their output. Similarly, the more pronounced the switching costs between firms, the more firms differ in the labor market.

Our model is also related to other theoretical approaches to the analysis of localized labor markets. In a series of papers, Kim (1989, 1990, 1991) analyzes the potential benefits of larger local labor markets when both firms and workers are heterogeneous. In his model, firms’ choice of technology is represented by their location in a space of product variants. The technological alternatives are located along the circumference of a Salop circle, along which workers are uniformly distributed by ability. Among other things, he shows that competition for workers becomes more intensive in larger regions and that workers have an incentive to invest more (less) in specific (general) human

\(^1\)Alternatively speaking, Krugman’s model may be interpreted as involving only firm specific demand shocks.
capital. Wages turn out to be higher in the larger local labor market.\(^2\) Helsley and Strange (1990) adopt a similar setting for two regions. They argue that mobile workers can expect a better match when migrating to the larger market. Thisse and Zhenou (1995), using a related set up, address a different issue. They ask for the socially optimal level of human capital formation costs, and its division between individual workers and the government. They also show that workers tend to be better off in larger labor markets, experiencing a higher wage. All these studies, however, consider symmetric firms and leave aside the issue of the resulting regional industry mix, as well as the effect of local labor market characteristics on this mix.

More specifically, we develop a model with two regions and two industries. In each industry there are two firms deciding upon their location as well as their output quantities, in the face of both, firm- and industry-specific demand shocks. Firms are price- and wage-takers, with prices and wages exogenously given. Both regions are populated by the same number of ex-ante identical workers. In the absence of shocks, full employment prevails as long as the firms are symmetrically distributed across the regions. There are three relevant periods in time. In the first period, firms decide upon their location. In the second period, nature decides about the demand shocks. In the final period, firms choose their profit maximizing output levels, by taking into account worker retraining costs.

The sequel of the paper is organized as follows. In the next section we outline the basic structure of the model and derive the optimal output decisions contingent on the realizations of shocks. In section 3 we analyze the location decisions of firms and the firms’ payoffs resulting from location decisions in different situations. This allows us in section 4 to look into the characteristics of the spatial equilibrium and the determinants of the industrial structure. In the concluding section 5 we speculate about possible extensions of our model.

\(^2\)Similar results follow from a simple reinterpretation of Schulz and Stahl’s (1996) model. They additionally consider differences in local market organizations.
2 Model

2.1 Ingredients

Our model economy consists of two regions and two industries involving two representative firms each. Each region \( k, k = I, II \) is populated by an identical number \( \bar{L} \) of immobile workers, each supplying inelastically one unit of labor. The two firms \( i, i = 1, 2 \) in each industry \( m, m = A, B \) produce for competitive world markets and thus take output prices \( p_i^m \) as given. The price fetched by any firm is subject to two shocks: a firm-specific, and an industry specific shock, denoted by \( \epsilon_i^m \) and \( \epsilon^m \), respectively. These shocks are assumed to change prices in an additively-linear way. To facilitate the analysis we assume that in the absence of shocks the price is the same across firms. Thus,

\[
p_i^m = p + \epsilon^m + \epsilon_i^m. \tag{1}
\]

Each firm produces only with labor according to the production function

\[
x_i^m = (2L_i^m)^{0.5}, \tag{2}
\]

where \( L_i^m \) denotes the number of workers employed in firm \( i \) of industry \( m \), to produce \( x_i^m \) units of output in that firm. The exogenous wage rate is normalized to unity.

For the sake of tractability we collapse the potentially many time periods and corresponding realizations of the shock variables into one single period of time. Without loss of generality this can be interpreted as the "representative" period, as location decisions are long term decisions, made on the basis of expected output and profit realizations. This short-cut immensely facilitates the analysis.

Against this background, we analyze the firms’ production and location decisions in a two-stage game. In its first stage, firms decide on their location anticipating the distribution, but not knowing the actual realizations of the demand price shocks. Between the first and the second stage of the game, nature reveals those. Thus, in the second stage, firms choose their optimal output levels conditional upon the realization
of the shocks. Before we analyze the two stages in more detail, it is necessary to specify the nature of the various shocks.

2.2 Shock structure

To simplify matters, we assume that there are three realizations to each shock that are identical for firm-specific as well as for the industry-specific shocks, namely $+a$, 0 and $-a$. Since the two shocks enter additively, we obtain five possible price realizations for each firm, differentiated by $2a$, $a$, 0, $-a$ and $-2a$.

However, the probabilities for the realizations of firm- and industry-specific shocks are allowed to differ. The latter are given by $prob^m(a) = \pi \alpha$, $prob^m(0) = 1 - \pi$, and $prob^m(-a) = \pi (1 - \alpha)$. With $\alpha = 0.5$, the expected value of the shock variable is just zero. With $\alpha > 0.5$ ($< 0.5$), the expected value becomes positive (negative). Therefore, variations in $\alpha$ indicate variations in the expected growth or decline of the respective industry and/or firm prices: Large (small) $\alpha$’s stand for price expectations in growing (declining) industries.

The probabilities for the firm-specific shocks are given by $prob_i(a) = \tilde{\pi} \alpha$, $prob_i(0) = 1 - \tilde{\pi}$, and $prob_i(-a) = \tilde{\pi} (1 - \alpha)$ respectively. With $\tilde{\pi} = \pi$ the firm-specific and the industry-specific shocks have just the same stochastic nature. If $\tilde{\pi} = 0$ the firm-specific shock is absent. In all, we can work with a sufficiently rich stochastic structure, to discuss their implications on a region’s industry mix.

2.3 The output decision stage

We now solve the model by starting with the firms’ determination of output quantities, when the price shocks are realized, under the (preliminary) assumption that it faces a perfectly elastic labor supply at unit wage.$^3$ More precisely, we analyze the impact of a shock realization in one "representative" period starting from a situation in which

---

$^3$Since firms act here in competitive markets, the second stage is not really a game in the sense of strategic interaction among the players.
firms' output, and thus labor demand decisions have been realized in the absence of shocks.

Taking (2) into account, the profits of each firm are

$$\Pi_i^m = \tilde{p}_i^m x_i^m - I_i^m = p^m x_i^m - 0.5(x_i^m)^2. \tag{3}$$

Assuming an interior solution, profit maximization yields the optimal output level

$$\hat{x}_i^m = p + \tilde{e}_i^m + \tilde{e}_i^m, \tag{4}$$

where $\tilde{e}_i^m$ and $\tilde{e}_i^m$ denote the realizations of the underlying random variables. The corresponding labor demand of each firm is

$$\hat{I}_i^m = 0.5(\hat{x}_i^m)^2 = 0.5(p + \tilde{e}_i^m + \tilde{e}_i^m)^2. \tag{5}$$

As a useful reference point (this will become clear below) we assume that, in the absence of shocks, labor demand per region with two firms in that region just equals labor supply, i.e.:

$$\bar{L} = p^2. \tag{6}$$

This implies that the pairing of two firms in one labor market region is a natural equilibrium outcome on which we build the ensuing discussion. \footnote{This does not imply, though, that the location stage of the game is trivial: Firms may pair in two ways.} However, with positive shocks and hence with $\tilde{p}_i^m > p$ for some firm $i$, its labor demand may exceed the supply available to it, and hence, with inelastic labor supply, that firm may be rationed in the labor market. In contrast, with negative shocks total supply may exceed total demand: even if the expected value of the shock variable is zero, the realized employment rate is not equal to unity. Indeed, as we will see later, many combinations of shocks may yield unemployment.

In the case the firms are rationed we adopt the following plausible rationing rules. If one of the firms is blessed with a more favourable demand realization than the other
firm, the former can fully realize its plans whence the less favoured firm is rationed. If both firms enjoy identical positive demand shocks and thus exercise equal excess labor demand, they are equally rationed and thus will continue to hire one half of the local labor force at the given wage rate.

2.4 Retraining costs

If a firm enjoys a positive demand shock, it will try to expand its output (see (4)), and to employ additional workers previously employed with the other firm. This transfer is subject to retraining costs that are asymmetric between workers that move between firms belonging to the same industry, and firms belonging to different industries. As only the difference matters, we standardize retraining costs to zero when workers are hired away from the firm in the same industry. However, an expansion of the work force when absorbing labor from the other industry is accompanied with costs that the expanding firm has to bear. The costs per worker recruited from a firm of the other industry is denoted by $c$. Total retraining costs $CR$ are proportional to the number of workers newly hired by firm $i$ in industry $m$, $\Delta L_i^m$:

$$CR(\Delta L_i^m) = c\Delta L_i^m. \quad (7)$$

Obviously, retraining costs affect the optimal output decision of expanding firms. With positive retraining costs, such a firm’s profits are

$$\Pi_i^m = p_i^m x_i^m - L_i^m - CR_i^m = p_i^m x_i^m - 0.5(x_i^m)^2 - c \left( \max(0.5(x_i^m)^2 - \tilde{L}/2), 0 \right), \quad (8)$$

whereby the last expression stems from (7) and the fact that the number of additional workers employed is just equal to the difference between the currently employed workers and $\tilde{L}/2$, the number of workers employed in the status quo ex ante.

Profit-maximization yields

$$\hat{x}_i^m = \begin{cases} \frac{p + \tilde{\ell}_i^m + \tilde{e}_i^m}{1+c} & \text{if } \frac{p + \tilde{\ell}_i^m + \tilde{e}_i^m}{1+c} > p \\ p + \tilde{\ell}_i^m + \tilde{e}_i^m & \text{otherwise.} \end{cases} \quad (9)$$
This gives us the following labor demand functions:

\[
\hat{l}_{i}^{m} = \begin{cases} 
0.5 \left( \frac{p + \hat{\epsilon}_{i}^{m} + \hat{\epsilon}_{m}}{1+c} \right)^{2} & \text{if } \frac{p + \hat{\epsilon}_{i}^{m} + \hat{\epsilon}_{m}}{1+c} > p \\
0.5 [p + \hat{\epsilon}_{i}^{m} + \hat{\epsilon}_{m}]^{2} & \text{otherwise.} 
\end{cases}
\]  

(10)

Eqs. (9) and (10) specify the typical firm’s optimal output level and thus its labor demand. Whether the firm’s plan can actually be realized depends on the particular shock realization for the other firm. If the sum of shocks is sufficiently positive firms are willing to employ more workers than are available. The plans of all firms can not be realized at the same time. Once again, in these situations we follow the notion that the more successful firm (i.e. the firm with the largest positive shock) will be able to attract all desired workers, whence the other firm is rationed in the labor market. Also, if firms face the same positive shocks, they receive the same number of workers each.\(^5\)

3 Location Stage

The firms’ location decisions are based on their expected profits when locating in the region under consideration. Before we address these, it is helpful to understand that there are only two potential equilibrium configurations we have to analyze. With the assumption that total labor supply per region just satisfies the expected labor demand for two firms, we consider only configurations such that firms will settle in equal numbers in the two regions. Suppose to the contrary that there were three firms locating in one region and only one in the other region. Then one of the three firms has an incentive to deviate, for this allows the firm to expand its output and profits. Hence, this spatial configuration as well as complete concentration of all firms in one region do not constitute equilibria in the first stage of our game. In both cases, the agglomeration disadvantage (the scarcity of immobile labor) can never be compensated

\(^5\)We used alternative rationing schemes (e.g. proportional rationing according to excess demand) and found that this does not alter our qualitative results.
for by an agglomeration advantage.

Thus, there remain only two configurations for the locational equilibrium in the first stage of the game. In both configurations each region is populated by two firms. The crucial question is on the conditions under which we observe industrial specialization vs. industry mix. Using our argument from above, it is straightforward to see that both situations are Nash equilibria. Given that two firms are located in each region, it never pays for a single firm to deviate. We will consider, however, as focal the equilibrium offering the highest expected profits.

We will refer to a situation in which both firms of the same industry locate in the same region as (2,0)-case and the situation in which firms of the same industry are dispersed across regions as (1,1) case. Since allocation decisions are perfectly symmetric, it suffices to compare the expected profits of a representative firm in the two alternative situations.

3.1 The (1,1)-case

We start by looking at the typical firm’s expected profits in the (1,1)-case. Before calculating them, which proves to be tedious, it is helpful to take a closer look at the structure of shock realizations arising in that configuration. Table 1 provides an overview.
We proceed in two steps. First, we calculate all the elements, i.e. profit levels conditional upon the shock realizations, in table 1. Second, we calculate the expected profits as weighted sum of these profit levels, with weights given by the underlying probabilities.

Let us first recall our rationing rule, according to which the firm with the higher demand can fully realize its plans whence the less favoured firm is rationed; and both firms are equally rationed if both exercise identical excess demand. In the present case (1,1), retraining costs may dampen the demand for labor in the case of an expansionary shock. If retraining costs are low, then demand will exceed total supply even if the successful firm enjoys the positive realization of just one shock variable, whence the other firm is confronted with an additive negative shock. It is this what we assume: in the sequel:

\[ L^i(p + a, c) + L^j(p - 2a) \geq \bar{L} \quad \text{with} \quad i \neq j \quad (11) \]

Eq. (10) gives us the critical \( \bar{c} \) for which the above expression holds with equality:

\[ 0.5 \left( \frac{p + a}{1 + \bar{c}} \right)^2 + 0.5(p - 2a)^2 = \bar{L}. \quad (12) \]

Recalling that, by assumption, \( p^2 = \bar{L} \), we obtain

\[ \bar{c} = \sqrt{\frac{(p + a)^2}{p^2 + 4ap - 4a^2}} - 1. \quad (13) \]

In what follows, we assume that \( 0 < c < \bar{c} \) always holds so that rationing is relatively likely to occur. We are now able to compute the profit levels of table 1. Tedium but straightforward calculations give us the typical firm’s profit levels. All profits refer to
Table 1: Shock structure and realizations in the (1,1) case

<table>
<thead>
<tr>
<th>Shock</th>
<th>Firm 1/Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1/F</td>
</tr>
<tr>
<td></td>
<td>1/F</td>
</tr>
<tr>
<td>Probabilities</td>
<td>( \pi \hat{r} )</td>
</tr>
<tr>
<td>Demand</td>
<td>( p + 2a )</td>
</tr>
<tr>
<td>1/F</td>
<td>( \pi \hat{r} \alpha^2 )</td>
</tr>
<tr>
<td>1/0</td>
<td>( \pi (1 - \hat{r})</td>
</tr>
<tr>
<td>0/F</td>
<td>( (1 - \pi)</td>
</tr>
<tr>
<td>-1/F</td>
<td>( \pi \hat{r} (1 - \alpha)</td>
</tr>
<tr>
<td>1/-F</td>
<td>( \pi \hat{r} (1 - \alpha)</td>
</tr>
<tr>
<td>0/0</td>
<td>( (1 - \pi)</td>
</tr>
<tr>
<td>-1/0</td>
<td>( \pi (1 - \hat{r})</td>
</tr>
<tr>
<td>-F/0</td>
<td>( (1 - \pi)</td>
</tr>
<tr>
<td>-1/-F</td>
<td>( \pi \hat{r} (1 - \alpha)^2 )</td>
</tr>
</tbody>
</table>

Remarks: 1: industry-specific shock; F: firm-specific shock; \( \Pi_1 \): Profit functions in [1,1] case
those for the first firm.

\[
\Pi(1,1) = (p + 2a)(p^2)^{1/2} - \frac{p^2}{2} = 0.5p^2 + 2ap \equiv \Pi_1
\]

\[
\Pi(1,2) = \frac{(p + 2a)^2}{2(1 + c)} + \frac{c^2}{2} \equiv \Pi_2
\]

\[
\Pi(1,2) = \Pi_1(1,v) \quad \forall v = (3,\ldots,9)
\]

\[
\Pi(2,1) = (p + a) \left[ 2p^2 - 0.5 \left( \frac{p + 2a}{1 + c} \right)^2 \right]^{0.5} - \left( p^2 - 0.5 \left( \frac{p + 2a}{1 + c} \right)^2 \right) \equiv \Pi_3
\]

\[
\Pi(2,2) = \Pi_1(2,3) = (p + a)p - 0.5p^2 = 0.5p^2 + ap \equiv \Pi_4
\]

\[
\Pi(2,4) = 0.5 \frac{(p + a)^2}{1 + c} + 0.5cp^2 \equiv \Pi_5
\]

\[
\Pi(2,4) = \Pi_1(2,5) = \Pi_1(2,6) = \Pi_1(2,7) = \Pi_1(2,8) = \Pi_1(2,9)
\]

\[
\Pi(2,k) = \Pi_1(3,k) \quad \forall k = (1,\ldots,9)
\]

\[
\Pi(4,1) = p \left[ 2(p^2 - 0.5 \left( \frac{p + 2a}{1 + c} \right)^2 )^{0.5} - \left( p^2 - 0.5 \left( \frac{p + 2a}{1 + c} \right)^2 \right) \right. \equiv \Pi_6
\]

\[
\Pi(4,2) = \Pi_1(4,3) = p \left[ 2p^2 - 0.5 \left( \frac{p + a}{1 + c} \right)^2 \right]^{0.5} - \left( p^2 - 0.5 \left( \frac{p + a}{1 + c} \right)^2 \right) \equiv \Pi_8
\]

\[
\Pi(4,2) = \Pi_1(4,w) = p(p^2)^{0.5} - 0.5p^2 = 0.5p^2 \quad \forall w = (4,\ldots,9)
\]

\[
\Pi(4,k) = \Pi_1(5,k) = \Pi_1(6,k) \quad \forall k = (1,\ldots,9)
\]

\[
\Pi(7,1) = (p - a) \left[ 2p^2 - 0.5 \left( \frac{p + 2a}{1 + c} \right)^2 \right]^{0.5} - \left( p^2 - 0.5 \left( \frac{p + 2a}{1 + c} \right)^2 \right) \equiv \Pi_9
\]

\[
\Pi(7,2) = (p - a) \left[ 2p^2 - 0.5 \left( \frac{p + a}{1 + c} \right)^2 \right]^{0.5} - \left( p^2 - 0.5 \left( \frac{p + a}{1 + c} \right)^2 \right) \equiv \Pi_{10}
\]

\[
\Pi(7,4) = \Pi_1(7,5) = \Pi_1(7,6) = \Pi_1(7,7) = \Pi_1(7,8) = \Pi_1(7,9) = 0.5(p - a)^2 \equiv \Pi_{11}
\]

\[
\Pi(7,k) = \Pi_1(8,k) \quad \forall k = (1,\ldots,9)
\]

\[
\Pi(9,1) = (p - 2a) \left[ 2p^2 - 0.5 \left( \frac{p + 2a}{1 + c} \right)^2 \right]^{0.5} - \left( p^2 - 0.5 \left( \frac{p + 2a}{1 + c} \right)^2 \right) \equiv \Pi_{12}
\]

\[
\Pi(9,2) = \Pi_1(9,3) = (p - 2a) \left[ 2p^2 - 0.5 \left( \frac{p + a}{1 + c} \right)^2 \right]^{0.5} - \left( p^2 - 0.5 \left( \frac{p + a}{1 + c} \right)^2 \right) \equiv \Pi_{13}
\]

\[
\Pi(9,4) = \Pi_1(9,5) = \Pi_1(9,6) = \Pi_1(9,7) = \Pi_1(9,8) = \Pi_1(9,9) = 0.5(p - 2a)^2 \equiv \Pi_{14}
\]
Weighting these profits with the probabilities as stated in table 1 yields the expected profits for the \((1,1)\) case:

\[
E \Pi(1, 1) = \sum_{m=1}^{9} \sum_{k=1}^{9} q_{mk} \Pi_1(m, q). \tag{15}
\]

### 3.2 The \((2,0)\) Case

This case differs from the \((1,1)\) case in that firms are not hedged against industry-specific shocks. In the now specialized region firms always experience the same industry-specific shock. While this represents the disadvantage of concentrating alike firms in one region, the advantage is that in the case of asymmetric firm specific shocks across firms, the winner can costlessly absorb labor from the loser. That is, firms can capitalize more easily on an expansionary shock.

Since in this case regions are specialized, the shock is much less complex than that in the \((1,1)\)-case. Table 2 gives an overview of these \(3 \times 3 \times 3\) realizations.

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
</tr>
<tr>
<td>Shocks</td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Probabilities</td>
<td>(\bar{\pi} \alpha)</td>
</tr>
<tr>
<td>Demand</td>
<td>(p + 2a)</td>
</tr>
<tr>
<td>F</td>
<td>(\bar{\pi} \alpha)</td>
</tr>
<tr>
<td>0</td>
<td>((1 - \bar{\pi}))</td>
</tr>
<tr>
<td>-F</td>
<td>(\bar{\pi}(1 - \alpha))</td>
</tr>
</tbody>
</table>

Remarks: \(\Pi^+_2\): Profit functions in the \((2,0)\) case with a positive industry-specific shock

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Table 2b: A zero industry-specific shock \((e^m = 0)\)

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
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<tbody>
<tr>
<td>Shocks</td>
<td>F</td>
</tr>
<tr>
<td>Probabilities</td>
<td>(\tilde{p} \alpha)</td>
</tr>
<tr>
<td>Demand</td>
<td>(p + a)</td>
</tr>
<tr>
<td>F</td>
<td>(\tilde{p} \alpha)</td>
</tr>
<tr>
<td>0</td>
<td>((1 - \tilde{p}))</td>
</tr>
<tr>
<td>-F</td>
<td>(\tilde{p}(1 - \alpha))</td>
</tr>
</tbody>
</table>

Remarks: \(\Pi_2^0\): Profit functions in the (2,0) case with a zero industry-specific shock

Table 2c: A negative industry-specific shock \((e^m = -a)\)

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shocks</td>
<td>F</td>
</tr>
<tr>
<td>Probabilities</td>
<td>(\tilde{p} \alpha)</td>
</tr>
<tr>
<td>Demand</td>
<td>(p)</td>
</tr>
<tr>
<td>F</td>
<td>(\tilde{p} \alpha)</td>
</tr>
<tr>
<td>0</td>
<td>((1 - \tilde{p}))</td>
</tr>
<tr>
<td>-F</td>
<td>(\tilde{p}(1 - \alpha))</td>
</tr>
</tbody>
</table>

Remarks: \(\Pi_2^-\): Profit functions in the (2,0) case with a negative industry-specific shock

As in the (1,1) case, we are now prepared to calculate the relevant profits, where \(\Pi^+, \Pi^0\) and \(\Pi^-\), respectively, denote firm 1’s profits realized under the relevant industry
specific shocks:

\[
\begin{align*}
\Pi^+ (1, 1) & = 0.5p^2 + 2ap = \Pi_1 \\
\Pi^+ (1, 2) & = \Pi^+_2 (1, 3) = 0.5(p + 2a)^2 = \Pi_{15} \\
\Pi^+ (2, 1) & = (p + a) [2p^2 - 0.5(p + 2a)^2]^{0.5} - (p^2 - 0.5(p + 2a)^2) \equiv \Pi_{16} \\
\Pi^+ (2, 2) & = 0.5p^2 + ap = \Pi_4 \\
\Pi^+ (2, 3) & = 0.5(p + a)^2 = \Pi_{17} \\
\Pi^+ (3, 1) & = p [2p^2 - 0.5p + 2a^2]^{0.5} - (p^2 - 0.5(p + 2a)^2) \equiv \Pi_{18} \\
\Pi^+ (3, 2) & = p [2p^2 - 0.5(p + a)^2]^{0.5} - (p^2 - 0.5(p + a)^2) \equiv \Pi_{19} \\
\Pi^+ (3, 3) & = 0.5p^2 = \Pi_8 \\
\Pi^0 (1, 1) & = 0.5p^2 + ap = \Pi_4 \\
\Pi^0 (1, 2) & = \Pi^0_2 (1, 3) = 0.5(p + a)^2 = \Pi_{17} \\
\Pi^0 (2, 1) & = p [2p^2 - 0.5p + a^2]^{0.5} - (p^2 - 0.5(p + a)^2) = \Pi_{19} \\
\Pi^0 (2, 2) & = \Pi^0_2 (2, 3) = 0.5p^2 = \Pi_8 \\
\Pi^0 (3, 1) & = (p - a) [2p^2 - 0.5p + a^2]^{0.5} - (p^2 - 0.5(p + a)^2) \equiv \Pi_{20} \\
\Pi^0 (3, 2) & = \Pi^0_2 (3, 3) = 0.5(p - a)^2 \equiv \Pi_{21} \\
\Pi^- (1, 1) & = \Pi^-_2 (1, 2) = \Pi^-_2 (1, 3) = 0.5p^2 = \Pi_8 \\
\Pi^- (2, 1) & = \Pi^-_2 (2, 2) = \Pi^-_2 (2, 3) = 0.5(p - a)^2 = \Pi_{11} \\
\Pi^- (3, 1) & = \Pi^-_2 (3, 2) = \Pi^-_2 (3, 3) = 0.5(p - 2a)^2 = \Pi_{14}
\end{align*}
\]

Weighting these profits with the probabilities as stated in table 2 yields the expected profits for the (2,0) case:

\[
E \Pi (2, 0) = \sum_{z=1}^{3} \sum_{y=1}^{3} q_{zk} \Pi^+_2 (zk) + \sum_{z=1}^{3} \sum_{y=1}^{3} q_{zk} \Pi^0_2 (zk) + \sum_{z=1}^{3} \sum_{y=1}^{3} q_{zk} \Pi^-_2 (zk).
\]

(17)
3.3 The mechanisms behind location choices

Once again, we consider the equilibrium as focal which yields the higher expected profits. Before we discuss in more detail the determinants of the profit differential between the (1,1) and the (2,0) cases, it proves helpful to investigate the main mechanisms that generate this profit differential.

Prima facie, there are two mechanisms at play. First, in the (1,1) case firms benefit from asymmetric industry-specific shocks in a more efficient manner. Unlike in the (2,0) case, the firm blessed with a positive industry specific shock can increase its profits by absorbing labor from the firm in the less successful industry. We call this the industry specific hedging effect. It works in favor of the (1,1) case. This hedging effect has, however, to be contrasted with the retraining cost effect also effective only in the (1,1) case. Ceteris paribus, it reduces the attractiveness of that case.

There is also a firm specific hedging effect, arising in the case where there are asymmetric firm specific shocks in one region. However, due to the retraining cost effect, that hedging effect is dampened in the (1,1) case, whence it is in full force in the (2,0) case.

We now pursue an analysis of the profit differentials arising in the two locational equilibria under the same shock structure. Starting with the simplest one, it is obvious that without any shocks the two situations are exactly equal and the difference between expected profit levels is just zero. Plugging both shock probabilities $\pi = \bar{\pi} = 0$ into (15) and (17) yields just:

$$\Gamma \equiv E\Pi(1,1) - E\Pi(2,0) = \Pi_{s} - \Pi_{s} = 0$$

Since retraining is never an issue in the absence of industry specific shocks, the level of retraining costs is irrelevant.

Thus, one could expect that in the absence of industry-specific shocks and with it, the absence of the industry specific hedging effect the (2,0) situation in which the regions specialize is always preferred by the firms, for the retraining cost effect makes industry mix, i.e. the (1,1) situation less attractive. However, computing the profit
differential by setting the industry specific shock probability \( \tilde{\pi} = 0 \) gives us

\[
\Gamma \equiv E\Pi(1,1) - E\Pi(2,0) = \tilde{\pi}(1 - \tilde{\pi})[\Pi_5 - \Pi_{17}] + (1 - \tilde{\pi})\tilde{\pi}[\Pi_7 - \Pi_{19}] + \\
+\tilde{\pi}(1 - \alpha)\tilde{\pi}[\Pi_{11} - \Pi_{20}].
\] (19)

Since \( \Pi_5 < \Pi_{17}, \Pi_7 > \Pi_{19} \) and \( \Pi_{11} > \Pi_{20} \), the profit differential cannot be signed. Hence, our initial intuition proves to be incorrect. Figure 1 gives a numerical example.

Here, the expected profit difference \( \Gamma \) is plotted against variations in \( \tilde{\pi} \), the probability by which a firm specific shock arises. The other parameters are fixed at \( c = 0.1, a = 0.5, p = 3 \), and \( \alpha = 0.5 \).

Figure 1: The impact of firm-specific shocks (with \( \pi = 0 \))

Thus, \( \Gamma \) can be positive as well as negative. Contrary to first expectations, it can be positive because of the rationing effect, for the following reason. The downside to a positive shock a firm experiences is that the other firm in the region then tends to be rationed in the labor market, and thus may be forced to contract output and profits. That negative rationing effect is not internalized by the expanding firm when deciding to expand it’s output. While the rationing effect arises in both, the (1,1) and the (2,0) cases, it tends to be less severe in the (1,1) case by the retraining costs that reduce the
fortunate firm’s expansion. Thus, while retraining costs reduce the expanding firm’s profits, they have a positive effect on the profits of the contracting firm. The net effect is ambiguous and depends on the parameters of the model. Figure 1 illustrates that if the firm-specific shock arises with low probability, the rationing effect dominates (and hence $\Gamma > 0$), whereas for higher $\bar{\pi}$s the retraining cost effect becomes more and more important, leading to a negative $\Gamma$.

The effect of rationing becomes clearer if we suppose for the moment that retraining costs are absent $(c = 0)$ but both, firm and industry specific shocks arise (with certainty, to facilitate the analysis). Our initial thinking would suggest that the hedging effect then calls for $\Gamma > 0$, as with industry mix, the firms can perfectly hedge against industry specific shocks. Yet once again this intuition proves to be too simple. Setting $c = 0$ into (15) and (17) and $\pi = \bar{\pi} = 1$ yields

$$\Gamma = E\Pi(1, 1) - E\Pi(2, 0) = \alpha^3(1 - \alpha)(\Pi_2 - \Pi_1) + \alpha^2(1 - \alpha)(2\alpha - 1)(\Pi_6 - \Pi_8).$$

As $\Pi_2 > \Pi_1$ and $\Pi_6 < \Pi_8$, the sign of the profit differential is not unambiguous. As in the above case, this is due to the existence of the rationing effect, that interacts with the hedging effect. The realization of positive shocks (e.g. a positive firm- and industry-specific shock) for one firm is to its advantage but quite disadvantageous for the other firm because it then tends to be rationed. Therefore, the (1,1)-situation is not unambiguously more attractive.

Hence, *seconda facie*, we have in total three effects governing the locational choice of firms in our framework, namely

- the hedging effect
- the retraining cost effect
- the rationing effect

With the help of these three effects, we will rationalize the impact of the various parameters on the spatial equilibrium in our model in the next section.
4 The Determinants of the Industry Mix

There are basically five crucial parameters that determine the spatial equilibrium: the likelihood of industry-specific shocks (measured by $\pi$), the likelihood of firm-specific shocks (depicted by $\tilde{\pi}$), the size of the shock (see $a$), the level of retraining costs ($c$), and the relative importance of the positive shock (given by $\alpha$). It turns out that the level of the product market price cancels out and is therefore irrelevant for our problem. As a glance at (15) and (17) reveals, it is not possible to derive explicit solutions for the influence of the respective parameters on the differential of expected profits in the two cases. Therefore, we proceed by presenting graphical illustrations based on numerical examples which reflect broadly based simulations and robust relationships. In all cases a clear and interpretable picture emerges.

4.1 The impact of industry-specific shocks

How does the likelihood of the industry-specific shock affect the relative attractiveness of the (1,1) over the (2,0) situation? We address this question by computing the expected profit differential $\Gamma$ as a function of this likelihood ($\pi$). The result is depicted in Figure 2. The $\Gamma_1(\pi)$-line is drawn for $p = 3, c = 0.1, a = 0.5, \alpha = 0.5$ and $\tilde{\pi} = 0.5$. With an increasing probability of the the industry-specific shock the tendency for firms of different industries to locate together becomes stronger. This just reflects the hedging effect which increasingly dominates the other effects. The larger the industry-specific shocks the better the hedge position of the firm in (1,1) case. In the absence of retraining costs and firm-specific shocks the (1,1)-situation is always preferred. Positive retraining costs shift the $\Gamma(\pi)$-line downward leading, for sufficiently small probabilities of the industry-specific shocks, to a negative $\Gamma$ (i.e., the (2,0) situation is strictly preferred). With larger firm-specific shocks the relative attractiveness of the (1,1) case decreases and the $\Gamma$-line shifts downward. This is due to the rationing effect.
4.2 The impact of firm-specific shocks

Firm-specific shocks reflect the notion that besides general product market conditions that affect firms in the same industry in the same way, there are idiosyncratic influences which differ across firms (such as product-specific changes in demand, managerial mistakes etc). Obviously, the more often we observe firm-specific shocks the more pronounced is the retraining cost effect making the (1,1) solution less attractive. In both spatial equilibria firm-specific shocks occur, but only in the (1,1) case firms have to pay retraining costs in order to adjust to these shocks. The hedging effect now becomes relatively less important. But, in addition to invoking the retraining cost effect, the likelihood of firm-specific shocks has implications for the relative attractiveness of the two spatial equilibria via the rationing effect. The more likely firm-specific shocks the more important is the rationing effect, which calls – in the presence of retraining costs – for the (1,1) situation. There, the retraining costs have a dampening impact on the expanding firms’ demand for new workers thus cushioning the negative effects for the the firm that is hit by an unfavourable product market shock. Figure 3 illustrates this trade-off and reveals that for low levels of $\pi$ the impact via the retraining cost effect overcompensates the rationing cost effect. For high levels of $\pi$ the reverse is true. Figure 3 shows that in the absence of industry-specific shocks and with pronounced
retraining costs the (2,0) solution is most likely. That is, our model predicts that firms (of one industry) which share the same labor pool (i.e. have relatively low retraining costs among each other) but are rather independent in the goods market (no or only few industry-specific shocks) are most likely to locate together in one region.

Figure 3: The impact of firm-specific shocks on the profit differential

Remark: The $\Gamma_1(\tilde{\pi})$ is drawn for $c = 0.1$, $a = 0.5$, $p = 3$, $\alpha = 0.5$ and $\pi = 0$.

4.3 The impact of the level of retraining costs

Higher retraining cost weaken the attractiveness of the (1,1) situation due to the retraining cost effect ($\partial \Gamma / \partial c < 0$). Firms have to pay higher retraining cost in order to expand their labor force. At the same time, higher retraining costs dampen the optimal output expansion of firms experiencing the higher product demand. This in turn is good news for the firm which looses workers and it is rationed on the labor market. Figure 4 shows that for small (large) retraining costs the rationing effect (the retraining cost effect) dominates.
Figure 4: The impact of retraining costs on the profit differential

Remark: The $\Gamma_1(\bar{\pi})$ is drawn for $a = 0.5, p = 3, \alpha = 0.5, \bar{\pi} = 0.5$, and $\pi = 0$.

4.4 The impact of $\alpha$ and $a$

A larger $\alpha$ can be interpreted as an indicator for a growing industry. With $\alpha = 0.5$ the expected value of the shock variable is zero. With $\alpha > (\leq)0.5$ the expected shock realization is positive (negative) depicting a growing (declining) industry. We find that growing industries prefer a (1,1) solution even in the absence of industry-specific shocks. With a large $\alpha$ the hedging effect (which takes place in the event of a positive shock) dominates the rationing effect (which occurs with a negative shock). By definition, with $\alpha > 0.5$, the likelihood of a positive shock is larger than of a negative one. In growing industries hedging is the main impetus which drives firms of industries to locate together (see figure 5). The size of the shock (measured by $a$) has no clear-cut effect on the relative attractiveness of either spatial situation. It turns out that $\Gamma(a)$ is upward sloping for a sufficiently large $\pi$ and downward sloping if the likelihood of firm-specific shocks is rather pronounced. In the first case $\Gamma$ is negative
for small values of $a$ and positive for large ones. With the second case, we observe the opposite pattern.

Figure 5: The impact of $\alpha$ on the profit differential

Remark: The $\Gamma_1(\alpha)$ is drawn for $a = 0.5, p = 3, c = 0.1, \bar{\pi} = 0.5$, and $\pi = 0$.

5 Conclusion

With the help of a simple two-region-two-industry model we have been able to investigate the determinants of a region’s industrial structure. The combination of the labor market pooling argument (which reflects the impact of uncertainty and hedging against shocks on location choice) and a switching costs argument (which favors the concentration of alikes) gave us a rich picture of the interaction of different factors that combine to determine the spatial patterns of firms and industries. Our model yields a number of straightforward and testable hypotheses. For instance, our model predicts that firms of one industry are most likely to agglomerate if their demand on workers’ skills is quite different from the one of firms in the other industry and if at the same time these firms are rather independent from each other in the goods market.

There are a number of directions in which our model can be extend. Foremost, it would be interesting to look at the implications of a mobile labor force. This would
enable us to analyze the simultaneous location choice of workers and firms, and allow for an even richer picture of spatial equilibria. For example, one could ask whether and when a concentration of all economic activity in one single region constitutes an equilibrium. Other extension encompass the possibility of correlated shocks and a larger number of firms.
References


Jellal, Thisse, Zenou


