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On the relevance of monetary aggregates in monetary policy models

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ABSTRACT

This paper develops a business cycle model with a financial intermediation sector. Financial wealth is defined as a predetermined state variable. Both, the additional sector of financial intermediaries and predetermination of financial wealth, affect the demand for real financial wealth. If real financial wealth also enters the monetary policy rule, the conditions for stability and uniqueness of the macroeconomic equilibrium path change fundamentally compared to standard New Keynesian business cycle models. Here, real financial wealth is interpreted as a real broad monetary aggregate. Furthermore, different interest rate rules and their consequences for stability and uniqueness of the macroeconomic equilibrium path are considered. Two monetary policy rules are found to be feasible - i.e. if these monetary policy rules are applied there exists a stable and unique macroeconomic equilibrium path. Simulations of the model showed that the monetary policy rule considering inflation and broad money as indicators is optimal.

JEL classification: E41, E51, E52

Key words: broad money, macroeconomic stability, monetary policy.

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1 Introduction

During the last three decades, many central banks changed monetary policy considerably. This is paralleled in the discussion of monetary theory and policy:[1] In the late 1970s, money and the long-run effects of its movements on inflation were in the center-stage of economic policy. Policymakers believed in Milton Friedman's famous statement that "inflation is always and everywhere a monetary phenomenon".[2] Therefore, the control of money supply should result in lower inflation rates. As the attempts of central bankers to control the money supply turned out to be more complicated than expected, the interest in money and its movements vanished. Policymakers observed the developments in money further on and they took it indirectly into account when making monetary policy decisions. However, the direct control of money supply lost importance; today most of the central banks in the world target the actual or the expected future inflation rate. Just a few central banks, e.g. the German Bundesbank, held on to the control of money supply and emphasized the relevance of money as a monetary indicator and target. Even though, the European Central Bank (ECB) explicitly considers money supply in its monetary policy analysis of the longer-term risks to price stability, its policy decisions during the last years were not based on the growth rate of the relevant monetary aggregate M3, which expanded with a clearly higher rate than the desirable reference value of the ECB.

The theoretical discussion of monetary policy analysis changed in a similar way. First, monetaristic models characterized the discussion of monetary theory and policy. When the empirical problems of controlling the money supply became obvious, New Keynesian models moved into the center-stage of the theoretical discussion. These models replaced the equation for the exogenous money supply by a so-called "feedback rule" for interest rates,[3] so that money was determined endogenously. Additionally, in the equation for money demand, money was then modeled as a jump variable. Therefore, in every period, under the condition of a given interest rate, the desired or needed amount of money could be provided, so that money did not play an active role in determining inflation or real variables any longer.

In the New Keynesian business cycle models, money is also no longer relevant as an indicator or a target for monetary policy. However, these models are used for monetary policy analysis at present; a fact which seems to be a paradox, keeping in mind that inflation is broadly considered to be a monetary phenomenon - at least - in the long run. Still, the question remains whether it is really possible and reasonable to analyze monetary policy in models without money,[4] or whether money should return into the focus of monetary policy analyses.

The controversial role of money in monetary policy models is currently a hot topic in the discussion of monetary policy and theory;[5] the different views are stretching from no relevance of money to high relevance of money.

The aim of this paper is to develop a business cycle model - as an extension of a standard New Keynesian model - with financial intermediation, staggered prices and broad money as a predetermined state variable. If broad money is a predetermined state variable it contains more information about inflation and output - especially of former periods - than a jump

[1] See e.g. King (2002).
[2] Cited in: Nelson (2003), p. 1033 (appearing originally in Friedman (1963), p. 17).
[3] The most famous example of such a "feedback rule" is the Taylor-rule. See Taylor (1993).
[4] See e.g. McCallum (2001).
[5] See e.g. Meyer (2001) or von Hagen (2003), pp. 1 et seq.

variable. It is, therefore, important for the macroeconomic equilibrium path and its stability and uniqueness. The analysis of different monetary policy rules also shows that taking real broad monetary aggregates into account alters the conditions for stability and uniqueness of the macroeconomic equilibrium path and, therewith, also for optimal monetary policy.

The rest of the paper is organized as follows: In section 2, the business cycle model and its macroeconomic equilibrium path are presented. The analysis of several monetary policy rules, which are all interest rate rules, in section 3 develops different conditions for feasible monetary policy. Section 4 simulates the business cycle model to analyze the efficiency of feasible monetary policy rules. Section 5 concludes.

2 The model

In this section, a monetary business cycle model (with microfoundations) is developed, as an extension of a standard New Keynesian model.[6] This model includes four sectors[7]: households, firms, the public sector and a financial intermediation sector (banks). The role of banks for the transmission of monetary policy is important.[8] In this model, banks' behavior is also relevant for the demand function for real financial wealth. Financial wealth is designed as a predetermined state variable.[9] This paper analyzes several monetary policy rules to find an optimal one.

Nominal variables are denoted by upper-case letters, while real variables are denoted by lower-case letters.

2.1 On the behavior of households

Households' utility depends positively on the consumption of goods (C_t) and on holding financial wealth (A_t)[10] and negatively on labor (L_t). At the beginning of each period, households decide to divide their - predetermined[11] - financial wealth into cash (M_t^h), sight deposits (D_t^o) and savings deposits (D_t).[12] Sight deposits do not earn interests whereas interests are paid on savings deposits at a rate of i_t^d:

$$A_{t-1} = M_t^h + D_t^o + D_t. \tag{1}$$

[6]Examples for the derivation of standard New Keynesian models can be found e.g. in Yun (1996), King and Wolman (1996), Rotemberg and Woodford (1998), Clarida et al. (1999), and Woodford (2001).

[7]The introduction of the financial intermediation sector can also be found in Brückner and Schabert (2002).

[8]See e.g. von Hagen (2003), p. 23.

[9]See e.g. Woodford (1996), p.5.

[10]As financial wealth is interpreted as a broad monetary aggregate, it will be called broad money in the rest of the paper. Here, a money-in-the-utility function is used because of its generality. As Feenstra (1986) demonstrates, models including transaction costs can also be written as models with money-in-the-utility functions. Carlstrom and Fuerst (2001) therefore conclude that shopping-time models are equivalent to money-in-the-utility function models, and cash-in-advance economies could be interpreted as extreme versions of money-in-the-utility function economies.

[11]See also McCallum and Nelson (1999), Woodford (1990) or Carlstrom and Fuerst (2001) for examples for stock of money entering the utility function at the beginning of the period.

[12]It is assumed, that these three assets are imperfect substitutes.

The public sector pays lump-sum transfers (τ_t) to households and firms pay a wage for households' labor input (w_t). Households own banks and firms in equal shares.[13] Therefore, firms (superscript f) as well as banks (superscript b) transfer their profits (Ω_t) in equal amounts to households. Households' budget restriction is of the following form:[14]

$$A_t = \left(1 + i_t^d\right) A_{t-1} - i_t^d M_t^h - i_t^d D_t^o + P_t w_t l_t - P_t c_t + P_t \tau_t + P_t \Omega_t^b + P_t \Omega_t^f \tag{2}$$

where P_t represents the general price level. Nominal consumption is therefore restricted to the beginning-of-period broad money holdings.[15]

Households maximize the present value of utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U \left[c_t, l_t, \left(m_t^h, d_t^o, d_t\right)\right], \qquad \text{with} \quad 0 < \beta < 1. \tag{3}$$

Here, E is the expectation parameter and U denotes the utility function. β notifies a discount factor to compare future and present values of utility. As real variables are all denoted by lower-case letters, c_t, l_t, m_t^h, d_t^o, and d_t represent the real values of consumption, labor, cash, sight deposits, and savings deposits, respectively. The utility function is of the following form - separable in all arguments:[16]

$$U \left[c_t, l_t, \left(m_t^h, d_t^o, d_t\right)\right] = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\varphi}}{1+\varphi} + \frac{\left(m_t^h\right)^{1-\nu}}{1-\nu} + \frac{\left(d_t^o\right)^{1-\psi}}{1-\psi} + \frac{d_t^{1-\kappa}}{1-\kappa} \tag{4}$$

- with $\sigma, \varphi, \nu, \psi, \kappa > 0$, denoting the constant substitution elasticities of the real values of consumption, labor, cash, sight deposits, and savings deposits.

In the optimum, the transversality condition

$$\lim_{j \to \infty} \lambda_{t+j} \beta^{t+j} \frac{A_{t+j}}{P_{t+j}} = 0 \tag{5}$$

must also be satisfied. The transversality condition provides a terminal condition for the households' intertemporal behavior.

Maximizing the utility function (3) under consideration of its functional form (4) and the budget restriction (2) in real terms, leads to the following first order conditions for real consumption, real labor, and the real values of cash, sight deposits and broad money:

$$c_t^{-\sigma} = \lambda_t \tag{6}$$

$$\frac{l_t^{\varphi}}{w_t} = \lambda_t \tag{7}$$

$$\left(m_t^h\right)^{-\nu} - \lambda_t i_t^d = d_t^{-\kappa} \tag{8}$$

[13] See Erceg et al. (2000), p. 287. Households' shares of firms and banks are not contained in the broad monetary aggregate, they do not correspond to the conception of stocks.

[14] See Christiano and Rostagno (2001), p. 4.

[15] This restriction is the so-called Clower constraint (see Clower (1967)).

[16] It is a simplified form of the utility function of Erceg et al. (2000), p. 287.

$$(d_t^o)^{-\psi} - \lambda_t i_t^d = d_t^{-\kappa} \tag{9}$$

$$\frac{1}{\beta}\lambda_t - E_t\left(\lambda_{t+1}\frac{\left(1+i_{t+1}^d\right)}{\pi_{t+1}}\right) = E_t\left(d_{t+1}^{-\kappa}\frac{1}{\pi_{t+1}}\right) \tag{10}$$

where λ denotes the Lagrange multiplier for the budget constraint (2) and the gross inflation rate is defined by $\pi_{t+1} = \frac{P_{t+1}}{P_t}$.

2.2 On the behavior of firms

While households act on perfectly competitive markets, the firm sector is marked by sticky prices and monopolistic competition.[17] Production is structured by the following steps: First, intermediate goods are produced; second, all the different intermediate goods are aggregated to create the final consumption good which will be consumed by households. By modeling monopolistic competition in the intermediate goods sector, price setting behavior of the Calvo-type[18] is introduced. Then price rigidity allows the derivation of a New Keynesian Phillips curve.

As a first step, the optimization problem of the intermediate goods producers is presented.[19] They maximize the present value of their future profits:

$$\max_{P_t^*(j)} \sum_{k=0}^{\infty} (\theta\beta)^k \, E_t \frac{\lambda_{t+k}}{\lambda_t} y_{t+k}(j) \left[P_t^*(j) - CO_{t+k}\right], \tag{11}$$

with $P_t^*(j)$ and CO_t denoting the optimal price respectively the production costs per unit of the intermediate good j, $y_{t+k}(j)$ denoting the demand for the intermediate good j, and k denoting the period. Intermediate goods producers' profit is the difference between their earnings of $P_t^*(j)y_{t+k}(j)$ when they sell the intermediate good j to the aggregator and their production costs of the intermediate good j $(CO_{t+k}y_{t+k}(j))$. The profits are weighted with different factors. First of all, they are discounted by the factor β (which lies between zero and one). As discount factors of all compared periods are multiplied with each other, β^k results. The profits are also weighted with the relation of households' future consumption utility to their present consumption utility $\frac{\lambda_{t+k}}{\lambda_t}$,[20] because households use firms' profits only for consumption purposes. Finally, the price setting mechanism of Calvo (1983) influences intermediate goods producers' profits. With the probability of $1-\theta$, intermediate goods producers are allowed to set the optimal price $P_t^*(j)$. If they are not allowed to set an optimal price in period $t+1$, they have to keep the optimal price of period t fixed with the probability of θ. Here, the optimization problem considers that intermediate goods producers have to fix this optimal price of period t for the following k periods, so that the probabilities multiply to θ^k.[21] Therefore, intermediate goods producers' profits are weighted with these probabilities.

[17] A detailed discussion can be found in Schumacher (2002).

[18] See Calvo (1983); in every period, firms are allowed to set their own optimal price with an exogenous probability of $1-\theta$. With the probability of θ they are not allowed to adjust their price and have to keep the price of the former period $t-1$.

[19] See e.g. Kimball (1995), p. 1245.

[20] See first order condition of households for the real value of consumption (6).

[21] If they are again allowed to set a new optimal price after k periods, the same optimization problem - with a new optimal price $P_t^*(j)$ - will arise.

To solve the optimization problem of the intermediate goods producers, the demand for the intermediate good ($y_{t+k}(j)$) has to be derived. For the aggregation of the final consumption good, the following aggregating production function is used:

$$y_t = \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \tag{12}$$

- with $\epsilon > 1$, denoting the substitution elasticity between the different intermediate goods.[22] All the intermediate goods are indexed on a continuum between zero and one and are needed for the production of the aggregated final consumption good. For the derivation of the demand function, the aggregator's optimization problem has to be solved.[23] It is assumed, that the aggregator maximizes his real profit:

$$y_t - \int_0^1 \frac{P_t(j)}{P_t} y_t(j)\, dj \tag{13}$$

whereby y_t represents the real earnings of selling the final consumption good to households and the second term denotes the real costs to obtain the intermediate good. Maximizing the aggregator's profit function considering the aggregating production function (12) leads to the demand for the intermediate good:[24]

$$y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} y_t. \tag{14}$$

The demand for the intermediate good depends positively on the demand for the final consumption good. But, if an intermediate goods producer raises the price for his good j relative to the aggregated price P_t the demand for the intermediate good will decrease. $-\epsilon$ denotes thereby the elasticity of the demand for $y_t(j)$ in relation to the price $P_t(j)$. It is also possible to derive the aggregated price index P_t from the aggregator's optimization problem under consideration of the demand function for the intermediate good:[25]

$$P_t = \left[\int_0^1 P_t(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}. \tag{15}$$

It is shown that the aggregating production function (12) and the price index (15) connect the production of the intermediate goods with the production of the final consumption good.

The intermediate goods producers' profits are not only determined by the demand for the intermediate goods but also by the cost function and, therefore, by the production technology.

[22] See Woodford (1996), p. 5 or Dixit and Stiglitz (1977), p. 298 et seq.

[23] The aggregator creates the final consumption good by aggregating the intermediate goods.

[24] The derivation of the demand function for the intermediate good can be found in Appendix 6.1.1.

[25] See again Appendix 6.1.1 for the computations and Woodford (1996), p. 5.

It is assumed, that the production function for the intermediate goods is of the following form:[26]

$$\begin{aligned} y_t(j) &= z_t l_t(j), \\ \text{with} \quad \log z_t &= (1-\mu)\log \overline{z} + \mu \log z_{t-1} + \zeta_t, \\ 0 &\leq \mu < 1, \; and \; \zeta_t \sim i.i.d.N\left(0, \sigma_\zeta^2\right). \end{aligned} \tag{16}$$

The factor cost

$$CO_t(j) = w_t P_t l_t(j) \tag{17}$$

is only composed of the cost for labor, i.e. nominal wage times labor input. As households are perfectly competitive, they all receive the same wage for their labor input.[27] The factor cost can then be rewritten as

$$CO_t(y_t(j)) = w_t P_t y_t(j) \frac{1}{z_t} \tag{18}$$

if the production function is taken into account. The first derivative in accordance to $y_t(j)$ of this cost function is the following marginal cost function:

$$MC_t = w_t P_t \frac{1}{z_t}, \tag{19}$$

i.e. a rising nominal wage $w_t P_t$ increases the marginal costs MC_t, while technology raises the labor productivity and, therefore, decreases marginal costs. With the cost function and the demand function for the intermediate good, all components of the intermediate goods producers' optimization problem are known. Then, the optimal price $P_t^*(j)$ for each intermediate goods producer can be determined. Therefore, the optimization problem in equation (11) can be reformulated for real profit by using equations (14) and (19):[28]

$$\max_{P_t^*(j)} \sum_{k=0}^{\infty} (\theta\beta)^k E_t \frac{\lambda_{t+k}}{\lambda_t} \frac{1}{P_{t+k}} \left(\frac{P_t^*(j)}{P_{t+k}}\right)^{-\epsilon} y_{t+k} \left[P_t^*(j) - MC_{t+k}\right]. \tag{20}$$

Maximizing the profit function and remodeling the equation lead to the optimal price in log-linearized form[29]:

$$\widehat{P_t^*} = (1-\theta\beta)\widehat{MC}_t + \theta\beta E_t \widehat{P^*}_{t+1} \tag{21}$$

where $\widehat{x}_t = \log x_t - \log \overline{x}$, i.e. variables with a "circumflex ($\hat{}$)" denote logarithmic deviations of these variables from their constant long-run or steady-state values - all variables with a "bar ($\bar{}$)".[30] To derive the New Keynesian Phillips curve, the aggregated price level is needed, which can be calculated by reformulating the aggregated price index (15):[31]

$$P_t = \left[(1-\theta) P_t^{*^{1-\epsilon}} + \theta P_{t-1}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}. \tag{22}$$

[26] In this case, capital is set to be constant and equal to one, so that the intermediate goods are only produced by labor input (l) and a stochastic technology variable (z).

[27] See Yun (1996), p. 351.

[28] See Yun (1996), p. 352.

[29] Log-linearization replaces the nonlinear model by a linearized approximate model.

[30] The derivation of the optimal price for the intermediate good will be found in Appendix 6.1.2.

[31] The interested reader can find the derivation of the aggregated price level in Appendix 6.1.3. See also King and Wolman (1996), p. 89.

Here, the aggregated price level is a combination of the current optimal price level and the price level of the former period. Log-linearizing the aggregated price level by a first order Taylor-approximation leads to the New Keynesian Phillips curve:[32]

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \chi \widehat{mc}_t \tag{23}$$

with $\chi = (1 - \theta\beta) \frac{1-\theta}{\theta}$ and $mc_t = \frac{MC_t}{P_t}$ denoting the real marginal costs.[33] The current "gross inflation rate" is determined by expected future "gross inflation rate" and current marginal costs.

2.3 On the behavior of financial intermediaries

Financial intermediaries are assumed to be perfectly competitive. They take sight deposits (D_t^o) and savings deposits (D_t) from households and they pay interests at a rate of i_t^d for savings deposits. The deposited funds are invested into government liabilities, i.e. money (M_t^b) and bonds (B_t). In each period, their profit ($P_t \Omega_t^b$) is transferred to households, which are the owners of banks. In addition, banks have to hold a constant ratio (δ) of sight deposits and savings deposits as a minimum reserve requirement:

$$M_t^b \geq \delta(D_t^o + D_t), \qquad \text{with} \quad 0 < \delta < 1. \tag{24}$$

Banks maximize the present value of their future real profits:[34]

$$\max_{b_t, m_t^b, d_t^o, d_t} E_t \sum_{k=0}^{\infty} \beta^k \frac{\lambda_{t+k}}{\lambda_t} \Omega_{t+k}^b, \tag{25}$$

with b_t, m_t^b, d_t^o, d_t denoting the real values of government bonds, money demanded by banks, sight deposits, and savings deposits. The assumptions about the weighting factors are similar to those for the firms' profits: β again denotes the discount factor, the profits are weighted with the relation of households' future consumption utility to their present consumption utility $\frac{\lambda_{t+k}}{\lambda_t}$,[35] because households use banks' profits for consumption purposes only. Banks' nominal profit is given by the following equation:

$$P_t \Omega_t^b = (1 + i_t) B_t + M_t^b + D_{t+1}^o + D_{t+1} - B_{t+1} - M_{t+1}^b - D_t^o - \left(1 + i_t^d\right) D_t, \tag{26}$$

with i_t denoting the interest rate on government bonds. It is further assumed that the financial intermediaries have to satisfy the following solvency constraint:

$$\lim_{j \to \infty} \left(B_{t+j} + M_{t+j}^b - D_{t+j}^o - D_{t+j}\right) E_t \prod_{n=1}^{j} (1 + i_{t+n})^{-1} \geq 0. \tag{27}$$

[32] For the derivation of the New Keynesian Phillips curve, see again Appendix 6.1.3.

[33] See Galí and Gertler (1999), p. 200.

[34] Although banks are perfectly competitive, they earn profits because of the difference between the interest rate on savings deposits i_t^d and the interest rate on government bonds i_t. The difference results due to the minimum reserve requirement for savings deposits. This can also be seen in the first order condition (31) and equation (33).

[35] See first order condition of households for the real value of consumption (6).

Solving the optimization problem (25) under consideration of the profit function (26) and the minimum reserve requirement (24) leads to the following first order conditions for the real values of government bonds, money, and savings deposits:

$$E_t \left[\lambda_{t+1} \frac{1 + i_{t+1}}{\pi_{t+1}} \right] = \frac{1}{\beta} \lambda_t \tag{28}$$

$$E_t \left[\lambda_{t+1} \frac{1 + \eta_{t+1}}{\pi_{t+1}} \right] = \frac{1}{\beta} \lambda_t \tag{29}$$

$$E_t \left[\lambda_{t+1} \frac{1 + i^d_{t+1} + \delta \eta_{t+1}}{\pi_{t+1}} \right] = \frac{1}{\beta} \lambda_t \tag{30}$$

$$\eta_t \left[m^b_t - \delta \left(d^o_t + d_t \right) \right] = 0, \qquad \text{with} \quad \eta_t > 0 \tag{31}$$

where η_t denotes the Kuhn-Tucker multiplier referring to the minimum reserve requirement and equation (31) is the Kuhn-Tucker condition.

Combining the first order conditions (28) and (29) leads to:

$$i_t = \eta_t. \tag{32}$$

Inserting equation (32) into the first order condition (30) shows the relation between the two different interest rates of the economy:

$$i_t \left(1 - \delta\right) = i^d_t. \tag{33}$$

As $0 < \delta < 1$ applies, i^d_t is less than i_t.

2.4 On the behavior of the public sector

The public sector consists of two parts: the fiscal authority and the monetary authority. The fiscal authority issues one-period risk-free government bonds B_t which bear interests at a rate i_t. Government bonds are held by banks only because they represent an instrument of the interbank market. The monetary authority issues money to households and banks: $M_t = M^h_t + M^b_t$. The receipts from this money creation are allocated to households by paying lump-sum transfers $P_t \tau_t$:

$$P_t \tau_t = B_{t+1} + M_{t+1} - \left(1 + i_t\right) B_t - M_t. \tag{34}$$

Furthermore, the monetary and the fiscal authorities have to satisfy the following solvency constraint:

$$\lim_{j \to \infty} \left(B_{t+j} + M_{t+j}\right) E_t \prod_{n=1}^{j} \left(1 + i_{t+n}\right)^{-1} = 0. \tag{35}$$

Additionally, the monetary authority is assumed to control the short-term nominal interest rate on government bonds i_t. It thereto uses the gross short-term nominal interest rate $R_t \equiv 1 + i_t > 1 \quad \forall t$.[36]

[36] For the conception of these policy regimes see Benhabib et al. (2001), p. 178 et seq.

2.5 Macroeconomic equilibrium

In this model, there exists a dynamic sequence of market equilibria.[37] Each of the three markets - goods market, labor market, and money market - has to be in equilibrium. In the following, it will be analyzed whether the macroeconomic equilibrium path converges to the steady-state, or whether the economy gets on an explosive path.

The dynamic macroeconomic equilibrium consists of three equations. The first equation is a forward-looking IS-curve which is derived by combining the first order condition of households for real consumption (6) and the first order condition of financial intermediaries for the real value of government bonds (28). After log-linearizing by a first-order-Taylor-approximation and some remodeling, the following equation is obtained:[38]

$$\widehat{y}_t = E_t \widehat{y}_{t+1} - \frac{1}{\sigma} E_t \left(\widehat{R}_{t+1} - \widehat{\pi}_{t+1} \right) \tag{36}$$

with $\sigma > 0$ denoting the substitution elasticity of consumption. $\widehat{x}_t = \log x_t - \log \overline{x}$ is again the representation of a variable in log-linearized terms, i.e. in deviations from its constant steady-state value. Real production - or aggregated demand -, therefore, depends positively on expected future production - aggregated demand - and negatively on expected future real interest rate $E_t \left(\widehat{R}_{t+1} - \widehat{\pi}_{t+1} \right)$. This means, that increasing future aggregated demand raises current aggregated demand, whereas the expectation of rising real interest rates lowers the current real production.

Second, remodeling the New Keynesian Phillips curve with the help of the production function of the intermediate goods producers, leads to the following Phillips curve:[39]

$$\widehat{\pi}_t = \gamma \widehat{y}_t + \beta E_t \widehat{\pi}_{t+1} - \chi (1 + \varphi) \widehat{z}_t \tag{37}$$

with $\gamma = \chi (\varphi + \sigma)$, and $\varphi > 0$, $\sigma > 0$ denoting the constant substitution elasticities of labor and consumption. The ”gross inflation rate” is determined positively by the expected future ”gross inflation rate” and the actual production, and negatively by the stochastic technology variable. Therefore, rising expectations of future "gross inflation rate" or current aggregated demand are converted into an increasing current "gross inflation rate". In contrast, "gross inflation rate" sinks due to a positive technology shock.

The third equation for the macroeconomic equilibrium path is the real broad money demand function. To derive this equation, the first order conditions of households for the real values of cash, sight deposits, and broad money as well as the first order condition of financial intermediaries for the real value of government bonds have to be combined and log-linearized by a first order Taylor-approximation:[40]

$$\widehat{a}_{t-1} = \sigma v \widehat{y}_t + \widehat{\pi}_t - v \frac{\overline{R}}{\overline{R} - 1} \widehat{R}_t \tag{38}$$

[37] In the literature, there are several examples of the procedure to obtain a dynamic stochastic general equilibrium from the households' first-order conditions, the market-clearing conditions and a monetary policy rule, see e.g. Erceg et al. (2000), Smets and Wouters (2003), or Giannoni and Woodford (2003).

[38] The derivation of the IS-curve can be found in Appendix 6.2.1.

[39] You can also find the derivation of the Phillips curve in Appendix 6.2.2.

[40] See for the several computations Appendix 6.2.3.

with $\upsilon = \frac{1}{\nu} + \frac{1}{\psi} + \frac{1}{\kappa}$ and $\overline{R}$ denoting the steady-state value of the "gross interest rate". Equation (38) defines real broad money demand. As real broad money is a predetermined state variable and, therefore, given at the beginning of period t, it is already denominated in the previous period's price level, as $a_{t-1} = \frac{A_{t-1}}{P_{t-1}}$. Real broad money contains information about inflation and production of previous periods which are important for the current development of these real variables. So, real broad money has essential positive influence on inflation and production. Production is weighted by the different constant substitution elasticities of consumption, cash, sight deposits, and savings deposits ($\sigma > 0$, $\nu > 0$, $\psi > 0$, $\kappa > 0$). Real broad money is negatively related to the "gross interest rate" as that denotes a measure of opportunity costs of holding money. Equation (38) can also be interpreted in the way that households desire to raise their consumption of goods (which is always equal to the production y) if their real broad money deviates positively from its steady-state value, et vice versa.

3 Implications for monetary policy

Before analyzing different monetary policy rules it is important to define the conditions for feasible monetary policies. To find the optimal macroeconomic equilibrium path it needs to be stable and unique. So, the central bank should avoid policies that lead to (i) fluctuations on the macroeconomic equilibrium path and (ii) the existence of explosive paths. In the following, different monetary policy rules are examined for their feasibility. The problem of finding feasible monetary policy rules can be solved by analyzing the macroeconomic equilibrium path for determinacy. If the evolving system of equations is determinate the used monetary policy is feasible, et vice versa.[41]

The monetary policy rules analyzed in this section are all interest rate rules. They are divided into interest rate rules for pure inflation targeting, pure monetary targeting, and interest rate rules taking into account both changes in inflation and changes in money.

First of all, the three equations in section 2.5 are reduced to a two-equation-system. Therefore, the IS-curve and the Phillips curve are remodeled by taking real broad money demand into account:[42]

$$\frac{1}{\upsilon}\widehat{a}_t - \left(\frac{1}{\upsilon} - 1\right) E_t\widehat{\pi}_{t+1} = \frac{1}{\upsilon}\widehat{a}_{t-1} - \frac{1}{\upsilon}\widehat{\pi}_t + \frac{\overline{R}}{\overline{R}-1}\widehat{R}_t - \frac{1}{\overline{R}-1}E_t\widehat{R}_{t+1} \tag{39}$$

$$\beta E_t\widehat{\pi}_{t+1} = -\frac{\gamma}{\sigma\upsilon}\widehat{a}_{t-1} + \left(1 + \frac{\gamma}{\sigma\upsilon}\right)\widehat{\pi}_t - \frac{\gamma}{\sigma}\frac{\overline{R}}{\overline{R}-1}\widehat{R}_t. \tag{40}$$

[41]Further examples for methods to solve dynamic models can be found e.g. in Kydland and Prescott (1982), Sims (1984), King et al. (1988a, 1988b, 2002), Coleman II (1989), Novales (1990), Baxter (1991), Tauchen and Hussey (1991) and Judd (1991).

[42]The necessary remodeling and reformulation to derive the reduced model can be found in Appendix 6.3.1.

Proposition 1 *If the central bank applies pure inflation targeting, the policy rule can be of the following - simple Taylor-type - form:*[43]

$$\widehat{R}_t = \rho_\pi \widehat{\pi}_t \tag{41}$$

with $\rho_\pi > 0$ *denoting the reaction parameter of the central bank for changes in the gross inflation rate. Then there exists a stable and unique macroeconomic equilibrium path converging to the steady-state of the economy if and only if:*[44]

$$\rho_\pi < 1 \tag{42}$$

respectively

$$\rho_\pi > 1 + 2(1+\beta)\frac{\sigma}{\gamma}. \tag{43}$$

So, if the central bank applies pure inflation targeting, it should either cut or raise the gross interest rate on government bonds by less than one-for-one on changes in inflation or by much more than just one-for-one on changes in inflation to conduct a feasible monetary policy.[45] Otherwise, a central bank altering the gross interest rate on government bonds by more than one-for-one on changes in inflation ($\rho_\pi > 1$) would not pursue a feasible monetary policy.

Proposition 2 *If the central bank applies pure monetary targeting, the policy rule can be of the following form:*

$$\widehat{R}_t = \rho_a \widehat{a}_{t-1} \tag{44}$$

where $\rho_a > 0$ *denotes the reaction parameter of the central bank on changes in real broad money*[46]*. There exist multiple stable macroeconomic equilibrium paths converging to the steady-state of the economy, because the macroeconomic equilibrium is indeterminate in this case.*[47]

As there does not exist only one stable macroeconomic equilibrium path, in this model, a monetary policy rule to conduct pure monetary targeting is not feasible. Maybe, the central bank should always react on changes in inflation as well:

Proposition 3 *If the monetary policy rule is given by the following interest rate feedback rule:*

$$\widehat{R}_t = \rho_a \widehat{a}_{t-1} + \rho_\pi \widehat{\pi}_t \tag{45}$$

[43] Bernanke and Woodford (1997) as well as Carlstrom and Fuerst (2001) show that forecast-based policy rules are susceptible to real indeterminacy of rational expectations equilibria, and Svensson (2001) discovers time-inconsistency problems of these rules. Therefore, it is refrained from monetary policy rules including expected inflation.

[44] The proof of proposition 1 can be found in Appendix 6.3.2.

[45] Benhabib et al. (2001, p. 181 et seq.) name these monetary policies "passive" if $\rho_\pi < 1$, "active" if $\rho_\pi > 1$, and "hyperactive" if $\rho_\pi \gg 1$.

[46] The reaction parameter on real broad money (ρ_a) should always be positive, because a central bank reacting on changes in broad money should raise (cut) the interest rate if broad money increases (decreases). Therefore, the condition that $\rho_a > 0$ is given by definition.

[47] The proof of proposition 2 is found in Appendix 6.3.3.

- with $\rho_\pi > 0$, $\rho_a > 0$ denoting the reaction parameters of the central bank on changes in inflation respectively in real broad money - there exists a stable and unique macroeconomic equilibrium path converging to the steady-state of the economy if and only if:[48]

$$\rho_\pi < 1 + \rho_a \left[\frac{\sigma \upsilon}{\gamma} (\beta - 1) + \frac{1 + \overline{R} (\upsilon - 1)}{\overline{R} - 1} \right] \tag{46}$$

respectively

$$\rho_\pi > 1 + 2 (1 + \beta) \frac{\sigma}{\gamma} + \rho_a \left[\frac{\sigma \upsilon}{\gamma} \frac{\left(\overline{R} + 1\right) (\beta + 1)}{\overline{R} - 1} + \frac{1 + \overline{R} (\upsilon - 1)}{\overline{R} - 1} \right]. \tag{47}$$

In this case, the central bank can cut or raise the gross interest rate on government bonds by less than one-for-one on changes in inflation to conduct a feasible monetary policy. But it is also possible that the central bank cuts or raises the gross interest rate on government bonds by more than one-for-one on changes in inflation if $\rho_a > 0$, and if the term in square brackets in equation (47) is positive. Therefore, the central bank can conduct a more active monetary policy than in the case of pure inflation targeting.

Summarizing these results, one can conclude that it depends very much on the interest rate rule whether the central bank should change the gross interest rate on government bonds by more than one-for-one on changes in inflation or by less than one-for-one on changes in inflation to conduct a feasible monetary policy. In the case of pure inflation targeting, the central bank should cut or raise the gross interest rate on government bonds by less than one-for-one on changes in inflation. But in the case of an interest rate rule also containing a real broad monetary aggregate, the central bank can additionally change the gross interest rate on government bonds by more than one-for-one on changes in inflation. The range for the reaction parameter on changes in inflation (ρ_π) is greater than in the case of pure inflation targeting. Therefore, policy makers are able to react more actively on changes in inflation and the probability to react incorrectly is much smaller. Price stability - as a potential policy objective - could then be pursued more effectively than in the case of pure inflation targeting. This analysis shows that taking real broad monetary aggregates into account in monetary policy rules alters the reaction of the central bank on changes in inflation fundamentally and therefore also the feasible monetary policy.

4 Simulation of the model

In section 3, it was shown that there exist two feasible monetary policy rules for the business cycle model developed in this paper (ref. to Proposition 1 and Proposition 3). This implies the problem that a policy recommendation should be unambiguous. The monetary authority should choose a feasible policy rule which is optimal. The optimal policy rule for a New Keynesian business cycle model like this one is a Taylor-type interest rate rule considering changes in inflation and in the output gap.[49]

[48] The proof of proposition 3 can be found in Appendix 6.3.4.

[49] For the derivation of this instrument rule, please refer to the Appendix of Clarida et al. (1999).

Here, the feasible monetary policy rules do not take the output gap into consideration. Therefore, these two policy rules must be evaluated for their efficiency to find the optimal rule. The efficiency of a policy rule is usually measured by a welfare criterion.[50] The utility-based approach of welfare analysis derives this welfare criterion from the households' preferences.[51] This approach states that households' welfare is defined as a steady output growth in relation with price stability. So, a minimal variability in the output gap and price stability are relevant for welfare. In contrast, high variabilities of the output gap and inflation are producing welfare losses, e.g. caused by inefficiencies in the labor market, technology shocks or nominal rigidities as sticky prices. An efficient monetary policy rule should be one that minimizes welfare losses.[52]

A central bank should adopt this social welfare function as its policy objective function to pursue an efficient respectively optimal monetary policy, as it is obvious that a monetary policy rule should fit the central bank's policy objective. Therefore, the monetary authority's policy objective is to stabilize prices and output growth, or - in negation - to minimize inflation and output gap variability. This is e.g. the policy objective of the Board of Governors of the Federal Reserve System (FED).[53] The European Central Bank's (ECB) policy objective would be slightly different as its mandate is to preserve price stability only. In the following, the FED's policy objective is formulated as the minimization of an ad hoc-loss function which is the weighted sum of unconditional variances[54] of the inflation rate and the output gap[55]:

$$L_{FED} = Var\left(\widehat{\pi}_t\right) + \alpha Var\left(\widehat{y}_t\right), \tag{48}$$

with $\alpha > 0$ determining the central banker's preference to reduce output gap variability in relation to inflation variability. In the ECB's loss function $\alpha = 0$[56], so that[57]

$$L_{ECB} = Var\left(\widehat{\pi}_t\right). \tag{49}$$

The model was calibrated and simulated by solving it for the recursive law of motion by applying the method of undetermined coefficients.[58] Calibrating the model, the following values which are frequently used in the literature were applied: The discount factor $\beta = 0.99$ implies a

[50] Welfare is here defined as economic surplus, and welfare loss is loss in economic surplus (see Aizenman and Frenkel (1986)).

[51] See Woodford (2003) for several examples for the derivation of social welfare functions depending on slight changes in the households' utility function.

[52] See Ball (1999).

[53] See Levin et al. (2003), p. 623.

[54] As the central bank should treat positive and negative deviations from steady-state values equally, the objective or loss function should be of a quadratic form or should consider variances respectively (see Brainard (1967), Tobin (1990), Blinder (1997) or Chadha and Schellekens (1999)).

[55] In this model, the Phillips curve is formulated as a function of the inflation rate. Therefore, the policy objective is to stabilize inflation (see also Woodford (2003) for economic reasoning).

[56] A central banker who places no weight on the variance of the output gap is called an "inflation nutter" by King (1997), p. 89.

[57] Further examples of loss functions of central banks can be found in Taylor (1979), Erceg et al. (2000), Fuhrer (2000), and Amato and Laubach (2001). The interested reader can find the derivation of an optimal objective function for a central bank in Debelle (1996).

[58] For the derivation of the method of undetermined coefficients see Muth (1961), Sargent (1979) or Whiteman (1983).

rate of intertemporal preference of 1 % per quarter. This leads to a quite realistic 4 % real interest rate per year in the steady state.[59] The substitution elasticities for consumption, labor, cash, sight deposits, and savings deposits are all equal and set to 1.5 ($\sigma = \varphi = \nu = \psi = \kappa = 1.5$).[60] The probability to keep prices fixed in a period is $\theta = 0.75$. This could be interpreted as a price contract duration parameter implying an average contract duration of $\frac{1}{1-0.75} = 4$ (periods). Therefore, in a quarterly model, the price can be set optimally with the probability of $(1 - 0.75) = 0.25$.[61] The influence of the technology shock variable z_t should be quite persistent, therefore, $\mu = 0.95$ with $\sigma_\zeta^2 = 0.5069$.

The model was simulated by using the algorithm of Uhlig (1997) with MATLAB. First, the two monetary policy rules were evaluated by their value of the FED's loss function: For $\alpha = 0.5$, ρ_π was varied in the interval [0.5;0.8] (for the monetary policy rule without money) respectively in the interval [0.5;1.5] (for the monetary policy rule with money).[62] The following losses - calculated based on the frequency-domain method - occurred:

Table 1: Welfare losses of the FED

values for ρ_π		0.5	0.8	1.5
Rule 1:	$\widehat{R}_t = \rho_\pi \widehat{\pi}_t$	0.1407	0.1906	
Rule 2:	$\widehat{R}_t = \rho_a \widehat{a}_{t-1} + \rho_\pi \widehat{\pi}_t$	0.1361	0.1358	0.1353

It is shown, that for the monetary policy rule without money the welfare losses rise with increasing ρ_π. In contrast, welfare losses decrease with increasing ρ_π, if the monetary policy rule with money is used. Furthermore, welfare losses generated by the monetary policy rule considering also broad money are smaller than those arising from the monetary policy rule considering inflation only. The monetary policy rule with money is more efficient than the monetary policy rule without money even though the reaction parameter on changes in real broad money ρ_a is very small. Therefore, a policy recommendation for the FED would be to use a monetary policy rule with inflation and broad money as indicators and a relatively high value for ρ_π.

Then, the two monetary policy rules were compared by their value of the ECB's loss function: Again, ρ_π was varied in the interval [0.5;0.8] and [0.5;1.5] (see above).[63] The following losses - calculated based on the frequency-domain method - occurred:

Table 2: Welfare losses of the ECB

values for ρ_π		0.5	0.8	1.5
Rule 1:	$\widehat{R}_t = \rho_\pi \widehat{\pi}_t$	0.079	0.164	
Rule 2:	$\widehat{R}_t = \rho_a \widehat{a}_{t-1} + \rho_\pi \widehat{\pi}_t$	0.043	0.046	0.051

[59] See e.g. Canova (1993), Erceg et al. (2000), Smets and Wouters (2003) and Casares (2004).

[60] See e.g. Erceg et al. (2000) where it is shown that the substitution elasticities should be greater than 1.

[61] See Erceg et al. (2000).

[62] In the policy rule with money, the central bank's reaction parameter on money $\rho_a = 0.03$. The sensitivity analysis of ρ_a showed that the welfare loss function generated by the monetary policy rule with money is convex. In the range of $0.5 \leq \rho_\pi \leq 1.5$, the monetary policy rule with $\rho_a = 0.03$ produces the minimal monotonously decreasing welfare loss function. For $0 < \rho_a < 0.03$ as well as for $0.03 < \rho_a < 5.0$ the welfare losses are slightly higher but they are still smaller than those arising from the monetary policy rule without money. A parameter value of $\rho_a \geq 5.0$ seems to be implausible for being used by a central bank.

[63] In the policy rule with money, the central bank's reaction parameter on money is again $\rho_a = 0.03$. Here, the sensitivity analysis of ρ_a showed that the welfare losses decreased with raising ρ_a.

Here, the welfare losses are rising with increasing ρ_π for both monetary policy rules. And again, the welfare losses generated by the monetary policy rule with broad money are considerably smaller than those arising from the monetary policy rule for pure inflation targeting. Therefore, in this case, the monetary policy rule with money is also more efficient than the monetary policy rule without money. A policy recommendation for the ECB would therefore be to use a monetary policy rule with broad money as well, but this time with a relatively low value for ρ_π, even though the absolute differences of the loss values are quite small for this rule.

Here, the fundamental influence of real broad money demand and its positive impact on welfare losses was shown.[64] Therefore, for both central banks' policy objectives the monetary policy rule including a broad monetary aggregate as an indicator would be optimal, because it is more efficient than the policy rule for pure inflation targeting.

5 Conclusions

In this paper, it was shown that real broad money can have a substantial effect on the macroeconomic equilibrium path. The introduction of a financial intermediation sector as well as the definition of real broad money as a predetermined state variable affect real broad money demand. If real broad money is additionally considered as an indicator in the monetary policy rule of the central bank the conditions for stability and uniqueness of the macroeconomic equilibrium path change fundamentally. The optimal monetary policy rule in this model is one that includes the inflation rate as well as a broad monetary aggregate as indicators. It reduces the welfare loss considerably. This result points out that monetary aggregates (especially broad monetary aggregates) and their movements contain very important information for the development of output (in the short-run) and inflation (in the longer-run). This information would be lossed if only pure inflation targeting was used.

Summarizing the paper, one can conclude that real broad monetary aggregates seem to be very relevant for monetary policy models and that they should explicitly be taken into consideration by central banks when conducting monetary policy.

[64] This influence of real broad money is especially determined by the sum of the inverse substitution elasticities of cash, sight deposits and savings deposits ($\upsilon = \frac{1}{\nu} + \frac{1}{\psi} + \frac{1}{\kappa}$). Raising these elasticities generates slightly rising welfare losses. But, even if these elasticities were increased to such extreme values as $\nu = \psi = \kappa = 100$, the welfare losses were still smaller than those generated by the monetary policy rule without money.

6 Appendix

6.1 Derivation of the New Keynesian Phillips curve

6.1.1 The demand function for the intermediate good

For the derivation of the demand function for the intermediate good, the aggregator's optimization problem has to be solved. It is assumed, that he maximizes his real profit:

$$y_t - \int_0^1 \frac{P_t(j)}{P_t} y_t(j)\, dj \tag{50}$$

whereby y_t represents the real earnings of selling the final consumption good to households and the second term denotes the real costs to obtain the intermediate good. Maximizing the aggregator's profit function considering the aggregating production function (12):

$$\max_{y_t(j)} \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}}\, dj \right]^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 \frac{P_t(j)}{P_t} y_t(j)\, dj \tag{51}$$

leads to the following first order condition:

$$y_t(j)^{-\frac{1}{\epsilon}} = \frac{P_t(j)}{P_t} y_t^{-\frac{1}{\epsilon}}$$

which can be reformulated to the demand function for the intermediate good:

$$y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} y_t. \tag{52}$$

It is also possible to derive the aggregated price index P_t from the aggregator's optimization problem under consideration of the demand function for the intermediate good in the equation of the expenditures for the intermediate good:

$$\int_0^1 \frac{P_t(j)}{P_t} y_t(j)\, dj = \int_0^1 P_t(j)^{1-\epsilon}\, dj \frac{1}{P_t^{1-\epsilon}} y_t.$$

This should be the same as the real value of one unit y_t, if one defines the price index as the price of one unit final consumption good:

$$\begin{aligned} \int_0^1 P_t(j)^{1-\epsilon}\, dj \frac{1}{P_t^{1-\epsilon}} y_t &= y_t \\ \int_0^1 P_t(j)^{1-\epsilon}\, dj &= P_t^{1-\epsilon} \end{aligned}$$

so that the price index will be denoted as follows:

$$P_t = \left[\int_0^1 P_t(j)^{1-\epsilon}\, dj \right]^{\frac{1}{1-\epsilon}}. \tag{53}$$

6.1.2 The optimal price for the intermediate good

The modified intermediate goods producers' optimization problem has the following form (see equation (20)):

$$\max_{P_t^*(j)} \sum_{k=0}^{\infty} (\theta\beta)^k E_t \frac{\lambda_{t+k}}{\lambda_t} \frac{1}{P_{t+k}} \left(\frac{P_t^*(j)}{P_{t+k}} \right)^{-\epsilon} y_{t+k} \left[P_t^*(j) - MC_{t+k} \right]. \tag{54}$$

It is possible to obtain the first order condition by derivating the profit function and setting it equal to zero:

$$0 = \sum_{k=0}^{\infty} (\theta\beta)^k E_t \frac{\lambda_{t+k}}{\lambda_t} P_t^*(j)^{-\epsilon} P_{t+k}^{\epsilon-1} y_{t+k} \left[P_t^*(j) - \frac{\epsilon}{\epsilon-1} MC_{t+k} \right]$$

Setting $\frac{\epsilon}{\epsilon-1} = \mu$ (markup on the marginal costs) and reformulating lead to the following equation:

$$\sum_{k=0}^{\infty} (\theta\beta)^k E_t \frac{\lambda_{t+k}}{\lambda_t} P_{t+k}^{\epsilon-1} y_{t+k} P_t^*(j) = \sum_{k=0}^{\infty} (\theta\beta)^k E_t \frac{\lambda_{t+k}}{\lambda_t} P_{t+k}^{\epsilon-1} y_{t+k} \mu MC_{t+k}. \tag{55}$$

After log-linearizing by a first order Taylor-approximation, one obtains:

$$\begin{aligned} &\overline{P}^{\epsilon-1} \overline{y} \overline{P} \sum_{k=0}^{\infty} (\theta\beta)^k E_t \left[\widehat{\lambda}_{t+k} - \widehat{\lambda}_t + (\epsilon-1) \widehat{P}_{t+k} + \widehat{y}_{t+k} + \widehat{P_t^*} \right] \\ = \; &\overline{P}^{\epsilon-1} \overline{y} \mu \overline{MC} \sum_{k=0}^{\infty} (\theta\beta)^k E_t \left[\widehat{\lambda}_{t+k} - \widehat{\lambda}_t + (\epsilon-1) \widehat{P}_{t+k} + \widehat{y}_{t+k} + \widehat{MC}_{t+k} \right]. \end{aligned}$$

As in the steady-state all firms are allowed to adjust their prices, it is here assumed that $\overline{P} = \mu \overline{MC}$:

$$\frac{1}{1-\theta\beta} \widehat{P_t^*} = \sum_{k=0}^{\infty} (\theta\beta)^k E_t \widehat{MC}_{t+k}.$$

This leads to the following optimal price for the intermediate good:[65]

$$\widehat{P_t^*} = (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k E_t \widehat{MC}_{t+k} \tag{56}$$

$$= (1-\theta\beta) \widehat{MC}_t + (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^{k+1} E_t \widehat{MC}_{t+k+1}. \tag{57}$$

Rewriting equation (57) in forward-looking terms, multiplying it with $\theta\beta$, and then inserting it into the second term on the right-hand side of (57) lead to the optimal price level for the intermediate good:

$$\begin{aligned} E_t \widehat{P^*}_{t+1} &= (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k E_t \widehat{MC}_{t+k+1} \\ \theta\beta E_t \widehat{P^*}_{t+1} &= (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^{k+1} E_t \widehat{MC}_{t+k+1} \end{aligned}$$

[65] See Galí and Gertler (1999), p. 199.

$$\widehat{P_t^*} = (1-\theta\beta)\,\widehat{MC}_t + \theta\beta E_t \widehat{P^*}_{t+1}. \tag{58}$$

For the further computations, the equation (58) is rewritten - along the lines of the definition $\widehat{x}_t = \log x_t - \log \overline{x}$:

$$\log P_t^* - \log \overline{P} = (1-\theta\beta)\left(\log MC_t - \log \overline{MC}\right) + \theta\beta E_t \left(P_{t+1}^* - \log \overline{P}\right). \tag{59}$$

The steady-state condition for all firms $\overline{P} = \mu \overline{MC}$ looks like $\log \overline{P} = \log \mu + \log \overline{MC}$ in log-terms:

$$\log P_t^* = (1-\theta\beta)(\log MC_t + \log \mu) + \theta\beta E_t \log P_{t+1}^*.$$

Rewriting this equation in relation to the price level and working with a few tautologies lead to a first order difference equation:

$$\begin{aligned}
\log P_t^* - \log P_t &= (1-\theta\beta)(\log MC_t + \log\mu) + \theta\beta E_t \log P_{t+1}^* - \log P_t \\
&= (1-\theta\beta)(\log MC_t - \log P_t + \log\mu) + (1-\theta\beta)\log P_t \\
&\quad + \theta\beta E_t\left(\log P_{t+1}^* - \log P_{t+1}\right) + \theta\beta E_t \log P_{t+1} - \log P_t \\
&= (1-\theta\beta)\left(\log \frac{MC_t}{P_t} + \log\mu\right) + \theta\beta E_t\left(\log P_{t+1}^* - \log P_{t+1}\right) \\
&\quad + \theta\beta E_t\left(\log P_{t+1} - \log P_t\right)
\end{aligned}$$

$$\begin{aligned}
\log P_t^* - \log P_t &= \theta\beta E_t\left(\log P_{t+1}^* - \log P_{t+1}\right) + (1-\theta\beta)\left(\log \frac{MC_t}{P_t} + \log\mu\right) \\
&\quad + \theta\beta E_t \log \pi_{t+1}.
\end{aligned} \tag{60}$$

6.1.3 The aggregated price level

The aggregated price level can be calculated by reformulating the aggregated price index (15):

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon}\, di\right]^{\frac{1}{1-\epsilon}} \tag{61}$$

where the index i indicates the intermediate goods producers in this case. Therefore, the index j (for the intermediate good) was replaced. In the following, $P_{s,t}^*(i)$ denotes a price which was set in period s and has to be fixed until period t. Now, the price setting mechanism of Calvo (1983) has to be taken into account: The probability to set a price in period s and to keep it fixed until period t is denoted by $\Pr\left[P_{s,t}^*(i)\right]$.[66] As the probability to keep the price of the former period is equal to θ, the following definition of $\Pr\left[P_{s,t}^*(i)\right]$ results:

$$\begin{aligned}
\Pr\left[P_{s,t}^*(i)\right] &= \theta \Pr\left[P_{s,t-1}^*(i)\right] \\
&= \theta^2 \Pr\left[P_{s,t-2}^*(i)\right] \\
&= \ldots \\
&= \theta^{t-s} \Pr\left[P_{s,s}^*(i)\right].
\end{aligned}$$

[66] See Calvo (1983), p. 385.

Here, $\Pr\left[P^*_{s,s}(i)\right]$ denotes the probability that the price can be set in each starting period with the probability of $1-\theta$. Therefore, if $s \geq 1$ then $\Pr\left[P^*_{s,s}(i)\right] = 1-\theta$, and if $s = 0$ then $\Pr\left[P^*_{s,s}(i)\right] = 1$. The probability to set a price in period s and to keep it fixed until period t is then:

$$\Pr\left[P^*_{s,t}(i)\right] = \left\{ \begin{array}{cc} \theta^{t-s}(1-\theta) & for \quad \forall t > s \geq 1 \\ \theta^t & for \quad s = 0 \end{array} \right\}. \tag{62}$$

In a symmetric economy, all firms will set their prices equally if they are allowed to, so that $P^*_{s,t}(i) = P^*_{s,t}$. There exist two groups of firms: One group is allowed to set the price in a certain period, and one group is not allowed to set the price. The assumption of a symmetric economy implies that the probability to set the price is equal to the fraction of firms who are allowed to set the price. Therefore, a fraction of $1-\theta$ firms is allowed to set the price in a certain period, and a fraction of θ firms is not allowed to do it. So, the aggregated price index can be reformulated in dependence on former prices as:

$$\begin{aligned} P_t &= \left[\int_\theta^1 P^*_t(i)^{1-\epsilon}\,di + ... + \int_{\theta^t}^{\theta^t+(1-\theta)\theta^{t-1}} P^*_1(i)^{1-\epsilon}\,di + \int_0^{\theta^t} P^*_0(i)^{1-\epsilon}\,di\right]^{\frac{1}{1-\epsilon}} & (63)\\ &= \left[\int_\theta^1 P^{*1-\epsilon}_t di + ... + \int_{\theta^t}^{\theta^t+(1-\theta)\theta^{t-1}} P^{*1-\epsilon}_1 di + \int_0^{\theta^t} P^{*1-\epsilon}_0 di\right]^{\frac{1}{1-\epsilon}} & (64)\\ &= \left[(1-\theta)\,P^{*1-\epsilon}_t + ... + (1-\theta)\,\theta^{t-1}P^{*1-\epsilon}_1 + \theta^t P^{*1-\epsilon}_0\right]^{\frac{1}{1-\epsilon}}. & (65)\end{aligned}$$

The index i vanished, because all price setting firms behave equally in the particular periods. If the observed period is infinite the aggregated price level can be denoted in a compressed form:

$$P_t = \left[\sum_{j=0}^{\infty}\theta^j\,(1-\theta)\,P^{*1-\epsilon}_{t-j}\right]^{\frac{1}{1-\epsilon}} \tag{66}$$

$$P^{1-\epsilon}_t = (1-\theta)\,P^{*1-\epsilon}_t + \theta\,(1-\theta)\,P^{*1-\epsilon}_{t-1} + \theta^2\,(1-\theta)\,P^{*1-\epsilon}_{t-2} + ... \tag{67}$$

The price level is composed of former optimal prices. Taking the price level of period $t-1$ and multiplying it with θ leads to a simplified expression of the aggregated price level:

$$\begin{aligned} P_{t-1} &= \left[\sum_{j=0}^{\infty}\theta^j\,(1-\theta)\,P^{*1-\epsilon}_{t-j-1}\right]^{\frac{1}{1-\epsilon}} \\ P^{1-\epsilon}_{t-1} &= (1-\theta)\,P^{*1-\epsilon}_{t-1} + \theta\,(1-\theta)\,P^{*1-\epsilon}_{t-2} + \theta^2\,(1-\theta)\,P^{*1-\epsilon}_{t-3} + ... \\ \theta P^{1-\epsilon}_{t-1} &= \theta\,(1-\theta)\,P^{*1-\epsilon}_{t-1} + \theta^2\,(1-\theta)\,P^{*1-\epsilon}_{t-2} + \theta^3\,(1-\theta)\,P^{*1-\epsilon}_{t-3} + ... \\ P^{1-\epsilon}_t &= (1-\theta)\,P^{*1-\epsilon}_t + \theta P^{1-\epsilon}_{t-1} \\ P_t &= \left[(1-\theta)\,P^{*1-\epsilon}_t + \theta P^{1-\epsilon}_{t-1}\right]^{\frac{1}{1-\epsilon}}. \qquad (68)\end{aligned}$$

Now, the aggregated price level is a combination of the current optimal price level and the price level of the former period.[67] Log-linearizing this term by a first order Taylor-approximation,

[67] Ref. to King and Wolman (1996), p. 89.

leads to:

$$P_t - \overline{P} = (1-\theta)\left(P_t^* - \overline{P}\right) + \theta\left(P_{t-1} - \overline{P}\right)$$
$$P_t = \theta P_{t-1} + (1-\theta) P_t^*. \tag{69}$$

Rewriting equation (69) in log-terms and using some tautologies:

$$\begin{aligned}
\log P_t &= \theta \log P_{t-1} + (1-\theta)\log P_t^* \\
\log P_t + \theta \log P_t - \theta \log P_t &= \theta \log P_{t-1} + (1-\theta)\log P_t^* \\
\log P_t - \log P_{t-1} &= \frac{1-\theta}{\theta}\left(\log P_t^* - \log P_t\right) \\
\log \pi_t &= \frac{1-\theta}{\theta}\left(\log P_t^* - \log P_t\right)
\end{aligned}$$

$$\log P_t^* - \log P_t = \frac{\theta}{1-\theta}\log \pi_t.$$

Inserting this expression into equation (60):

$$\begin{aligned}
\frac{\theta}{1-\theta}\log \pi_t &= \theta\beta\frac{\theta}{1-\theta}E_t\left(\log \pi_{t+1}\right) + (1-\theta\beta)\left(\log\frac{MC_t}{P_t} + \log \mu\right) \\
&\quad +\theta\beta E_t \log \pi_{t+1} \\
\log \pi_t &= \beta E_t\left(\log \pi_{t+1}\right) + (1-\theta\beta)\frac{1-\theta}{\theta}\left(\log\frac{MC_t}{P_t} + \log \mu\right) \qquad (70)
\end{aligned}$$

leads to the New Keynesian Phillips curve in log-linearized form:

$$\widehat{\pi}_t = \beta E_t\left(\widehat{\pi}_{t+1}\right) + \chi \widehat{mc}_t \tag{71}$$

with $\chi = (1-\theta\beta)\frac{1-\theta}{\theta}$ and $mc_t = \frac{MC_t}{P_t}$.[68]

6.2 Derivation of the macroeconomic equilibrium path

6.2.1 Derivation of the IS-curve

To derive the forward-looking IS-curve, the first order condition of households for real consumption (6) and the first order condition of financial intermediaries for the real value of government bonds (28) are combined:

$$c_t^{-\sigma} = \beta E_t\left[\lambda_{t+1} R_{t+1} \pi_{t+1}^{-1}\right]. \tag{72}$$

Log-linearizing equation (72) by a first order Taylor-approximation leads to:

$$-\sigma\widehat{c}_t = E_t\widehat{\lambda}_{t+1} + E_t\widehat{R}_{t+1} - E_t\widehat{\pi}_{t+1}.$$

As $c_t^{-\sigma} = \lambda_t$ and $\overline{c}^{-\sigma} = \overline{\lambda}$ hold, the following expression is valid for this economy: $-\sigma\widehat{c}_t = \widehat{\lambda}_t$. The equation

$$\begin{aligned}
-\sigma\widehat{c}_t &= -\sigma E_t\widehat{c}_{t+1} + E_t\widehat{R}_{t+1} - E_t\widehat{\pi}_{t+1} \\
\widehat{c}_t &= E_t\widehat{c}_{t+1} - \frac{1}{\sigma}E_t\left[\widehat{R}_{t+1} - \widehat{\pi}_{t+1}\right] \qquad (73)
\end{aligned}$$

[68] See Galí and Gertler (1999), p. 200.

can be interpreted as an intertemporal consumption function. In this economy, no investment takes place, so that $c_t = y_t$, which means that $\overline{c} = \overline{y}$ and $\widehat{c}_t = \widehat{y}_t$ hold. Rewriting the intertemporal consumption function will lead to the IS-curve (36):

$$\widehat{y}_t = E_t \widehat{y}_{t+1} - \frac{1}{\sigma} E_t \left[\widehat{R}_{t+1} - \widehat{\pi}_{t+1} \right]. \tag{74}$$

6.2.2 Derivation of the Phillips curve

Remodeling the New Keynesian Phillips curve:

$$\widehat{\pi}_t = \chi \widehat{mc}_t + \beta E_t \widehat{\pi}_{t+1}$$

with the help of the intermediate goods producers' production function:

$$mc_t = w_t \frac{1}{z_t} = \frac{l_t^{\varphi}}{c_t^{-\sigma}} \frac{1}{z_t}$$

- after log-linearizing by a first order Taylor-approximation -

$$\widehat{mc}_t = \varphi \widehat{l}_t + \sigma \widehat{c}_t - \widehat{z}_t$$

- as the production function is: $y_t = z_t l_t$, the following holds: $\widehat{y}_t = \widehat{z}_t + \widehat{l}_t$ -

$$\widehat{mc}_t = (\varphi + \sigma) \widehat{y}_t - (1 + \varphi) \widehat{z}_t$$

leads to the following Phillips curve:

$$\begin{aligned} \widehat{\pi}_t &= \chi (\varphi + \sigma) \widehat{y}_t + \beta E_t \widehat{\pi}_{t+1} - \chi (1 + \varphi) \widehat{z}_t \\ \widehat{\pi}_t &= \gamma \widehat{y}_t + \beta E_t \widehat{\pi}_{t+1} - \vartheta \widehat{z}_t \end{aligned} \tag{75}$$

with $\gamma = \chi (\varphi + \sigma)$ and $\vartheta = \chi (1 + \varphi)$.

6.2.3 Derivation of real broad money demand

Log-linearizing the first order condition of households for real cash (8) ($\left(m_t^h\right)^{-\nu} - \lambda_t i_t^d = d_t^{-\kappa}$) by a first order Taylor-approximation, leads to:

$$\begin{aligned} -\nu \overline{m^h}^{-\nu} \widehat{m_t^h} - \left[\overline{i^d} \overline{\lambda} \widehat{\lambda}_t + \overline{i^d} \overline{\lambda} \widehat{i_t^d} \right] &= -\kappa \overline{d}^{-\kappa} \widehat{d}_t \\ -\nu \overline{m^h}^{-\nu} \widehat{m_t^h} - \overline{i^d} \overline{c}^{-\sigma} \left(-\sigma \widehat{c}_t + \widehat{i_t^d} \right) &= -\kappa \overline{d}^{-\kappa} \widehat{d}_t \end{aligned}$$

$$-\nu \overline{m^h}^{-\nu} \widehat{m_t^h} = -\kappa \overline{d}^{-\kappa} \widehat{d}_t - \overline{i^d} \overline{c}^{-\sigma} \sigma \widehat{c}_t + \overline{i^d} \overline{c}^{-\sigma} \widehat{i_t^d}.$$

As $i_t^d = R_t^d - 1$ and $\overline{i^d} = \overline{R^d} - 1$, the following holds true: $\widehat{i_t^d} = \widehat{R_t^d} \frac{\overline{R^d}}{\overline{R^d} - 1}$.

$$-\nu \overline{m^h}^{-\nu} \widehat{m_t^h} = -\kappa \overline{d}^{-\kappa} \widehat{d}_t - \sigma \left(\overline{R^d} - 1 \right) \overline{c}^{-\sigma} \widehat{c}_t + \overline{R^d} \overline{c}^{-\sigma} \widehat{R_t^d}. \tag{76}$$

Log-linearizing the first order condition of households for real sight deposits (9) ($(d_t^o)^{-\psi} - \lambda_t i_t^d = d_t^{-\kappa}$) by a first order Taylor-approximation leads to an analogous equation:

$$-\psi \overline{d^o}^{-\psi} \widehat{d_t^o} = -\kappa \overline{d}^{-\kappa} \widehat{d}_t - \sigma \left(\overline{R^d} - 1 \right) \overline{c}^{-\sigma} \widehat{c}_t + \overline{R^d}\,\overline{c}^{-\sigma} \widehat{R_t^d}. \tag{77}$$

Combining the first order condition of households for real broad money (10) with the first order condition of financial intermediaries for government bonds (28) leads to:

$$E_t \left[\lambda_{t+1} \frac{1 + i_{t+1}}{\pi_{t+1}} \right] - E_t \left(\lambda_{t+1} \frac{\left(1 + i_{t+1}^d\right)}{\pi_{t+1}} \right) = E_t \left(d_{t+1}^{-\kappa} \frac{1}{\pi_{t+1}} \right).$$

After log-linearizing this equation by a first order Taylor-approximation:

$$\begin{aligned} \left(\overline{R} - \overline{R^d} \right) \overline{\lambda} E_t \widehat{\lambda}_{t+1} + \overline{\lambda} \left(\overline{R} E_t \widehat{R}_{t+1} - \overline{R^d} E_t \widehat{R^d}_{t+1} \right) &= -\kappa \overline{d}^{-\kappa} E_t \widehat{d}_{t+1} \\ -\sigma \left(\overline{R} - \overline{R^d} \right) \overline{c}^{-\sigma} E_t \widehat{c}_{t+1} + \overline{c}^{-\sigma} \left(\overline{R} E_t \widehat{R}_{t+1} - \overline{R^d} E_t \widehat{R^d}_{t+1} \right) &= -\kappa \overline{d}^{-\kappa} E_t \widehat{d}_{t+1} \end{aligned}$$

and reformulating this equation, one obtains:

$$-\sigma \left(\overline{R} - \overline{R^d} \right) \overline{c}^{-\sigma} \widehat{c}_t + \overline{c}^{-\sigma} \left(\overline{R} \widehat{R}_t - \overline{R^d} \widehat{R^d}_t \right) = -\kappa \overline{d}^{-\kappa} \widehat{d}_t. \tag{78}$$

Inserting this expression into equation (76):

$$\begin{aligned} -\nu \overline{m^h}^{-\nu} \widehat{m_t^h} \;&=\; -\sigma \left(\overline{R} - \overline{R^d} \right) \overline{c}^{-\sigma} \widehat{c}_t + \overline{c}^{-\sigma} \left(\overline{R} \widehat{R}_t - \overline{R^d} \widehat{R^d}_t \right) - \sigma \left(\overline{R^d} - 1 \right) \overline{c}^{-\sigma} \widehat{c}_t \\ &\quad + \overline{R^d}\,\overline{c}^{-\sigma} \widehat{R_t^d} \end{aligned}$$

leads to the following cash demand function:

$$\widehat{m_t^h} = \frac{\sigma}{\nu} \widehat{c}_t - \frac{1}{\nu} \frac{\overline{R}}{\left(\overline{R} - 1\right)} \widehat{R}_t. \tag{79}$$

Cash demand is determined by the current consumption and the current gross interest rate of government bonds. Inserting the equation (78) into equation (77) leads to an analogous demand function for sight deposits:

$$\widehat{d_t^o} = \frac{\sigma}{\psi} \widehat{c}_t - \frac{1}{\psi} \frac{\overline{R}}{\left(\overline{R} - 1\right)} \widehat{R}_t. \tag{80}$$

Reformulating equation (78) leads to the savings deposits' demand function:

$$\widehat{d}_t = \frac{\sigma}{\kappa} \widehat{c}_t - \frac{1}{\kappa} \frac{\left(\overline{R} \widehat{R}_t - \overline{R^d} \widehat{R^d}_t \right)}{\left(\overline{R} - \overline{R^d} \right)}. \tag{81}$$

Savings deposits' demand is not only dependent on current consumption and on the current gross interest rate on government bonds but also on the current gross interest rate on savings deposits. The definition of real broad money is:

$$a_{t-1} \frac{1}{\pi_t} = m_t^h + d_t^o + d_t.$$

Log-linearizing this equation by a first order Taylor-approximation results in:

$$\widehat{a}_{t-1}-\widehat{\pi}_t=\frac{\overline{m^h}}{\overline{a\pi}^{-1}}\widehat{m_t^h}+\frac{\overline{d^o}}{\overline{a\pi}^{-1}}\widehat{d_t^o}+\frac{\overline{d}}{\overline{a\pi}^{-1}}\widehat{d}_t. \tag{82}$$

Inserting the different demand functions of the components of real broad money leads to the real broad money demand:

$$\widehat{a}_{t-1}=\sigma\upsilon\widehat{y}_t+\widehat{\pi}_t-\upsilon\frac{\overline{R}}{\left(\overline{R}-1\right)}\widehat{R}_t \tag{83}$$

with $\upsilon=\frac{1}{\nu}+\frac{1}{\psi}+\frac{1}{\kappa}$, the sum of the inverse substitution elasticities of the different components of real broad money. Real broad money of the previous period is determined by current production, the current gross inflation rate and the current gross interest rate on government bonds.

6.3 Proofs of the propositions for feasible monetary policy

6.3.1 Derivation of the reduced model

First of all, the three-equation system of inflation, output and real broad money has to be reduced into a two-equation system of only inflation and real broad money. Therefore, the reformulation of real broad money demand (38) leads to:

$$\begin{aligned}\widehat{y}_t &= \frac{1}{\sigma\upsilon}\widehat{a}_{t-1}-\widehat{\pi}_t+\frac{1}{\sigma}\frac{\overline{R}}{\overline{R}-1}\widehat{R}_t\\ E_t\widehat{y}_{t+1} &= \frac{1}{\sigma\upsilon}\widehat{a}_t-E_t\widehat{\pi}_{t+1}+\frac{1}{\sigma}\frac{\overline{R}}{\overline{R}-1}E_t\widehat{R}_{t+1}.\end{aligned}$$

Inserting this new equation into the IS-curve (36):

$$\begin{aligned}\frac{1}{\sigma\upsilon}\widehat{a}_{t-1}-\widehat{\pi}_t+\frac{1}{\sigma}\frac{\overline{R}}{\overline{R}-1}\widehat{R}_t &= \frac{1}{\sigma\upsilon}\widehat{a}_t-E_t\widehat{\pi}_{t+1}+\\ &\quad \frac{1}{\sigma}\frac{\overline{R}}{\overline{R}-1}E_t\widehat{R}_{t+1}-\frac{1}{\sigma}E_t\left(\widehat{R}_{t+1}-\widehat{\pi}_{t+1}\right)\end{aligned}$$

leads to a modified IS-curve (39):

$$\frac{1}{\upsilon}\widehat{a}_t-\left(\frac{1}{\upsilon}-1\right)E_t\widehat{\pi}_{t+1}=\frac{1}{\upsilon}\widehat{a}_{t-1}-\frac{1}{\upsilon}\widehat{\pi}_t+\frac{\overline{R}}{\overline{R}-1}\widehat{R}_t-\frac{1}{\overline{R}-1}E_t\widehat{R}_{t+1}. \tag{84}$$

The same reformulations of equation (37) are necessary to obtain a modified Phillips curve (40):

$$\begin{aligned}\widehat{\pi}_t &= \gamma\left(\frac{1}{\sigma\upsilon}\widehat{a}_{t-1}-\widehat{\pi}_t+\frac{1}{\sigma}\frac{\overline{R}}{\overline{R}-1}\widehat{R}_t\right)+\beta E_t\widehat{\pi}_{t+1}-\vartheta\widehat{z}_t\\ \beta E_t\widehat{\pi}_{t+1} &= -\frac{\gamma}{\sigma\upsilon}\widehat{a}_{t-1}+\left(1+\frac{\gamma}{\sigma\upsilon}\right)\widehat{\pi}_t-\frac{\gamma}{\sigma}\frac{\overline{R}}{\overline{R}-1}\widehat{R}_t-\vartheta\widehat{z}_t.\end{aligned} \tag{85}$$

For the following analysis of stability and uniqueness of the macroeconomic equilibrium path, only the non-stochastic part of the dynamic system has to be examined. Therefore, the results will also hold for any monetary policy rule that depends on current or past values of the exogenous technology process. In the following proofs, only the deterministic part of the two dimensional system (equations (39) and (40)) is investigated.

6.3.2 Proof of proposition 1

If the central bank applies pure inflation targeting, the monetary policy rule can be of the following - simple Taylor-type - form:

$$\widehat{R}_t = \rho_\pi \widehat{\pi}_t.$$

Inserting this policy rule into the equations (39) and (40) will lead to a modified two-equation system:

$$\frac{1}{\upsilon}\widehat{a}_t - \left(\frac{1}{\upsilon} - 1 - \frac{\rho_\pi}{\overline{R}-1}\right) E_t\widehat{\pi}_{t+1} = \frac{1}{\upsilon}\widehat{a}_{t-1} - \left(\frac{1}{\upsilon} - \frac{\rho_\pi \overline{R}}{\overline{R}-1}\right)\widehat{\pi}_t \tag{86}$$

$$\beta E_t\widehat{\pi}_{t+1} = -\frac{\gamma}{\sigma\upsilon}\widehat{a}_{t-1} + \left(1 + \frac{\gamma}{\sigma\upsilon} - \frac{\gamma}{\sigma}\frac{\rho_\pi \overline{R}}{\overline{R}-1}\right)\widehat{\pi}_t. \tag{87}$$

In matrix form it looks like:

$$\begin{pmatrix} \frac{1}{\upsilon} & -\frac{1}{\upsilon} + 1 + \frac{\rho_\pi}{\overline{R}-1} \\ 0 & \beta \end{pmatrix} \begin{pmatrix} \widehat{a}_t \\ E_t\widehat{\pi}_{t+1} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{1}{\upsilon} & -\frac{1}{\upsilon} + \frac{\rho_\pi \overline{R}}{\overline{R}-1} \\ -\frac{\gamma}{\sigma\upsilon} & 1 + \frac{\gamma}{\sigma\upsilon} - \frac{\gamma}{\sigma}\frac{\rho_\pi \overline{R}}{\overline{R}-1} \end{pmatrix} \begin{pmatrix} \widehat{a}_{t-1} \\ \widehat{\pi}_t \end{pmatrix}.$$

Since this system contains one forward-looking and one backward-looking variable, the system is determinate if one eigenvalue of the following matrix M is larger than one in absolute value and one is smaller than one in absolute value:[69]

$$M = \begin{pmatrix} \frac{1}{\upsilon} & -\frac{1}{\upsilon} + 1 + \frac{\rho_\pi}{\overline{R}-1} \\ 0 & \beta \end{pmatrix}^{-1} \begin{pmatrix} \frac{1}{\upsilon} & -\frac{1}{\upsilon} + \frac{\rho_\pi \overline{R}}{\overline{R}-1} \\ -\frac{\gamma}{\sigma\upsilon} & 1 + \frac{\gamma}{\sigma\upsilon} - \frac{\gamma}{\sigma}\frac{\rho_\pi \overline{R}}{\overline{R}-1} \end{pmatrix}.$$

The characteristic equation of M is given by:

$$f(X) = X^2 - \frac{1}{\beta}\left(1 + \beta + \frac{\gamma}{\sigma} - \frac{\gamma}{\sigma}\rho_\pi\right)X + \frac{1}{\beta}.$$

Since

$$f(0) = \frac{1}{\beta} > 0 \qquad as \quad 0 < \beta < 1,$$

there exists one eigenvalue between zero and one, and one eigenvalue larger than one if $f(1) < 0$:

$$f(1) = 1 - \frac{1}{\beta}\left(1 + \beta + \frac{\gamma}{\sigma} - \frac{\gamma}{\sigma}\rho_\pi\right) + \frac{1}{\beta}.$$

[69] See Blanchard and Kahn (1980).

This expression is less than zero if:

$$\frac{\gamma}{\sigma\beta}\rho_\pi - \frac{\gamma}{\sigma\beta} < 0$$

$$\rho_\pi < 1.$$

The second condition to ensure determinacy of the system proves that there exists one eigenvalue between minus one and zero, and one eigenvalue smaller than minus one if $f(-1) < 0$:

$$f(-1) = 1 + \frac{1}{\beta}\left(1 + \beta + \frac{\gamma}{\sigma} - \frac{\gamma}{\sigma}\rho_\pi\right) + \frac{1}{\beta}.$$

This expression is less than zero if:

$$\begin{aligned} -\frac{\gamma}{\sigma\beta}\rho_\pi + 2 + \frac{2}{\beta} + \frac{\gamma}{\sigma\beta} &< 0 \\ \rho_\pi &> 1 + 2(1+\beta)\frac{\sigma}{\gamma}. \end{aligned}$$

This completes the proof of proposition 1 - with a monetary policy rule for pure inflation targeting.

6.3.3 Proof of proposition 2

If the central bank applies pure monetary targeting, the monetary policy rule can be of the following form:

$$\widehat{R}_t = \rho_a \widehat{a}_{t-1}.$$

Inserting this policy rule into the equations (39) and (40) will lead to a modified two-equation system:

$$\left(\frac{1}{v} + \frac{\rho_a}{\overline{R}-1}\right)\widehat{a}_t - \left(\frac{1}{v} - 1\right)E_t\widehat{\pi}_{t+1} = \left(\frac{1}{v} + \frac{\rho_a\overline{R}}{\overline{R}-1}\right)\widehat{a}_{t-1} - \frac{1}{v}\widehat{\pi}_t \tag{88}$$

$$\beta E_t\widehat{\pi}_{t+1} = \left(-\frac{\gamma}{\sigma v} - \frac{\gamma}{\sigma}\frac{\rho_a\overline{R}}{\overline{R}-1}\right)\widehat{a}_{t-1} + \left(1 + \frac{\gamma}{\sigma v}\right)\widehat{\pi}_t. \tag{89}$$

In matrix form it looks like:

$$\begin{aligned} &\begin{pmatrix} \frac{1}{v} + \frac{\rho_a}{\overline{R}-1} & -\frac{1}{v} + 1 \\ 0 & \beta \end{pmatrix} \begin{pmatrix} \widehat{a}_t \\ E_t\widehat{\pi}_{t+1} \end{pmatrix} \\ = &\begin{pmatrix} \frac{1}{v} + \frac{\rho_a\overline{R}}{\overline{R}-1} & -\frac{1}{v} \\ -\frac{\gamma}{\sigma v} - \frac{\gamma}{\sigma}\frac{\rho_a\overline{R}}{\overline{R}-1} & 1 + \frac{\gamma}{\sigma v} \end{pmatrix} \begin{pmatrix} \widehat{a}_{t-1} \\ \widehat{\pi}_t \end{pmatrix}. \end{aligned}$$

Since this system contains one forward-looking and one backward-looking variable, the system is determinate if one eigenvalue of the following matrix M is larger than one in absolute value and one is smaller than one in absolute value:

$$M = \begin{pmatrix} \frac{1}{v} + \frac{\rho_a}{\overline{R}-1} & -\frac{1}{v} + 1 \\ 0 & \beta \end{pmatrix}^{-1} \begin{pmatrix} \frac{1}{v} + \frac{\rho_a\overline{R}}{\overline{R}-1} & -\frac{1}{v} \\ -\frac{\gamma}{\sigma v} - \frac{\gamma}{\sigma}\frac{\rho_a\overline{R}}{\overline{R}-1} & 1 + \frac{\gamma}{\sigma v} \end{pmatrix}.$$

The characteristic equation of M is given by:

$$\begin{aligned} f(X) &= X^2 \\ &\quad -\frac{1}{\beta}\frac{[\beta\sigma\upsilon-\gamma(1-\upsilon)]\left(\overline{R}-1+\overline{R}\upsilon\rho_a\right)+(\sigma\upsilon+\gamma)\left(\overline{R}-1+\upsilon\rho_a\right)}{\sigma\upsilon\left(\overline{R}-1+\upsilon\rho_a\right)}X \\ &\quad +\frac{1}{\beta}\frac{\overline{R}-1+\overline{R}\upsilon\rho_a}{\overline{R}-1+\upsilon\rho_a}. \end{aligned}$$

Since

$$f(0)=\frac{1}{\beta}\frac{\overline{R}-1+\overline{R}\upsilon\rho_a}{\overline{R}-1+\upsilon\rho_a}>0 \qquad for \quad \rho_a>0,$$

there exists one eigenvalue between zero and one, and one eigenvalue larger than one if $f(1)<0$:

$$\begin{aligned} f(1) &= 1 \\ &\quad -\frac{1}{\beta}\frac{[\beta\sigma\upsilon-\gamma(1-\upsilon)]\left(\overline{R}-1+\overline{R}\upsilon\rho_a\right)+(\sigma\upsilon+\gamma)\left(\overline{R}-1+\upsilon\rho_a\right)}{\sigma\upsilon\left(\overline{R}-1+\upsilon\rho_a\right)} \\ &\quad +\frac{1}{\beta}\frac{\overline{R}-1+\overline{R}\upsilon\rho_a}{\overline{R}-1+\upsilon\rho_a} \\ &= \frac{\rho_a\left[(1-\beta)\left(\overline{R}-1\right)\sigma\upsilon\upsilon+\gamma\upsilon\left(\overline{R}-1-\upsilon\overline{R}\right)\right]-\gamma\upsilon\left(\overline{R}-1\right)}{\beta\sigma\upsilon\left(\overline{R}-1+\upsilon\rho_a\right)}. \end{aligned}$$

This expression is less than zero if:

$$\rho_a\left[(1-\beta)\left(\overline{R}-1\right)\sigma\upsilon\upsilon+\gamma\upsilon\left(\overline{R}-1-\upsilon\overline{R}\right)\right]-\gamma\upsilon\left(\overline{R}-1\right)<0$$

$$\begin{aligned} \rho_a &< \frac{\gamma\left(\overline{R}-1\right)}{(1-\beta)\left(\overline{R}-1\right)\sigma\upsilon+\gamma\left(\overline{R}-1-\upsilon\overline{R}\right)} \\ \rho_a &> 0 \qquad if \quad (1-\beta)\left(\overline{R}-1\right)\sigma\upsilon+\gamma\left(\overline{R}-1-\upsilon\overline{R}\right)>0. \end{aligned}$$

The second condition to ensure determinacy of the system proves that there exists one eigenvalue between minus one and zero, and one eigenvalue smaller than minus one if $f(-1)<0$:

$$\begin{aligned} f(-1) &= 1 \\ &\quad +\frac{1}{\beta}\frac{[\beta\sigma\upsilon-\gamma(1-\upsilon)]\left(\overline{R}-1+\overline{R}\upsilon\rho_a\right)+(\sigma\upsilon+\gamma)\left(\overline{R}-1+\upsilon\rho_a\right)}{\sigma\upsilon\left(\overline{R}-1+\upsilon\rho_a\right)} \\ &\quad +\frac{1}{\beta}\frac{\overline{R}-1+\overline{R}\upsilon\rho_a}{\overline{R}-1+\upsilon\rho_a} \\ &= \frac{\begin{array}{c}\rho_a\left[(\beta+1)\left(\overline{R}+1\right)\sigma\upsilon\upsilon+\gamma\upsilon\left(1+(\upsilon-1)\overline{R}\right)\right] \\ +\upsilon\left[\gamma+2\sigma(\beta+1)\right]\left(\overline{R}-1\right)\end{array}}{\beta\sigma\upsilon\left(\overline{R}-1+\upsilon\rho_a\right)} \end{aligned}$$

This expression is less than zero if:

$$\rho_a\left[(\beta+1)\left(\overline{R}+1\right)\sigma\upsilon\upsilon+\gamma\upsilon\left(1+(\upsilon-1)\overline{R}\right)\right]+\upsilon\left[\gamma+2\sigma(\beta+1)\right]\left(\overline{R}-1\right)<0$$

$$
\begin{aligned}
\rho_a &< -\frac{[\gamma + 2\sigma(\beta+1)]\left(\overline{R}-1\right)}{(\beta+1)\left(\overline{R}+1\right)\sigma\upsilon + \gamma\left(1+(\upsilon-1)\overline{R}\right)} \\
\rho_a &< 0.
\end{aligned}
$$

As - by definition - $\rho_a > 0$ the system is not determinate for a monetary policy rule in the case of monetary targeting. This completes the proof of proposition 2.

6.3.4 Proof of proposition 3

If the central bank applies a policy rule represented by the following interest rate feedback rule:

$$
\widehat{R}_t = \rho_a \widehat{a}_{t-1} + \rho_\pi \widehat{\pi}_t
$$

the following modified two-equation system will exist:

$$
\begin{aligned}
\left(\frac{1}{\upsilon} + \frac{\rho_a}{\overline{R}-1}\right)\widehat{a}_t - \left(\frac{1}{\upsilon} - 1 - \frac{\rho_\pi}{\overline{R}-1}\right)E_t\widehat{\pi}_{t+1} &= \left(\frac{1}{\upsilon} + \frac{\rho_a\overline{R}}{\overline{R}-1}\right)\widehat{a}_{t-1} \\
&\quad -\left(\frac{1}{\upsilon} - \frac{\rho_\pi\overline{R}}{\overline{R}-1}\right)\widehat{\pi}_t \qquad (90)
\end{aligned}
$$

$$
\beta E_t\widehat{\pi}_{t+1} = \left(-\frac{\gamma}{\sigma\upsilon} - \frac{\gamma}{\sigma}\frac{\rho_a\overline{R}}{\overline{R}-1}\right)\widehat{a}_{t-1} + \left(1 + \frac{\gamma}{\sigma\upsilon} - \frac{\gamma}{\sigma}\frac{\rho_\pi\overline{R}}{\overline{R}-1}\right)\widehat{\pi}_t. \qquad (91)
$$

In matrix form it looks like:

$$
\begin{aligned}
&\begin{pmatrix} \frac{1}{\upsilon} + \frac{\rho_a}{\overline{R}-1} & -\frac{1}{\upsilon} + 1 + \frac{\rho_\pi}{\overline{R}-1} \\ 0 & \beta \end{pmatrix} \begin{pmatrix} \widehat{a}_t \\ E_t\widehat{\pi}_{t+1} \end{pmatrix} \\
= &\begin{pmatrix} \frac{1}{\upsilon} + \frac{\rho_a\overline{R}}{\overline{R}-1} & -\frac{1}{\upsilon} + \frac{\rho_\pi\overline{R}}{\overline{R}-1} \\ -\frac{\gamma}{\sigma\upsilon} - \frac{\gamma}{\sigma}\frac{\rho_a\overline{R}}{\overline{R}-1} & 1 + \frac{\gamma}{\sigma\upsilon} - \frac{\gamma}{\sigma}\frac{\rho_\pi\overline{R}}{\overline{R}-1} \end{pmatrix} \begin{pmatrix} \widehat{a}_{t-1} \\ \widehat{\pi}_t \end{pmatrix}.
\end{aligned}
$$

Since this system contains one forward-looking and one backward-looking variable, the system is determinate if one eigenvalue of the following matrix M is larger than one in absolute value and one is smaller than one in absolute value:

$$
M = \begin{pmatrix} \frac{1}{\upsilon} + \frac{\rho_a}{\overline{R}-1} & -\frac{1}{\upsilon} + 1 + \frac{\rho_\pi}{\overline{R}-1} \\ 0 & \beta \end{pmatrix}^{-1} \begin{pmatrix} \frac{1}{\upsilon} + \frac{\rho_a\overline{R}}{\overline{R}-1} & -\frac{1}{\upsilon} + \frac{\rho_\pi\overline{R}}{\overline{R}-1} \\ -\frac{\gamma}{\sigma\upsilon} - \frac{\gamma}{\sigma}\frac{\rho_a\overline{R}}{\overline{R}-1} & 1 + \frac{\gamma}{\sigma\upsilon} - \frac{\gamma}{\sigma}\frac{\rho_\pi\overline{R}}{\overline{R}-1} \end{pmatrix}.
$$

The characteristic equation of M is given by:

$$
\begin{aligned}
f(X) &= X^2 \\
&\quad -\frac{1}{\beta}\frac{\left(\overline{R}-1\right)(\gamma+\beta\sigma+\sigma) + \rho_a\left[\overline{R}\left(\gamma(\upsilon-1)+\beta\sigma\upsilon\right)+\sigma\upsilon+\gamma\right] - \rho_\pi\gamma\left(\overline{R}-1\right)}{\sigma\left(\overline{R}-1+\upsilon\rho_a\right)}X \\
&\quad +\frac{1}{\beta}\frac{\left(\overline{R}-1\right)+\upsilon\overline{R}\rho_a}{\overline{R}-1+\upsilon\rho_a}.
\end{aligned}
$$

Since

$$f(0)=\frac{1}{\beta}\frac{\left(\overline{R}-1\right)+\upsilon\overline{R}\rho_a}{\overline{R}-1+\upsilon\rho_a}>0 \qquad for \quad \rho_a>0,$$

there exists one eigenvalue between zero and one, and one eigenvalue larger than one if $f(1)<0$:

$$\begin{aligned} f(1) &= 1-\frac{1}{\beta}\frac{\begin{array}{c}\left(\overline{R}-1\right)(\gamma+\beta\sigma+\sigma)+\rho_a\left[\overline{R}\left(\gamma\left(\upsilon-1\right)+\beta\sigma\upsilon\right)+\sigma\upsilon+\gamma\right]\\ -\rho_\pi\gamma\left(\overline{R}-1\right)\end{array}}{\sigma\left(\overline{R}-1+\upsilon\rho_a\right)} \\ &\quad +\frac{1}{\beta}\frac{\left(\overline{R}-1\right)+\upsilon\overline{R}\rho_a}{\overline{R}-1+\upsilon\rho_a} \\ &= \frac{\begin{array}{c}\gamma\left(\overline{R}-1\right)\rho_\pi+\left[\sigma\upsilon\left(\overline{R}-1\right)(1-\beta)-\gamma\left(\left(\upsilon-1\right)\overline{R}+1\right)\right]\rho_a\\ -\gamma\left(\overline{R}-1\right)\end{array}}{\beta\sigma\left(\overline{R}-1+\upsilon\rho_a\right)}. \end{aligned}$$

This expression is less than zero if:

$$\gamma\left(\overline{R}-1\right)\rho_\pi+\left[\sigma\upsilon\left(\overline{R}-1\right)(1-\beta)-\gamma\left(\left(\upsilon-1\right)\overline{R}+1\right)\right]\rho_a-\gamma\left(\overline{R}-1\right)<0$$

$$\rho_\pi<1+\rho_a\left[\frac{\sigma\upsilon}{\gamma}(\beta-1)+\frac{1+\overline{R}\left(\upsilon-1\right)}{\overline{R}-1}\right].$$

The second condition to ensure determinacy of the system proves that there exists one eigenvalue between minus one and zero, and one eigenvalue smaller than minus one if $f(-1)<0$:

$$\begin{aligned} f(-1) &= 1+\frac{1}{\beta}\frac{\begin{array}{c}\left(\overline{R}-1\right)(\gamma+\beta\sigma+\sigma)+\rho_a\left[\overline{R}\left(\gamma\left(\upsilon-1\right)+\beta\sigma\upsilon\right)+\sigma\upsilon+\gamma\right]\\ -\rho_\pi\gamma\left(\overline{R}-1\right)\end{array}}{\sigma\left(\overline{R}-1+\upsilon\rho_a\right)} \\ &\quad +\frac{1}{\beta}\frac{\left(\overline{R}-1\right)+\upsilon\overline{R}\rho_a}{\overline{R}-1+\upsilon\rho_a} \\ &= \frac{\begin{array}{c}-\gamma\left(\overline{R}-1\right)\rho_\pi+\left[(\beta+1)\left(\overline{R}+1\right)\sigma\upsilon+\gamma\left(\left(\upsilon-1\right)\overline{R}+1\right)\right]\rho_a\\ +\left(\overline{R}-1\right)(2\sigma+2\beta\sigma+\gamma)\end{array}}{\beta\sigma\left(\overline{R}-1+\upsilon\rho_a\right)}. \end{aligned}$$

This expression is less than zero if:

$$\begin{aligned} 0 &> -\gamma\left(\overline{R}-1\right)\rho_\pi+\left[(\beta+1)\left(\overline{R}+1\right)\sigma\upsilon+\gamma\left(\left(\upsilon-1\right)\overline{R}+1\right)\right]\rho_a \\ &\quad +\left(\overline{R}-1\right)(2\sigma+2\beta\sigma+\gamma) \\ \rho_\pi &> 1+2(1+\beta)\frac{\sigma}{\gamma}+\rho_a\left[\frac{\sigma\upsilon}{\gamma}\frac{\left(\overline{R}+1\right)(\beta+1)}{\overline{R}-1}+\frac{1+\overline{R}\left(\upsilon-1\right)}{\overline{R}-1}\right]. \end{aligned}$$

This completes the proof of proposition 3 - with a monetary policy rule containing inflation and real broad money.

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