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THE DEMAND FOR MONEY
UNDER RATIONAL EXPECTATIONS
OF INFLATION:
FIML ESTIMATES FOR BRAZIL

by

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1. Introduction

Ever since the important contribution by Cagan [1956], a large part of

the empirical literature on the demand for money has been based on the experience

of economies operating under highly inflationary conditions.[2] At the theore-

tical level, it is generally postulated that the amount of real money balances

demanded by the public depends on the level of real income and on the opportu-

nity cost of holding money relative to other assets and goods. The opportunity

cost consists of the yield on alternative assets, measured by their expected

rates of interest, and the return on holding goods, measured by the expected

rate of inflation. Yet empirically, when studying economies with substantial

inflation, most researchers have utilized the expected rate of inflation as

the only relevant opportunity cost of holding money. This particular specifi-

cation can be, and has been, justified on the basis of at least two arguments:

first, in an inflationary environment, variations in the yields of other finan-

cial assets are likely to be dominated by variations in expected inflation.[3]

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[2]   For a review of this literature, see Laidler [1977].

[3]   An (extreme) example of this is in the case of full indexation of nominal
interest rates. To the extent that such indexation is implemented through a
Fisher-type equation, nominal interest rates would vary with movements in expected
(or actual) inflation so as to make real interest rates independent of inflation.

Second, the advent of high inflation has been accompanied in many countries by governmental imposition of financial controls, which in fact limited the availability of financial assets. In this case, the relevant substitution is between money and goods, or real assets, and it is captured mostly by the expected rate of inflation.

"Expected inflation" is generally an unobservable variable. Therefore, in order to empirically implement a money demand equation that includes this variable, an assumption about the formation of expectations is required. In the early literature on the subject, the common assumption was the one of adaptive expectations. As is well known by now, this assumption has at least two serious limitations: first, only information about past rates of inflation is assumed to affect agents' expectations of current and future inflation; second, the adaptive formula is arbitrarily postulated, without paying explicit attention to the actual process followed by the inflation rate. To overcome these difficulties, recent studies have postulated the rational-expectations hypothesis[4/] (due to Muth [1961]). According to this hypothesis, economic agents' expectations of inflation are assumed to be equivalent to optimal forecasts obtained by exploiting any systematic relationship between inflation and other relevant economic variables.

This paper reports new empirical evidence on the demand for money under rational expectations of inflation. This evidence is based on a sample of

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[4/]   See Sargent and Wallace [1973], Frenkel [1977], Sargent [1977], Evans [1978], Salemi and Sargent [1979], and Salemi [1979]. Interestingly, all these studies are based on data for the European hyperinflations.

quarterly data for Brazil covering the period from 1970 to 1977. Although
money demand in Brazil has previously been studied, this has been done for
earlier sample periods and only under the assumption of adaptive expectations.[5]
From a methodological point of view, this study extends most previous empiri-
cal studies of money demand under rational expectations in at least two impor-
tant directions. First, rather than postulating the rational-expectations
assumption and holding it as a maintained hypothesis, this study presents a
methodology for empirically testing the restrictions that rational-expectations
imposes on data.[6] Moreover, rather than proceeding with a unique a priori
specification of the rational expectations hypothesis, this study considers
the sensitivity of empirical evidence on the demand for money to alternative
versions of this hypothesis, and compares these findings with those obtained
by application of a simple version of adaptive expectations. Section 2 derives
the specifications of money demand to be estimated and discusses the estimation
procedure. Empirical results are reported in Section 3, and Section 4 presents
the main conclusions from this investigation.

2. Specifications and Methodology

As in most models of the demand for money in developing, and/or high-
inflation, countries we postulate that the amount of real cash balances demanded
by the public is a positive function of real income, and a negative function of
the expected rate of inflation.[7] Assuming a semi-log functional form, the

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5/   Previous estimates of money demand in Brazil are reported by Silveira
[1973], Khan [1977,1980] and Pastore et al [1977].

6/   Lucas [1972] has emphasized the emergence of cross-equation restrictions
in models embodying rational expectations; see also Hansen and Sargent [1980].

7/   For a recent discussion of the specification of money demand in the context
of developing countries, see Khan [1980]. The latter also reports empirical
estimates for 11 developing countries (including Brazil). The suitability of
equation (1) for the case of Brazil is discussed in Section 3.

the demand for money is written as:

$$(1) \quad m^d = \alpha + \beta y + \gamma \Pi^e + \mu$$

where m^d is the logarithm of real-money demand; y is the logarithm of real income; Π^e is the expected rate of inflation, and μ is a random variable with mean zero. As discussed above, the standard hypothesis is that $\beta > 0$ and $\gamma < 0$.

When non-negligible adjustment costs are present, it is likely that economic agents adjust gradually their actual real-money balances to the level implied by (1). A commonly specified adjustment process takes the form:[8]

$$(2) \quad (1-L)m = \lambda(m^d - m_{-1})$$

where L is the lag operator, which is defined by the operation $L^i X = X_{-i}$; and $0 \leq \lambda \leq 1$ is the adjustment coefficient. Substituting equation (1) into (2), and rearranging terms, yields:

$$(3) \quad m = \alpha\lambda + \beta\lambda y + \gamma\lambda \Pi^e + (1-\lambda)m_{-1} + \varepsilon$$

where $\varepsilon = \lambda\mu$.

Under the assumption that ε has classical properties, formally stated below, expression (3) can be regarded as a regression equation. To empirically implement it, however, a scheme for the formation of expected inflation, Π^e, is required. The assumption of rational expectations implies the condition[9]

$$(4) \quad \pi^e = E(\pi|\Omega) = \pi - \delta$$

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[8]   It is well to recognize that the choice of this particular form for the adjustment process is somewhat arbitrary, and it is mainly done here to gain comparability with previous studies. An interesting discussion of the limitations of this form, as well as of other popular adjustment processes, from the viewpoint of studying the dynamics of inflation is presented by Khan [1980].

[9]   The discussion below is similar to that presented by Sargent [1973] and McCallum [1977] in another context.

where $E(\pi|\Omega)$ denotes the expectation of π conditional on the past values of the set of variables included in the information set Ω, and δ is a random term orthogonal to Ω (i. e. $E(\delta|\Omega) = 0$). The specific composition of Ω will be discussed in section 3. Futhermore, under the assumption that the conditional expectation in (4) is linear, it follows that

(5) $E(\pi|\Omega) = \theta\Omega$

and hence

(6) $\pi = \theta\Omega + \delta$

where θ is a vector of least-squares coefficients conformable to Ω. That is, π^e is in effect formed as the prediction from a least-squares regression of π on Ω.

Using expressions (4) and (5), we can now eliminate the unobservable variable π^e appearing in (3), thus yielding the equation

(7) $m = \alpha\lambda + \beta\lambda y + \gamma\lambda\theta\Omega + (1-\lambda) m_{-1} + \varepsilon.$

Expression (7) is the real-money holdings equation to be estimated. Notice that the equation contains, in addition to the parameters entering the original money demand equation (1) and the process (2), the θ parameters that enter the inflation equation (6); these are the cross-equation (testable) restrictions implied by the rational expectations hypothesis.

For estimation purposes, the model to be implemented is composed by equations (6) and (7). The estimation method utilized here is as follows: given a sample of size T, and assuming that the error vectors $(\delta_\tau, \varepsilon_\tau)$, $\tau = 1,\ldots, T$, are drawn from a bivariate normal distribution, maximum likelihood estimates of the parameters can be obtained by minimizing $|\hat{V}|$, the determinant

of the covariance matrix of the residuals, over the unknown parameters (see

Dhrymes [1974, pp. 322-31]). Since the parameters enter $|\hat{V}|$ in a nonlinear

fashion, I have used Wymer's [1978] program RESIMUL, which is designed to

obtain full-information-maximum-likelihood (FIML) estimates of systems con-

taining nonlinear restrictions across and within equations. [10]

In estimating and testing the model, I proceeded as follows. For a

given specification of the information set Ω, the model is estimated twice:

first, subject to the rational expectations restrictions, and second, without

imposing these restrictions (that is, without imposing on (7) the θ coeffi-

cients from (6)). The empirical validity of the restrictions can be assessed

by a likelihood ratio test, based on the likelihood ratio statistic

$$LR=-2Log(Lr/Lu)$$

where Lr denotes the maximal value of the likelihood function under the

restrictions and Lu denotes its value when the restrictions are not imposed.

Under the null hypothesis that the restrictions are correct, LR is asymptotically

distributed as chi-square with q degrees of freedom, where q is the number of

restrictions imposed. In addition, since the different specifications of Ω

utilized below are nested, I will perform tests of exclusion restrictions on

the information set. Last, the system will also be estimated under the restric-

tions imposed by a version of adaptive expectations, and pertinent tests and

comparisons of parameter estimates will be done .

10/   The program uses a Newton-Raphson iterative procedure, beginning with
arbitrarily given initial parameter values.

3. Empirical Results

The estimates reported below are based on quarterly time series for Brazil for the 1970-1977 period (32 observations). During this period Brazil experienced a relatively high rate of inflation: 6% per-quarter on average (standard deviation = 0.026). While there is a strong presumption that equation (1) is a suitable money demand representation for developing, and high-inflation, countries, and despite the fact that previous studies for Brazil have postulated similar specifications (see Silveira [1973], Khan [1977, 1980], and Pastore et. al. [1977]), it is important to discuss its applicability for the sample in hand. During most of the 1970-77 period, financial markets in Brazil were characterized by a high degree of government intervention in the setting of key nominal interest rates, especially through the institutionalization of indexation of these interest rates.[11] It appears that the main objective of such indexation has been to make real rates of return on financial assets independent of fluctuations in inflation, and examination of the relevant evidence suggests that indeed

"...indexation played a prominent direct role in maintaining

the real value of some selected key financial instruments,

including government bonds, and indirectly maintained the

real value of virtually the whole spectrum of obligations

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[11]   Indexation of interest rates on financial assets started in 1964, and in 1975-76 complete liberalization of interest rates was accomplished. The following are some of the assets whose interest rates were indexed: certificates of deposit, bills of exchange, time deposits, treasure bonds, and savings accounts. For extensive discussions of financial-market indexation in Brazil, see Lemgruber [1977, pp. 413-17], Pastore [1977], and Galbis [1979].

of financial institutions and instruments outside the financial system." (Galbis [1979, p.346]). [12]

These considerations suggest that during the period under study Brazilians had the opportunity and economic incentive to hold a variety of indexed and nonindexed financial assets, in addition to money and goods (or real assets). Thus, it would seem more appropriate to include interest rates on these assets in the right hand side of equation (1). This was not done here for two practical reasons: first, inclusion of the relevant interest rates jointly with expected inflation is likely to lead to problems of multicollinearity. This is so because, as indicated above - and given the relatively high level and variability of inflation throughout the period under study, movements in nominal interest rates appear to have closely followed fluctuations in inflation. Second, there is a lack of readily available quarterly series on effective interest rates that would be reliable and consistent. (See Galbis [1979; Appendix]). Thus, the money demand estimates to be reported are based on equation (1), and obviously caution is suggested in their interpretation. In particular, the above discussion implies that the γ coefficient reflects not only the money/goods (or real assets) margin of substitution, but also, at least partially, the money/financial assets margin.

As previously indicated, equations (6) and (7) are estimated using a FIML procedure. To implement this procedure, it is required to specify the variables that are included in the information set Ω. Rather than a-priori specifying a unique set of variables entering Ω, I have considered several alternative estimates of the model. These alternative estimates are refered to below as options I through IV, and they correspond to the following plausible

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12/   For similar conclusions, see Lemgruber [1977, pp.416-7].

specifications of Ω:

$$\text{Option I} \quad - \Omega: (\Pi_{-i})$$

$$\text{Option II} \quad - \Omega: (\Pi_{-i}, \rho_{-i})$$

$$\text{Option III} - \Omega: (\Pi_{-i}, \eta_{-i})$$

$$\text{Option IV} \quad - \Omega: (\Pi_{-i}, \eta_{-i}, \rho_{-i})$$

$$i = 1,..,4$$

In the first option, only lagged values of inflation (Π_{-i}) are included; in other words, inflation is in this case "predicted" on the basis of its own history. The second option includes not only lagged inflation, but also lagged measures of external inflation (ρ_{-i}) which presumable affect inflation in the Brazilian open economy. The third option consists of inclusion of both lagged inflation and lagged measures of money growth (ρ_{-i}) in Ω. In the fourth option, both lagged external inflation and lagged money growth are jointly entered with lagged inflation. It can be seen that the different options here used are nested. In fact, any one of Options I, II, and III results from imposing exclusion restrictions on Option IV - which is the most general option - and testing these restrictions can provide some information on the relative empirical accuracy of the different options.

Table 1 reports the results of estimating equations (6) and (7) for each option by FIML. Most variables are represented by readily available series.[13/] In general, relatively good fits were obtained for all the options.

13/   Real money balances are represented by the sum of currency and demand deposits deflated by the consumer price index. Inflation is represented by the logarithmic first difference of the consumer price index. External inflation is measured by the logarithmic first difference of the U.S. C.P.I The source for all the above: International Financial Statistics, various issues. y is represented by the quarterly real income series constructed by Wachter [1976] (The series were extended through 1977 using exactly Wachter's method).

Table 1 - Alternative FIML Estimates of Equations

(6) and (7) for Brazil (1970-77, quarterly)*

Option I

(6) $\Pi_t = -0.01 + (0.81 - 0.47L + 0.52L^2 + 0.06L^3)\Pi_{t-1}$
$\quad\quad (0.01) \quad (0.17) \quad (0.20) \quad (0.20) \quad\quad (0.18)$

(7) $m_t = 0.66 + 0.22y_t - 2.09\Pi_t^e + 0.88m_{t-1}$
$\quad\quad (0.25) \quad (0.09) \quad (0.41) \quad\quad (0.07)$

$\quad\quad\quad \hat{R}^2(\Pi) = 0.65 \quad\quad\quad\quad \hat{R}^2(m) = 0.98$

Log likelihood Value = 247.28

Log Likelihood Value for Unrestricted System = 247.71

Likelihood Ratio Statistic to Test Rational Expectations = 0.86

Critical $\chi^2(3, 0.1)$ Value = 6.25

Option II

(6) $\Pi_t = 0.002 + (0.64 - 0.46L + 0.57L^2 + 0.23L^3)\Pi_{t-1} +$
$\quad\quad (0.01) \quad\quad (0.18) \quad (0.19) \quad (0.19) \quad\quad (0.22)$

$\quad\quad\quad\quad + (0.55 - 0.14L - 0.48L^2 + 0.34L^3)\rho_{t-1}$
$\quad\quad\quad\quad\quad (0.26)(0.30) \quad\quad (0.31) \quad\quad (0.22)$

(7) $m_t = 0.55 + 0.18y_t - 1.88\Pi_t^e + 0.92m_{t-1}$
$\quad\quad (0.24) \quad\quad (0.09) \quad\quad (0.39) \quad\quad (0.07)$
$\quad \hat{R}^2(\Pi) = 0.72 \quad\quad\quad\quad \hat{R}^2(m) = 0.98$

Log likelihood Value = 250.01

Log likelihood Value for Unrestricted System = 253.94

Likelihood Ratio Statistic to Test Rational Expectations = 7.86

Critical $\chi^2(7, 0.1)$ Value = 12.02

Option III

(6) $\Pi_t = 0.04 + (0.64 - 0.40L + 0.60L^2 + 0.09L^3)\Pi_{t-1} +$
$\quad\quad (0.01) \quad (0.16) \ (0.16) \ (0.16) \quad (0.16)$
$\quad\quad\quad\quad + (0.11 + 0.20L + 0.24L^2 + 0.05L^3)\eta_{t-1}$
$\quad\quad\quad\quad\quad (0.05) \ (0.05) \ (0.06) \quad (0.06)$

(7) $m_t = 0.68 + 0.24y_t - 2.29\Pi_t^e + 0.90m_{t-1}$
$\quad\quad (0.24) \ (0.08) \ (0.41) \quad (0.07)$

$\hat{R}^2(\Pi) = 0.81 \quad\quad\quad\quad \hat{R}^2(m) = 0.98$

Log likelihood Value = 258.10

Log likelihood Value for Unrestricted System = 259.76

Likelihood Ratio Statistic to Test Rational Expectations = 3.32

Critical $\chi^2(7, 0.1)$ Value = 12.02

Option IV

(6) $\Pi_t = 0.04 + (0.53 - 0.43L + 0.73L^2 + 0.16L^3)\Pi_{t-1} +$
$\quad\quad (0.01) \quad (0.14) \ (0.13) \ (0.14) \quad (0.15)$
$\quad\quad\quad\quad + (0.07 + 0.20L + 0.25L^2 + 0.01L^3)\eta_{t-1} +$
$\quad\quad\quad\quad\quad (0.04) \ (0.04) \ (0.05) \quad (0.05)$
$\quad\quad\quad\quad + (0.42 + 0.04L - 0.68L^2 + 0.48L^3)\rho_{t-1}$
$\quad\quad\quad\quad\quad (0.16) \ (0.19) \ (0.19) \quad (0.14)$

(7) $m_t = 0.64 + 0.21y_t - 2.20\Pi_t^e + 0.92m_{t-1}$
$\quad\quad (0.24) \ (0.08) \ (0.42) \quad (0.07)$

$\hat{R}^2(\Pi) = 0.88 \quad\quad\quad\quad \hat{R}^2(m) = 0.98$

Log likelihood Value = 265.24

Log likelihood Value for Unrestricted System = 270.46

Likelihood Ratio Statistic to Test Rational Expectations = 10.44

Critical $\chi^2(11, 0.1)$ Value = 17.28

(See note to Table 1 in next page)

Note to Table 1:

* The notation and explanation of the different options is presented

in the text. Figures in parentheses beneath regression coefficients

are asymptotic standard errors. L is the lag operator.

$R^2(z)$ is 1-(SSR/SST), where SSR: sum of squared residuals in equation

z, and SST: sample size times sample variance of dependent variable z.

Furthermore, examination of the residuals's autocorrelation function for each equation and option indicated that in all cases the residuals are white noise, so that no adjustment for serial correlation was required.[14/] It should be noted that seasonal dummies were included in equation (7), but for brevity their coefficients are not reported in the table. (No seasonal adjustments were required for the different versions of equation (6)).

The results for money demand (equation (7)) indicate that the estimated coefficients have the hypothesized signs and are significantly different from zero. The estimated coefficients for the real-income variable (that is, short-run income elasticities) are in the [0.18, 0.24] range, and are quite similar to those obtained in previous studies. The implied long-run income elasticities of money demand are [1.83,2.25,2.40,2.62] for Options I through IV respectively. The results also indicate that the expected rate of inflation has significant negative effects on real money holdings. The estimated coefficients on this variable are in the [-1.88,-2.29] range. These results imply long-run expected inflation semi-elasticities of [17.4,23.5,22.9,27.5], in absolute values, for options I through IV respectively, which (given the inflation rate sample mean) can be translated into long-run expected inflation elasticities of [1.0,1.4, 1.3, 1.6]. These estimates are much greater (in absolute value) than those generally found in previous studies. [15/] The fact, to be reported below, that estimations under adaptive expectations yield point estimates similar to the above for the present sample would seem to suggest that this difference in the estimated

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14/   These autocorrelation functions, and the F-statistics pertinent for testing the hypothesis of white noise residuals, are included in an Appendix which is available from the author upon request.

15/   For example, for quarterly results Silveira [1973] found a semi-elasticity of 1.2, and Khan [1977,1980] reports semi-elasticities of 1.3 (for the period 1960-70) and 4.0 (for the period 1962-76).

coefficient of expected inflation is presumably due to differences in the
sample periods utilized. It appears then that the degree of responsiveness
of money holders to fluctuations in expected inflation is an increasing
function of their familiarity with the inflationary process.[16]/ Regarding
the estimated coefficient of lagged real money balances, the implied mean
time lags [17]/ in the adjustment of real-money holdings to changes in income
and expected inflation range from a low of 7 quarters to a high of 11
quarters. That is, there appears to be a rather slow adjustment of actual
money holdings to changes in demand. This result is somewhat puzzling for
the case of a high-inflation country like Brazil; yet similar empirical evi-
dence has emerged from previous studies of money demand for developing as well
as developed economies (see Khan [1980, p.270]).

An important feature of these results for money demand is that they are
derived under the imposition of the restrictions implied by the rational
expectations hypothesis. Specifically, the components of Ω are assumed to
affect real-money holdings only through their effects on the "prediction" of
inflation, i.e. through the θ coefficients of equation (6). The compatibility
of the expectational assumption with the sample information can be assessed
by testing the implied restrictions. To do so, it is appropriate to estimate
also, for each option, in addition to (6), an unrestricted equation for real-
money holdings. The latter relaxes the restriction that the elements of Ω
enter as regressors only pre-multiplied by the θ coefficients. The restrictions
can then be tested by a likelihood-ratio test, based on a comparison of the fit

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16/ For some cross-sectional evidence on this issue, see Khan [1980, p. 271].

17/ These are calculated as $(1-\lambda)/\lambda$.

of the restricted and unrestricted equations, as explained in Section 2 above. Table 1 reports the likelihood ratios pertinent for testing the null hypothesis that the cross-equation restrictions imposed by rational expectations hold in the present sample. For all the options considered the test statistics obtain values that are smaller than the corresponding critical values. Thus, the results indicate that the null hypothesis is not rejected by the sample information at the usual 0.10 significance level.

Given that the estimation of equations (6) and (7) was performed for several alternative options, and that for each option the rational expectations' restrictions are not statistically rejected, it is interesting to compare the results of table 1 across options in terms of their main differentiating factor: the specification of equation (6) - the equation used in generating rational expectations of inflation.[18] The first option considers actual inflation as a function of four own lagged values. The results indicate that only the first three lags have in this case regression coefficients which significantly differ from zero, so that here an AR3 process is suggested by the data. Next, option II postulates for equation (6) a bivariate autoregression, where lagged values of external inflation are entered as regressors jointly with lagged inflation rates, and the results are as follows: while three own lags of inflation still obtain parameter estimates which are significant, only

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[18] In using equation (6) to generate expected inflation, the implicit assumption is that the θ coefficients are constant through time. I have checked this assumption by performing Chow-tests based on a mid-sample split (and on OLS versions of equation (6) and for all the options the pertinent F-tests indicated nonrejection of the null hypothesis. Obviously, due to the small number of degrees of freedom these tests should be interpreted with caution.

the first lag of external inflation has a significant, and positive, regression coefficient. A different result emerges for option III, where lagged money growth is entered instead of lagged external inflation. In this case, the coefficients on the first three lags of money growth are significant and positive, thus suggesting that in Brazil money growth is an important predictor of inflation. As before, the first three lags of inflation appear with significant regression coefficients. Option lV is the most general formulation in that it postulates the appearance of both lagged money growth and lagged external inflation in equation (6), in addition to lagged inflation. Here we find that for each one of these variables considered as regressors, there are at least three regression coefficients that are significantly different from zero.

What is the option most compatible with the sample information? As the different options are uniquely ordered (or nested), it is possible to provide a formal (statistical) answer to this question. Consider, first, the case of option I. This option results from imposing zero restrictions on the coefficients corresponding to the η and ρ variables of equation (6) - option IV. To test these restrictions, a LR statistic can be calculated on the basis of the log likelihood values obtained for these options and reported in Table 1. In the case of testing option I vs. option IV, LR obtains a value of 35.92, with 8 degrees of freedom. As the critical $\chi^2(8,0.1) = 13.36$, this test results in rejection of the restrictions imposed by option I on the more general formulation of option IV. When options II and III are similarly tested against option IV, the resulting LR's are 30.46 and 14.28 respectively, with 4 degrees of freedom in each case. The critical $\chi^2(4,0.1)=7.78$, so that these options are also rejected by the data. Summing up, the results indicate that, in addition to lagged inflation rates, lagged values of money growth and foreign

inflation appear to be important predictors of Brazilian inflation. Thus, these findings support the notion that these variables should be included in the information set used by agents in determining their rational forecast inflation. However, whether agents will <u>actually</u> use these variables in their information sets or not, remains as an open question, to be resolved only by exploring the specific costs and benefits associated with these informational items.[19]

The empirical evidence reported up to this point is based on specifications that embody the rational expectations hypothesis. In order to assess the sensitivity of the above results with respect to changes in this expectational assumption, I now turn to a discussion of the empirical evidence that emerges under an alternative hypothesis, the one of adaptive expectations. The model to be estimated now consists of equations (3) and (6), except that the inflation-expectations (Π^e) variable appearing in (3) is here replaced by a standard version of adaptive expectations, denoted by Π^{ae},

$$(8) \quad \Pi_t^{ae} = \psi \sum_{i=0}^{n} (1-\psi)^i \Pi_{t-i-1},$$

where ψ is the coefficient of adaptation of expectations. To gain comparability with the rational-expectations formulation in terms of the information set used, I have set for empirical purposes $n=3$ so that only four lagged values of Π are used in the expectational variable (as in the case of option I above). With these specifications, equations (3) and (6) were jointly estimated by FIML for different values of ψ varying over the (0,1) interval. It turns out that the $\psi=0.3$ value maximized the log likelihood function, and the resulting estimates are reported in table 2.

19/ For a discussion of this issue, and of the concept of economically rational expectations, see Feige and Pearce [1976].

Table 2 - FIML Estimates of Equations (6) and (3)

Under Adaptive Expectations (Eq. (8)) for

Brazil (1970-77, quarterly)*

(6) $\Pi_t = -0.01 + (0.72 - 0.31L + 0.48L^2 - 0.02L^3)\Pi_{t-1}$

$\qquad$ (0.01) $\quad$ (0.16) (0.19) $\quad$ (0.19) $\quad$ (0.17)

(3) $m_t = 0.85 + 0.31y_t - 2.74\Pi_t^{ae} + 0.81m_{t-1}$

$\qquad$ (0.32) (0.12) (0.70) $\quad$ (0.09)

$\qquad$ $\hat{R}^2(\Pi) = 0.64$ $\qquad\qquad$ $\hat{R}^2(m) = 0.97$

Log likelihood Value = 244.18

Log likelihood Value for Unrestricted System = 247.71

Likelihood Ratio Statistic to Test Adaptive Expectations = 7.06

Critical $\chi^2(3,0.1)$ Value = 6.25

Note to Table 2:

* The notation and explanations are provided in the text. The estimates

reported are obtained by imposing the $\psi = 0.3$ value for the adaptation

coefficient. Figures in parentheses beneath regression coefficients are

asymptotic standard errors. L is the lag operator, and $\hat{R}^2$ has been defi-

ned in the Note to Table 1.

Comparison of the above results for money demand with those obtained under rational expectations suggests that the main differences in parameter estimates are as follows: income and expected inflation elasticities are greater, in absolute value, for adaptive than for rational expectations. Also, under adaptive expectations the coefficient on the lagged dependent variable is smaller than those of Table 1, thus implying that the adjustment of actual money balances to changes in desired balances is shorter under the adaptive-expectations formulation .

Despite these differences, the overall impression from these findings is that the parameter estimates of Brazilian money demand under adaptive expectations are quite similar to those based on rational expectations. However, it should be noted that the log likelihood value for the system of table 2 is smaller than its rational-expectations counterpart of option I - table 1. Furthermore, the assumption of adaptive expectations imposes restrictions on data, which are readily testable. Concretely, this assumption asserts that lagged inflation rates enter in equation (3) only through the ψ coefficients of equation (8). The system can also be estimated under relaxation of this restriction, and a LR statistic can then be calculated. In this case, LR obtains a value of 7.06 (reported in table 2), and since the critical $\chi^2(3,0.1) = 6.25$, the likelihood ratio test results in rejection of the restrictions imposed by adaptive expectations on the data.

4. Conclusions

This paper presented a methodology for estimating and testing the demand for money under rational expectations of inflation. As usually implemented, the assumption of rational expectations requires specifying the information set used by agents in forming their expectations. An important feature of the analysis above is that empirical evidence was presented for several plausible

specifications of the information set, which amount to alternative operational
versions of the rational expectations hypothesis.

At the empirical level, the econometric analysis was based on quarterly
data for the high-inflation economy of Brazil (1970-77). The parameter esti-
mates reported for money demand turn out to imply expected-inflation elastici-
ties that are substantially larger than those found in previous studies of
Brazil. This finding holds for specifications using rational as well as
adaptive expectations of inflation, and therefore it appears that this di-
fference in the results is mainly due to differences in the sample periods
utilized. On the other hand, the reported estimates of the other coefficients
entering money demand are quite similar to those appearing in previous work.
Regarding the alternative specifications (or options) considered for the infor-
mation set used in forming expectations, some evidence was presented in support
of the notion that lagged money growth and external inflation contain, in addi-
tion to lagged inflation rates, important information for predicting future
inflation in Brazil.

Particular emphasis was given in the empirical analysis to testing the
cross-equation restrictions implied by the rational-expectations hypothesis.
The empirical findings on this issue indicated that for each one of the four
options postulated for the information set, these restrictions were not statis-
tically rejected by the data. Yet when comparisons were made across options,
the results indicated support to the broadest option utilized (that is, the
one that specifies that inflation expectations are formed on the basis of infor-
mation about the past behavior of inflation, money growth, and external infla-
tion). In addition to considering evidence under rational expectations, the
basic system was also estimated under the assumption of adaptive expectations.

In this case, econometric tests of the cross-equation restrictions implied by this assumption resulted in rejection of these restrictions.

All in all, then, the Brazilian data considered appear to support a specification of the demand for money that embodies rational expectations of inflation. However, caution is suggested in using the results reported in this paper to draw firm conclusions on the debate of adaptive vs. rational expectations. This is so because it is well to recognize that, for purposes of empirical tractability, the expectational specifications used above (both for adaptive and rational expectations) are very simple ones. For example, in some cases it may be plausible to assume that the adaptation coefficient, used under adaptive expectations, is not a constant, but endogenously varies through time (see e.g. Khan [1977]). Also, it may well be that, although having rational expectations of inflation, agents learn only gradually about the form of the optimal inflation predictor (equation (6)). Only by considering these more complex formulations, as well as data for several countries, one would be able to reach final judgements on the accuracy of adaptive vs. rational expectations.

Appendix- Residuals' Autoregressions for the Equation of Table 1

(Estimated equation: $\xi_t = \rho_1 \xi_{t-1} + \rho_2 \xi_{t-2} + \rho_3 \xi_{t-3} + \rho_4 \xi_{t-4}$)

	Option I		Option II		Option III		Option IV	
	EQ. (6)	EQ. (7)	EQ. (6)	EQ. (7)	EQ. (6)	EQ. (7)	EQ. (6)	EQ. (7)
Parameter								
ρ_1	0.07 (0.20)	-0.37 (0.21)	0.12 (0.20)	-0.31 (0.21)	0.02 (0.20)	-0.34 (0.21)	0.16 (0.19)	-0.32 (0.21)
ρ_2	0.08 (0.20)	-0.12 (0.21)	0.04 (0.20)	-0.16 (0.20)	-0.17 (0.21)	-0.10 (0.22)	-0.20 (0.20)	-0.03 (0.21)
ρ_3	-0.14 (0.20)	-0.30 (0.22)	-0.25 (0.20)	-0.41 (0.20)	-0.17 (0.21)	-0.24 (0.23)	-0.05 (0.20)	-0.29 (0.22)
ρ_4	0.02 (0.20)	-0.09 (0.22)	0.10 (0.20)	-0.08 (0.21)	-0.10 (0.21)	-0.08 (0.33)	-0.06 (0.19)	-0.16 (0.22)
Statistic								
R^2	0.03	0.20	0.08	0.21	0.06	0.15	0.07	0.17
DW	2.01	2.03	1.96	2.01	1.95	2.04	1.95	2.11
$F_{(4,24)}$	0.18	1.50	0.48	1.50	0.36	1.20	0.42	1.20

Note: ξ denotes the residual in a given equation of Table 1. Figures in parentheses are standard errors of regression coefficients.

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