CARIBBEAN FOOD CROPS SOCIETY

PROCEEDINGS

ELEVENTH ANNUAL MEETING

THE ANALYSIS OF AN ARRANGEMENT DESIGNED FOR LIMITED RESOURCES

by

F. B. LAUCKNER[+]

and

B. G. F. SPRINGER

The University of the West Indies
St. Augustine, Trinidad.

SUMMARY

A modified composite design is described. This design has been specially developed for experimentation under conditions found in the Caribbean. An example of the analysis of such a design is shown.

INTRODUCTION

In many experiments in the Commonwealth Caribbean, the experimenter operates under limited resources with regard to land and finance available. Yet the nature of the problems to be solved require consideration of a number of factors over a wide range of levels for these factors. One type of design suited to this sort of problem is a composite design which is a full or fractional replicate of a factorial system with additional treatment combinations within and outside the treatment range chosen for the factorial system. A discussion on composite designs can be found in Cochran & Cox (1966). These designs suffer from the drawback that the treatment combinations outside the factorial system are fixed by

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the choice of combinations within the system. Also some of these outer treatment combinations may not be of immediate interest to the experimenter. As an illustration figure 1 represents one replicate of a central composite design in three factors.

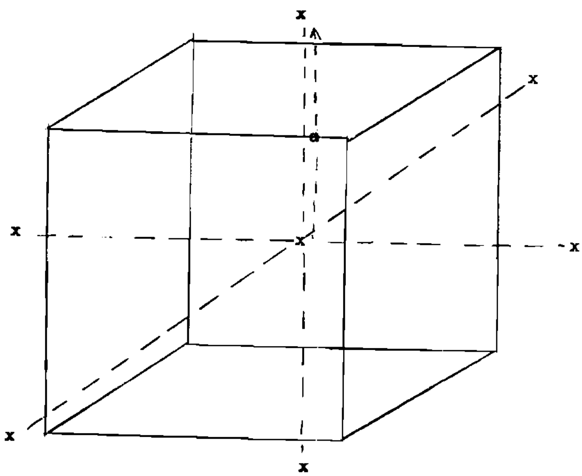

Figure 1

The design consists of eight points at the corners of the cube (as in a 2^3 factorial) plus an additional point at the centre of the cube and six further points each distant *a* from the centre at the end of lines drawn perpendicular to the six faces. This distance *a* is determined by the size of the cube. Now the experimenter may well wish to consider points further away than *a* from the centre of the cube; further he is likely to have chosen his lower levels of the three factors in the 2^3 such that he is not interested in experimenting at levels still lower. Thus three of the points outside the face of the cube in Figure 1 are not likely to produce results of interest.

MODIFIED COMPOSITE DESIGN

To overcome these difficulties Springer (1972) suggested a modified version of the above design. The modified composite design is shown in Figure 2.

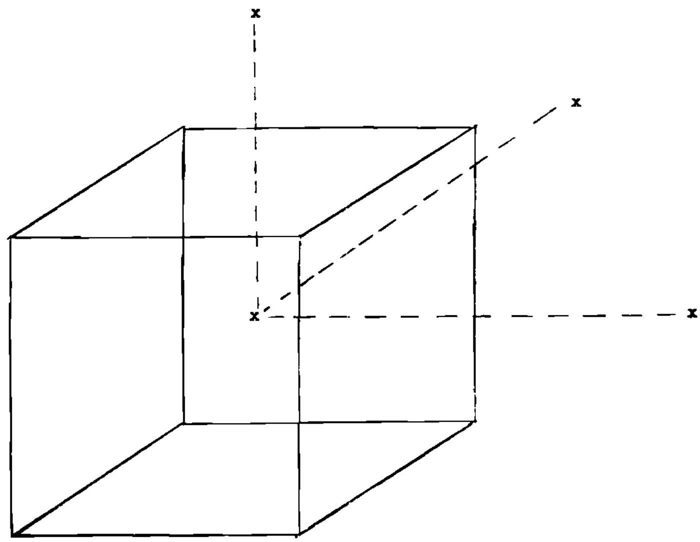

Figure 2

Here we have eight points of the 2^3 factorial as before, but in addition, this time we have four observations at the centre and two observations at three points on the outside of the cube. These three points are at a distance from the centre determined by the experimenter and are chosen, at points along the perpendicular to the faces, which are likely to be of interest. The three points outside the other faces are discarded as being of little or no interest. We regard the treatment levels at the corners of the cube as levels 0 and 2, the level at the centre level 1, and the peripheral treatment levels as level 3. Springer (1972) suggested that in an experiment to investigate the response of a crop to three nutrient levels, level 0 should be such as to permit some growth of the crop under investigation, level 2 should be the experimenters guess at the optimum level of the factors, level 1 is the mean of levels 0 and 2 and level 3 should be chosen not too close to level 2, but not so far away as to induce a toxic effect.

ANALYSIS OF A MODIFIED COMPOSITE DESIGN

The data in Table 1 refer to a modified composite design analysed at U.W.I. St. Augustine. The experiment was to investigate the effect of

Nitrogen (N), Phosphorous (P) and Potassium (K) on the yield of corn. The treatment code levels are as described above. The eighteen points of the modified composite design were split into two sets or fractions and there were two replicates of each making four blocks in all.

The analysis of this experiment is shown in Table 3. The blocks sum of squares is found from the four block totals and the grand total.

These are shown in table 1.

$$\text{Blocks sum of squares} = \frac{9328^2 + 10128^2 + 9897^2 + 10903^2}{9} - \frac{40256^2}{36}$$

We next consider the main effects and interactions of N, P and K over levels 0 and 2. Over these levels, the NPK interaction is completely confounded with blocks so this does not appear in the table. These main effects and interactions are calculated in the usual way for a 2^3 factorial, but we consider only the four treatments at levels 0 and 2 in each block. To calculate the relevant sums of squares we refer to table 2 and the treatment totals in Table 1. We find the factorial effect totals by adding or subtracting the successive treatment totals according to table 2. Then we square the factorial effect total and divide by 16 to find the sum of squares. For example the sum of squares for N is given by

$$1/_{16}[-1029 + 205 - 863 + 2552 - 749 + 576 - 1037 + 3233]^2$$

We can define four orthogonal contrasts associated with the treatments at the centre and outside the cube. We attempt to define contrasts which are likely to be most useful for the analysis. These contrasts must be orthogonal to each other as well as to the main effects and interactions already computed. In this example only three contrasts were calculated, the remaining contrast was not of interest.

Table 1

Results of modified composite design to investigate effects of nutrients on yield of corn

Treatment Code			Yield (Kilos/Acre)		
N	P	K	Rep 1	Rep 2	Total (Rep 1 + Rep 2)
0	0	0	843	186	1029
0	2	2	635	402	1037
2	0	2	217	359	576
2	2	0	1302	1250	2552
3	1	1	1839	2286	4125
1	3	1	1508	1089	2597
1	1	3	1315	1836	3151
1	1	1	403	1425	1828
1	1	1	1266	1295	2561
Block totals			9328	10128	
2	0	0	106	99	205
0	2	0	681	182	863
0	0	2	134	615	749
2	2	2	1677	1556	3233
3	1	1	2044	1894	3938
1	3	1	912	1833	2745
1	1	3	1834	1519	3353
1	1	1	1696	1801	3497
1	1	1	813	1404	2217
Block totals			9897	10903	

Grand total = 40256

The curvature effect compares the centre cube treatment (1, 1, 1) with the treatments in the 2^3 factorial to see whether the N, P, K relationship is linear with the cube. It is calculated from the 2^3 factorial treatment totals and the total of all replicates at the cube which is 10103 kilos per acre. The computation is:

$$\text{Curvature sum of squares} = \frac{(1029 + 205 + 863 + 2552 + 749 + 576 + 1037 + 3233 - 2 \times 10103)^2}{48}$$

Table 2

Main effect and interactions of N, P and K expressed in terms of individual treatment totals over levels 0 and 2

Factorial Effect	Treatment Combination							
	(0,0,0)	(2,0,0)	(0,2,0)	(2,2,0)	(0,0,2)	(2,0,2)	(0,2,2)	(2,2,2)
N	–	+	–	+	–	+	–	+
P	–	–	+	+	–	–	+	+
K	–	–	–	–	+	+	+	+
NP	+	–	–	+	+	–	–	+
NK	+	–	+	–	–	+	–	+
PK	+	+	–	–	–	–	+	+

The other two orthogonal contrasts investigate whether there is a linear or quadratic relationship between the outer points. Here we need to calculate the treatment totals for the outer three points which are 8063, 5342 and 6504. The computations are:

$$\text{Linear level 3 sum of squares} = \frac{(8063 - 6504)^2}{8}$$

$$\text{Quadratic level 3 sum of squares} = \frac{(8063 + 6504 - 2 \times 5342)^2}{24}$$

Table 3

Analysis of Variance of data in Table 1

	Source	D.F.	Sum of Squares	Mean Square	F-ratio
	Blocks	3	141956	47318.6	0.14
These main effects and interactions are over levels 0 and 2.	N	1	521284	521284	1.51
	P	1	1642240	1642240	4.77
	K	1	55932.3	55932.3	0.16
	NP	1	1489620	1489620	4.32
	NK	1	83810.3	83810.3	0.24
	PK	1	36481.0	36481.0	0.11
Orthogonal contrasts associated with the peripheral and centre treatments.	Curvature	1	2067530	2067530	6.00
	Linear Level 3	1	303810	303810	0.88
	Quad. level 3	1	628237	628237	1.82
	Residual	23	7925900	344604	
	Total	35	14896700		

Estimate of Error Standard Deviation on 23 d.f. = 587 . 03

Coefficient of Variation (Percentage) = 52 . 5

5% significance level of F with 1 and 23 d.f. = 4 . 28

Table 4

Table of Means (Kilos per acre) for data in Table 1

Level	Factor		
	N	P	K
0	460	320	581
1	1263	1263	1263
2	821	961	699
3	2016	1335	1626

LSD (5%) for differences between levels 0 and 1, and 1 and 2 = 527.2

LSD (5%) for differences between levels 2 and 3 = 745.6

If a different order of treatment combinations were decided upon, say N, K, P instead of N, P, K, the above computations would be different but the interpretation would be the same.

If desired, the curvature contrast can be replaced with a contrast which measures the mean of the treatments (1, 1, 1) against the mean of the treatments (3, 1, 1), (1, 3, 1) and (1, 1, 3). This is given by:

Average treatments 1
Average treatments 3 sum of squares $= \dfrac{2 \times (8063 + 5342 + 6504) - 3 \times 10103}{136}$

However, this contrast cannot be included in the same analysis as the curvature contrast as these two contrasts are not orthogonal to each other.

The total sum of squares is found in the usual way by subtracting the correction for the mean $\frac{40256^2}{36}$ from the sum of squares of all the observations; then the residual sum of squares is found by subtraction.

The error standard deviation is the square root of the residuals mean square and this is divided by the overall mean of all the treatments to give the coefficient of variation. In this example, the coefficient of variation is very high; this suggests that our analysis is unlikely to be very useful. However, for illustration, we will demonstrate how to compare the factor levels for individual treatments. The table of means of N, P and K at each of the four levels is then drawn up in the usual way (Table 4). The least significant difference between two entries in the table is given by

$$587.03 \times t_{22} \frac{1}{\sqrt{(1/r_1 + 1/r_2)}}$$

where t_{22} is the students t distribution with 22 degrees of freedom chosen at the appropriate confidence level. r_1 and r_2 are the number of replications for the entries in the table which are being compared.

Extension of modified composite design

If it is desired to test over a wider range of treatment levels than is afforded even by a modified composite design, then two or more such designs can be laid out to test different factor levels. The analysis of variance would then contain a term for difference between the different designs.

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