44

Boston University

Center for Latin American Development Studies

THE DEMAND FOR MONEY UNDER RATIONAL EXPECTATIONS OF INFLATION: FIML ESTIMATES FOR BRAZIL

by

Leonardo Leiderman*

*Boston University and Tel-Aviv University

Discussion Paper Series
Number 44
August 1980

THE DEMAND FOR MONEY UNDER RATIONAL EXPECTATIONS OF INFLATION: FIML ESTIMATES FOR BRAZIL

by

Leonardo Leiderman*

* The author is a Research Associate at the Center for Latin American Development Studies of Boston University, whose support is gratefully acknowledged.

The Demand for Money Under Rational Expectations of Inflation: FIML Estimates for Brazil

by

Leonardo Leiderman [1/]

1. Introduction

Ever since the important contribution by Cagan [1956], a large part of the empirical literature on the demand for money has been based on the experience of economies operating under highly inflationary conditions.[2/] At the theoretical level, it is generally postulated that the amount of real money balances demanded by the public depends on the level of real income and on the opportunity cost of holding money relative to other assets and goods. The opportunity cost consists of the yield on alternative assets, measured by their expected rates of interest, and the return on holding goods, measured by the expected rate of inflation. Yet empirically, when studying economies with substantial inflation, most researchers have utilized the expected rate of inflation as the only relevant opportunity cost of holding money. This particular specification can be, and has been, justified on the basis of at least two arguments: first, in an inflationary environment, variations in the yields of other financial assets are likely to be dominated by variations in expected inflation.[3/]

1/ I would like to thank Phillip Cagan, and especially an anonymous referee for many valuable comments. Any errors are mine.

2/ For a review of this literature, see Laidler [1977].

3/ An (extreme) example of this is in the case of full indexation of nominal interest rates. To the extent that such indexation is implemented through a Fisher-type equation, nominal interest rates would vary with movements in expected (or actual) inflation so as to make real interest rates independent of inflation.

Second, the advent of high inflation has been accompanied in many countries by governmental imposition of financial controls, which in fact limited the availability of financial assets. In this case, the relevant substitution is between money and goods, or real assets, and it is captured mostly by the expected rate of inflation.

"Expected inflation" is generally an unobservable variable. Therefore, in order to empirically implement a money demand equation that includes this variable, an assumption about the formation of expectations is required. In the early literature on the subject, the common assumption was the one of adaptive expectations. As is well known by now, this assumption has at least two serious limitations: first, only information about past rates of inflation is assumed to affect agents' expectations of current and future inflation; second, the adaptive formula is arbitrarily postulated, without paying explicit attention to the actual process followed by the inflation rate. To overcome these difficulties, recent studies have postulated the rational-expectations hypothesis[4/] (due to Muth [1961]). According to this hypothesis, economic agents' expectations of inflation are assumed to be equivalent to optimal forecasts obtained by exploiting any systematic relationship between inflation and other relevant economic variables.

This paper reports new empirical evidence on the demand for money under rational expectations of inflation. This evidence is based on a sample of

---

4/ See Sargent and Wallace [1973], Frenkel [1977], Sargent [1977], Evans [1978], Salemi and Sargent [1979], and Salemi [1979]. Interestingly, all these studies are based on data for the European hyperinflations.

quarterly data for Brazil covering the period from 1970 to 1977. Although money demand in Brazil has previously been studied, this has been done for earlier sample periods and only under the assumption of adaptive expectations.[5/] From a methodological point of view, this study extends most previous empirical studies of money demand under rational expectations in at least two important directions. First, rather than postulating the rational-expectations assumption and holding it as a maintained hypothesis, this study presents a methodology for empirically testing the restrictions that rational-expectations imposes on data.[6/] Moreover, rather than proceeding with a unique a priori specification of the rational expectations hypothesis, this study considers the sensitivity of empirical evidence on the demand for money to alternative versions of this hypothesis, and compares these findings with those obtained by application of a simple version of adaptive expectations. Section 2 derives the specifications of money demand to be estimated and discusses the estimation procedure. Empirical results are reported in Section 3, and Section 4 presents the main conclusions from this investigation.

2. Specifications and Methodology

As in most models of the demand for money in developing, and/or high-inflation, countries we postulate that the amount of real cash balances demanded by the public is a positive function of real income, and a negative function of the expected rate of inflation.[7/] Assuming a semi-log functional form, the

---

5/ Previous estimates of money demand in Brazil are reported by Silveira [1973], Khan [1977,1980] and Pastore <u>et al</u> [1977].

6/ Lucas [1972] has emphasized the emergence of cross-equation restrictions in models embodying rational expectations; see also Hansen and Sargent [1980].

7/ For a recent discussion of the specification of money demand in the context of developing countries, see Khan [1980]. The latter also reports empirical estimates for 11 developing countries (including Brazil). The suitability of equation (1) for the case of Brazil is discussed in Section 3.

the demand for money is written as:

(1) $m^d = \alpha + \beta y + \gamma \Pi^e + \mu$

where m^d is the logarithm of real-money demand; y is the logarithm of real income; Π^e is the expected rate of inflation, and μ is a random variable with mean zero. As discussed above, the standard hypothesis is that $\beta > 0$ and $\gamma < 0$.

When non-negligible adjustment costs are present, it is likely that economic agents adjust gradually their actual real-money balances to the level implied by (1). A commonly specified adjustment process takes the form:[8/]

(2) $(1-L)m = \lambda(m^d - m_{-1})$

where L is the lag operator, which is defined by the operation $L^i X = X_{-i}$; and $0 \leq \lambda \leq 1$ is the adjustment coefficient. Substituting equation (1) into (2), and rearranging terms, yields:

(3) $m = \alpha\lambda + \beta\lambda y + \gamma\lambda \Pi^e + (1-\lambda)m_{-1} + \varepsilon$

where $\varepsilon = \lambda\mu$.

Under the assumption that ε has classical properties, formally stated below, expression (3) can be regarded as a regression equation. To empirically implement it, however, a scheme for the formation of expected inflation, Π^e, is required. The assumption of rational expectations implies the condition[9/]

(4) $\pi^e = E(\pi|\Omega) = \pi - \delta$

8/ It is well to recognize that the choice of this particular form for the adjustment process is somewhat arbitrary, and it is mainly done here to gain comparability with previous studies. An interesting discussion of the limitations of this form, as well as of other popular adjustment processes, from the viewpoint of studying the dynamics of inflation is presented by Khan [1980].

9/ The discussion below is similar to that presented by Sargent [1973] and McCallum [1977] in another context.

where $E(\pi|\Omega)$ denotes the expectation of π conditional on the past values of the set of variables included in the information set Ω, and δ is a random term orthogonal to Ω (i. e. $E(\delta|\Omega) = 0$). The specific composition of Ω will be discussed in section 3. Futhermore, under the assumption that the conditional expectation in (4) is linear, it follows that

(5) $E(\pi|\Omega) = \theta\Omega$

and hence

(6) $\pi = \theta\Omega + \delta$

where θ is a vector of least-squares coefficients conformable to Ω. That is, π^e is in effect formed as the prediction from a least-squares regression of π on Ω.

Using expressions (4) and (5), we can now eliminate the unobservable variable π^e appearing in (3), thus yielding the equation

(7) $m = \alpha\lambda + \beta\lambda y + \gamma\lambda\theta\Omega + (1-\lambda)\, m_{-1} + \varepsilon.$

Expression (7) is the real-money holdings equation to be estimated. Notice that the equation contains, in addition to the parameters entering the original money demand equation (1) and the process (2), the θ parameters that enter the inflation equation (6); these are the cross-equation (testable) restrictions implied by the rational expectations hypothesis.

For estimation purposes, the model to be implemented is composed by equations (6) and (7). The estimation method utilized here is as follows: given a sample of size T, and assuming that the error vectors $(\delta_\tau, \varepsilon_\tau)$, $\tau = 1,\ldots, T$, are drawn from a bivariate normal distribution, maximum likelihood estimates of the parameters can be obtained by minimizing $|\hat{V}|$, the determinant of the covariance matrix of the residuals, over the unknown parameters (see Dhrymes [1974, pp. 322-31]). Since the parameters enter $|\hat{V}|$ in a nonlinear fashion, I have used Wymer's [1978] program RESIMUL, which is designed to obtain full-information-maximum-likelihood (FIML) estimates of systems containing nonlinear restrictions across and within equations. 10/

In estimating and testing the model, I proceeded as follows. For a given specification of the information set Ω, the model is estimated twice: first, subject to the rational expectations restrictions, and second, without imposing these restrictions (that is, without imposing on (7) the θ coefficients from (6)). The empirical validity of the restrictions can be assessed by a likelihood ratio test, based on the likelihood ratio statistic

$$LR=-2\text{Log}(Lr/Lu)$$

where Lr denotes the maximal value of the likelihood function under the restrictions and Lu denotes its value when the restrictions are not imposed. Under the null hypothesis that the restrictions are correct, LR is asymptotically distributed as chi-square with q degrees of freedom, where q is the number of restrictions imposed. In addition, since the different specifications of Ω utilized below are nested, I will perform tests of exclusion restrictions on the information set. Last, the system will also be estimated under the restrictions imposed by a version of adaptive expectations, and pertinent tests and comparisons of parameter estimates will be done .

10/ The program uses a Newton-Raphson iterative procedure, beginning with arbitrarily given initial parameter values.

3. Empirical Results

The estimates reported below are based on quarterly time series for Brazil for the 1970-1977 period (32 observations). During this period Brazil experienced a relatively high rate of inflation: 6% per-quarter on average (standard deviation = 0.026). While there is a strong presumption that equation (1) is a suitable money demand representation for developing, and high-inflation, countries, and despite the fact that previous studies for Brazil have postulated similar specifications (see Silveira [1973], Khan [1977, 1980], and Pastore *et. al.* [1977]), it is important to discuss its applicability for the sample in hand. During most of the 1970-77 period, financial markets in Brazil were characterized by a high degree of government intervention in the setting of key nominal interest rates, especially through the institutionalization of indexation of these interest rates.[11/] It appears that the main objective of such indexation has been to make real rates of return on financial assets independent of fluctuations in inflation, and examination of the relevant evidence suggests that indeed

> "...indexation played a prominent direct role in maintaining the real value of some selected key financial instruments, including government bonds, and indirectly maintained the real value of virtually the whole spectrum of obligations

---

11/ Indexation of interest rates on financial assets started in 1964, and in 1975-76 complete liberalization of interest rates was accomplished. The following are some of the assets whose interest rates were indexed: certificates of deposit, bills of exchange, time deposits, treasure bonds, and savings accounts. For extensive discussions of financial-market indexation in Brazil, see Lemgruber [1977,pp. 413-17], Pastore [1977], and Galbis [1979].

of financial institutions and instruments outside the financial system." (Galbis [1979, p.346]). 12/

These considerations suggest that during the period under study Brazilians had the opportunity and economic incentive to hold a variety of indexed and nonindexed financial assets, in addition to money and goods (or real assets). Thus, it would seem more appropriate to include interest rates on these assets in the right hand side of equation (1). This was not done here for two practical reasons: first, inclusion of the relevant interest rates jointly with expected inflation is likely to lead to problems of multicollinearity. This is so because, as indicated above - and given the relatively high level and variability of inflation throughout the period under study, movements in nominal interest rates appear to have closely followed fluctuations in inflation. Second, there is a lack of readily available quarterly series on effective interest rates that would be reliable and consistent. (See Galbis [1979; Appendix]). Thus, the money demand estimates to be reported are based on equation (1), and obviously caution is suggested in their interpretation. In particular, the above discussion implies that the γ coefficient reflects not only the money/goods (or real assets) margin of substitution, but also, at least partially, the money/financial assets margin.

As previously indicated, equations (6) and (7) are estimated using a FIML procedure. To implement this procedure, it is required to specify the variables that are included in the information set Ω. Rather than a-priori specifying a unique set of variables entering Ω, I have considered several alternative estimates of the model. These alternative estimates are refered to below as options I through IV, and they correspond to the following plausible

---

12/ For similar conclusions, see Lemgruber [1977, pp.416-7].

specifications of Ω:

$$\text{Option I} - \Omega: (\Pi_{-i})$$

$$\text{Option II} - \Omega: (\Pi_{-i}, \rho_{-i})$$

$$\text{Option III} - \Omega: (\Pi_{-i}, \eta_{-i})$$

$$\text{Option IV} - \Omega: (\Pi_{-i}, \eta_{-i}, \rho_{-i})$$

$$i = 1,..,4$$

In the first option, only lagged values of inflation (Π_{-i}) are included; in other words, inflation is in this case "predicted" on the basis of its own history. The second option includes not only lagged inflation, but also lagged measures of external inflation (ρ_{-i}) which presumable affect inflation in the Brazilian open economy. The third option consists of inclusion of both lagged inflation and lagged measures of money growth (ρ_{-i}) in Ω. In the fourth option, both lagged external inflation and lagged money growth are jointly entered with lagged inflation. It can be seen that the different options here used are nested. In fact, any one of Options I, II, and III results from imposing exclusion restrictions on Option IV - which is the most general option - and testing these restrictions can provide some information on the relative empirical accuracy of the different options.

Table 1 reports the results of estimating equations (6) and (7) for each option by FIML. Most variables are represented by readily available series.[13/] In general, relatively good fits were obtained for all the options.

13/ Real money balances are represented by the sum of currency and demand deposits deflated by the consumer price index. Inflation is represented by the logarithmic first difference of the consumer price index. External inflation is measured by the logarithmic first difference of the U.S. C.P.I The source for all the above: International Financial Statistics, various issues. y is represented by the quarterly real income series constructed by Wachter [1976] (The series were extended through 1977 using exactly Wachter's method).

Table 1 - Alternative FIML Estimates of Equations (6) and (7) for Brazil (1970-77, quarterly)*

Option I

$$(6)\quad \Pi_t = \underset{(0.01)}{-0.01} + (\underset{(0.17)}{0.81} - \underset{(0.20)}{0.47L} + \underset{(0.20)}{0.52L^2} + \underset{(0.18)}{0.06L^3})\Pi_{t-1}$$

$$(7)\quad m_t = \underset{(0.25)}{0.66} + \underset{(0.09)}{0.22y_t} \underset{(0.41)}{-2.09\Pi_t^e} + \underset{(0.07)}{0.88m_{t-1}}$$

$\hat{R}^2(\Pi) = 0.65$ $\qquad$ $\hat{R}^2(m) = 0.98$

Log likelihood Value = 247.28

Log Likelihood Value for Unrestricted System = 247.71

Likelihood Ratio Statistic to Test Rational Expectations = 0.86

Critical $\chi^2(3, 0.1)$ Value = 6.25

Option II

$$(6)\quad \Pi_t = \underset{(0.01)}{0.002} + (\underset{(0.18)}{0.64} - \underset{(0.19)}{0.46L} + \underset{(0.19)}{0.57L^2} + \underset{(0.22)}{0.23L^3})\Pi_{t-1} + (\underset{(0.26)}{0.55} \underset{(0.30)}{-0.14L} - \underset{(0.31)}{0.48L^2} + \underset{(0.22)}{0.34L^3})\rho_{t-1}$$

$$(7)\quad m_t = \underset{(0.24)}{0.55} + \underset{(0.09)}{0.18y_t} - \underset{(0.39)}{1.88\Pi_t^e} + \underset{(0.07)}{0.92m_{t-1}}$$

$\hat{R}^2(\Pi) = 0.72$ $\qquad$ $\hat{R}^2(m) = 0.98$

Log likelihood Value = 250.01

Log likelihood Value for Unrestricted System = 253.94

Likelihood Ratio Statistic to Test Rational Expectations = 7.86

Critical $\chi^2(7, 0.1)$ Value = 12.02

Option III

$$(6)\quad \Pi_t = \underset{(0.01)}{0.04} + (\underset{(0.16)}{0.64} - \underset{(0.16)}{0.40L} + \underset{(0.16)}{0.60L^2} + \underset{(0.16)}{0.09L^3})\Pi_{t-1} + (\underset{(0.05)}{0.11} + \underset{(0.05)}{0.20L} + \underset{(0.06)}{0.24L^2} + \underset{(0.06)}{0.05L^3})\eta_{t-1}$$

$$(7)\quad m_t = \underset{(0.24)}{0.68} + \underset{(0.08)}{0.24y_t} - \underset{(0.41)}{2.29\Pi^e_t} + \underset{(0.07)}{0.90m_{t-1}}$$

$\hat{R}^2(\Pi) = 0.81$ $\hat{R}^2(m) = 0.98$

Log likelihood Value = 258.10

Log likelihood Value for Unrestricted System = 259.76

Likelihood Ratio Statistic to Test Rational Expectations = 3.32

Critical χ^2(7,0.1) Value = 12.02

Option IV

$$(6)\quad \Pi_t = \underset{(0.01)}{0.04} + (\underset{(0.14)}{0.53} - \underset{(0.13)}{0.43L} + \underset{(0.14)}{0.73L^2} + \underset{(0.15)}{0.16L^3})\Pi_{t-1} + (\underset{(0.04)}{0.07} + \underset{(0.04)}{0.20L} + \underset{(0.05)}{0.25L^2} + \underset{(0.05)}{0.01L^3})\eta_{t-1} + (\underset{(0.16)}{0.42} + \underset{(0.19)}{0.04L} - \underset{(0.19)}{0.68L^2} + \underset{(0.14)}{0.48L^3})\rho_{t-1}$$

$$(7)\quad m_t = \underset{(0.24)}{0.64} + \underset{(0.08)}{0.21y_t} - \underset{(0.42)}{2.20\Pi^e_t} + \underset{(0.07)}{0.92m_{t-1}}$$

$\hat{R}^2(\Pi) = 0.88$ $\hat{R}^2(m) = 0.98$

Log likelihood Value = 265.24

Log likelihood Value for Unrestricted System = 270.46

Likelihood Ratio Statistic to Test Rational Expectations = 10.44

Critical χ^2(11,0.1) Value = 17.28

(See note to Table 1 in next page)

Note to Table 1:

* The notation and explanation of the different options is presented in the text. Figures in parentheses beneath regression coefficients are asymptotic standard errors. L is the lag operator.

$R^2(z)$ is 1-(SSR/SST), where SSR: sum of squared residuals in equation z, and SST: sample size times sample variance of dependent variable z.

Furthermore, examination of the residuals's autocorrelation function for each equation and option indicated that in all cases the residuals are white noise, so that no adjustment for serial correlation was required. 14/ It should be noted that seasonal dummies were included in equation (7), but for brevity their coefficients are not reported in the table. (No seasonal adjustments were required for the different versions of equation (6)).

The results for money demand (equation (7)) indicate that the estimated coefficients have the hypothesized signs and are significantly different from zero. The estimated coefficients for the real-income variable (that is, short-run income elasticities) are in the [0.18, 0.24] range, and are quite similar to those obtained in previous studies. The implied long-run income elasticities of money demand are [1.83,2.25,2.40,2.62] for Options I through IV respectively. The results also indicate that the expected rate of inflation has significant negative effects on real money holdings. The estimated coefficients on this variable are in the [-1.88,-2.29] range. These results imply long-run expected inflation semi-elasticities of [17.4,23.5,22.9,27.5], in absolute values, for options I through IV respectively, which (given the inflation rate sample mean) can be translated into long-run expected inflation elasticities of [1.0,1.4, 1.3, 1.6]. These estimates are much greater (in absolute value) than those generally found in previous studies. 15/ The fact, to be reported below, that estimations under adaptive expectations yield point estimates similar to the above for the present sample would seem to suggest that this difference in the estimated

---

14/ These autocorrelation functions, and the F-statistics pertinent for testing the hypothesis of white noise residuals, are included in an Appendix which is available from the author upon request.

15/ For example, for quarterly results Silveira [1973] found a semi-elasticity of 1.2, and Khan [1977,1980] reports semi-elasticities of 1.3 (for the period 1960-70) and 4.0 (for the period 1962-76).

coefficient of expected inflation is presumably due to differences in the sample periods utilized. It appears then that the degree of responsiveness of money holders to fluctuations in expected inflation is an increasing function of their familiarity with the inflationary process.[16/] Regarding the estimated coefficient of lagged real money balances, the implied mean time lags [17/] in the adjustment of real-money holdings to changes in income and expected inflation range from a low of 7 quarters to a high of 11 quarters. That is, there appears to be a rather slow adjustment of actual money holdings to changes in demand. This result is somewhat puzzling for the case of a high-inflation country like Brazil; yet similar empirical evidence has emerged from previous studies of money demand for developing as well as developed economies (see Khan [1980, p.270]).

An important feature of these results for money demand is that they are derived under the imposition of the restrictions implied by the rational expectations hypothesis. Specifically, the components of Ω are assumed to affect real-money holdings only through their effects on the "prediction" of inflation, i.e. through the θ coefficients of equation (6). The compatibility of the expectational assumption with the sample information can be assessed by testing the implied restrictions. To do so, it is appropriate to estimate also, for each option, in addition to (6), an unrestricted equation for real-money holdings. The latter relaxes the restriction that the elements of Ω enter as regressors only pre-multiplied by the θ coefficients. The restrictions can then be tested by a likelihood-ratio test, based on a comparison of the fit

---

16/ For some cross-sectional evidence on this issue, see Khan [1980, p. 271].

17/ These are calculated as $(1-\lambda)/\lambda$.

of the restricted and unrestricted equations, as explained in Section 2 above. Table 1 reports the likelihood ratios pertinent for testing the null hypothesis that the cross-equation restrictions imposed by rational expectations hold in the present sample. For all the options considered the test statistics obtain values that are smaller than the corresponding critical values. Thus, the results indicate that the null hypothesis is not rejected by the sample information at the usual 0.10 significance level.

Given that the estimation of equations (6) and (7) was performed for several alternative options, and that for each option the rational expectations' restrictions are not statistically rejected, it is interesting to compare the results of table 1 across options in terms of their main differentiating factor: the specification of equation (6) - the equation used in generating rational expectations of inflation.[18/] The first option considers actual inflation as a function of four own lagged values. The results indicate that only the first three lags have in this case regression coefficients which significantly differ from zero, so that here an AR3 process is suggested by the data. Next, option II postulates for equation (6) a bivariate autoregression, where lagged values of external inflation are entered as regressors jointly with lagged inflation rates, and the results are as follows: while three own lags of inflation still obtain parameter estimates which are significant, only

---

18/ In using equation (6) to generate expected inflation, the implicit assumption is that the θ coefficients are constant through time. I have checked this assumption by performing Chow-tests based on a mid-sample split (and on OLS versions of equation (6) and for all the options the pertinent F-tests indicated nonrejection of the null hypothesis. Obviously, due to the small number of degrees of freedom these tests should be interpreted with caution.

the first lag of external inflation has a significant, and positive, regression coefficient. A different result emerges for option III, where lagged money growth is entered instead of lagged external inflation. In this case, the coefficients on the first three lags of money growth are significant and positive, thus suggesting that in Brazil money growth is an important predictor of inflation. As before, the first three lags of inflation appear with significant regression coefficients. Option lV is the most general formulation in that it postulates the appearance of both lagged money growth and lagged external inflation in equation (6), in addition to lagged inflation. Here we find that for each one of these variables considered as regressors, there are at least three regression coefficients that are significantly different from zero.

What is the option most compatible with the sample information? As the different options are uniquely ordered (or nested), it is possible to provide a formal (statistical) answer to this question. Consider, first, the case of option I. This option results from imposing zero restrictions on the coefficients corresponding to the η and ρ variables of equation (6) - option IV. To test these restrictions, a LR statistic can be calculated on the basis of the log likelihood values obtained for these options and reported in Table 1. In the case of testing option I vs. option IV, LR obtains a value of 35.92, with 8 degrees of freedom. As the critical $\chi^2(8,0.1) = 13.36$, this test results in rejection of the restrictions imposed by option I on the more general formulation of option IV. When options II and III are similarly tested against option IV, the resulting LR's are 30.46 and 14.28 respectively, with 4 degrees of freedom in each case. The critical $\chi^2(4,0.1)=7.78$, so that these options are also rejected by the data. Summing up, the results indicate that, in addition to lagged inflation rates, lagged values of money growth and foreign inflation appear to be important predictors of Brazilian inflation. Thus, these findings support the notion that these variables should be included in the information set used by agents in determining their rational forecast inflation. However, whether agents will <u>actually</u> use these variables in their information sets or not, remains as an open question, to be resolved only by exploring the specific costs and benefits associated with these informational items.[19/]

The empirical evidence reported up to this point is based on specifications that embody the rational expectations hypothesis. In order to assess the sensitivity of the above results with respect to changes in this expectational assumption, I now turn to a discussion of the empirical evidence that emerges under an alternative hypothesis, the one of adaptive expectations. The model to be estimated now consists of equations (3) and (6), except that the inflation-expectations (Π^e) variable appearing in (3) is here replaced by a standard version of adaptive expectations, denoted by Π^{ae},

$$(8) \quad \Pi_t^{ae} = \psi \sum_{i=0}^{n} (1-\psi)^i \Pi_{t-i-1},$$

where ψ is the coefficient of adaptation of expectations. To gain comparability with the rational-expectations formulation in terms of the information set used, I have set for empirical purposes $n=3$ so that only four lagged values of Π are used in the expectational variable (as in the case of option I above). With these specifications, equations (3) and (6) were jointly estimated by FIML for different values of ψ varying over the (0,1) interval. It turns out that the $\psi=0.3$ value maximized the log likelihood function, and the resulting estimates are reported in table 2.

19/ For a discussion of this issue, and of the concept of economically rational expectations, see Feige and Pearce [1976].

Table 2 - FIML Estimates of Equations (6) and (3) Under Adaptive Expectations (Eq. (8)) for Brazil (1970-77, quarterly)*

(6) $\Pi_t = -0.01 + (0.72 - 0.31L + 0.48L^2 - 0.02L^3)\Pi_{t-1}$

(0.01) (0.16) (0.19) (0.19) (0.17)

(3) $m_t = 0.85 + 0.31y_t - 2.74\Pi_t^{ae} + 0.81m_{t-1}$

(0.32) (0.12) (0.70) (0.09)

$\hat{R}^2(\Pi) = 0.64$ $\hat{R}^2(m) = 0.97$

Log likelihood Value = 244.18

Log likelihood Value for Unrestricted System = 247.71

Likelihood Ratio Statistic to Test Adaptive Expectations = 7.06

Critical $\chi^2(3,0.1)$ Value = 6.25

Note to Table 2:

* The notation and explanations are provided in the text. The estimates reported are obtained by imposing the $\psi = 0.3$ value for the adaptation coefficient. Figures in parentheses beneath regression coefficients are asymptotic standard errors. L is the lag operator, and $\hat{R}^2$ has been defined in the Note to Table 1.

Comparison of the above results for money demand with those obtained under rational expectations suggests that the main differences in parameter estimates are as follows: income and expected inflation elasticities are greater, in absolute value, for adaptive than for rational expectations. Also, under adaptive expectations the coefficient on the lagged dependent variable is smaller than those of Table 1, thus implying that the adjustment of actual money balances to changes in desired balances is shorter under the adaptive-expectations formulation .

Despite these differences, the overall impression from these findings is that the parameter estimates of Brazilian money demand under adaptive expectations are quite similar to those based on rational expectations. However, it should be noted that the log likelihood value for the system of table 2 is smaller than its rational-expectations counterpart of option I - table 1. Furthermore, the assumption of adaptive expectations imposes restrictions on data, which are readily testable. Concretely, this assumption asserts that lagged inflation rates enter in equation (3) only through the ψ coefficients of equation (8). The system can also be estimated under relaxation of this restriction, and a LR statistic can then be calculated. In this case, LR obtains a value of 7.06 (reported in table 2), and since the critical $\chi^2(3,0.1) = 6.25$, the likelihood ratio test results in rejection of the restrictions imposed by adaptive expectations on the data.

4. Conclusions

This paper presented a methodology for estimating and testing the demand for money under rational expectations of inflation. As usually implemented, the assumption of rational expectations requires specifying the information set used by agents in forming their expectations. An important feature of the analysis above is that empirical evidence was presented for several plausible specifications of the information set, which amount to alternative operational versions of the rational expectations hypothesis.

At the empirical level, the econometric analysis was based on quarterly data for the high-inflation economy of Brazil (1970-77). The parameter estimates reported for money demand turn out to imply expected-inflation elasticities that are substantially larger than those found in previous studies of Brazil. This finding holds for specifications using rational as well as adaptive expectations of inflation, and therefore it appears that this difference in the results is mainly due to differences in the sample periods utilized. On the other hand, the reported estimates of the other coefficients entering money demand are quite similar to those appearing in previous work. Regarding the alternative specifications (or options) considered for the information set used in forming expectations, some evidence was presented in support of the notion that lagged money growth and external inflation contain, in addition to lagged inflation rates, important information for predicting future inflation in Brazil.

Particular emphasis was given in the empirical analysis to testing the cross-equation restrictions implied by the rational-expectations hypothesis. The empirical findings on this issue indicated that for each one of the four options postulated for the information set, these restrictions were not statistically rejected by the data. Yet when comparisons were made across options, the results indicated support to the broadest option utilized (that is, the one that specifies that inflation expectations are formed on the basis of information about the past behavior of inflation, money growth, and external inflation). In addition to considering evidence under rational expectations, the basic system was also estimated under the assumption of adaptive expectations.

In this case, econometric tests of the cross-equation restrictions implied by this assumption resulted in rejection of these restrictions.

All in all, then, the Brazilian data considered appear to support a specification of the demand for money that embodies rational expectations of inflation. However, caution is suggested in using the results reported in this paper to draw firm conclusions on the debate of adaptive vs. rational expectations. This is so because it is well to recognize that, for purposes of empirical tractability, the expectational specifications used above (both for adaptive and rational expectations) are very simple ones. For example, in some cases it may be plausible to assume that the adaptation coefficient, used under adaptive expectations, is not a constant, but endogenously varies through time (see e.g. Khan [1977]). Also, it may well be that, although having rational expectations of inflation, agents learn only gradually about the form of the optimal inflation predictor (equation (6)). Only by considering these more complex formulations, as well as data for several countries, one would be able to reach final judgements on the accuracy of adaptive vs. rational expectations.

Appendix- Residuals' Autoregressions for the Equation of Table 1

(Estimated equation: $\xi_t = \rho_1 \xi_{t-1} + \rho_2 \xi_{t-2} + \rho_3 \xi_{t-3} + \rho_4 \xi_{t-4}$)

	Option I		Option II		Option III		Option IV	
	EQ. (6)	EQ. (7)	EQ. (6)	EQ. (7)	EQ. (6)	EQ. (7)	EQ. (6)	EQ. (7)
Parameter								
ρ_1	0.07 (0.20)	-0.37 (0.21)	0.12 (0.20)	-0.31 (0.21)	0.02 (0.20)	-0.34 (0.21)	0.16 (0.19)	-0.32 (0.21)
ρ_2	0.08 (0.20)	-0.12 (0.21)	0.04 (0.20)	-0.16 (0.20)	-0.17 (0.21)	-0.10 (0.22)	-0.20 (0.20)	-0.03 (0.21)
ρ_3	-0.14 (0.20)	-0.30 (0.22)	-0.25 (0.20)	-0.41 (0.20)	-0.17 (0.21)	-0.24 (0.23)	-0.05 (0.20)	-0.29 (0.22)
ρ_4	0.02 (0.20)	-0.09 (0.22)	0.10 (0.20)	-0.08 (0.21)	-0.10 (0.21)	-0.08 (0.33)	-0.06 (0.19)	-0.16 (0.22)
Statistic								
R^2	0.03	0.20	0.08	0.21	0.06	0.15	0.07	0.17
DW	2.01	2.03	1.96	2.01	1.95	2.04	1.95	2.11
F(4,24)	0.18	1.50	0.48	1.50	0.36	1.20	0.42	1.20

Note: ξ denotes the residual in a given equation of Table 1. Figures in parentheses are standard errors of regression coefficients.

REFERENCES

Cagan, P., "The Monetary Dynamics of Hyperinflation," in, M. Friedman, ed., Studies in the Quantity Theory of Money (Chicago: Univ of Chicago Press, 1956).

Dhrymes, P. J., Econometrics (New York: Praeger, 1974).

Evans, P., "Time-Series Analysis of the German Hyperinflation," International Economic Review, 19 (February, 1978) 195-210

Feige, E. L., and D. K. Pearce, "Economically Rational Expectations: Are Innovations in the Rate of Inflation Independent of Innovations in Measures of Monetary and Fiscal Policy?," Journal of Political Economy, 84 (June, 1976), 499 -522.

Frenkel, J. A., "The Forward Rate, Expectations, and the Demand for Money: The German Hyperinflation," American Economic Review, 67 (September, 1977), 653-70

Galbis, V., "Inflation and Interest Rate Policies in Latin America, 1967-76," IMF Staff Papers , 26 (June, 1979), 334-66.

Hansen, L. P., and T.J. Sargent, "Formulating and Estimating Dynamic Linear Rational Expectations Models," Journal of Economic Dynamics and Control, forthcoming 1980.

Khan, M. S., "Variable Expectations and the Demand for Money in High-Inflation Countries," Manchester School, 45 (September, 1977), 270-93.

__________, "Monetary Shocks and the Dynamics of Inflation," IMF Staff Papers, 27 (June, 1980), 250-84.

Laidler, D. E. W. The Demand for Money: Theories and Evidence (New York : Harper and Row, second edition, 1977).

Lemgruber, A. C., "Inflation in Brazil," in, L. B. Krause and W. S. Salant, eds., Worldwide Inflation (Washington: Brookings, 1977), 413-41

Lucas, R. E., Jr., "Econometric Testing of the Natural Rate Hypothesis," in, O. Eckstein, ed., The Econometrics of Price Determination (Washington, 1972).

McCallum, B. T., "Rational Expectations and the Natural Rate Hypothesis : Some Consistent Estimates," Econometrics, 44 (January, 1976), 43-52.

Muth, J. F., "Rational Expectations and the Theory of Price Movements," Econometrica, 29 (July, 1961), 315-35.

Pastore, A. C., et.al., "Reflection of the Brazilian Experience with Indexation," Explorations in Economic Research, 4 (Winter, 1977), 8-55.

Salemi, M. K., "Adaptive Expectations, Rational Expectations, and Money Demand in Hyperinflation Germany," Journal of Monetary Economics, 5 (,1979), 593-604.

__________ ,and T. J. Sargent, "The Demand for Money During Hyperinflation Under Rational Expectations: II," International Economic Review, 20 (October, 1979), 741-58.

Sargent, T. J., "Rational Expectations, The Real Rate of Interest, and the Natural Rate of Unemployment," Brookings Papers, 4(1973), 429-80.

__________,"The Demand for Money During Hyperinflations under Rational Expectations," International Economic Review, 18(February, 1977),59-82.

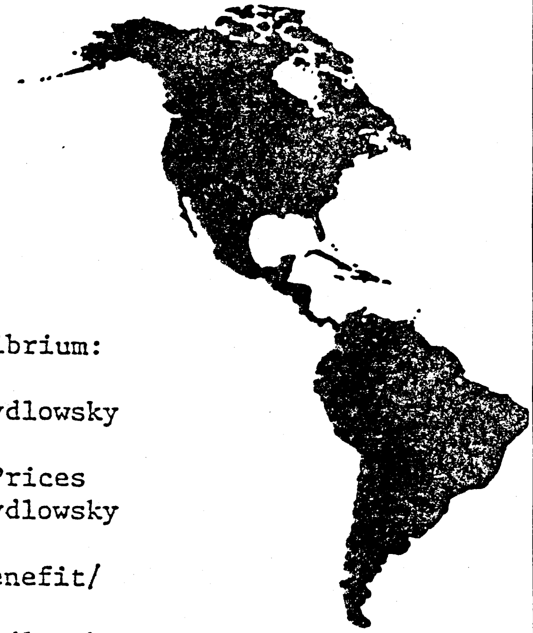
__________, and N. Wallace, "Rational Expectations and the Dynamics of Hyperinflation," International Economic Review, 14(June, 1973), 328-50.

Silveira, A. M., "The Demand for Money: The Evidence from the Brazilian Economy," Journal of Money, Credit, and Banking, 5(February,1973), 113-40.

Wachter, S. M. Latin American Inflation (Lexington: Lexington Books, 1976).

Wymer, C. R., "Computer Programs: Resimul Manual," International Monetary Fund, (March, 1978).

BOSTON UNIVERSITY
CENTER FOR LATIN AMERICAN DEVELOPMENT STUDIES



DISCUSSION PAPER SERIES

(*) _____1. Project Evaluation in Economies in General Disequilibrium: An Application of Second Best Analysis
March 1973 46 pages - Daniel M. Schydlowsky

(*) _____2. Methodology for the Empirical Estimation of Shadow Prices
April 1973 49 pages - Daniel M. Schydlowsky

(*) _____3. A Note on Distributional Considerations in Social Benefit/ Cost Analysis
April 1973 17 pages - Daniel M. Schydlowsky

_____4. Chile Under Allende
May 1973 20 pages - P. N. Rosenstein-Rodan

_____5. International Trade Policy in the Economic Growth of Latin America
May 1973 38 pages - Daniel M. Schydlowsky

_____6. Influencia del Mercado Financiero sobre la Utilizacion de Capacidad Instalada
May 1973 23 pages - Daniel M. Schydlowsky

_____7. Discussion on Chile: Diagnosis and Prognosis
November 1973 7 pages - P. N. Rosenstein-Rodan

_____8. Consistency in Measuring Capacity Utilization
March 1974 30 pages - Patricio Millan

_____9. Multiple Shifts in the Pure Investment Decision
December 1974 95 pages - Patricio Millan

_____10. In Support of Democracy in Latin America
April 1975 10 pages - Carlos Lleras Restrepo

_____11. Price and Scale Obstacles to Export Expansion in LDC's
April 1975 31 pages - Daniel M. Schydlowsky

_____12. La Ampliación de Turnos en la Industria Chilena: La Factibilidad de una Política de Empleo Productivo
April 1975 69 pages - Joseph Ramos

_____13. Guidelines for Policies to Increase the Use of Capital Through Multiple Shifts in Industrial Plants
June 1975 28 pages - Patricio Millan

_____14. The Theory of Capital Utilization: Some Extensions
October 1975 31 pages - Christopher Clague

(*) Available also in Spanish

____15. A Statistical Shift-Choice Model of Capital Utilization
November 1975 64 pages - Roberto Abusada-Salah

____16. The Theory of Capital Utilization and the Putty-Rubber Production Function
May 1975 21 pages - Christopher Clague

____17. Criterios Analiticos para la Formulacion de una Politica Economica Subregional Andina
March 1976 49 pages - Daniel M. Schydlowsky

____18. The Determinants of Capital Utilization in Labor-Managed Enterprises
March 1976 34 pages - Roger Betancourt
Christopher Clague

____19. Accelerated Modernization and the Population Explosion
June 1976 38 pages - Emilio Casetti

____20. The Theory of the Costs and Benefits from Foreign Direct Investment and Economic Integration
December 1976 35 pages - Ernesto Tironi

____21. Customs Union Theory and Foreign Direct Investment
December 1976 48 pages - Ernesto Tironi

____22. Capital Utilization, Growth, Employment, Balance of Payments and Price Stabilization
December 1976 72 pages - Daniel M. Schydlowsky

____23. The Andean Pact: A Model of Economic Integration for Developing Countries
December 1976 53 pages - Ricardo Ffrench-Davis

____24. Towards a Change in the Economic Paradigm Through the Experience of Developing Countries
February 1977 58 pages - Marcelo Diamond

____25. Real Wages and Economic Growth in Peru, 1900-1940
March 1977 56 pages - Shane Hunt

____26. The Subsidy and Countervailing Duties Negotiations and the Developing Countries
March 1977 53 pages - Daniel M. Schydlowsky

____27. Monetary Aspects of the Black-Market Exchange Rate Determination
July 1977 24 pages - Mario I. Blejer

____28. From Pre-Keynes to Post-Keynes
September 1977 35 pages - Abba P. Lerner

(*)_____29. Design of Benefit/Cost Analysis of Investment Projects in Peru: A Country-Specific View
October 1977 118 pages - Daniel M. Schydlowsky

_____30. A Monetary Approach to the Crawling-Peg System: Theory and Evidence.
January 1979 33 pages - Mario I. Blejer
Leonardo Leiderman

_____31. Strike Activity and Wage Determination Under Rapid Inflation: A Quantitative Study of the Chilean Case
January 1979 22 pages - Mario I. Blejer

_____32. The Anatomy of an Economic Failure: Peru 1968-78
February 1979 116 pages - Daniel M. Schydlowsky
Juan J. Wicht

_____33. The Effects of Unanticipated Money Growth on Prices, Output and its Composition in a Fixed-Exchange-Rate Open Economy: The Mexican Experience
July 1979 38 pages - Mario I. Blejer
Roque B. Fernandez

_____34. Relationships Between Macroeconomic Time Series in a Fixed-Exchange-Rate-Economy
July 1979 52 pages - Leonardo Leiderman

_____35. Inflation and Relative Price Variability in the Open Economy.
September 1979 41 pages - Mario I. Blejer
Leonardo Leiderman

_____36. Containing the Costs of Stabilization in Semi-Industrialized LDC's: A Marshallian Approach
December 1979 102 pages - Daniel M. Schydlowsky

_____37. Why an Initial Increase in Exports of Labor Intensive Manufactured Goods May Not Create Much Additional Employment: A Hypothesis
June 1980 25 pages - David Morawetz

_____38. Production Efficiency and the Transferability of Input-Output Coefficients
June 1980 39 pages - David Wheeler

_____39. Benefit-Cost Analysis Methodologies: Untangling Their Equivalences and Discrepancies
June 1980 86 pages - Luis Ramirez

(*)_____40. A Policymaker's Guide to Comparative Advantage
June 1980 34 pages - Daniel M. Schydlowsky

_____41. The Vulnerability of Small Semi-Industrialized Economies to Export Shocks: A Simulation Analysis Based on Peruvian Data
June 1980 27 pages - Daniel M. Schydlowsky
Martha Rodriguez

(*) Available also in Spanish

____42. The Short Run Potential for Employment Generation on Installed Capacity in Latin America
June 1980 73 pages - Daniel M. Schydlowsky

____43. Relative Price Variability and Output-Inflation Tradeoffs in Mexican Economy
July 1980 20 pages - Leonardo Leiderman
Mario I. Blejer

____44. The Demand for Money under rational Expectations of Inflation FIML Estimates for Brazil
August 1980 24 pages - Leonardo Leiderman

Institutional exchange may be arranged or single copies purchased as noted below. Send check or money order made out to: Boston University, CLADS; attention Librarian, 745 Commonwealth Avenue, Boston, Massachusetts 02215. Prices:

10-30	pages	$1.00
31-60	pages	$2.25
61-100	pages	$3.50
101-200	pages	$4.50

(Papers will be sent surface rate unless request is made for first-class or air-mail delivery, in which case an additional charge will be made.)

Boston University

Department of Economics
College of Liberal Arts
270 Bay State Road
Boston, Massachusetts 02215
617/353-4440

Cable: BUECONOMIC

BOSTON UNIVERSITY DEPARTMENT OF ECONOMICS

DISCUSSION PAPER SERIES

The following Discussion Papers are available for a nominal charge of $1.00 each. Orders or inquiries should be addressed to: Librarian, Department of Economics, Boston University, 270 Bay State Road, Boston, Massachusetts, 02215. Please make checks payable to Boston University, Department of Economics.

1. Capital Utilization, Growth, Employment and Balance of Payments and Price Stabilization
   March 1977 - Daniel M. Schydlowsky

2. Hedonic Wage Equations and Psychic Wages in the Returns to Schooling
   March 1977 - Robert E. B. Lucas

3. Is There Excess Health Insurance? Comments on Feldstein
   March 1977 - Thomas G. McGuire

4. National Health Insurance and the Market for Private Psychiatric Services
   March 1977 - Thomas G. McGuire

5. The Black Market for Foreign Exchange and the Domestic Demand for Money: Some Theoretical and Empirical Issues
   March 1977 - Mario I. Blejer

6. The Subsidy and Countervailing Duties Negotiations and the Developing Countries
   March 1977 - Daniel M. Schydlowsky

7. Economic Growth, Income Distribution, and the Political Process in Less Developed Countries *
   June 1977 - Gustav F. Papanek

8. Variances in Returns to Human Capital
   June 1977 - Robert E. B. Lucas

9. Measurement of Private Efficiency Under Diverse Technologies in Indian Agriculture *
   June 1977 - Prem S. Vashishtha

10. Sharing, Monitoring and Incentives: Marshallian Misallocation Reassessed.
June 1977 - Robert E. B. Lucas

11. A Human Capital Quandary
June 1977 - Robert E. B. Lucas

12. The Stabilization of a World Economy: A Simple Model
June 1977 - Henry Y. Wan, Jr.

13. Three Variations on a Theme of Samuelson
June 1977 - Henry Y. Wan, Jr.

14. Shumpeterian Waves of Innovation and Infrastructure Development in Great Britain and the United States: The Kondratieff Cycle Revisited
June 1977 - David Wheeler and Raymond S. Hartman

15. Budget-Maximizing Agencies and Efficiency in Government
December 1977 - Thomas G. McGuire, Michael Coiner, and Larry Spancake

16. Sex, Marriage, Children and Statistical Discrimination
May 1978 - Paul Osterman

17. Racial Differentials in Male Youth Unemployment
May 1978 - Paul Osterman

18. Education and Labor Markets at the Turn of the Century
May 1978 - Paul Osterman

19. Expectations and Output - Inflation Tradeoffs in a Fixed Exchange Rate Economy
June 1978 - Leonardo Leiderman

20. I Waive My Right to Read This Recommendation: A Theoretical Analysis of the Buckley Amendment
(Bell Laboratories and New York University)
June 1978 - Michael Manove and J. Ordover

21. On the Theory of DRC Criteria
August 1978 - Robert E. B. Lucas

22. The Real Value of International Financial Assets: An Application to Non-Oil LDC Debt
August 1978 - Millard Long and Frank Veneroso

23. The Debt-related Problems of the Non-Oil Less Developed Countries
September 1978 - Millard Long and Frank Veneroso

24. Migration, Employment and Earnings in Indonesia *
October 1978 - Bisrat Aklilu and John Harris

25. Balance of Payments Disturbances and the Debt of the Non-Oil Less Developed Countries: Retrospect and Prospect
October 1978 - Millard Long and Frank Veneroso

26. Youth Labor Market Structure
November 1978 - Paul Osterman

27. Patients' Trust and the Quality of Physicians
January 1979 - Thomas G. McGuire

28. On the Effects of Inflation and Relative Price Variability on Output and Unemployment: Some Empirical Evidence
January 1979 - Gustav F. Papanek

29. Real Wages, Growth, Inflation, Income Distribution, and Politics in Pakistan, India, Bangladesh, Indonesia *
January 1979 - Gustav F. Papanek

30. Methodological and Statistical Appendix to Real Wages, Growth, Inflation, Income Distribution and Politics in Pakistan, India, Bangladesh, Indonesia *
January 1979 - Gustav F. Papanek

31. The Kibbutz as a Model for Developing Countries, or On Maintaining Full Economic Equality in Practice
May 1979 - David Morawetz

32. The Fate of the Least Developed Member of an LDC Integration Scheme: Bolivia in the Andean Group
May 1979 - David Morawetz

33. Empirical Analysis and Minimum Optimum Scale: Some New Results
May 1979 - Peter F. Cory

34. The International Transmission of Inflation: India in the 1970's *
May 1979 - Dileep M. Wagle

35. Black and White Youth Employment: A Cross-Sectional Analysis
June 1979 - Paul Osterman

36. Laissez-Faire, Growth and Equity: Hong Kong *
June 1979 - Steven Chow and Gustav F. Papanek

37. Macroeconometric Testing of the Rational Expectations and Structural Neutrality Hypothesis for the United States
July 1979 - Leonardo Leiderman

38. Output Supply in the Open Economy: Some International Evidence
July 1979 - Leonardo Leiderman

39. Unanticipated Movements in Aggregate Demand and the Business Cycle: Results From Variance Decompositions
July 1979 - Leonardo Leiderman

40. Black Youth Unemployment: A Review of the Evidence and Some Policy Recommendations
July 1979 - Paul Osterman

41. Moral Hazard, Bandwagon Effects and Psychiatrists' Influence on Demand: An Empirical Study of Insurance and Demand for Private Psychiatric Care
August 1979 - Thomas G. McGuire

42. Optimal Patents with Compulsory Licensing
August 1979 - Pankaj Tandon

43. The Corporation Tax and Incentives to Undertake Risky Investments
August 1979 - Pankaj Tandon

44. Rivalry and the Excessive Allocation of Resources to Research
August 1979 - Pankaj Tandon

45. The Rate of Technology Diffusion to Developing Countries - Yugoslavia and Mexico
August 1979 - Peter Cory

46. Education, Training and Employment in Worcester
Part I - Objectives and Methods
May 1979 - Donna Olszewski and Hubert Smith

47. Education, Training and Employment in Worcester
Part 2 - High School Evaluation
May 1979 - Donna Olszewski and Hubert Smith

48. Education, Training and Employment in Worcester
Part 3 - Education and Training History
May 1979 - Donna Olszewski and Hubert Smith

49. Education, Training and Employment in Worcester
Part 4 - Job Characteristics
May 1979 - Donna Olszewski and Hubert Smith

50. Improving School-to-Career Linkages: Policy Options for Occupational Education and Training
August 1979 - Bruce Vermeulen

51. Projecting Labor Market Trends for Education and Training
August 1979 - Susan Hudson-Wilson

52. Private Sector Employment Strategies: How Employers Hire and Train
August 1979 - Michael Diggins and Bruce Vermeulen and Susan Hudson-Wilson

53. Career Paths and Vocational Education
August 1979 - Donna Olszewski

54. The U.S. System of Occupational Education and Training: Profile, Performance and Policy
August 1979 - Patricia Flynn Pannell

55. International Migration: Economic Causes, Consequences, Evaluation and Policies
August 1979 - Robert E. B. Lucas

56. Budget-Maximizing Governmental Agencies: An Empirical Test
September 1979 - Thomas G. McGuire

57. On the Non-Existence of Index-Linked Corporate Debentures
July 1979 - Paul Beckerman

* These Papers also are available through the Center for Asian Development Studies Discussion Paper Series.