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## Discussion Paper

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# OPTIMAL CRITICAL VALUES OF A PRELIMINARY TEST FOR LINEAR RESTRICTIONS IN A REGRESSION MODEL WITH MULTIVARIATE STUDENT-t DISTURBANCES

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Abstract

Applied researchers typically pre-test to decide the final estimator of the coefficients in a regression model. Brook (1976) shows that the optimal critical value for the preliminary test is approximately two in value, regardless of the degrees of freedom, according to a mini-max (risk) regret criterion, when the prior test is of the validity of restrictions on the model's coefficients. Brook's result depends on the often unrealistic assumption of normally distributed disturbances. We consider the wider family of multivariate Student-t disturbances.

Key Words: Spherical Symmetry; Conditional Inference; F-Test; Mini-max Rule

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# 1. INTRODUCTION

Applied statisticians and econometricians frequently choose the ultimate specification of a multiple linear regression model on the basis of preliminary tests on that model. For instance, they may test the validity of exact linear restrictions on the model's parameters and then, depending on the outcome of the prior test, use either the Ordinary Least Squares estimator (OLSE) or the Restricted Least Squares estimator (RLSE). This procedure leads to the use of a "preliminary test" estimator (PTE). The PTE is the OLSE if we reject the hypothesis that the restrictions are correct or it is the RLSE if we cannot reject this hypothesis. Unfortunately, however, many researchers fail to recognize that they are reporting a PTE, which has <u>different</u> sampling properties than either of its component estimators.

The use of pre-test estimators is common in a range of statistical applications, as is evidenced by the extensive bibliographies of Bancroft and Han (1977) and Han *et al.* (1988). The finite sample properties of many pre-test estimators (typically their biases and risks under quadratic loss) have been examined in the literature. In each case, the sampling properties of the PTE depend on, among other factors, the size (and hence critical value) selected for the preliminary test. This suggests, if a researcher is going to pre-test, that he should use a critical value that is "optimal" in some sense, rather than use arbitrary test sizes of say 1% or 5%.

One such procedure is to use a critical value according to the mini-max regret criterion used by, for example, Gun (1967), Sawa and Hiromatsu (1973), Ohtani and Toyoda (1978), and Giles *et al.* (1991). In particular, assuming a correctly specified model and normally distributed disturbances, Brook (1976) examines the choice of optimal critical value

(OCV) according to the mini-max regret criterion when the pre-test is of the validity of exact linear restrictions on the model's coefficients. He finds that the OCV is approximately equal to two irrespective of the number of restrictions under test or the degrees of freedom of the model. This result has obvious practical appeal.

This result, of course, does not imply a constant test size, as this will vary across different degrees of freedom and numbers of restrictions. Further, the OCV of two rarely results in a test size of 1% or 5% - the optimal test size can be as high as 30%, and for less than five restrictions it is always more than 10%.

Brook's result depends on the often unrealistic assumption that the disturbances are normally distributed. Frequently we use data series which exhibit more kurtosis than implied under normality. This paper addresses this issue by extending Brook's analysis to the case of Multivariate Student-t disturbances.

## 2. MODEL FRAMEWORK AND RISK FUNCTIONS

We consider the classical linear regression model $y = X\beta + e$, where $y$ is a $(T\times 1)$ vector of observations on the dependent variable, $X$ is a $(T\times k)$ full rank matrix of non-stochastic regressors $(k<T)$, and $e$ is a $(T\times 1)$ vector of disturbance terms. We test $m$ independent linear restrictions expressed by the hypotheses, $H_0 : R\beta = r$ against $H_A : R\beta \neq r$, where $R$ is $(m\times k)$, of rank $m$; $r$ is $(m\times 1)$; and both $R$ and $r$ are non-stochastic. This testing structure includes, for example, testing the individual significance of one or more regressors and testing the joint significance of the regressors. We assume $e$ has a multivariate Student-t (Mt) distribution with degrees of freedom $\nu$ and scale parameter $\sigma^2 = 1$, for simplicity (and without any loss of generality). Then $f(e) =$

$\left[\nu^{\nu/2}\Gamma\left((\nu+T)/2\right)\right]\left[\pi^{\nu/2}\ \Gamma(\nu/2)\right]^{-1}\left[\nu+e'e\right]^{-(\nu+T)/2}$. The mean and covariance matrix of e, for $\nu > 2$, are $E(e) = 0$ and $E(ee') = \nu/(\nu-2)I_T$. Normal errors correspond to $\nu = \infty$. For $\nu < \infty$ the marginal distributions have more kurtosis than when $\nu = \infty$.

The validity of the restrictions is tested using the usual F-statistic, $f = (Rb-r)'[RS^{-1}R']^{-1}(Rb-r)/ms^2$, where $S = X'X$; $b = S^{-1}X'y$ is the OLSE of $\beta$; $s^2 = (y-Xb)'(y-Xb)/v$; and $v = (T-k)$. This is the uniformly most powerful invariant size - $\alpha$ test of $H_0$ when e is distributed according to any member of the elliptically symmetric family (see King (1979)), of which the Mt distribution is a member. It is straightforward to show that $f \sim F_{(m,v)}$ under $H_0$ and that its distribution under $H_A$ for Mt disturbances is

$$f_{Mt}(f) = \sum_{r=0}^{\infty} \frac{(2\lambda/\nu)^r\Gamma(\frac{\nu}{2}+r)m^{\frac{m}{2}+r}\ v^{\frac{v}{2}}f^{\frac{m}{2}+r-1}}{r!(1+2\lambda/\nu)^{\frac{\nu}{2}+r}\ B\left(\frac{m}{2}+r;\frac{v}{2}\right)\Gamma\left(\frac{\nu}{2}\right)\left(v+mf\right)^{\frac{m+v}{2}+r}}$$

(see Ullah and Phillips (1986), Sutradhar (1988), or Giles (1991)). $\lambda = (R\beta-r)'[RS^{-1}R'](R\beta-r)/2$ is the usual non-centrality parameter under normal errors and is a measure of the hypothesis error ($\lambda = 0$ when $R\beta = r$), and $B(.;.)$ is the Beta function. When $e \sim N(0,I)$, $f \sim F'_{(m,v;\lambda)}$.

In practice, the researcher tests the validity of the restrictions <u>prior</u> to deciding whether to use the OLSE, b, or the RLSE, $b^* = b + S^{-1}R'\left[RS^{-1}R'\right]^{-1}(r-Rb)$. Consequently, the estimator we use is conditional on the preliminary test of $H_0$ and we are actually reporting the PTE:

$$\hat{b} = \begin{cases} b & \text{if } f > c(\alpha) \\ b^* & \text{if } f \le c(\alpha) \end{cases}$$

where $c(\alpha)$ is the critical value of the test associated with an $\alpha\%$

significance level. Thus, a researcher has three options for estimation in this model framework: a) ignore the *a priori* restrictions and apply OLS; b) impose the *a priori* restrictions and apply RLS; or c) apply OLS or RLS depending on the outcome of the pre-test.

We follow the usual approach in the associated literature and compare the sampling properties of $\hat{b}$, b, and b* on the basis of predictive risk under squared error loss. This enables the analysis to be explicitly independent of the regressor data. For any estimator, $\tilde{b}$, of $\beta$, we define this risk as $\rho(X\tilde{b},E(y)) = E\left[(X\tilde{b}-E(y))'(X\tilde{b}-E(y))\right]$, which is equal to the risk of $\tilde{b}$ itself if the regressors are orthonormal.

Giles (1991) derives the risk of Xb, Xb*, and $X\hat{b}$ for Mt disturbances:

$$\rho_{Mt}(Xb,E(y)) = k\nu/(\nu-2) \qquad (2.1)$$

$$\rho_{Mt}(Xb^*,E(y)) = \left((k-m)\nu+2\lambda(\nu-2)\right)/(\nu-2) \qquad (2.2)$$

$$\rho_{Mt}(X\hat{b},E(y)) = \left(k\nu-m\nu P_{21}+2\lambda(\nu-2)\left(2P_{22}-P_{42}\right)\right)/(\nu-2) \qquad (2.3)$$

where $$P_{ij} = \sum_{r=0}^{\infty} \frac{\left(2\lambda/\nu\right)^r \Gamma\left(\frac{\nu}{2}+r+j-2\right)}{r!\left(1+2\lambda/\nu\right)^{\frac{\nu}{2}+r+j-2} \Gamma\left(\frac{\nu}{2}+j-2\right)} I_x\left(\frac{1}{2}(m+i)+r;\frac{\nu}{2}\right)$$

for $i,j = 0,1,2, \ldots$ . $I_x(.,.)$ is Pearson's incomplete beta function with $x = cm/(v+cm)$.

When $\nu = \infty$ the disturbances are normally distributed, and (2.1) - (2.3) collapse to the expressions given by, for instance, Brook (1976) and Judge and Bock (1978). Figure 1 illustrates the risk functions when $\nu = 5$, $m = 3$, $v = 24$ and $k = 5$. This figure is qualitatively similar to that which results under normal errors, though there are some quantitative differences (see Giles (1991)).

One of the key results under normal disturbances is that it is <u>never</u> preferable to pre-test; that is, the risk of the pre-test estimator never dominates <u>both</u> of its component estimators. Consequently, if $\lambda$ were known, the optimal strategy in terms of minimizing risk under quadratic loss, is to use the RLSE for $\lambda \in \left[0, \frac{m}{2}\right]$ and the OLSE for $\lambda > \frac{m}{2}$. The risk so traced is termed the "minimum risk boundary".

As Figure 1 illustrates, there are many cases with Mt disturbances, for which these results qualitatively carry over. However this need not occur. When $\nu < \infty$ there are situations for which the pre-test estimator can dominate <u>both</u> of its component estimators. This is possible as first, $\rho_{Mt}(Xb,E(y)) = \rho_{Mt}(Xb^*,E(y))$ when $\lambda = m\nu/(2(\nu-2))$, secondly $\rho_{Mt}(Xb,E(y)) = \rho_{Mt}(X\hat{b},E(y))$ for $\lambda_1$ such that $\lambda_1 \in \left(\frac{m\nu}{4(\nu-2)}, \frac{2\nu P_{21}}{2(\nu-2)P_{22}}\right)$, and as $P_{21}$ can be greater than $P_{22}$, $2\nu P_{21}/(2(\nu-2)P_{22})$ can be greater than $2\nu/(2(\nu-2))$. This implies that for those cases for which $P_{21} > P_{22}$ there exists a region over which the pre-test estimator can dominate both of its component estimators.[1]

Figure 2 illustrates such an example. Our numerical evaluations of the risk functions suggest that the existence and magnitude of the dominating region for the pre-test estimator depends on the values of m and $\nu$ (but not v). In particular, the value of m for which the dominating region occurs, decreases as $\nu$ decreases. For example, when $\nu = 50$ we find that the pre-test estimator can dominate both of its component estimators for $m > 10$. However, when $\nu = 5$ it can result for $m \geq 4$. Further, (a) the dominating $\lambda$-range, say $[\lambda^*.\lambda^+]$, increases as m increases; (b) the minimum risk boundary is no longer given by $\min_{\lambda}\left\{\rho(Xb,(E(y)),\rho(Xb^*,E(y))\right\}$, though the potential risk gain of the pre-test estimator over this boundary is relatively small; (c) there is no strictly dominating pre-test estimator over $[\lambda^*, \lambda^+]$. That is, the risk functions of the pre-test estimator for $c \in (0,\infty)$ cross within $[\lambda^*, \lambda^+]$.

## 3. MINI-MAX REGRET CRITERION

Unfortunately, $\lambda$ is typically unobservable and the restricted estimator can have an infinite risk. This suggests that the optimal strategy is to pre-test and use a critical value which draws the risk of the pre-test estimator "as close as possible" to the minimum risk boundary. In this paper we use Brook's (1976) mini-max regret criterion as our notion of "as close as possible". This is similar to the criteria used by Gun (1964) and Sawa and Hiromatsu (1973) and is equivalent to that used by Giles *et al.* (1991). We define the regret associated with using the pre-test estimator as

$$R(X\hat{b}) = \begin{cases} \rho(X\hat{b}, E(y)) - \rho(Xb^*, E(y)); & \lambda \le m\nu/(2(\nu-2)) \\ \rho(X\hat{b}, E(y)) - \rho(Xb, E(y)); & \lambda > m\nu/(2(\nu-2)) \end{cases} .$$

We define $\lambda_L$ as the value of $\lambda \le m\nu/(2(\nu-2))$ such that $R(X\hat{b})$ is maximized and $d_L$ as that value of $R(X\hat{b})$. We define $\lambda_U$ as the value of $\lambda > m\nu/(2(\nu-2))$ such that $R(X\hat{b})$ is maximized and $d_U$ as that value of $R(X\hat{b})$.

It is well known that an increase in $c(\alpha)$ decreases $d_L$ but increases $d_U$ while the opposite results from a decrease in $c(\alpha)$. The mini-max regret procedure exploits this result and finds the critical value $c(\alpha) = c^*(\alpha)$ such that $d_U = d_L$ and both are simultaneously minimized. That is, the OCV must satisfy:

$$\min_{c} \left\{ \max_{\lambda \le \frac{m\nu}{2(\nu-2)}} R(X\hat{b}) \right\} = \min_{c} \left\{ \max_{\lambda > \frac{m\nu}{2(\nu-2)}} R(X\hat{b}) \right\} .$$

Figure 3 depicts the criterion. Brook (1976) obtains the OCVs according to this procedure for normal disturbances. Giles (1991) shows that Brook's OCV is not invariant to the values of $\nu$. This is clear from both of Figures 1 and 2 - $d_L \neq d_U$ for Brook's OCV and in fact $d_L$ is always greater than $d_U$ for finite $\nu$. Consequently, the OCV for finite $\nu$ will be higher than Brook's OCV as we wish to decrease $d_L$ and increase $d_U$ until they are equal and simultaneously minimized.

The use of this criterion for Mt disturbances has the same justification as that for normal disturbances when there is no region of the dominance of the pre-test estimator. When the pre-test estimator can dominate both Xb and Xb*, the minimum risk boundary is no longer given by $\min_{\lambda}\left\{\left(\rho(Xb,E(y)\right), \rho\left(Xb^*,E(y)\right)\right\}$. Nevertheless, given the aforementioned points (b) and (c), the criterion still achieves (in an overall sense) our desired aim of bringing the pre-test estimators risk function "as close as possible" to the minimum risk boundary.

## 4. OCV'S AND DISCUSSION OF THE RESULTS

OCV's, c*, are reported in Table 1 for several values of m,v and $\nu$. These were calculated using a FORTRAN program written by the authors and executed on a VAX 6340. Subroutines from Press *et al.* (1986) were used to evaluate the gamma and incomplete beta functions. Once the OCV was determined, its corresponding size ($\alpha^*$) was calculated using Davies' (1980) algorithm. Table 1 also gives the normal errors case ($\nu = \infty$) for comparative purposes.

The results suggest first, that the OCVs are not constant over all values of $\nu$. For a given m and v, the OCV is higher the lower is $\nu$. This implies $\alpha^*$ decreases with decreases in $\nu$, *ceteris paribus*. Optimally, the pre-test estimator will select the restricted estimator more often when $\nu < \infty$ than for when $\nu = \infty$. Nevertheless, we still typically maintain that pre-testing at the 1% and 5% levels is not recommended. At best, the arbitrary choice of a 5% significance level is only approximately appropriate for relatively high m (say, $m \geq 4$) and high v (say, $v \geq 60$) with small $\nu$, while the use of a 1% significance level is never appropriate for the range of arguments investigated in Table 1.

Secondly, the results suggest that for a given $\nu$, the OCVs are relatively invariant to m and v, and that Brook's rule of thumb of an OCV of approximately two holds reasonably well for $\nu \geq 20$. Thus, if $\nu$ is known, the results are as easy to apply in practice as is Brook's rule. For example, use $c^* \simeq 2.4$ for $\nu = 5$, $c^* \simeq 2.1$ for $\nu = 10$, and $c^* \simeq 2.0$ for $\nu \geq 20$ regardless of m and v.

## 5. RISK COMPARISONS

We have calculated the OCV's according to the mini-max regret criterion, and we have suggested rules that are straightforward to apply in practice when $\nu$ is known. What if $\nu$ is unknown? We could estimate $\nu$ and assume that our results are still approximately valid. Alternatively, we consider whether there is a critical value that will be approximately optimal according to the mini-max regret criterion for all values of $\nu$. The obvious candidate is Brook's OCV. Accordingly, we have evaluated the risk functions for each of the cases presented in Table 1. Figure 4 illustrates a typical result. Our evaluations suggest that there is relatively little difference between the risk functions using the OCVs we arrived at for finite $\nu$ and those computed by Brook for normal errors. Thus if $\nu$ is unknown, the applied researcher could be (practically) content with using Brook's OCV for all values of $\nu$.

## 6. FINAL REMARKS

Pre-testing is typically practised by applied researchers. It is therefore important to investigate the consequences of this practice and, given the impact of $c(\alpha)$ on the sampling properties of the pre-test estimator, to determine an optimal choice of critial value. We have focussed our attention on this issue using the mini-max regret criterion when the disturbances are Mt.

Our results show that Brook's practically appealing rule of thumb, of a critical value of approximately two in value, is not invariant to the value of $\nu$. Nevertheless, we offer equally practical prescriptions when $\nu$ is known. Further, if $\nu$ is unknown, our results suggest that a researcher could be (practically) content to continue to use Brook's OCV.

## FOOTNOTES

1. Giles' (1991) bounds for the Mt case are incorrect. Her bounds imply that the pre-test estimator can never dominate both of its component estimators. This was also supported by her limited numerical evaluations.

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**Table 1 Optimal Critical Values**

| | | $\nu = 5$ | | $\nu = 10$ | | $\nu = 15$ | | $\nu = 20$ | | $\nu = 50$ | | $\nu = 100$ | | $\nu = \infty$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| m | v | c* | α* | c* | α* | c* | α* | c* | α* | c* | α* | c* | α* | c* | α* |
| 1 | 2 | 2.535 | 0.252 | 2.187 | 0.277 | 2.104 | 0.284 | 2.068 | 0.287 | 2.008 | 0.292 | 1.990 | 0.294 | 1.990 | 0.294 |
| | 4 | 2.445 | 0.193 | 2.122 | 0.219 | 2.046 | 0.226 | 2.012 | 0.229 | 1.956 | 0.235 | 1.939 | 0.236 | 1.939 | 0.236 |
| | 8 | 2.394 | 0.160 | 2.089 | 0.186 | 2.016 | 0.193 | 1.984 | 0.197 | 1.931 | 0.202 | 1.914 | 0.204 | 1.910 | 0.204 |
| | 16 | 2.366 | 0.144 | 2.071 | 0.169 | 2.000 | 0.177 | 1.970 | 0.180 | 1.918 | 0.185 | 1.902 | 0.187 | 1.900 | 0.187 |
| | 24 | 2.357 | 0.138 | 2.065 | 0.164 | 1.995 | 0.171 | 1.965 | 0.174 | 1.914 | 0.179 | 1.898 | 0.181 | 1.890 | 0.182 |
| | 60 | 2.345 | 0.131 | 2.058 | 0.157 | 1.990 | 0.164 | 1.959 | 0.167 | 1.908 | 0.172 | 1.893 | 0.174 | 1.890 | 0.174 |
| | 120 | 2.341 | 0.129 | 2.055 | 0.154 | 1.988 | 0.161 | 1.957 | 0.164 | 1.907 | 0.170 | 1.891 | 0.172 | 1.890 | 0.172 |
| 2 | 2 | 2.675 | 0.272 | 2.317 | 0.302 | 2.233 | 0.309 | 2.195 | 0.313 | 2.134 | 0.319 | 2.115 | 0.321 | 2.090 | 0.324 |
| | 4 | 2.541 | 0.194 | 2.210 | 0.226 | 2.132 | 0.234 | 2.098 | 0.238 | 2.041 | 0.245 | 2.023 | 0.247 | 2.000 | 0.250 |
| | 8 | 2.454 | 0.148 | 2.143 | 0.180 | 2.070 | 0.189 | 2.038 | 0.193 | 1.985 | 0.200 | 1.968 | 0.202 | 1.960 | 0.203 |
| | 16 | 2.412 | 0.122 | 2.109 | 0.154 | 2.038 | 0.163 | 2.006 | 0.167 | 1.954 | 0.174 | 1.938 | 0.176 | 1.930 | 0.177 |
| | 24 | 2.399 | 0.112 | 2.098 | 0.145 | 2.028 | 0.154 | 1.996 | 0.158 | 1.945 | 0.165 | 1.929 | 0.167 | 1.920 | 0.168 |
| | 60 | 2.383 | 0.101 | 2.087 | 0.133 | 2.017 | 0.142 | 1.986 | 0.146 | 1.935 | 0.153 | 1.919 | 0.156 | 1.910 | 0.157 |
| | 120 | 2.378 | 0.097 | 2.083 | 0.129 | 2.014 | 0.138 | 1.983 | 0.142 | 1.932 | 0.149 | 1.916 | 0.152 | 1.900 | 0.154 |
| 3 | 4 | 2.582 | 0.191 | 2.246 | 0.225 | 2.168 | 0.235 | 2.133 | 0.239 | 2.075 | 0.246 | 2.057 | 0.249 | 2.040 | 0.251 |
| | 8 | 2.490 | 0.135 | 2.169 | 0.170 | 2.093 | 0.180 | 2.059 | 0.184 | 2.004 | 0.192 | 1.987 | 0.195 | 1.970 | 0.197 |
| | 16 | 2.444 | 0.102 | 2.135 | 0.136 | 2.062 | 0.146 | 2.029 | 0.150 | 1.974 | 0.159 | 1.958 | 0.161 | 1.942 | 0.164 |
| | 24 | 2.429 | 0.090 | 2.125 | 0.124 | 2.052 | 0.133 | 2.020 | 0.138 | 1.966 | 0.146 | 1.950 | 0.149 | 1.934 | 0.151 |
| | 60 | 2.411 | 0.076 | 2.114 | 0.108 | 2.043 | 0.117 | 2.011 | 0.122 | 1.958 | 0.130 | 1.942 | 0.133 | 1.926 | 0.135 |
| | 120 | 2.405 | 0.071 | 2.110 | 0.103 | 2.040 | 0.112 | 2.008 | 0.116 | 1.956 | 0.124 | 1.940 | 0.127 | 1.924 | 0.129 |
| 4 | 4 | 2.607 | 0.188 | 2.267 | 0.224 | 2.188 | 0.234 | 2.152 | 0.238 | 2.094 | 0.246 | 2.076 | 0.248 | 2.060 | 0.251 |
| | 8 | 2.512 | 0.125 | 2.188 | 0.161 | 2.112 | 0.171 | 2.077 | 0.176 | 2.020 | 0.184 | 2.003 | 0.187 | 1.990 | 0.189 |
| | 16 | 2.463 | 0.087 | 2.153 | 0.121 | 2.079 | 0.131 | 2.046 | 0.136 | 1.991 | 0.145 | 1.974 | 0.147 | 1.960 | 0.149 |
| | 24 | 2.446 | 0.074 | 2.143 | 0.107 | 2.070 | 0.116 | 2.037 | 0.121 | 1.983 | 0.129 | 1.966 | 0.132 | 1.950 | 0.135 |
| | 60 | 2.426 | 0.058 | 2.131 | 0.088 | 2.060 | 0.097 | 2.028 | 0.102 | 1.975 | 0.110 | 1.959 | 0.112 | 1.940 | 0.115 |
| | 120 | 2.418 | 0.052 | 2.127 | 0.082 | 2.057 | 0.091 | 2.026 | 0.095 | 1.973 | 0.103 | 1.957 | 0.106 | 1.940 | 0.108 |
| 5 | 8 | 2.528 | 0.117 | 2.202 | 0.154 | 2.125 | 0.164 | 2.090 | 0.169 | 2.033 | 0.178 | 2.015 | 0.181 | 1.998 | 0.183 |
| | 16 | 2.476 | 0.076 | 2.166 | 0.110 | 2.092 | 0.120 | 2.058 | 0.125 | 2.003 | 0.133 | 1.985 | 0.136 | 1.969 | 0.139 |
| | 24 | 2.457 | 0.062 | 2.154 | 0.093 | 2.082 | 0.103 | 2.049 | 0.108 | 1.994 | 0.116 | 1.977 | 0.119 | 1.961 | 0.121 |
| | 60 | 2.434 | 0.045 | 2.142 | 0.073 | 2.071 | 0.082 | 2.040 | 0.086 | 1.986 | 0.094 | 1.970 | 0.096 | 1.954 | 0.099 |
| | 120 | 2.426 | 0.039 | 2.138 | 0.066 | 2.068 | 0.074 | 2.037 | 0.078 | 1.984 | 0.086 | 1.968 | 0.088 | 1.952 | 0.091 |
| 8 | 8 | 2.555 | 0.103 | 2.227 | 0.139 | 2.149 | 0.150 | 2.114 | 0.155 | 2.055 | 0.164 | 2.037 | 0.167 | 2.020 | 0.170 |
| | 16 | 2.497 | 0.057 | 2.187 | 0.087 | 2.113 | 0.097 | 2.079 | 0.101 | 2.023 | 0.110 | 2.005 | 0.112 | 1.990 | 0.115 |
| | 24 | 2.474 | 0.041 | 2.174 | 0.068 | 2.101 | 0.076 | 2.069 | 0.081 | 2.014 | 0.088 | 1.997 | 0.091 | 1.980 | 0.094 |
| | 60 | 2.445 | 0.023 | 2.158 | 0.044 | 2.089 | 0.051 | 2.058 | 0.055 | 2.005 | 0.061 | 1.988 | 0.063 | 1.970 | 0.066 |
| | 120 | 2.434 | 0.018 | 2.152 | 0.036 | 2.085 | 0.042 | 2.054 | 0.046 | 2.003 | 0.052 | 1.986 | 0.054 | 1.970 | 0.056 |

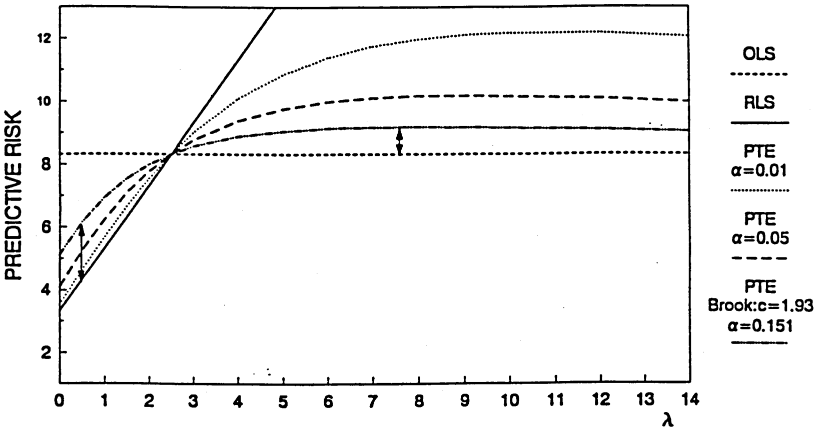


Figure 1. Predictive Risk Functions - $\nu$=5, m=3, v=24, k=5.

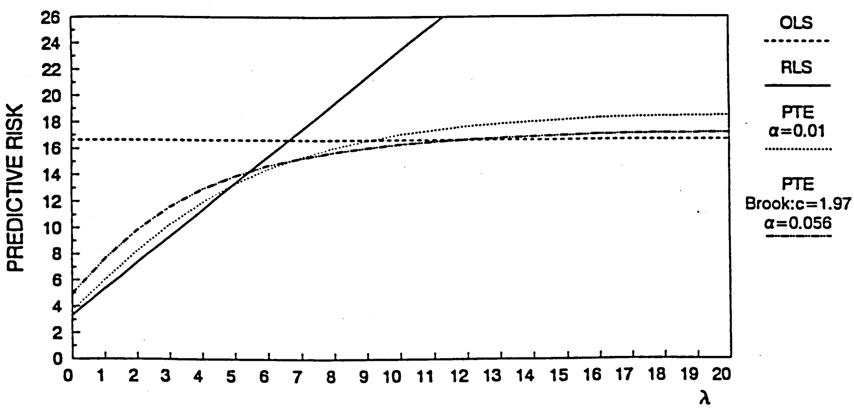


Figure 2. Predictive Risk Functions - $\nu$=5, m=8, v=120, k=10.

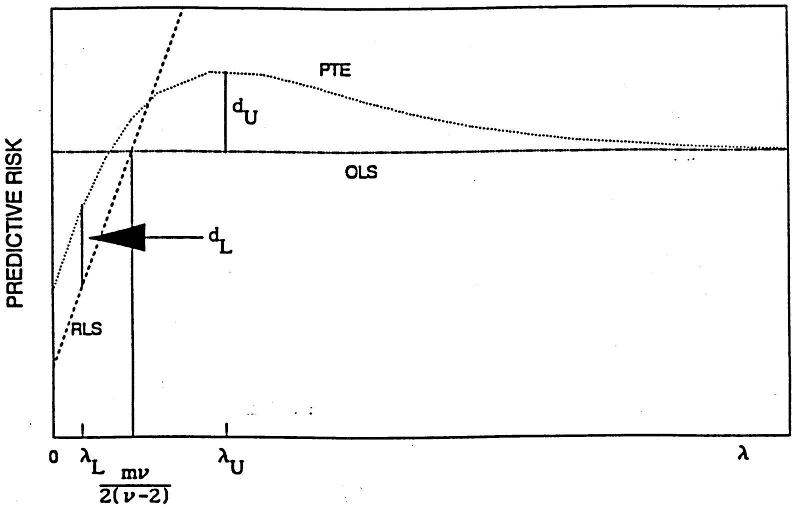


Figure 3. Mini-max Regret Criterion

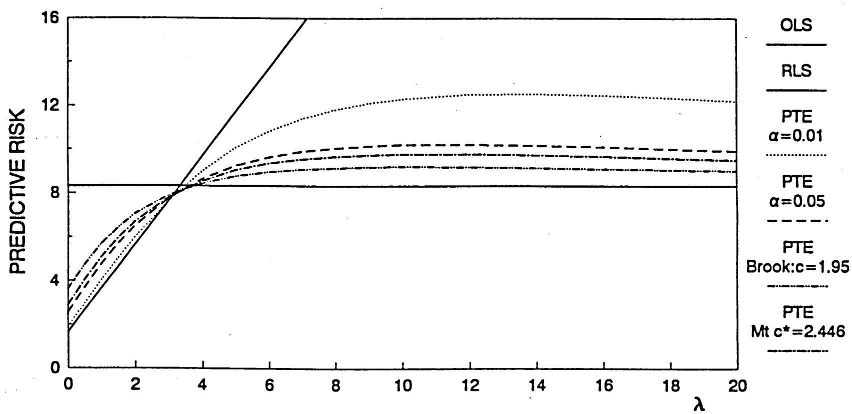


Figure 4. Predictive Risk Functions - $\nu$=5, m=4, v=24, k=5.

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