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Key Words and Phrases: pre-test estimation; mis-specified regression models; spherical symmetry; multivariate Student-t.

ABSTRACT

The risk properties of estimators of the scale parameter after a pre-test for homogeneity of the error variances in the two sample linear regression model has received quite an amount of attention in the literature. This literature typically assumes normal disturbances and a properly specified model. In this paper we consider that both equations may be mis-specified by the omission of relevant regressors and that the error distributions may belong to a wider class than the normal distribution. We derive and analyse the exact risk (under quadratic loss) of the pre-test estimator of the scale parameter for the first sub-sample.

1. INTRODUCTION AND MODEL FRAMEWORK

We consider a simple heteroscedastic linear regression model in which the error variance is constant within each sample but it may be different between the samples,

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \qquad (1)$$

or

$$y = X\beta + Z\gamma + e$$

where for $i=1,2$, y_i is a $(T_i \times 1)$ vector of observations on the dependent variable, X_i is a known $(T_i \times k_i)$ non-stochastic design matrix of rank k_i $(<T_i)$, Z_i is a fixed $(T_i \times p_i)$ matrix of full rank, γ_i is a

$(p_i \times 1)$ coefficient vector, and β_i is a $(k_i \times 1)$ vector of unknown parameters. Let $T=T_1+T_2$.

We assume that $E(e_i)=0$, and that $E(e_i e_i')=\sigma^2_{e_i} I_{T_i}$. Let $\phi=\sigma^2_{e_1}/\sigma^2_{e_2}$ so that

$$E(ee') = \begin{bmatrix} \sigma^2_{e_1} I_{T_1} & 0 \\ 0 & \sigma^2_{e_2} I_{T_2} \end{bmatrix} = \sigma^2_{e_2} \begin{bmatrix} \phi I_{T_1} & 0 \\ 0 & I_{T_2} \end{bmatrix} = \sigma^2_{e_2} \Sigma \quad . \tag{2}$$

We suppose that e has a non-normal distribution of the form $f(e)=\int_0^\infty f_N(e) f(\tau) d\tau$, where $f_N(e)$ is $f(e)$ when $e \sim N(0,\tau^2\Sigma)$, and $f(\tau)$ is supported on $[0,\infty)$. Then $\sigma^2_{e_2}=E(\tau^2)$, $\sigma^2_{e_1}=\phi E(\tau^2)$, and e has an elliptically symmetric distribution (ESD_N) when $\phi \neq 1$ but a spherically symmetric distribution (SSD_N) when the error variances are equal.[1,2] This is the commonly called scale mixture of normal distributions family, of which the multivariate Student-t (Mt) distribution is the most well known member. The Mt family arises if $f(\tau)$ is an inverted gamma density with, say, scale parameter σ^2_2 and degrees of freedom parameter ν. We then write $e \sim Mt\left(0,(\nu\sigma^2_2/(\nu-2))\Sigma\right)$. For this member the marginal distributions are univariate Student-t with thicker tails than under a corresponding normality assumption for $\nu<\infty$. The kurtosis increases as ν decreases and $\nu=\infty$ corresponds to normality.

Rather than model (1) being estimated we suppose that the proposed model is

$$y = X\beta + u \quad , \quad u \sim N\left(0,\sigma^2_{e_2}\Sigma\right) \tag{3}$$

which the researcher assumes is correctly specified. In fact, (3) is

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[1] A discussion of this family of distributions is beyond the scope of *this paper. See, for example,* Kelker (1970), Muirhead (1982), and Dickey and Chen (1985).

[2] It would be relatively straightforward to extend our analysis to the case of different mixing distributions for each sample when we have independent mixing distributions. It is unclear, however, how we would proceed if they are dependent.

mis-specified as $u \sim ESD_N(Z\gamma, \sigma^2_{e_2}\Sigma)$. Note that (3) reflects the fact that $\sigma^2_{u_i} = \sigma^2_{e_i}$, i=1,2. The researcher is interested in estimating $\sigma^2_{e_1}$ but he is uncertain of the homogeneity of the error variances and so conducts a pre-test of

$$H_0 : \phi = 1 \quad vs \quad H_1 : \phi < 1 \quad , \tag{4}$$

where ϕ is a measure of the hypothesis error and we assume, for simplicity, a one-sided alternative hypothesis though the analysis could be easily extended to the two-sided case.

Assuming the usual least squares estimators of the error variances, the reseacher, mistakenly proceeding as if (3) is properly specified, has three options for the estimation of $\sigma^2_{e_1}$:

(1) He could assume that the variances are equal and use the so-called <u>always pool</u> estimator of $\sigma^2_{e_1}$, s^2_A :

$$s^2_A = \left(v_1 s^2_1 + v_2 s^2_2\right)/(v_1+v_2) \tag{5}$$

where $s^2_i = (y_i - X_i b_i)'(y_i - X_i b_i)/v_i$, $v_i = T_i - k_i$, and $b_i = (X_i'X_i)^{-1}X_i'y_i$, i=1,2.

(2) He could proceed as if the error variances are unequal, ignore the second sample, and use the so-called <u>never pool</u> estimator of $\sigma^2_{e_1}$, s^2_N :

$$s^2_N = s^2_1 \quad . \tag{6}$$

(3) He could undertake a preliminary test of the validity of H_0 and use s^2_A if he accepts H_0 or s^2_N if he rejects H_0. As the researcher assumes that model (3) is correctly specified, he uses the usual J test for homoscedasticity, given by $J = s^2_2/s^2_1$, and proceeds as if $f(J) = \phi^{-1} f(F_{(v_2,v_1)})$, where $F_{(v_2,v_1)}$ is a central F variate with v_2 and v_1 degrees of freedom. Under this option, he is in fact using the pre-test estimator s^2_P:

$$s^2_P = \begin{cases} s^2_N & \text{if } J > c \\ s^2_A & \text{if } J \le c \end{cases} = I_{[0,c]}(J)s^2_A + I_{(c,\infty)}(J)s^2_N \tag{7}$$

where $I_{[.,.]}(J)$ equals one if J lies within the subscripted range, zero otherwise, and c is the critical value of the pre-test corresponding to a <u>nominal</u> test size of α such that $\int_c^\infty f(F_{(v_2,v_1)})=\alpha$.

Assuming normal disturbances, $e\sim N(0,\sigma_2^2\Sigma)$, and a correctly specified design matrix, the risk of s_P^2, defined as $\rho(\sigma_{e_1}^2,s_P^2)=E(s_P^2-\sigma_{e_1}^2)^2$, is considered by Bancroft (1944), Toyoda and Wallace (1975), Ohtani and Toyoda (1978), and Bancroft and Han (1983).[3] Figure 1 presents a typical relative risk function under their assumptions, where the relative risk is $R(s_P^2)=\rho(\sigma_{e_1}^2,s_P^2)/\sigma_2^4$.[4]

We see from Figure 1 that it is only preferable to pool the samples around the neighbourhood of the null hypothesis and that we should never use the never pool estimator, as there is a family of pre-test estimators with $c\in(0,2)$ which strictly dominate this estimator (see Toyoda and Wallace (1975)). Ohtani and Toyoda (1978) prove that the pre-test estimator which uses c=1 has the smallest risk of this dominating family.

Giles (1992) extends these aforementioned studies to the error distribution framework considered in this paper, though she still maintains the (unrealistic) assumption of a correctly specified design matrix. In the case that she examines we know that the test statistic J maintains the same null and non-null distributions as under normal disturbances from the results of King (1979) and Chmielewski (1981). Figure 2 illustrates typical relative risk functions under Mt disturbances $\left[e\sim Mt\left(0,(\nu\sigma_2^2/(\nu-2))\Sigma\right)\right]$ when $\nu=5$.

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[3] A related, though not identical, pre-test problem is considered by Yancey *et al.* (1983) and Ohtani (1987). Both of these studies also assume normal errors.

[4] We lose no loss in generality in considering relative risk; the results could equally be interpreted as the risk functions when $\sigma_2^2=1$.

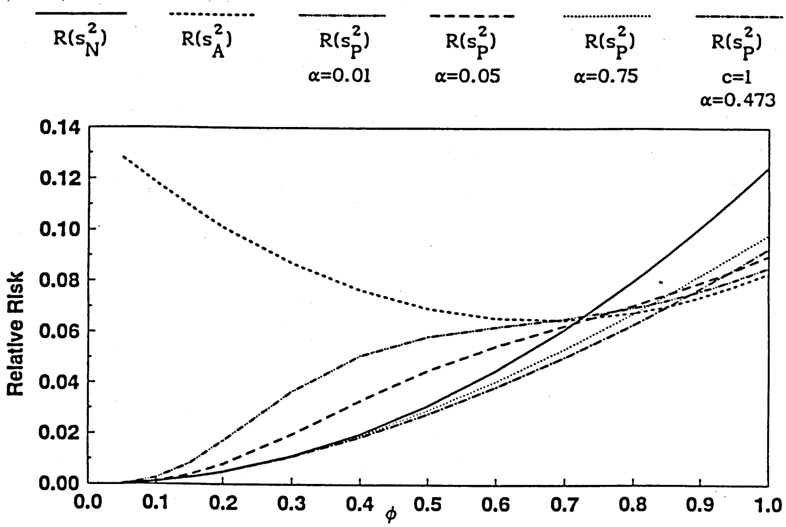

Figure 1. Relative risk functions for s_N^2, s_A^2, and s_P^2 when $e \sim N(0, \sigma_2^2\Sigma)$, $v_1=16$, $v_2=8$, $k_1=k_2=3$.

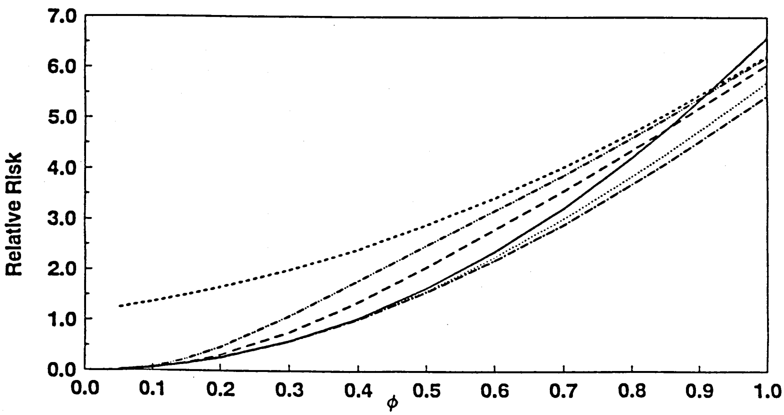

Figure 2. Relative risk functions for s_N^2, s_A^2, and s_P^2 when $e \sim Mt\left(0, (\nu\sigma_2^2/(\nu-2))\Sigma\right)$, $v_1=16$, $v_2=8$, $k_1=k_2=3$, $\nu=5$.

We see from Figure 2 that for non-normal disturbances we should sometimes always pre-test, *even if the error variances are equal.* Giles (1992) proves that the optimal critical value to use in these circumstances is c=1.

In this paper we recognise that in reality departures from the standard regression assumptions are likely to occur simultaneously and we, accordingly, examine the risk properties of s_N^2, s_A^2, and s_P^2 when the disturbances are elliptically symmetric and we have omitted relevant regressors from the design matrix. The impact of excluding regressors on the risk functions of the estimators of the error variance for this particular pre-test problem has yet to be examined in the pre-test literature, even under normality. Other studies, though, have investigated the effect of this mis-specification after other pre-tests (see, for example, Ohtani (1983), Mittelhammer (1984), Giles (1986), Ohtani (1987), Giles and Clarke (1989), and Giles (1991b)).

To undertake this task we need to first examine the distribution of the test statistic J, which is now no longer a function of a central F variate under either the null or the alternative hypotheses. We consider this in the next section. We follow in section 3 with the derivation of the exact risk functions, and then in section 4 with some numerical evaluations of these risk functions, assuming for the purpose of this discussion that the variance mixing distribution is inverted gamma so that our errors are Mt. The final section provides some concluding remarks.

2. THE DISTRIBUTION OF J WITH OMITTED REGRESSORS

If the model is mis-specified in the way investigated here then both the null and the non-null distributions of J depend on v_1, v_2, the degree of mis-specification of the design matrix, and the variance mixing distribution, $f(\tau)$. This is shown by Theorem 1 and Corollary 1.

Theorem 1. Under the stated assumptions, the density function of $J=s_2^2/s_1^2$ is

$$f(J) = \phi^{-1} \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \frac{\theta_1^s \theta_2^r v_1^{v_1/2+s} v_2^{v_2/2+r} J^{v_2/2+r-1}}{r!s!B\left(\frac{v_2}{2}+r;\frac{v_1}{2}+s\right) \left(v_1+v_2J\right)^{(v_1+v_2)/2+r+s}}$$

$$\int_0^{\infty} e^{-(\theta_1+\theta_2)/\tau^2}\left(\tau^2\right)^{-(r+s)} f(\tau)d\tau \quad , \qquad (8)$$

where $\theta_1=\gamma_1' Z_1' M_1 Z_1 \gamma_1/(2\phi)$, $\theta_2=\gamma_2' Z_2' M_2 Z_2 \gamma_2/2$, $M_i=I_{T_i}-X_i(X_i'X_i)^{-1}X_i'$, and $B(.;.)$ is the beta function.

Proof. See the appendix.

Corollary 1. Under the null hypothesis, $\phi=1$, and

$$f(J) = \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \frac{\theta_{10}^s \theta_2^r v_1^{v_1/2+s} v_2^{v_2/2+r} J^{v_2/2+r-1}}{r!s!B\left(\frac{v_2}{2}+r;\frac{v_1}{2}+s\right) \left(v_1+v_2J\right)^{(v_1+v_2)/2+r+s}}$$

$$\times \int_0^{\infty} e^{-(\theta_{10}+\theta_2)/\tau^2}\left(\tau^2\right)^{-(r+s)} f(\tau)d\tau \quad , \qquad (9)$$

where $\theta_{10}=\gamma_1' Z_1' M_1 Z_1 \gamma_1/2$.

Proof. As $\theta_1=\theta_{10}$ when $\phi=1$, (9) follows from (8).

If $e\sim N(0,\sigma_2^2\Sigma)$ and, say, $\sigma_{e_2}^2=\sigma_2^2$ and $\sigma_{e_1}^2=\sigma_1^2$, then from (8)

$$f_N(J) = \phi^{-1} f\left(F''_{(v_2,v_1;\lambda_2,\lambda_1)}\right) \qquad (10)$$

where $\lambda_i=\theta_i/\sigma_2^2$, $i=1,2$. Note that (10) does not collapse to a central F density under H_0; that is, the standard assumption that J has a central F density under H_0 is invalid if we have omitted relevant regressors, irrespective of our assumption on the distribution of the disturbances. So, there will be a difference between the nominal and true sizes of the test.

In this section we have established the distribution of the test statistic J which we use to test for the homogeneity of the error variances. In the next section we derive the exact risk functions of the never pool, always pool, and pre-test estimators of $\sigma^2_{e_1}$.

3. THE RISK FUNCTIONS

Theorem 2. If we use the mis-specified model (3) rather than the true model (1) when $e \sim ESD_N(0,\sigma^2_{e_2}\Sigma)$ and the pre-test is of H_0 in (4) then

$$\rho\left(\sigma^2_{e_1}, s^2_N\right) = \phi^2\left(v_1(v_1+2)E(\tau^4)-v_1^2\left(E(\tau^2)\right)^2+8\theta_1 E(\tau^2)+4\theta_1^2\right)/v_1^2 \tag{11}$$

$$\rho\left(\sigma^2_{e_1}, s^2_A\right) = \left(\phi^2\left[v_1(v_1+2)E(\tau^4)+4(v_1+2)\theta_1 E(\tau^2)-2v_1(v_1+v_2)\left(E(\tau^2)\right)^2\right.\right.$$
$$\left.+\left(2\theta_1-E(\tau^2)(v_1+v_2)\right)^2\right]+2\phi\left[v_1v_2E(\tau^4)-v_2(v_1+v_2)\left(E(\tau^2)\right)^2\right.$$
$$\left.\left.+2v_2E(\tau^2)(\theta_1-\theta_2)+4\theta_1\theta_2\right]+v_2(v_2+2)E(\tau^4)+4(v_2+2)\theta_2E(\tau^2)+4\theta_2^2\right)$$
$$/(v_1+v_2)^2 \tag{12}$$

$$\rho\left(\sigma^2_{e_1}, s^2_P\right) = \left(\phi^2(v_1+v_2)^2\left[v_1(v_1+2)E(\tau^4)-v_1^2\left(E(\tau^2)\right)^2+8\theta_1E(\tau^2)+4\theta_1^2\right]\right.$$
$$+\int_0^\infty\left(\phi^2v_2\left[-(2v_1+v_2)\left(v_1(v_1+2)\tau^4Q^\tau_{04}+4(v_1+2)\theta_1\tau^2Q^\tau_{06}+4\theta_1^2Q^\tau_{08}\right)\right.\right.$$
$$\left.\left.+2E(\tau^2)v_1(v_1+2)\left(v_1\tau^2Q^\tau_{02}+2\theta_1Q^\tau_{04}\right)\right]+2v_1^2\phi\left[v_1v_2\tau^4Q^\tau_{22}\right.$$
$$-v_2(v_1+v_2)E(\tau^2)\tau^2Q^\tau_{20}+2v_1\theta_2\tau^2Q^\tau_{42}-2(v_1+v_2)\theta_2E(\tau^2)Q^\tau_{40}$$

$$+2v_2\theta_1\tau^2Q^{\tau}_{24}+4\theta_1\theta_2\tau^2Q^{\tau}_{44}\Big]+v_1^2\Big[v_2(v_2+2)\tau^4Q^{\tau}_{40}$$

$$+4(v_2+2)\tau^2\theta_2Q^{\tau}_{60}+4\theta_2^2Q^{\tau}_{80}\Big]\Big)f(\tau)d\tau\Big)\Big/\Big(v_1^2(v_1+v_2)^2\Big) \qquad (13)$$

where

$$Q^{\tau}_{ij} = \Pr.\Big[F''_{(v_2+i,v_1+j;\lambda_{2\tau},\lambda_{1\tau})}\le\Big(v_2(v_1+j)c\phi\Big)\Big/\Big(v_1(v_2+i)\Big)\Big]$$

$i,j=0,1,2,\ldots$ and $\lambda_{i\tau}=\theta_i/\tau^2$, $i=1,2$.

Proof. See the appendix.

Remarks:

(1) Aside from depending on the arguments of the model v_1 and v_2, the risk functions depend on first, the true error variances $\sigma^2_{e_1}$ and $\sigma^2_{e_2}$ via ϕ; secondly, they depend on $f(\tau)$; thirdly, $\rho\left(\sigma^2_{e_1},s^2_P\right)$ depends on the nominal significance level of the pre-test; and finally, the risk functions depend on the degree of mis-specification in each sample, via θ_1 and θ_2.

(2) The data enter the risk functions only through θ_1 and θ_2.

(3) It is straightforward to show that (11)-(13) collapse first to the appropriate expressions derived by Giles (1992) when the design matrix is in fact correctly specified, and secondly, to the risk functions derived by Toyoda and Wallace (1975), for example, when (3) is the valid model specification.

(4) If $\alpha=0$ then $c=\infty$ and $Q^{\tau}_{ij}=1$ so that we never reject H_0. Then the risk of the pre-test estimator collapses to that of the always pool estimator. Conversely, if $\alpha=1$ then $c=0$ and $Q^{\tau}_{ij}=0$ so that we always reject H_0 and the risk of the pre-test estimator equals that of the never pool estimator.

(5) $\lim_{\phi\to0}\left[\rho\left(\sigma^2_{e_1},s^2_P\right)\right]=\lim_{\phi\to0}\left[\rho\left(\sigma^2_{e_1},s^2_N\right)\right]=0$ while $\lim_{\phi\to0}\left[\rho\left(\sigma^2_{e_1},s^2_A\right)\right]=\Big(v_2(v_2+2)E(\tau^4)+4(v_2+2)\theta_2E(\tau^2)+4\theta_2^2\Big)/(v_1+v_2)^2>0$. Pre-testing leads us to follow the appropriate strategy of ignoring the prior information when

that information is very false.

(6) $\rho\left(\sigma^2_{e_1},s^2_N\right)$ is independent of θ_2 and therefore bounded as $\theta_2\to\infty$, for a given value of θ_1. However, this risk function is bounded as $\theta_1\to\infty$, for a given value of θ_2. Similarly, $\rho\left(\sigma^2_{e_1},s^2_P\right)$ is unbounded as $\theta_1\to\infty$, given θ_2, but it is bounded (by $\rho\left(\sigma^2_{e_1},s^2_N\right)$) as $\theta_2\to\infty$, given θ_1. Intuitively, if the model for the second sample is badly mis-specified relative to the model for the first sample, then pre-testing will lead us to ignore the second sample, which is the appropriate strategy. $\rho\left(\sigma^2_{e_1},s^2_A\right)$, on the other hand, is unbounded as $\theta_1\to\infty$, given θ_2, or as $\theta_2\to\infty$, given θ_1.

Further, $\left[\rho\left(\sigma^2_{e_1},s^2_A\right)-\rho\left(\sigma^2_{e_1},s^2_N\right)\right]$ and $\left[\rho\left(\sigma^2_{e_1},s^2_A\right)-\rho\left(\sigma^2_{e_1},s^2_P\right)\right]$ are unbounded as $\theta_1\to\infty$, given θ_2, or as $\theta_2\to\infty$, given θ_1, while $\left[\rho\left(\sigma^2_{e_1},s^2_N\right)-\rho\left(\sigma^2_{e_1},s^2_P\right)\right]$ is bounded (and equal to zero) as $\theta_2\to\infty$, given θ_1, but it is unbounded as $\theta_1\to\infty$, given θ_2. These results imply, in particular, that the risk of the always pool estimator can be infinitely higher than that of the never pool estimator and the pre-test estimator <u>even if the error variances are equal</u>. That is, there is no guarantee of a reduction in risk by imposing valid prior information if we have omitted relevant regressors. This accords with the results of, for instance, Mittelhammer (1984) and Giles (1991b) in the case of estimating the coefficient vector in the classical linear regression model after a pre-test for exact linear restrictions.

(7) For any given degree of mis-specification through the omitted variables, there exists a family of pre-test estimators which strictly dominate the never pool estimator, and of this family of dominating estimators the pre-test estimator which uses c=1 has the smallest risk.[5] This generalises the result of Ohtani and Toyoda (1978) and it holds for all θ_1 and θ_2, and for all feasible members of

[5]Though not included in this paper, it is straightforward to analytically prove this result using the approach outlined by Giles (1991a,b, 1992). Details are available from Giles (1990) or the author.

the ESD_N family.

Further, these same pre-test estimators can also strictly dominate the always pool estimator, given θ_1 and θ_2. The condition under which this result occurs depends on v_1, v_2, θ_1, θ_2, α, and on the variance mixing distribution $f(\tau)$.

(8) The risks of s_N^2 and s_A^2 have two possible ϕ intersections:[6]

$$\phi_i = (v_1/v_2)\Bigg\{v_1\Big[v_1v_2E(\tau^4)-v_2(v_1+v_2)\Big(E(\tau^2)\Big)^2+2v_2E(\tau^2)(\theta_1-\theta_2)+4\theta_1\theta_2\Big]$$
$$\pm\Bigg[v_1^2\Big[v_1v_2E(\tau^4)-v_2(v_1+v_2)\Big(E(\tau^2)\Big)^2+2v_2E(\tau^2)(\theta_1-\theta_2)+4\theta_1\theta_2\Big]^2$$
$$+v_2\Big[(2v_1+v_2)\Big(v_1(v_1+2)E(\tau^4)+4\theta_1(v_1+2)E(\tau^2)+4\theta_1^2-2v_1(v_1+v_2)$$
$$\times E(\tau^2)\Big(v_1E(\tau^2)+2\theta_1\Big)\Big]\Big[v_2(v_2+2)E(\tau^4)+4(v_2+2)\theta_2E(\tau^2)-4\theta_2^2\Big]\Bigg]^{1/2}\Bigg\}$$
$$\Big/\Big[(2v_1+v_2)\Big(v_1(v_1+2)E(\tau^4)+4\theta_1(v_1+2)E(\tau^2)+4\theta_1^2\Big)$$
$$-2v_1(v_1+v_2)E(\tau^2)\Big(v_1E(\tau^2)+2\theta_1\Big)\Big]$$
$$= \omega\pm\kappa \,, \tag{14}$$

$i=1,2$. Let $\phi_1=\omega+\kappa$ and $\phi_2=\omega-\kappa$. Giles (1992) shows that if the disturbances are ESD_N but the design matrix is correctly specified then there always exists two possible events but only one feasible intersection - neither the always pool estimator nor the never pool estimator can strictly dominate each other. However, when relevant regressors have been excluded there are four possible events: (i) $0<\phi_1<1$, $\phi_2>1$; (ii) $\phi_1>1$, $\phi_2<0$; (iii) $0<\phi_1<1$, $\phi_2<0$; (iv) $\phi_1>1$, $\phi_2>1$. If v_1, v_2, θ_1, and θ_2 are such that cases (ii) or (iv) result then the always pool estimator is strictly dominated by the never pool estimator for all $\phi\in(0,1]$. We discuss this result further in the following section.

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[6] We require $\Big[(2v_1+v_2)\Big(v_1(v_1+2)E(\tau^4)+4\theta_1(v_1+2)E(\tau^2)+4\theta_1^2\Big)-2v_1(v_1+v_2)\times E(\tau^2)\Big(v_1E(\tau^2)+2\theta_1\Big)\Big] \neq 0$.

4. NUMERICAL EVALUATIONS OF THE RISK FUNCTIONS

To illustrate the risk functions we have numerically evaluated them for the special case of Mt disturbances, which arise when τ is an inverted gamma variate. Then $e \sim Mt\left(0,\ (\nu\sigma_2^2/(\nu-2))\Sigma\right)$ and

$$\rho_{Mt}\left(\sigma_{e_1}^2, s_N^2\right) = 2\phi^2\sigma_2^4\left(\nu^2 v_1(v_1+\nu-2)+4\lambda_1\nu(\nu-2)(\nu-4) + 2\lambda_1^2(\nu-2)^2(\nu-4)\right)/\left(v_1^2(\nu-2)^2(\nu-4)\right) \qquad (15)$$

$$\begin{aligned}\rho_{Mt}\left(\sigma_{e_1}^2, s_A^2\right) = \sigma_2^4\Big(&\phi^2\Big[v_2^2\nu^2(\nu-4)+2v_1\nu^2(v_1+\nu-2)-4\lambda_1\nu(\nu-2)(\nu-4)(v_2-2)\\ &+4\lambda_1^2(\nu-2)^2(\nu-4)\Big]+2\phi\Big[v_2\nu^2\left(2v_1-v_2(\nu-4)\right)+2v_2\nu(\nu-2)(\nu-4)(\lambda_1-\lambda_2)\\ &+4\lambda_1\lambda_2(\nu-2)^2(\nu-4)\Big]+v_2(v_2+2)\nu^2(\nu-2)+4(v_2+2)\lambda_2\nu(\nu-2)(\nu-4)\\ &+4\lambda_2^2(\nu-2)^2(\nu-4)\Big)/\left((v_1+v_2)^2(\nu-2)^2(\nu-4)\right)\end{aligned} \qquad (16)$$

$$\begin{aligned}\rho_{Mt}\left(\sigma_{e_1}^2, s_P^2\right) = \sigma_2^4\Big(&2\phi^2(v_1+v_2)^2\Big[v_1\nu^2(v_1+\nu-2)+4\lambda_1\nu(\nu-2)(\nu-4)\\ &+4\lambda_1^2(\nu-2)^2(\nu-4)\Big]+\phi^2 v_2\Big[-(\nu-2)(2v_1+v_2)\Big(v_1(v_1+2)\nu^2 Q_{040}\\ &+4(v_1+2)\nu(\nu-4)Q_{061}+4\lambda_1^2(\nu-2)(\nu-4)Q_{082}\Big)+2\nu(\nu-4)v_1(v_1+v_2)\\ &\times\left(v_1\nu Q_{021}+2\lambda_1(\nu-2)Q_{042}\right)\Big]+2v_1^2\phi\Big[v_1v_2\nu^2(\nu-2)Q_{220}-v_2(v_1+v_2)\nu^2\\ &\times(\nu-4)Q_{201}+2v_1\lambda_2\nu(\nu-2)(\nu-4)Q_{421}-2(v_1+v_2)\lambda_2\nu(\nu-2)(\nu-4)Q_{402}\\ &+2v_2\lambda_1\nu(\nu-2)(\nu-4)Q_{241}+4\lambda_1\lambda_2(\nu-2)^2(\nu-4)Q_{442}\Big]\\ &+v_1^2(\nu-2)\Big[v_2(v_2+2)\nu^2 Q_{400}+4(v_2+2)\lambda_2\nu(\nu-4)Q_{601}\\ &+4\lambda_2^2(\nu-2)(\nu-4)Q_{802}\Big]\Big)/\left(v_1^2(v_1+v_2)^2(\nu-2)^2(\nu-4)\right)\end{aligned} \qquad (17)$$

where

$$Q_{ijn} = \sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{(2\lambda_2/\nu)^r(2\lambda_1/\nu)^s\Gamma(\nu/2+r+s+n-2)}{r!s!\left(1+2(\lambda_1+\lambda_2)/\nu\right)^{\nu/2+r+s+n-2}\Gamma(\nu/2+n-2)}$$

$$\times I_w\left((v_2+i)/2+r;(v_1+j)/2+s\right) \qquad i,j,n=0,1,2,\ldots ,$$

$\lambda_i=\theta_i/\sigma_2^2$ (i=1,2) and $I_w(.;.)$ is Pearson's incomplete beta function with $w=c\phi v_2/(v_1+c\phi v_2)$.

We have evaluated (15)-(17) for a wide range of the arguments : $v_1=v_2=20$; $v_1=16$, $v_2=8$; $v_1=25$, $v_2=10$; $k_1=k_2=3$, 4, 5; $\alpha=0.01$, 0.05, 0.30, 0.50, 0.75 and those values of α associated with a critical value of unity; $\nu=5$, 10, 100, ∞; $\lambda_1\in[0,20]$; $\lambda_2\in[0,20]$; and $\phi\in[0.05,1]$. The FORTRAN computer programs, executed on a VAX 6230 computer, used various subroutines from Press *et al.* (1986) and Davies' (1980) algorithm. Figures 3 to 8 present some typical results; full details of which are available on request. These figures consider $\nu=5$ and $\nu=\infty$ (normal errors) for three degrees of mis-specification: first, when regressors are excluded from the model for sample one but not from that for sample two ($\lambda_1=3$, $\lambda_2=0$); secondly, when the design matrix for sample one is correctly specified but that for sample two is not ($\lambda_1=0$, $\lambda_2=3$); and thirdly, when both models are mis-specified to the same degree ($\lambda_1=\lambda_2=3$). As in Figures 1 and 2, we consider risk relative to σ_2^4 and parameterise with respect to λ_1 and λ_2 rather than with respect to θ_1 and θ_2 to eliminate the scale parameter σ_2^2. Equivalently, the figures represent the risks of the estimators when $\sigma_2^2=1$.

The figures illustrate the features discussed in the previous section. In particular, they highlight that the always pool estimator can be strictly dominated by both the never pool estimator and the pre-test estimator. Then, as the never pool estimator is itself always strictly dominated by at least the pre-test estimator which uses c=1, it is always preferable to pre-test and to use a critical value of unity.

Giles (1992), in the correctly specified design matrix case, shows that the pre-test estimator which uses c=1 can strictly dominate <u>both</u> of its component estimators for relatively small ν. Our numerical evaluations support her findings but they also suggest that the always pool estimator will be strictly dominated by the pre-test estimator which uses c=1 if the models for either sample are, or for both samples is, sufficiently mis-specified, irrespective of the value of ν.

Regarding the possibility of the strict dominance of the always pool estimator by the never pool estimator, which is infeasible in the properly specified model, our numerical evaluations suggest that this result depends not only on the degree of mis-specification but also on the other arguments in the problem. Specifically, it will typically be observed if $v_1 \geq v_2$, $\lambda_2 \gg \lambda_1$, and ν is relatively large. Intuitively, the gain in information from the second sample in terms of additional degrees of freedom is outweighed by the loss in the 'quality' of the information due to the relatively higher specification error in the second sample. For small values of ν we find there is still a small ϕ-range, in the neighbourhood of H_0, over which it is better to always pool the samples than to never pool them.

If, however, the mis-specification in the first sample is significantly higher than that in the second sample then there is a ϕ-range over which the always pool estimator has smaller risk than the never pool estimator, irrespective of the value of ν. *Ceteris paribus*, the width of this range increases with ν and with λ_1.

5. CONCLUDING REMARKS

In this paper we have considered the risk properties of estimators of the error variance after a pre-test for homogeneity, when the joint distribution of the unobservable errors in each sample is SSD_N but it is assumed to be normal, and there is simultaneously a possible mis-specification of the design matrix. We showed first that the classical test for homoscedasticity in our model is no longer valid when regressors have been omitted. Then, both the null and non-null distributions of J depend on v_1, v_2, the degree of mis-specification of the design matrix, and the variance mixing distribution, $f(\tau)$.

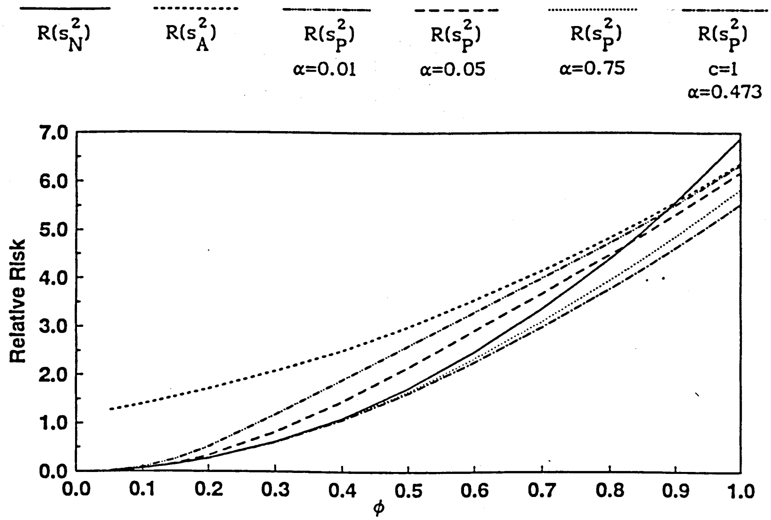

Figure 3. Relative risk functions for s_N^2, s_A^2, and s_P^2 when $e \sim Mt\left(0, (\nu\sigma_2^2/(\nu-2))\Sigma\right)$, $v_1=16$, $v_2=8$, $k_1=k_2=3$, $\nu=5$, $\lambda_1=3$, $\lambda_2=0$.

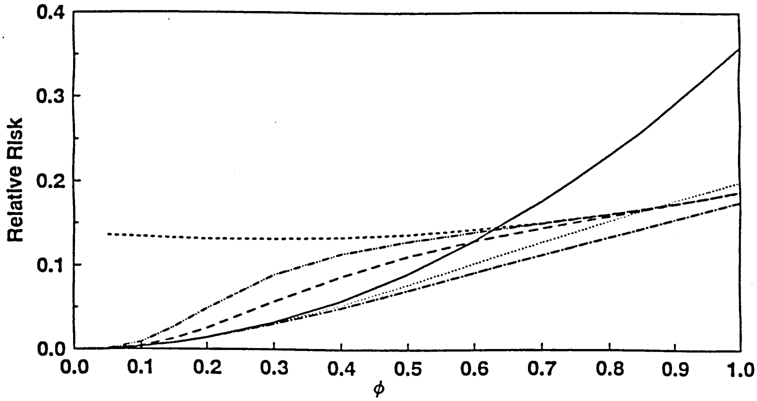

Figure 4. Relative risk functions for s_N^2, s_A^2, and s_P^2 when $e \sim N(0, \sigma_2^2\Sigma)$, $v_1=16$, $v_2=8$, $k_1=k_2=3$, $\lambda_1=3$, $\lambda_2=0$.

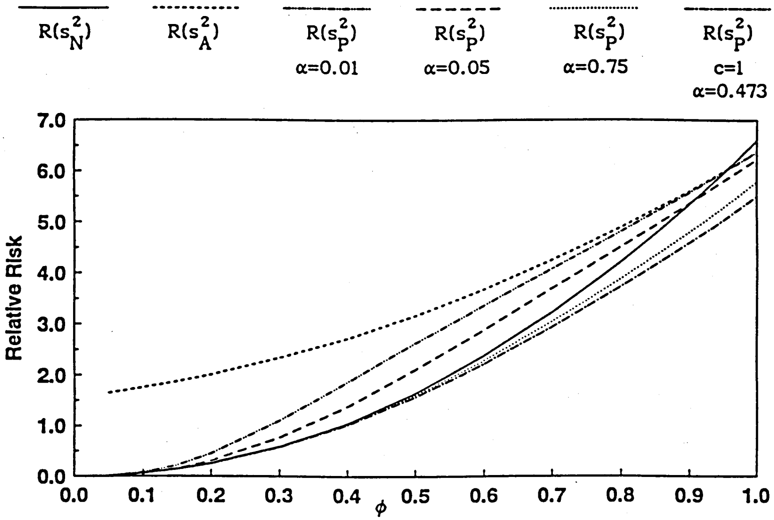

Figure 5. Relative risk functions for s_N^2, s_A^2, and s_P^2 when $e \sim Mt\left(0, (\nu\sigma_2^2/(\nu-2))\Sigma\right)$, $v_1=16$, $v_2=8$, $k_1=k_2=3$, $\nu=5$, $\lambda_1=0$, $\lambda_2=3$.

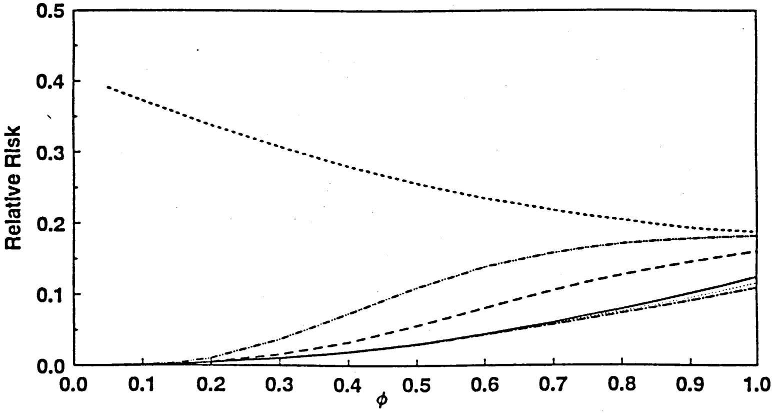

Figure 6. Relative risk functions for s_N^2, s_A^2, and s_P^2 when $e \sim N(0, \sigma_2^2\Sigma)$, $v_1=16$, $v_2=8$, $k_1=k_2=3$, $\lambda_1=0$, $\lambda_2=3$.

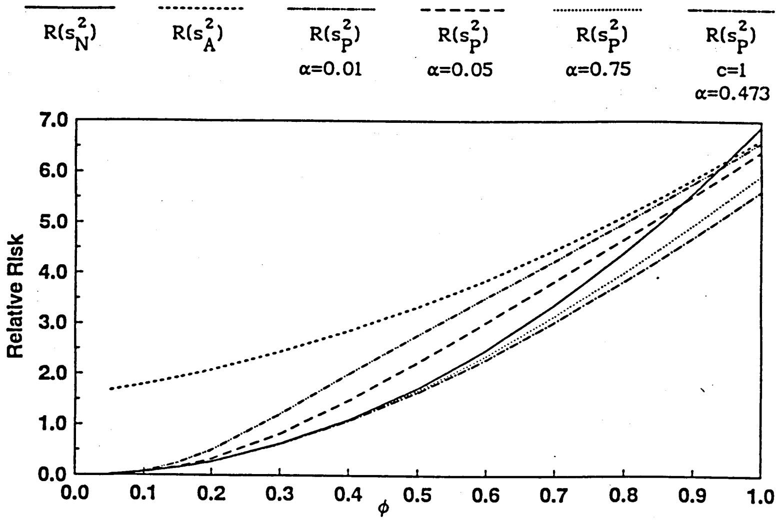

Figure 7. Relative risk functions for s_N^2, s_A^2, and s_P^2 when $e \sim Mt\left(0,(\nu\sigma_2^2/(\nu-2))\Sigma\right)$, $v_1=16$, $v_2=8$, $k_1=k_2=3$, $\nu=5$, $\lambda_1=\lambda_2=3$.

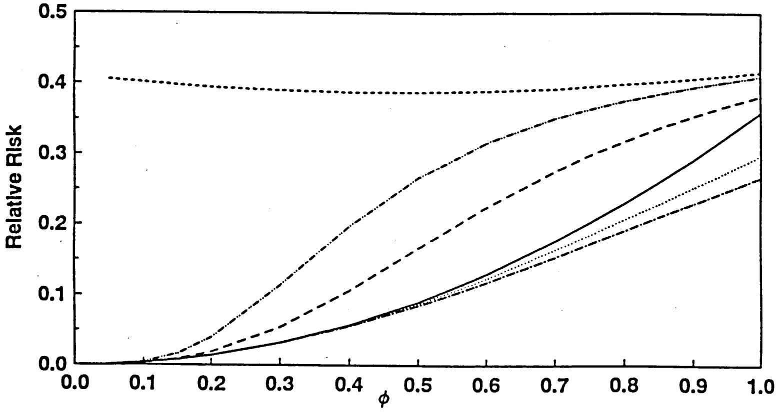

Figure 8. Relative risk functions for s_N^2, s_A^2, and s_P^2 when $e \sim N(0,\sigma_2^2\Sigma)$, $v_1=16$, $v_2=8$, $k_1=k_2=3$, $\lambda_1=\lambda_2=3$.

Our analysis of the risk functions of the never pool, always pool, and pre-test estimators of $\sigma^2_{e_1}$ showed first that the dominance of the never pool estimator by a family of pre-test estimators is robust to the mis-specification of the design matrix and of the error distribution. It is straightforward to show that the pre-test estimator which uses c=1 has the smallest risk of this family of dominating pre-test estimators irrespective of the degree of mis-specification of the design matrix or of the form of the variance mixing distribution.

We showed secondly that if we have excluded variables then the never pool estimator can also strictly dominate the always pool estimator. This result is impossible when we have properly specified the model, and it suggests that the risk of the always pool estimator is (qualitatively) less robust to the mis-specification than is the never pool estimator.

Practically, our analysis suggests, given that the degrees of model mis-specification and hypothesis error are unknown, that it is generally preferable to pre-test rather than to impose or ignore the prior information without testing. Then the optimal critical value is unity irrespective of the degrees of freedom of the model. Typically this critical value results in a (nominal) size that is far greater than the usual test sizes of 1% or 5%.

It remains for future research to consider the extension of this analysis to the two-sided alternative hypothesis case. The sensitivity of the results to the particular form of non-normality also requires attention. In particular, it is unclear whether they will extend to the case of iid non-normal disturbances.

APPENDIX

Proof of Theorem 1.

$$f(J) = \int_0^\infty f_N(J) f(\tau) d\tau, \tag{A.1}$$

where $f_N(J)$ is the density function of J when $e \sim N(0,\tau^2\Sigma)$. Under this normality assumption

$$e^* \equiv \begin{bmatrix} (Z_1\gamma_1+e_1)/\sqrt{\phi} \\ (Z_2\gamma_2+e_2) \end{bmatrix} \sim N\left(\begin{bmatrix} Z_1\gamma_1/\sqrt{\phi} \\ Z_2\gamma_2 \end{bmatrix}, \tau^2 I_T \right)$$

and so

$$J = \frac{s_2^2}{s_1^2} = \frac{v_1(Z_2\gamma_2+e_2)'M_2(Z_2\gamma_2+e_2)}{v_2(Z_1\gamma_1+e_1)'M_1(Z_1\gamma_1+e_1)} = \frac{v_1 e^{*\prime}M_2^* e^*}{v_2 e^{*\prime}M_1^* e^*}.$$

where M_1^* and M_2^* are (T×T) idempotent matrices partitioned as $M_1^* = \begin{bmatrix} M_1 & 0 \\ 0 & 0 \end{bmatrix}$ and $M_2^* = \begin{bmatrix} 0 & 0 \\ 0 & M_2 \end{bmatrix}$ with $r(M_i^*)=r(M_i)=v_i$, i=1,2. Under the normality assumption, it is straightforward to show that the quadratic forms $(e^{*\prime}M_2^*e^*/\tau^2)$ and $(e^{*\prime}M_1^*e^*/\tau^2)$ are independent with $(e^{*\prime}M_i^*e^*/\tau^2) \sim\chi^2_{v_i;\lambda_{i\tau}}$ and $\lambda_{i\tau}=\theta_i/\tau^2$, i=1,2. So, $f_N(J)=\phi^{-1}f\left(F''_{(v_2,v_1;\theta_2/\tau^2,\theta_1/\tau^2)}\right)$. Given the density function of a doubly non-central F variate and using (A.1) equation (8) follows directly. ■

Proof of Theorem 2.

$$\rho\left(\sigma^2_{e_1},s_N^2\right) = E\left(s_N^4\right)-2\phi E(\tau^2)E\left(s_N^2\right)+\phi^2\left(E(\tau^2)\right)^2 \qquad (A.2)$$

and $E\left(s_N^2\right)=\int_0^\infty E_N\left(s_N^2\right)f(\tau)d\tau$, where $E_N\left(s_N^2\right)=E\left(s_N^2\right)$ when $e\sim N(0,\tau^2\Sigma)$. Then, $e^*\sim N(0,\tau^2 I_T)$, $e^{*\prime}M_1^*e^*/\tau^2\sim\chi^2_{v_1;\lambda_{1\tau}}$, $E_N(e^{*\prime}M_1^*e^*/\tau^2)=v_1+2\lambda_{1\tau}$, and so $E\left(s_N^2\right) =\phi\left(v_1E(\tau^2) + 2\theta_1\right)/v_1$. Using the same approach, $E\left(s_N^4\right)= \phi^2\left(v_1(v_1+2)E(\tau^4) + 4(v_1+2)\theta_1E(\tau^2) + 4\theta_1^2\right)/v_1^2$. Substituting these expressions into (A.2) yields $\rho\left(\sigma^2_{e_1},s_N^2\right)$.

$$\rho\left(\sigma^2_{e_1},s_A^2\right) = E\left(s_A^4\right)-2\phi E(\tau^2)E\left(s_A^2\right)+\phi^2\left(E(\tau^2)\right)^2 \qquad (A.3)$$

and $E\left(s_A^2\right)=\int_0^\infty E_N\left(s_A^2\right)f(\tau)d\tau$, where $E_N\left(s_A^2\right)=E\left(s_A^2\right)$ when $e\sim N(0,\tau^2\Sigma)$. Then, $e^*\sim N(0,\tau^2 I_T)$, $e^{*\prime}M_i^*e^*/\tau^2\sim\chi^2_{v_i;\lambda_{i\tau}}$, $E_N(e^{*\prime}M_i^*e^*/\tau^2)=v_i+2\lambda_{i\tau}$ (i=1,2), and so as $s_A^2=(\phi e^{*\prime}M_1^*e^*+e^{*\prime}M_2^*e^*)/(v_1+v_2)$ we have $E_N(s_A^2)=\tau^2\left(\phi(v_1+2\lambda_{1\tau})+v_2+2\lambda_{2\tau}\right)/(v_1+v_2)$. Integrating this expression with respect to τ gives $E(s_A^2)=\left[\phi\left(v_1E(\tau^2)+2\theta_1\right) + v_2E(\tau^2) + 2\theta_2\right]/(v_1+v_2)$. Similarly, $E(s_A^4) = \left(\phi^2\left[v_1(v_1+2)E(\tau^4) + 4(v_1+2)\theta_1E(\tau^2) + 4\theta_1^2\right]+2\phi\left[v_1v_2E(\tau^4) +2v_1\theta_2E(\tau^2) + 2v_2\theta_1E(\tau^2) + 4\theta_1\theta_2\right] + v_2(v_2+2)E(\tau^4) + 4(v_2+2)\theta_2E(\tau^2) + 4\theta_2^2\right) /(v_1+v_2)^2$. Substituting these expressions into (A.3) completes the derivation of $\rho\left(\sigma_{e_1}^2,s_A^2\right)$.

Finally,

$$\rho\left(\sigma_{e_1}^2,s_P^2\right) = E\left(s_P^4\right)-2\phi E(\tau^2)E\left(s_P^2\right)+\phi^2\left(E(\tau^2)\right)^2 \quad . \qquad \text{(A.4)}$$

Using the aforementioned notation we write $s_P^2=\left(\phi(v_1+v_2)(e^{*\prime}M_1^*e^*) + \left[v_1e^{*\prime}M_2^*e^* - \phi v_2e^{*\prime}M_1^*e^*\right]I_{[0,c\phi]}\left((v_1e^{*\prime}M_2^*e^*)/(v_2e^{*\prime}M_1^*e^*)\right)\right)/\left(v_1(v_1+v_2)\right)$. Using Lemma 1 of Clarke *et al.* (1987) $E_N\left[(e^{*\prime}M_2^*e^*/\tau^2) \times I_{[0,c\phi]}\left((v_1e^{*\prime}M_2^*e^*)/(v_2e^{*\prime}M_1^*e^*)\right)\right]=v_2Q_{20}^{\tau} + 2\lambda_{2\tau}Q_{40}^{\tau}$ and $E_N\left[(e^{*\prime}M_1^*e^*/\tau^2) \times I_{[0,c\phi]}\left((v_1e^{*\prime}M_2^*e^*)/(v_2e^{*\prime}M_1^*e^*)\right)\right] = v_1Q_{02}^{\tau} + 2\lambda_{1\tau}Q_{04}^{\tau}$. So, $E_N\left(s_P^2\right)=\left(\phi(v_1+v_2)(v_1\tau^2+2\theta_1) + v_1v_2\tau^2(Q_{20}^{\tau}-\phi Q_{02}^{\tau}) + 2v_1\theta_2Q_{40}^{\tau} - 2v_2\theta_1\phi Q_{04}^{\tau}\right)/\left(v_1(v_1+v_2)\right)$ from which we obtain $E\left(s_P^2\right)$ by integrating with respect to τ. Likewise, $E_N\left(s_P^4\right) = \Big(\phi^2(v_1+v_2)^2\left[v_1(v_1+2)\tau^4 + 4(v_1+2)\theta_1\tau^2 + 4\theta_1^2\right]- \phi^2v_2(2v_1+v_2)\left[v_1(v_1+2)\tau^4Q_{04}^{\tau} + 4(v_1+2)\theta_1\tau^2Q_{06}^{\tau} + 4\theta_1^2Q_{08}^{\tau}\right] + v_1^2\left[v_2(v_2+2)\tau^4Q_{40}^{\tau} + 4(v_2+2)\theta_2\tau^2Q_{60}^{\tau} + 4\theta_2^2Q_{80}^{\tau}\right] + 2\phi v_1^2\left[v_1v_2\tau^4Q_{22}^{\tau} + 2v_1\theta_2\tau^2Q_{42}^{\tau} + 2v_2\theta_1Q_{24}^{\tau} + 4\theta_1\theta_2Q_{44}^{\tau}\right]\Big)/\left(v_1(v_1+v_2)\right)^2$ which we integrate with respect to τ to obtain $E\left(s_P^4\right)$. Substituting these expressions into (A.4) completes the proof. ∎

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