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**Estimating large-scale factor models  
for economic activity in Germany:  
Do they outperform simpler models?**

**Christian Dreger  
Christian Schumacher**

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# **Estimating large-scale factor models for economic activity in Germany: Do they outperform simpler models?**

**Christian Dreger  
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## **HWWA DISCUSSION PAPER**

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## **Abstract**

This paper discusses a large-scale factor model for the German economy. Following the recent literature, a data set of 121 time series is used via principal component analysis to determine the factors, which enter a dynamic model for German GDP. The model is compared with alternative univariate and multivariate models. These models are based on regression techniques and considerably smaller data sets. Out-of-sample forecasts show that the prediction errors of the factor model are smaller than the errors of the rival models. However, these advantages are not statistically significant, as a test for equal forecast accuracy shows. Therefore, the efficiency gains of using a large data set with this kind of factor models seem to be limited.

## **Zusammenfassung**

Diese Arbeit diskutiert ein großes Faktorenmodell für die deutsche Wirtschaft. Der jüngeren Literatur folgend werden aus einem umfangreichen Datensatz von 121 Zeitreihen mit einer Hauptkomponentenanalyse gemeinsame Faktoren extrahiert, welche in ein dynamisches Modell zur Erklärung des deutschen Bruttoinlandsprodukts eingehen. Das Modell wird mit alternativen univariaten und multivariaten Modellen verglichen, die auf Regressionsansätzen und deutlich kleineren Datensätzen beruhen. Vergleiche von Prognosen außerhalb des Schätzzeitraums zeigen, dass die Prognosefehler des großen Faktorenmodells kleiner als bei den alternativen Modellen sind. Jedoch sind diese empirischen Vorteile nicht statistisch signifikant, wie Tests auf paarweise Gleichheit der Prognosegüte zeigen. Demzufolge scheinen die Effizienzvorteile des auf einem großen Datensatz beruhenden Faktorenmodells lediglich gering zu sein.

**Key Words:** Factor models, Principal components, forecasting accuracy

**JEL Classification:** E32, C51, C43

# 1 Introduction

The idea that fluctuations in a large number of economic variables can be modelled by a small number of factors is appealing and is used in many economic analyses. For example, the seminal work of Burns/Mitchell (1946) assumes that the business cycle phenomenon is characterised by simultaneous comovement in many economic activities. Following this idea, empirical approaches as in Stock/Watson (1991) often model the business cycle as a factor extracted from a number of economic variables. Behind this literature, there is the implicit assumption that the essential characteristics of the business cycle are captured by few relevant aggregate variables and that the information contained in all the potentially available aggregate time series are individually less useful to understand macroeconomic behaviour.

In the past, the empirical literature about factor models was restricted to use relatively small panels of time series to determine the common factors.<sup>1</sup> For example, Stock/Watson (1991) estimate a state-space model with an unobserved factor using four variables. Computational difficulties make it necessary to abandon information on many series even though they are available. Hence, the inclusion of a broader data set is hardly possible in those approaches. The main feature of recent factor models is the use of a larger cross-section of time series. The idea is to use many time series simultaneously and hence use the available information more efficiently. Stock/Watson (2002) and Forni et al. (2001) use principal component analysis to estimate large-scale factor models. The estimation method based on principal components is non-parametric and does not face the problem that a growing cross-section dimension leads to an increased number of parameters and higher uncertainty of coefficient estimates as in state-space models and regression approaches. The extensions of these techniques to large cross-sections can therefore be viewed as an efficient way of extracting information from a large number of data series. From an economic perspective, the large-scale factor models follow the famous definition of Burns and Mitchell (1946) closely, because many variables should better reflect the comovement of many economic activities than only a few.

This paper applies the large-scale factor model recently proposed by Stock/Watson (2002) and Artis et al. (2001) to German data. An application could be interesting, because large-scale factor-models have recently been successfully applied to forecast US macroeconomic variables.<sup>2</sup> To our knowledge, this is the first application of large-scale factor models to the German economy. The application is an extension of other recent studies that investigate mainly single indicators or small scale factor models for German economic activity. Bandholz/Funke (2001) estimate a multivariate state-space model with and without regime-switching to determine a business cycle factor. Breitung/Jagodziniski (2002) give an overview about other composite indicators. Both papers discuss factor models based on a small cross-section of time series. Fritsche/Stephan (2002) investigate the indicator properties of single economic time series using spectral analysis. Hüfner/Schröder (2002) compare the forecasting accuracy of the ifo climate index and the ZEW business expectations, both obtained from survey data. Compared with these papers, the present work questions whether the use of a larger data set in a factor model framework leads to better empirical results than smaller scaled models. Especially, we follow the recent literature and investigate the gains of predictive accuracy when using a large-scale

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<sup>1</sup>See Geweke (1977), Sargent/Sims (1977), Stock/Watson (1991), Camba-Mendez et al. (2001).

<sup>2</sup>See Stock/Watson (1999, 2002).



factor model.

Therefore, a large number of macroeconomic time series is collected to provide an exhaustive description of the German economy. The broad data set is used to estimate the factor model, and to forecast German GDP. To highlight the differences arising from the size of the information used, the forecasts of the large-scale factor model are compared with alternative forecasts from a simple univariate time series model, a vector autoregressive model, and a single time series indicator obtained from surveys.

The paper proceeds as follows: The following section contains a description of the factor model and how it can be estimated via principal components analysis. Section 3 briefly describes the German data set. In section 4, the forecasting methodology is introduced, and the factor model and its competitors are applied to German data. The calculations include recursive quasi-real time estimations and forecasts. Section 5 concludes.

## 2 The factor model

In factor models, each variable is represented as the sum of two mutually orthogonal unobservable components: the common component and the idiosyncratic component. The common component is driven by a small number of factors common to all of the variables in the model. The idiosyncratic component on the other hand is driven by variable-specific shocks. Let  $X_t$  be the  $(N \times 1)$  dimensional vector of stationary time series with observations for  $t = 1, \dots, T$ . The dynamic factor model representation is for a variable  $X_{it}$  as an element of the vector  $X_t$  given by

$$X_{it} = \lambda_i(L)f_t + \varepsilon_{it}, \quad (1)$$

where  $f_t$  is the  $(q \times 1)$  vector of factors which has to be determined from the data. The idea of the factor model is that the dimension of the factors is lower than the dimension of the data, so  $q \ll N$ . However, the small number of factors should be able to replicate most of the variance of the variables  $X_{it}$  for  $i = 1, \dots, N$ .  $\lambda_i(L)$  is a lag polynomial with non-negative power of the lag operator  $L$ , with  $Lx_t = x_{t-1}$ . Hence, it is allowed that lags of the factors affect the current movement of the variables. The  $(1 \times q)$  vector  $\lambda_i(L)$  shows how the factors and their lags determine the variable  $X_{it}$ .  $\varepsilon_{it}$  is the idiosyncratic component of the variable. It is that residual part which is not explained by the factors. The key problem of the estimation of the factor model is the determination of the factors from the model's variables. For estimation purposes, it is convenient to reformulate the model. If the lag polynomial has a finite lag order  $p$ ,  $\lambda_i(L) = \lambda_{i0} + \lambda_{i1}L + \dots + \lambda_{ip}L^p$ , and the model can be rewritten as

$$X_t = \Lambda F_t + \varepsilon_t, \quad (2)$$

where  $X_t$  is the vector of time series at time  $t$  and  $F_t = (f'_t, \dots, f'_{t-p})'$  is a  $r = q(p+1)$  dimensional vector of stacked factors. The  $(N \times r)$  dimensional parameter matrix  $\Lambda$  holds the reorganized coefficients of  $\lambda_i(L)$  for  $i = 1, \dots, N$ .<sup>3</sup> The main advantage of this static representation is that its factors can be estimated using principal component analysis.<sup>4</sup> Principal

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<sup>3</sup>See the technical appendix A for details of the derivation of (2).

<sup>4</sup>See Artis et al. (2001), Stock/Watson (2002).

component analysis allows for the estimation of factor weights based on a very large cross-section of the data set and is computationally convenient. In the present case, the principal component analysis leads to an eigenvalue problem of the variance covariance matrix of the time series vector  $X_t$  and the corresponding eigenvectors form the parameter matrix  $\Lambda$  and the weights of the factors  $F_t$ .<sup>5</sup> Bai/Ng (2002) derive the principal component estimators under fairly mild conditions of the idiosyncratic component when both the cross-section and time dimension become large. It should be noted that the  $r$  common factors are not uniquely identified because for a non-singular matrix  $Q$  of dimension  $(r \times r)$ ,  $\Lambda F_t = \Lambda Q Q^{-1} F_t = \Lambda^* F_t^*$  holds. However, we are primarily interested in the part of the time series fluctuations that are explained by all the common factors and not in each common factor separately, so this problem can be neglected. While this lack of identification is not problematic for forecasting, it should be taken into consideration when interpreting the factors in a structural way.

Crucial to the analysis of the factor model is the determination of the number of factors. Bai/Ng (2002) derive information criteria to determine the number of static factors  $r$  in (2). The information criteria represent the usual trade-off between goodness-of-fit and overfitting. The information criteria can be seen as extensions to the familiar Bayes or Akaike criteria. The main difference of the factor model information criteria is the consideration of the cross-section unit  $N$  which is absent in the pure time series environment.<sup>6</sup> The formal criteria to determine the number of factors only apply to the number of static vectors. Via principal component analysis, we can determine the components of the model (2) where the relationship between the variables is static. Up to now, there is no econometric theory for the decomposition of the static factors into lagged dynamic factors as in (1).<sup>7</sup> But the empirical literature shows that the inclusion of lags of the factors may improve the forecasting ability of the models.<sup>8</sup> Therefore, in order to obtain such a dynamic factor model, the literature suggests to use empirical criteria such as forecasting accuracy to determine the number of dynamic factors  $q$  and the corresponding lags  $p$ . The maximum combination of dynamic factors and their lags is, however, given by the number of static factors  $r = q(p + 1)$ . Hence, the information criteria of Bai/Ng (2002) give an upper bound of the number of factors in the dynamic model.<sup>9</sup> The strategy of our empirical investigation will be based on both approaches: First, we use the factor model specified according to the criteria of Bai/Ng (2002). Second, we estimate a large variety of models with differing parameter values and select the best ones according to their empirical forecast performance.

### 3 German data

Since the main task of this paper is to evaluate the gains of using a large data set compared with a small data set to predict key variables of economic activity, we should collect a sufficiently large data set. The collected data set for Germany, which is explained in the data appendix F, contains 121 quarterly series over the sample period 1978:2-2002:1. We choose quarterly time

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<sup>5</sup>Further details about the estimation can be found in the appendix B.

<sup>6</sup>See Bai/Ng (2002), pp. 199-201.

<sup>7</sup>See Forni et al. (2001, 2002).

<sup>8</sup>See Artis et al. (2001), Stock/Watson (2002) and for state-space models, Camba-Mendez et al. (2001).

<sup>9</sup>See Bai/Ng (2002), p. 212.

series because we want to discuss the empirical properties of the factor model with respect to GDP which is available at the quarterly frequency. In addition, with this data set we are able to describe the economy on a broad basis because sectoral data can be taken into consideration.

In order to have a balanced and as exhaustive as possible representation of the German economy, we include a variety of variables, which are described in the data appendix. For example, we use GDP and its expenditure components such as consumption and fixed capital formation, and gross value added by sectors. Industrial production, received orders and turnover, disaggregated by sectors are included. Labour market variables are employment, unemployment and wages. Several disaggregated price time series, interest rates and spreads as well as exchange rates are considered. Additionally, we use ifo survey time series such as business situation and expectations, assessment of stocks and capacity utilization, and other series.

As is typical for the empirical indicator literature, the vector of time series will be preprocessed. First, the time series are corrected for outliers and then seasonally adjusted as explained in the appendix. Moreover, since the principal component analysis requires stationary time series for estimation, non-stationary time-series were appropriately differenced.<sup>10</sup> Finally, the series were normalized to have sample mean zero and unit variance.

## 4 Predicting German GDP

To evaluate the empirical performance of the described factor model, we discuss out-of-sample forecasts of German GDP. As reference models, we choose three alternatives:

- Univariate autoregressive (AR) model: Such a simple model serves as a benchmark model in our forecasting exercise. Although sometimes labelled as "naive" model, it is often difficult for other econometric models to produce better forecasts.
- Forecasts based on the ifo business climate indicator: The ifo business climate index is a business cycle indicator obtained from surveys. It receives much attention in the public, and has some information content for real activity in Germany.<sup>11</sup> The ifo climate index is also included in the large data set of the factor model described above. Hence, the comparison between the large-scale approach and the ifo climate index highlights the importance of the other variables and the difference between the two alternative modelling strategies.
- Forecasts using a vector autoregressive (VAR) model: VAR models are a widely used tool in empirical research. Here, we use a six variable model according to a leading indicator model used by Grasmann/Keereman (2001). The data vector consists of GDP, retail business situation, construction business expectations, terms of trade, interest rate spread and car registrations. While the factor model described before will rely on over a hundred time series, the VAR model has a considerably smaller data set which is the outcome of an explicit preselection. A comparison of forecasts of the factor model and the VAR model will shed some light on the relative efficiency of such preselections.

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<sup>10</sup>See, for example, Altissimo et al. (2002), Forni et al. (2001).

<sup>11</sup>See Hüfner/Schröder (2002), p. 317.

The alternative models vary concerning the extent of external information which is used to predict German GDP.<sup>12</sup> While the autoregressive model uses only past GDP information, the ifo business climate index is one additional time series used for forecasting. Together with the VAR model with a data set of six time series, a competitive group of alternatives for the factor model is given. A forecast comparison gives insight into the empirical gains of the factor model in comparison with other leading indicators of economic activity in Germany. The forecasting model follows Stock/Watson (1999, 2002), Forni et al. (2002) and Artis et al. (2001) who use a dynamic or multi-step estimation approach.<sup>13</sup> All forecasting models are specified and estimated as a linear projection of an  $h$ -step ahead transformed variable,  $y_{t+h}$  onto  $t$ -dated predictors. More precisely, the forecasting models have the form

$$y_{t+h} = \alpha(L)y_t + \beta(L)D_t + \varepsilon_t, \quad (3)$$

where  $D_t$  is the external information or the predictor variables used for forecasting in addition to  $y_t$  itself.  $\alpha(L)$  and  $\beta(L)$  are lag polynomials of finite order. Of the large-scale factor models, we investigate two specifications as has been discussed in the methodological section: First, the static factor model (2) is specified. We use the static form of the factor model and determine the number of factors with the Bai/Ng (2002) information criteria.<sup>14</sup> Then  $D_t$  is a  $(r \times 1)$  dimensional vector of static factors, and the coefficients are collected in a time independent vector  $\beta$  with dimensions  $(1 \times r)$  which is estimated via OLS. Second, we estimate a dynamic factor model according to (1). Following Stock/Watson (2002) and Artis et al. (2001), the  $q$  dynamic factors are estimated via principal components, and  $D_t$  is a  $(q \times 1)$  dimensional vector of dynamic factors. In this case, the factors' parameters are collected in the  $(1 \times q)$  dimensional vector polynomial  $\beta(L) = \sum_{i=0}^p \beta_i L^i$  with  $p$  lags and the effect of the contemporaneous factors. The estimation method of the lag polynomials is OLS. To determine the lag order  $p$  and the number of dynamic factors  $q$ , we will estimate a large variety of models and choose the specification with the lowest mean square forecast error for each forecast horizon.<sup>15</sup> In addition to the external information, the forecasting equation includes lagged values of the variable of interest  $y_t$  and their effect is measured via  $\alpha(L)$ , a scalar finite order lag polynomial. The left hand side of the forecasting equation,  $y_{t+h}$ , is the forecast of the growth in the GDP series between period  $t$  and period  $t+h$ . The original series of GDP,  $Y_t$ , is in natural logarithms, so the growth rate becomes  $y_{t+h} = \log(Y_{t+h}/Y_t) = \sum_{i=1}^h \Delta \log(Y_{t+i})$ . The autoregressive term on the right hand side is defined as  $y_t = \log(Y_t/Y_{t-1})$ . The dynamic estimation approach above differs from the standard one-step ahead approach. To forecast  $h$  periods ahead within the standard approach, one estimates the model with one lag, and then iterating that model forward to obtain  $h$ -step ahead predictions.<sup>16</sup> The dynamic estimation approach is different, because the left hand side variables are specified  $h$  periods ahead of the explanatory variables

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<sup>12</sup>In preliminary steps of this investigation, we also included the OECD leading indicator, and different VAR models. We used the famous trivariate monetary VAR model of Cochrane (1998), in which GDP, prices and an interest rate form the data vector. The OECD leading indicator was included as an indicator in an equal way as the ifo indicator. Empirical results using these alternatives can be obtained from the authors upon request. However, these models performed worse than the alternatives presented here.

<sup>13</sup>See chapter 11 of Clements/Hendry (1998) for a detailed treatment of multi-step estimation.

<sup>14</sup>See appendix C for the exact formulas.

<sup>15</sup>Such a specification strategy is proposed by Forni et al. (2002).

<sup>16</sup>For a discussion, see Clements/Hendry (1998), pp. 243-246.

as can be seen from equation (3).

This dynamic approach has two main advantages. First, additional equations for simultaneously forecasting the indicators  $D_t$  are not needed. This can be useful because the stochastic process governing  $D_t$  is not known in general. Second, the potential impact of specification error in the one-step ahead model can be reduced by using the same horizon for estimation as for forecasting. The forecasts of the alternative models are also computed using the multi-step estimation. In the univariate autoregressive model, there is no external information and  $D_t$  and its coefficients are zero. The ifo climate index is a single indicator, so  $D_t$  is a scalar. The VAR model is estimated accordingly. The equation for GDP for example is equal to equation (3), with  $D_t$  including five other variables than GDP as described above. Again, to specify the lag orders of the different models, we use a variety of parameter combinations to identify that specification that minimizes the mean square error. This strategy follows Forni et al. (2002) who estimate their models over a broad range of parameters and choose coefficient combinations which minimize a forecast error criterion. The models with these parameter values yield a minimal value of the mean square forecast error. This mean square error is thus the best forecasting performance obtainable, at the horizon  $h$ , via the dynamic forecast procedure based on each of the models.

To evaluate the quantitative forecasts from the factor model and its competitors, we perform a simulated forecasting experiment. The forecasts are out-of-sample and only in-sample information is used to estimate the parameters of the models. For statistical forecast tests, the models are estimated recursively. The first sample period covers one third of the total time series sample. Forecasts are computed with a forecast horizon of one to eight quarters and forecast errors are stored. Then the sample size is increased by one, the model is reestimated, forecasts are computed and so on. At the end of the sample, we have a total of 57 quarters of forecasts for each horizon which can be evaluated. The recursive forecasts are computed to shed light on the real-time properties of the empirical model. In applications, policy makers have primarily a forward-looking perspective so the out-of-sample estimates of the indicators are of main interest. Recursive estimations and forecasts show how the model would have predicted in the past if only real-time data had been available.

To get a first impression about the forecasting accuracy of the competing models, we first report relative mean square errors which are relative to the naive autoregressive model. A relative mean square error less than one indicates a superior forecasting performance of a model. Table 1 shows the relative mean square errors and the resulting ranking. The relative mean square error of the dynamic factor model is smaller than one at all forecast horizons. Hence, the dynamic factor model outperforms the simple autoregressive model in our setup. The forecasting gains of using estimated factors as predictors compared with the autoregressive model are higher at shorter horizons than at longer horizons, and the relative forecast improvement lies between 8% and 26% according to the mean square error. The static model determined with the criteria of Bai/Ng (2002) performs worse than the dynamic version of the factor model. As compared with the ifo indicator and the VAR model, the relative mean square error of the dynamic multifactor model is again smaller. The static factor model has often larger mean square errors than the ifo indicator and the VAR model. In the light of this poor results, we don't report further results of the static factor model. In the following tables the factor model always refers to the dynamic version of the factor model. At horizons from four to eight quarters, the

Table 1: Relative MSE

	Relative MSE at forecast horizon							
	1	2	3	4	5	6	7	8
static factor model	1.003	0.956	0.961	1.115	1.084	1.045	1.088	1.134
dynamic factor model	0.772	0.741	0.734	0.886	0.861	0.866	0.891	0.921
ifo climate	0.864	0.784	0.779	0.957	1.000	1.004	1.012	1.002
VAR model	0.953	0.949	0.941	1.194	1.349	1.316	1.317	1.383

  

	Ranking							
	1	2	3	4	5	6	7	8
static factor model	4	4	4	3	3	3	4	4
dynamic factor model	1	1	1	1	1	1	1	1
ifo climate	2	2	2	2	2	2	2	2
VAR model	3	3	3	4	4	4	3	3

Notes: The table shows the mean-square errors (MSE) of the various models relative to the MSE of the autoregressive model.

ifo climate indicator have larger forecast errors than the autoregressive model. Although the dynamic factor model performs best overall, the magnitude of the forecasting improvements of the dynamic factor model are quite small at various forecast horizons. For example, the forecasting power relative to the ifo indicator is often below 10%.

In order to test whether the forecast gains of the large-scale factor model are systematic, we apply the Diebold/Mariano (1996) test of equal forecasting accuracy. This test is based on the difference of squared forecast errors of two competing forecast models.<sup>17</sup> Under the null hypothesis, the forecasting accuracy is not statistically different. In table 2, we report significance levels for this test. A small significance level below 0.10 or 0.05 indicates a rejection of the null of equal forecast accuracy. The results of the tests show that the forecasts of the large-scale factor model are only in a few cases significantly better than the forecasts of the other models. Especially the forecasts of the simple autoregressive model without any exogeneous indicators are not significantly worse at horizons four and eight. Moreover, the advantages over the ifo indicator are significant only at significance levels larger than 10%. This implies that although the factor model performs best in the chosen small sample according to the relative mean square error in table 1, the advantages are not systematic.

The finding that the forecasts of a preferred model are better than those of a rival model should not be the final conclusion of a model comparison. As noted by Harvey et al. (1998), it could rather be the case that the competing models have some additional information content for the variable to be predicted absent in the preferred forecast model. Hence, a combination of two rival forecasts could improve the preferred forecast although the preferred forecast model has smaller forecast errors. This question of forecast combination of two rival models can be

<sup>17</sup>For further details, see appendix D.

Table 2: Test for equal forecasting accuracy

forecast horizon		Diebold/Mariano (1995) test			
		factor model	ifo climate	VAR model	AR model
1	factor model	-	0.14	0.10	0.06
	ifo climate		-	0.22	0.15
	VAR model			-	0.24
	AR model				-
4	factor model	-	0.11	0.17	0.21
	ifo climate		-	0.25	0.40
	OECD			-	0.21
	AR model				-
8	factor model	-	0.15	0.15	0.37
	ifo climate		-	0.19	0.50
	VAR model			-	0.10
	AR model				-

Notes: The table shows significance levels. The null hypothesis is pairwise equal forecast accuracy. The test is symmetric. Further information about the computation of the test is given in the appendix D.

tested with the simple regression

$$y_t = (1 - \lambda) y_{A,t|t-h} + \lambda y_{B,t|t-h} + \varepsilon_t, \quad (4)$$

where  $y_{A,t|t-h}$  and  $y_{B,t|t-h}$  are the rival forecasts of the variable  $y_t$  computed using information available at  $t - h$ .<sup>18</sup> The parameter  $\lambda$  can be estimated using OLS and with some corrections for possible heteroscedasticity and autocorrelation. A forecast combination parameter  $\lambda$  which is significantly larger than zero implies an additional information content of the competing forecast model  $B$  that is absent in the forecast of model  $A$ . If the coefficient is equal to zero, then the preferred model's forecasts cannot be improved and the preferred model  $A$  encompasses its rival. In table 3, we report the results of a forecast encompassing test of Harvey et al. (1998) for the whole variety of model combinations. Under the null hypothesis, the model standing in row of the table (model  $A$ ) encompasses the model standing in the column of the table (rival forecast  $B$ ). The table shows that the rival models have almost no information content for the multifactor model forecasts. For example, at a forecast horizon of four quarters, the significance level that the factor model encompasses the ifo indicator is 15%. The only exception is the VAR model which is not encompassed by the factor model at horizon one (significance level 9%). On the other hand, the test results indicate that the factor forecasts can improve all rival forecasts at the 10% significance level.

<sup>18</sup>See Clements/Hendry (1998), p. 228 or Harvey et al. (1998), p. 254.

Table 3: Test for forecast encompassing errors

forecast horizon		Harvey et al. (1998) test			
		factor model	ifo climate	VAR model	AR model
1	factor model	-	0.12	0.09	0.11
	ifo climate	0.00	-	0.06	0.07
	VAR model	0.00	0.00	-	0.27
	AR model	0.00	0.00	0.03	-
4	factor model	-	0.15	0.44	0.47
	ifo climate	0.00	-	0.40	0.16
	VAR model	0.08	0.09	-	0.06
	AR model	0.07	0.06	0.47	-
8	factor model	-	0.27	0.38	0.27
	ifo climate	0.01	-	0.46	0.22
	VAR model	0.08	0.09	-	0.03
	AR model	0.09	0.20	0.26	-

Notes: The table shows significance levels. The null hypothesis is that the model standing in the row encompasses the model standing in the column. The test is not symmetric. The test leads to the estimation of forecast combination weights  $\lambda$  from the equation  $y_t = (1 - \lambda) y_{A,t|t-h} + \lambda y_{B,t|t-h} + \varepsilon_t$ , where  $y_{B,t|t-h}$  is the forecast of the model from the table's column and  $y_{A,t|t-h}$  is the forecast of a model standing in the row of the table. If the null is true, then a small significance level indicates that model  $B$  adds no additional forecasting power and model  $A$  encompasses model  $B$ . Further information about the computation of the test is given in the appendix D.

## 5 Discussion of the results and conclusions

The results of the paper give insights into the usefulness of the empirical applicability of a large-scale factor model proposed by Stock/Watson (2002) and Artis et al. (2001). The large-scale factor model is applied to German data. To evaluate the empirical performance of the factor model, recursive out-of-sample GDP forecasts of the model are compared with rival forecasts from a simple univariate model, the ifo business climate index, and a VAR model.

The empirical results of the paper show that although the dynamic large-scale factor model outperforms the smaller sized rival models, the forecasting gains are limited and not systematic. On the other hand, the forecasting performance of the factor model cannot be improved by the alternative models. Moreover, the large-scale factor model always has some information content for the ifo business climate index, the VAR model and the univariate autoregressive model investigated here. Hence, using the rival models for forecasting purposes would be ineffective, since the factor model forecasts can always improve their forecasts. However, because the forecasting errors of the factor model are not significantly smaller than the errors of the rival models, the efficiency of using such a large panel of data can be questioned. One has to keep in mind that the better forecasting properties of the large-scale model come at the costs of



the need to keep a larger data set up to date which can be quite resource consuming. This is another drawback of such a large-scale model compared with the smaller sized models.

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# A Deriving the static representation of the factor model

In the dynamic form of the factor model (1),  $X_{it} = \lambda_i(L)f_t + \varepsilon_{it}$ , the parameters are time dependent. Our goal is to make the model static in the parameters, so that it is possible to estimate the model via principal components.<sup>19</sup> First, transform the vector of factors  $f_t$  into a dynamic one. As in the main text, the factors are lagged and reordered to obtain  $F_t = (f'_t, \dots, f'_{t-p})'$  with dimensions  $(q(p+1) \times 1)$ . This implies the transformed model

$$X_{it} = \Lambda_i F_t + \varepsilon_{it}, \quad (5)$$

where  $\Lambda_i$  is a  $(1 \times q(p+1))$  dimensional vector of reorganized parameters of  $\lambda_i(L)$ . Let the number of static factors be defined as  $r = q(p+1)$ . The aim is to determine the parameters of  $\Lambda_i$  to fulfill

$$\Lambda_i F_t = \lambda_i(L)f_t. \quad (6)$$

The lag polynomial  $\lambda_i(L)f_t$  can be rewritten as

$$\lambda_i(L)f_t = (\lambda_{i1}(L), \dots, \lambda_{iq}(L)) f_t, \quad (7)$$

where the polynomial  $\lambda_{ij}(L) = \lambda_{ij0} + \lambda_{ij1}L + \dots + \lambda_{ijp}L^p$  shows how the  $j$ -th factor and its lags are related to the  $i$ -th variable. Define the coefficients of this polynomial as

$$\lambda_{ij} = (\lambda_{ij0}, \lambda_{ij1}, \dots, \lambda_{ijp}). \quad (8)$$

Inserting this into the total effect of all factors on the variable  $i$  gives

$$\begin{aligned} \lambda_i(L)f_t &= \lambda_{i1}(L)f_{1t} + \dots + \lambda_{iq}(L)f_{qt} \\ &= \lambda_{i1} \begin{pmatrix} f_{1t} \\ \vdots \\ f_{1,t-p} \end{pmatrix} + \dots + \lambda_{iq} \begin{pmatrix} f_{qt} \\ \vdots \\ f_{q,t-p} \end{pmatrix} \\ &= (\lambda_{i10}, \dots, \lambda_{i1p}) \begin{pmatrix} f_{1t} \\ \vdots \\ f_{1,t-p} \end{pmatrix} + \dots + (\lambda_{iq0}, \dots, \lambda_{iqp}) \begin{pmatrix} f_{qt} \\ \vdots \\ f_{q,t-p} \end{pmatrix}. \end{aligned} \quad (9)$$

Since this is a sum of matrices, the terms can be combined as the product of a vector of coefficients and a vector of lagged factors.

$$\lambda_i(L)f_t = (\lambda_{i10}, \dots, \lambda_{i1p}, \dots, \lambda_{iq0}, \dots, \lambda_{iqp}) \begin{pmatrix} f_{1t} \\ \vdots \\ f_{1,t-p} \\ \vdots \\ f_{qt} \\ \vdots \\ f_{q,t-p} \end{pmatrix}. \quad (10)$$

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<sup>19</sup>See Stock/Watson (2002).

Within this matrix product, it is possible to reorder the elements freely. Reordering to obtain the components of  $F_t$  in the column vector gives

$$\lambda_i(L)f_t = (\lambda_{i10}, \lambda_{i20}, \dots, \lambda_{iq0}, \dots, \lambda_{i1p}, \dots, \lambda_{iqp}) \begin{pmatrix} f_{1t} \\ f_{2t} \\ \vdots \\ f_{qt} \\ \vdots \\ f_{1,t-p} \\ \vdots \\ f_{q,t-p} \end{pmatrix} = \Lambda_i F_t. \quad (11)$$

It can now be verified, that  $\Lambda_i = (\lambda_{i10}, \lambda_{i20}, \dots, \lambda_{iq0}, \dots, \lambda_{i1p}, \dots, \lambda_{iqp})$ , can simply be obtained from a vertical concatenation of the coefficients  $\lambda_{ij}$ , which is

$$\begin{pmatrix} \lambda_{i1} \\ \vdots \\ \lambda_{iq} \end{pmatrix} = \begin{pmatrix} \lambda_{i10} & \dots & \lambda_{i1p} \\ \vdots & \ddots & \vdots \\ \lambda_{iq0} & \dots & \lambda_{iqp} \end{pmatrix}. \quad (12)$$

The stacked columns or vectorization of this matrix is simply the transpose of  $\Lambda_i$ :

$$\Lambda'_i = \text{vec} \begin{pmatrix} \lambda_{i1} \\ \vdots \\ \lambda_{iq} \end{pmatrix}. \quad (13)$$

Since the parameters of the static form of the model are now determined, we can also derive it for all variables  $i = 1, \dots, N$ . The model can be rewritten as

$$X_t = \Lambda F_t + \varepsilon_t, \quad (14)$$

where  $X_t$  is the  $(N \times 1)$  vector of time series at time  $t$  and  $F_t = (f'_t, \dots, f'_{t-p})'$  as before. The  $(N \times r)$  dimensional matrix of coefficients is defined as  $\Lambda = (\Lambda'_1, \dots, \Lambda'_N)'$ , so equation (2) in the main text is derived.

## B Principal component analysis of the factor model

The goal of principal component analysis is to reduce the dimensionality of a data set comprised of a large number of interrelated variables, while retaining as much as possible of the variation present in the data. In our case, the aim is to choose the parameters and factors of the model  $X_t = \Lambda F_t + \varepsilon_t$  in order to maximize the explained variance of the original variables for a given number of factors  $r \leq N$ . The resulting factors are called the principal components. The  $(r \times 1)$  dimensional vector of factors  $F_t$  is assumed to be a linear combination of the observed data,  $F_t = \Psi X_t$  where  $\Psi = (\psi'_1, \dots, \psi'_r)'$  is a  $(r \times N)$  matrix of coefficients and  $\psi_i = (\psi_{i1}, \dots, \psi_{iN})$  for  $i = 1, \dots, r$ . The intuitive reasoning behind the specification for the factors is that a linear

combination of variables exhibiting maximum variation will capture most of the variability in the original dataset. The variance of the factors is given by

$$\text{Var}(F_t) = \text{E}(\Psi X_t X_t' \Psi') = \Psi \text{E}(X_t X_t') \Psi' = \Psi \Omega \Psi', \quad (15)$$

where  $\Omega$  is the variance covariance matrix of the vector of time series. The maximization of the variance of the principal components or

$$\max_{\Psi} \text{Var}(F_t) = \Psi \Omega \Psi' \quad (16)$$

is subject to the normalization  $\Psi \Psi' = I$  in order to avoid a solution where  $\Psi$  is arbitrarily large. For the first factor  $f_{1t} = \psi_1 X_t$ , the Lagrangian is  $\psi_1 \Omega \psi_1' - \gamma (\psi_1 \psi_1' - 1)$  and optimizing with respect to  $\psi_1$  gives the necessary first order conditions  $\Omega \psi_1' = \gamma \psi_1'$  and  $\psi_1 \psi_1' = 1$ . The first condition can be transformed into  $(\Omega - \gamma I) \psi_1' = 0$  and is equal to an eigenvalue problem. The two conditions are satisfied only if the  $\psi_1$  is the eigenvector of the eigenvalue problem for the matrix  $\Omega$  and the eigenvalue  $\gamma$ . Repeating the optimization for all the  $r$  largest eigenvalues and stacking the optimality conditions gives

$$\Omega \Psi' = \Psi' \Gamma, \quad (17)$$

where  $\Gamma = \text{diag}(\gamma_1, \dots, \gamma_r)$  is a diagonal matrix that contains the largest  $r$  eigenvalues. It can then be seen that the solution to the problem given in (16) subject to the normalization constraint is given by the eigen decomposition of  $\Omega$ . The desired coefficient matrix  $\Psi$  in  $F_t = \Psi X_t$  consists of the stacked eigenvectors of this eigen decomposition. After premultiplying by  $\Psi$  and applying the normalization  $\Psi \Psi' = I$  one gets the result  $\Psi \Omega \Psi' = \Gamma = \text{Var}(F_t)$ , which is variance of the principal components. Hence, the variance of each factor is equal to its corresponding eigenvalue. This implies that, for a given number of factors  $r$ , the maximization of the variance of the factors amounts to use the  $r$  largest eigenvalues and their corresponding eigenvectors.

In order to fully estimate the factor model in the main text, we have to determine the  $(N \times r)$  coefficients  $\Lambda$  in the model  $X_t = \Lambda F_t + \varepsilon_t$  where the factors have been derived in the steps before. To estimate the coefficient matrix, rewrite the model as  $X = F \Lambda' + \varepsilon$  with  $X = (X_1, \dots, X_N)$  and  $X_i = (X_{i1}, \dots, X_{iT})'$  for  $i = 1, \dots, N$ , and  $F = (f_1, \dots, f_r)$  with  $f_i = (f_{i1}, \dots, f_{iT})'$  for  $i = 1, \dots, r$ . For given factors, the minimization of the idiosyncratic component leads to the optimality condition  $\Lambda' = (F' F)^{-1} F' X$  where  $F = X \Psi'$  holds. This implies

$$\begin{aligned} \Lambda' &= (F' F)^{-1} F' X = (\Psi X' X \Psi')^{-1} \Psi X' X \\ &= (\Psi X' X \Psi')^{-1} \Psi X' X \left[ \Psi' \Psi (\Psi' \Psi)^{-1} \right] = \Psi (\Psi' \Psi)^{-1}, \end{aligned} \quad (18)$$

and premultiplying by  $\Psi'$  and then by  $\Psi$  on both sides gives

$$\begin{aligned} \Psi' \Lambda' &= I \\ \Psi \Psi' \Lambda' &= \Psi \\ \Lambda' &= \Psi. \end{aligned} \quad (19)$$

From the second to the third line, the normalization  $\Psi \Psi' = I$  was used. Hence, the coefficient matrix of the factors is the transpose of the eigenvectors obtained from the maximization problem. For further details, see Bai/Ng (2002), Forni et al. (2002), and Brillinger (1981), theorem 9.2.1.

## C Bai/Ng (2002) information criteria

To determine the number of factors in the model described in the main text, Bai/Ng (2002) suggest the use of information criteria. We report only two criteria out of six which performed best in the simulations of Bai/Ng (2002):

$$IC_1(r) = \ln(V(r, F)) + r \left( \frac{N+T}{NT} \right) \ln \left( \frac{NT}{N+T} \right), \quad (20)$$

$$IC_2(r) = \ln(V(r, F)) + r \left( \frac{N+T}{NT} \right) \ln(\min\{N, T\})^2. \quad (21)$$

The information criteria reflect the trade-off between goodness-of-fit on the one hand and overfitting on the other. The first term on the right hand side shows the goodness-of-fit which is given by the residual sum of squares

$$V(r, F) = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T (X_{it} - \Lambda_i F_t)^2, \quad (22)$$

which depends on the estimates of the factors and the number of factors. The residuals are given by  $\varepsilon_{it} = X_{it} - \Lambda_i F_t$ , where  $\Lambda_i$  is a  $(1 \times q(p+1))$  dimensional row vector of the parameter matrix  $\Lambda$  of the static model. If the number of factors  $r$  is increased, the variance of the factors  $F_t$  increases, too, and the sum of squared residuals decreases. Hence, the information criteria have to be minimized in order to determine the number of factors. The penalty of overfitting in both criteria,  $g(N, T)$ , which is the second term on the right hand side, is an increasing function of the cross-section size  $N$  and time series length  $T$ . Although both information criteria share the same asymptotic properties for large  $N$  and  $T$ , their small sample behaviour can be different. In comparison with criteria based on alternative penalties,  $IC_1(r)$  and  $IC_2(r)$  perform relatively better and more stable. In empirical applications, one has to fix a maximum number of factors, say  $r_{\max}$ , and estimate the model for all number of factors  $r = 1, \dots, r_{\max}$ . The optimal number of factors minimizes  $IC_1(r)$  and  $IC_2(r)$ .

## D Tests for equal forecast accuracy and forecast encompassing

### D.1 Diebold/Mariano (1996) test for equal forecast accuracy

Consider two models  $A$  and  $B$  which produce forecasts of variable  $y$  at period  $t$ . The forecasts  $h$  periods ahead are conditional on information available in period  $t-h$ , so the forecast is the application of the conditional expectation operator  $y_{A,t|t-h}$  and  $y_{B,t|t-h}$  for model  $A$  and  $B$ , respectively. Calculate a sequence of  $T_f$  forecast errors for both models  $e_{i,t}(h) = y_t - y_{i,t|t-h}$  for  $i = A, B$  and  $t = 1, \dots, T_f$ . The Diebold/Mariano (1996) test for equal forecast accuracy is based on the time series of differences of the squared forecast errors,  $d_t(h) = e_{A,t}^2(h) - e_{B,t}^2(h)$ . Under the null hypothesis, the sample mean of  $d_t(h)$ ,  $\bar{d}(h) = \frac{1}{T_f} \sum_t d_t(h)$ , is not significantly

different from zero. The statistic is defined as

$$DM(h) = \frac{\sqrt{T_f} \bar{d}(h)}{\sqrt{(1/T_f) \sum_{\tau=-(T_f-1)}^{(T_f-1)} (h/(|\tau| - h)) \sum_{t=|\tau|+1}^{T_f} (d_t(h) - \bar{d}(h)) (d_{t-|\tau|}(h) - \bar{d}(h))}}, \quad (23)$$

where the denominator includes a heteroscedasticity and autocorrelation consistent estimate of the variance of  $d_t$  assuming that the  $h$ -step ahead forecast errors are at most  $(h-1)$ -dependent.<sup>20</sup> The weighting scheme of the autocovariances follows Newey/West (1987). The statistic  $DM(h)$  is standard normal distributed. Harvey et al. (1997) show simulation results that suggest to use a small sample correction for the statistic  $DM(h)$ . The modified statistic is defined as  $MDM(h) = \kappa DM(h)$  with  $\kappa = T_f^{-0.5} [T_f + 1 - 2h + T_f^{-1}h(h-1)]^{0.5}$  and its critical values should be taken from the  $t(T_f - 1)$  distribution rather than the normal distribution. The tests in the main text use this small sample correction.

## D.2 Harvey et al. (1998) tests for forecast encompassing

Harvey et al. (1998) modify the tests of equal forecast accuracy to forecast encompassing. The issue of forecast encompassing can be tested with the simple combination of two rival forecasts

$$y_t = (1 - \lambda) y_{A,t|t-h} + \lambda y_{B,t|t-h} + \varepsilon_t, \quad (24)$$

where  $y_{A,t|t-h}$  and  $y_{B,t|t-h}$  are the rival forecasts of the variable  $y_t$  observed at period  $t$  and  $\varepsilon_t$  is the error of the combined forecast.<sup>21</sup> The aim is to determine the forecast combination parameter  $\lambda$ , so the coefficients in the above equation should sum to one. If  $\lambda$  is equal to zero, then the alternative model has no additional information content and model  $A$  encompasses model  $B$ . Hence, the null hypothesis is that model  $A$  encompasses model  $B$  and  $\lambda = 0$ . The alternative hypothesis is  $\lambda > 0$ . In order to implement the restriction on both parameters, define  $e_{i,t}(h) = y_t - y_{i,t|t-h}$  for  $i = A, B$  and  $t = 1, \dots, T_f$ . We can rewrite the combination equation as

$$e_{1,t}(h) = \lambda (e_{1,t}(h) - e_{2,t}(h)) + \varepsilon_t, \quad (25)$$

simply after inserting the definition for the error. In this equation, the parameter  $\lambda$  can be estimated using OLS which is the basis of the Harvey et al. statistic. Harvey et al. (1998) define  $d_t(h) = (e_{A,t}^2(h) - e_{B,t}^2(h)) e_{A,t}^2(h)$ , and construct the mean  $\bar{d}(h) = \frac{1}{T_f} \sum_t d_t(h)$  and its variance as in the Diebold/Mariano (1996) test before. Hence, the test statistic  $HLN(h)$  for forecast encompassing is equal to the statistic  $DM(h)$  except that  $d_t(h)$  is different. Again, we use a small sample correction proposed by Harvey et al. (1998).

## E Specifications of empirical models

To specify the lag orders of the different models, we use a variety of parameter combinations to identify that specification that minimizes the mean square error. This strategy follows Forni et

<sup>20</sup>See Diebold/Mariano (1996), p. 254.

<sup>21</sup>See Clements/Hendry (1998), p. 228 or Harvey et al. (1998), p. 254.

al. (2002) who estimate their models over a broad range of parameters and choose coefficient combinations which minimize a forecast error criterion. The models with these parameter values yield a minimal value of the mean square forecast error. This mean square error is thus the best forecasting performance obtainable, at the horizon  $h$ , via the dynamic forecast procedure based on each of the models. The following table gives the order of the key parameters.<sup>22</sup>

Table 4: Specifications

Model	Parameter description	Number of parameters at horizon...							
		1	2	3	4	5	6	7	8
dynamic factor model	number of dynamic factors	2	1	1	1	1	1	1	1
	number of AR lags	2	1	0	1	1	0	3	3
	number of lags of each factor	1	2	1	0	0	0	0	0
ifo climate	number of AR lags	1	1	1	0	0	0	0	0
	number of lags of indicator	1	1	3	2	0	0	0	0
VAR model	number of VAR lags	0	0	0	0	0	0	0	1
AR model	number of AR lags	0	0	2	3	2	0	0	0

Notes: The table shows the parameter combinations of each model for various forecast horizons. The underlying dynamic equation behind each model is  $y_{t+h} = \alpha_p(L)y_t + \beta_q(L)D_t + \varepsilon_t$ . Hence, lag order of zero implies that only the contemporaneous values of  $y_t$  or  $D_t$  enter the equation. A lag order  $p > 0$  or  $q > 0$  means that  $p$  or  $q$  lags and the contemporaneous value enter the equation.

The table shows the number of lags of the factors, external indicators and autoregressive lags of the models. Although the models vary somewhat for the different forecast horizons, the effects of choosing one model specification for all horizons changes the empirical results only slightly. The forecasting performance differs only gradually for alternative combination of lags. In addition to the specification selection described above, we compared models with the same specification for each forecast horizon. The empirical results were essentially the same. Again, the factor model outperformed its competitors according to its relative mean square error compared with the autoregressive model.

## F German data set

This appendix describes the panel of time series for the German economy. To have a balanced and as exhaustive as possible representation of the German economy, we include a huge variety of variables. Because GDP is the reference series, all time series are quarterly or transformed via averaging into quarterly series. Moreover, natural logarithms were taken for all time series except interest rates, unemployment ratios, and capacity utilization. Stationarity was obtained

<sup>22</sup>The static factor model of Bai/Ng (2002) is estimated with five static factors which were determined according to both information criteria described in appendix C.



via first differencing if necessary. Outlier correction was done using TRAMO, seasonal fluctuations were eliminated using Census-X12 if necessary. To eliminate scale effects, the series were centered around zero mean and standardized to have unit variance.

Time series for unified Germany are available only for the time period after 1991. We have therefore combined time series of West Germany and unified Germany after having rescaled the West German data to the unified German time series. Moreover, the Western Germany series are not measured according to the ESA 95 (European System of National Accounts), and it was necessary to rebase these series before joining them to the unified Germany data. Preliminary stability tests, where the factor model described in the main text was estimated in recursive manner, showed that there were no structural breaks left in the underlying data set. The combination scheme is equal to the method used by the ECB to provide long time series of monetary aggregates.<sup>23</sup>

The whole data set for Germany contains 121 quarterly series, over the sample period 1978:2-2002:1. The sources of the time series are the HWWA time series database for the majority of time series of unified Germany, the National Accounts database of the Federal German Statistical Office, the Bundesbank database for labour market and interest rates, the DIW database for the gross value added time series, and Datastream.

## Use of GDP and gross value added

1. Gross domestic product
2. Private consumption expenditure
3. Government consumption expenditure
4. Gross fixed capital formation: machinery & equipment
5. Construction
6. Exports
7. Imports
8. Gross value added: Mining and fishery
9. Gross value added: Producing sector excluding construction
10. Gross value added: Construction
11. Gross value added: Wholesale and retail trade, restaurants, hotels and transport
12. Gross value added: Financing and rents
13. Gross value added: Services

## Prices

1. Consumer price index
2. Export prices
3. Import prices
4. Terms of trade

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<sup>23</sup>See also Scharnagl (1998) or Bandholz/Funke (2002).

5. Deflator of GDP
6. Deflator of private consumption expenditure
7. Deflator of government consumption expenditure
8. Deflator of machinery & equipment
9. Deflator of construction

## **Manufacturing turnover, production and received orders**

1. Domestic turnover industry
2. Domestic turnover intermediate goods industry
3. Domestic turnover capital goods industry
4. Domestic turnover durable and non-durable consumer goods industry
5. Domestic turnover mechanical engineering
6. Domestic turnover electrical engineering
7. Domestic turnover vehicle engineering
8. Export turnover industry
9. Export turnover intermediate goods industry
10. Export turnover capital goods industry
11. Export turnover durable and non-durable consumer goods industry
12. Export turnover mechanical engineering
13. Export turnover electrical engineering
14. Export turnover vehicle engineering
15. Production industry
16. Production intermediate goods industry
17. Production capital goods industry
18. Production durable and non-durable consumer goods industry
19. Production mechanical engineering
20. Production electrical engineering
21. Production vehicle engineering
22. Orders received by the industry from the domestic market
23. Orders received by the intermediate goods industry from the domestic market
24. Orders received by the capital goods industry from the domestic market
25. Orders received by the durable and non-durable consumer goods industry from the domestic market
26. Orders received by the mechanical engineering from the domestic market
27. Orders received by the electrical engineering from the domestic market
28. Orders received by the vehicle engineering from the domestic market
29. Orders received by the industry from abroad
30. Orders received by the intermediate goods industry from abroad

31. Orders received by the capital goods industry from abroad
32. Orders received by the durable and non-durable consumer goods industry from abroad
33. Orders received by the mechanical engineering from abroad
34. Orders received by the electrical engineering from abroad
35. Orders received by the vehicle engineering from abroad

## **Construction**

1. Production of construction sector
2. Orders received by the construction sector
3. Orders received by the construction sector: Building construction
4. Orders received by the construction sector: Residential building
5. Orders received by the construction sector: Non-residential building construction
6. Orders received by the construction sector: Civil engineering
7. Orders received by the construction sector: Road construction
8. Orders received by the construction sector: Rest, industrial, and governmental civil engineering excluding roads
9. Man-hours worked: Building construction
10. Man-hours worked: Civil engineering
11. Man-hours worked: Residential building
12. Man-hours worked: Industrial building
13. Man-hours worked: Public building
14. Turnover: Building construction
15. Turnover: Civil engineering
16. Turnover: Residential building
17. Turnover: Industrial building
18. Turnover: Public building

## **Surveys**

1. Business situation manufacturing
2. Assessment of stocks manufacturing
3. Business expectations next six months manufacturing
4. Capacity utilization manufacturing
5. Business situation capital goods producers
6. Assessment of stocks capital goods producers
7. Business expectations next six months capital goods producers
8. Capacity utilization capital goods producers
9. Business situation basic & producer goods
10. Assessment of stocks basic & producer goods

11. Business expectations next six months basic & producer goods
12. Business situation producers non-durable consumption goods
13. Assessment of stocks producers non-durable consumption goods
14. Business expectations next six months producers non-durable consumption goods
15. Capacity utilization producers non-durable consumption goods
16. Business situation foodstuff & tobacco producers
17. Assessment of stocks foodstuff & tobacco producers
18. Business expectations next six months foodstuff & tobacco producers
19. Capacity utilization foodstuff & tobacco producers
20. Business situation retail trade
21. Assessment of stocks retail trade
22. Business expectations next six months retail trade
23. Business situation construction
24. Business expectations next six months construction
25. Capacity utilization construction

## **Labour market**

1. Short-term employed
2. Vacancies
3. Unemployment, level
4. Unemployment rate, as percent of labour force
5. Productivity per employee
6. Productivity per hour
7. Wages and salaries per employee
8. Wages and salaries per hour
9. Employees (residence concept)
10. Unemployment, level, male
11. Unemployment rate, male
12. Unemployment, level, female
13. Unemployment rate, female

## **Interest rates**

1. Money market rate, overnight deposits
2. Money market rate, 3 months deposits
3. Bond yields on public and non-public long term bonds with average rest maturity from 1 to 2 years
4. Bond yields on public and non-public long term bonds with average rest maturity from 9 to 10 years
5. Yield spread: Average 9 to 10 year bond yields minus 3 month deposit rate

## Miscellaneous indicators

1. New car registrations
2. Current account: External trade
3. Current account: Services