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Abstract

We consider the true size of the Chow Test for the structural stability of a regression model when the disturbances are autocorrelated. We show that there may be considerable size distortion in the case of either AR(1) or MA(1) errors.

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1. Introduction

This paper presents preliminary results which show that the Chow (1960) test size is not robust to AR(1) or MA(1) autocorrelation in the regression errors. We provide exact evidence which supports other results in the case of AR(1) errors, and offers new insights in the MA(1) case. The Chow test considers shifts in the relationship generating the data, so it is widely applied with time series data. Ironically, more attention has been paid to the consequences of having heteroscedastic errors than of having autocorrelated errors. For example, see MacKinnon (1989) and the references therein.

The (exact) effects of AR(1) errors on the size of the Chow test have been considered in a limited way by Consigliere (1981) and Kramer (1989). Corsi *et al.* (1982) provide some Monte Carlo evidence and Kiviet's (1980) approach can be used to construct bounds on the test's critical value under ARMA errors. However, there are no exact results based on realistic data sets in the AR(1) case, or on the robustness of the Chow test to MA(1) errors.

2. Notation and Theory

Consider two sub-samples with n_1 and n_2 $(=n-n_1)$ observations, and the models

$$y_i = X_i\beta_i + u_i \ ; \qquad i = 1,2 \qquad (1)$$

where y_i and u_i are $(n_i\times 1)$, β_i is a $(k\times 1)$, and X_i is $(n_i\times k)$, of rank k, and non-stochastic. Equations (1) may be written as

$$y \equiv \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \equiv X^*\beta^* + u. \qquad (2)$$

Under the null hypothesis, H_0: $\beta_1 = \beta_2 = \beta$, we have

$$y = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \beta \equiv X\beta + u. \qquad (3)$$

The Chow test rejects H_0 if $f > c(\alpha)$, where $f = [(e'e - e^{*\prime}e^*)/k] / [e^{*\prime}e^*/(n-2k)]$, e^* and e are the least squares residuals vectors for (2) and (3), and $c(\alpha)$ is the critical value for a (nominal) test size of α. If H_0 is true and $u \sim N(0,\sigma^2 I_n)$, f is F-distributed with k and (n-2k) degrees of freedom, and $c(\alpha)$ is chosen accordingly.

It is well known that if the covarince matrix of u is non-scalar, then f is *not* F-distributed under H_0. However, the true size of the Chow test can be calculated by noting that[1]

$$\Pr.\left[f > c(\alpha)\right] = 1 - \Pr.\left[u'(dB - M^*c(\alpha))u \leq 0\right] \qquad (4)$$

where $d = (n-2k)/k$, $B = (M-M^*)$, $M = I - X(X'X)^{-1}X'$, and $M^* = I - X^*(X^{*\prime}X^*)^{-1}X^{*\prime}$. If $u \sim N(0,\Omega)$ then[2]

$$\Pr.\left[f > c(\alpha) \,|\, \Omega\right] = 1 - \Pr.\left[\sum_{j=1}^{n} \lambda_j z_j^2 \leq 0\right] \qquad (5)$$

where the λ_j's are the eigenvalues of $\Omega^{1/2}\left(dB - M^*c(\alpha)\right)\Omega^{1/2}$ and the z_j^2 are independent Chi-square variables, each with one degree of freedom. The *true* size of the Chow test, in (5), depends on the regressors and the form[3] of Ω when $\Omega \neq \sigma^2 I$. It is readily calculated using Davies' (1980) algorithm.

3. The Study

If the disturbances are AR(1) then $u_t = \rho u_{t-1} + \varepsilon_t$, and if they are MA(1) then $u_t = \varepsilon_t + \rho\varepsilon_{t-1}$, where $|\rho| < 1$ and $\varepsilon \sim IN(0,\sigma_\varepsilon^2)$. Values of n = 20, 60 and various sample splits were considered with the following regressor matrices:[4] X1 comprises the annual "spirits" income and price data of Durbin and Watson (1951); X2 comprises the quarterly Australian Consumers Price Index, and its lag; X3, X4 and X5 each include a linear time trend and (respectively) a Normal (30,4), log-Normal (2.23, 19.58) and Uniform (0,10) regressor; X6 comprises the orthogonal regressors $(a_2 + a_n)/\sqrt{2}$ and $(a_3+a_{n-1})/\sqrt{2}$, where the a_i's are the eigenvectors of the usual "differencing" matrix[5], A. Test sizes of 1%, 5% and 10% were considered.

All computations were undertaken with Davies' routine in the SHAZAM package (White *et al.* (1990)) on a VAX 6340. Our code was verified by replicating Consigliere's (1981, p.130) results for her linear trend model.

4. Results

Our results with AR(1) errors accord with those in earlier studies - the size of the Chow test is distorted when the errors are autocorrelated. Generally, we find that the true size of the test is greater (less) than its nominal size for $\rho > 0$ (< 0), as shown in Figures 1 and 2. As most of our regressors are positively autocorrelated, this is consistent with the analytical results of Corsi *et al.* (1982) for a single regressor model. (An exception[6] is with X6 - in that case the true size exceeds the nominal size regardless of the sign of ρ.) The size distortion can be substantial, especially if $\rho > 0$.

With a nominal test size of 5% for $\rho = 0.9$ the true size ranges from 30% - 60% (50% - 80%) for n=20 (60) with our data. The increase in size distortion as n increases reflects the consistency of the test, even in the presence of model misspecification. Generally, the greater the imbalance in the sample split, the less the size distortion, especially for large n. Similar results are reported by Consigliere (1981) and Corsi *et al.* (1982) in simpler models.

There are no previous results for MA(1) errors. We find that the Chow test is more robust in this case - the degree of size distortion is 30% - 50% as much as with AR(1) errors and the same data. This reflects the non-zero elements off the three leading diagonals of Ω in the AR(1) case. Generally, the patterns noted above apply in the MA(1) case, as in Figures 3 and 4. The only exception is that with MA(1) errors there is far less difference between the size distortions with n=20 and n=60, compared with AR(1) errors. The Chow test is more robust to MA(1) autocorrelation than AR(1) autocorrelation for larger sample sizes.

5. Conclusions

The limited previous evidence on the effects of autocorrelated errors on the Chow test for structural change relates to simple models, artificial data, and AR(1) errors. We show that the size of this test is distorted by such autocorrelation for a range of realistic data sets. A similar distortion arises if the errors follow an MA(1) process, through there is (relatively) greater robustness in this case if the sample size is reasonably large.

In practice, positive autocorrelation of both the errors and the regressors is likely. Then the Chow test is biased towards rejecting the null, and the researcher will "detect" structural change when none has occurred. This will be a severe problem with realistic sample sizes if the errors follow a positive AR(1) process.

The power of the test will be similarly distorted. Work in progress considers this, and the effects of "seasonal" autocorrelation in more detail.

FIGURE 1

CHOW TEST SIZE WITH AR(1) ERRORS

X2 DATA , n=60, nominal size=5%

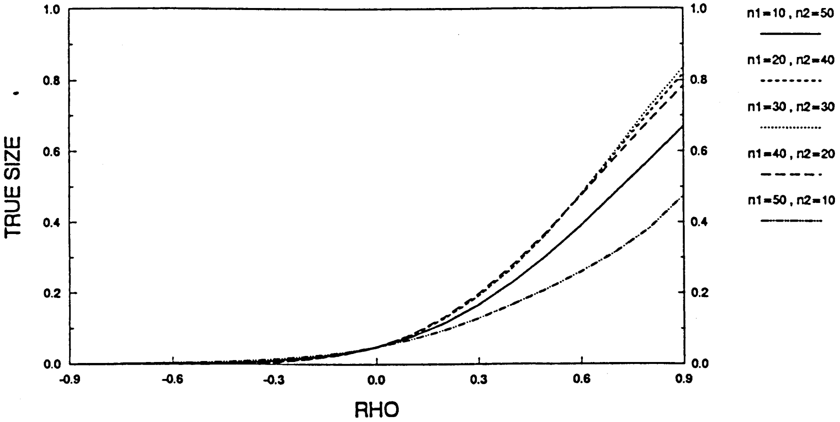


FIGURE 2

CHOW TEST SIZE WITH AR(1) ERRORS

X2 DATA , n=20, nominal size=5%

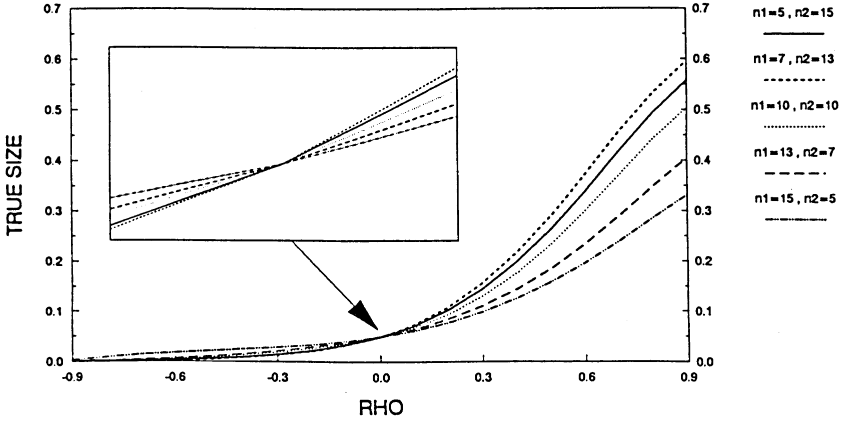


FIGURE 3

CHOW TEST SIZE WITH MA(1) ERRORS

X3 DATA , n=60, nominal size=5%

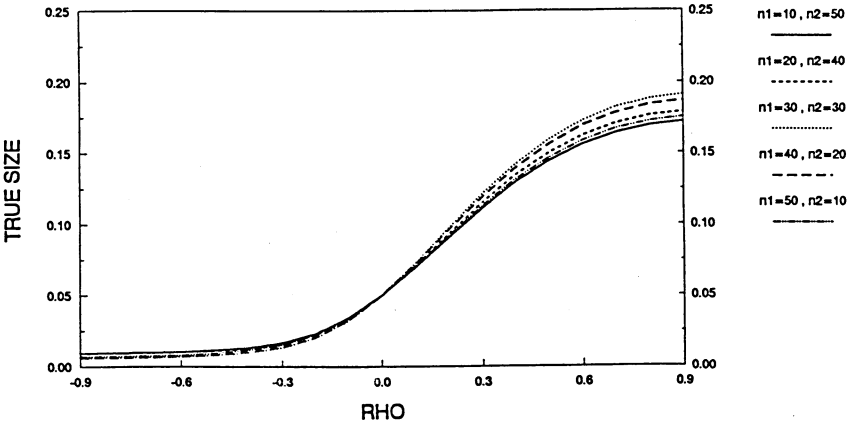
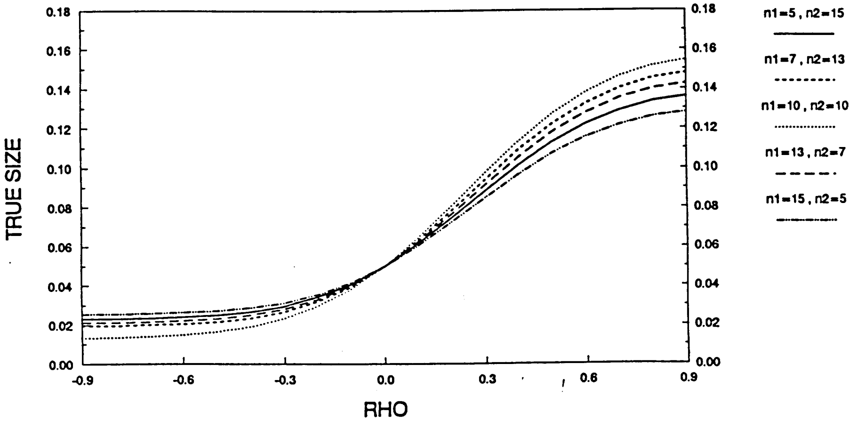


FIGURE 4

CHOW TEST SIZE WITH MA(1) ERRORS

X3 DATA , n=20, nominal size=5%



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FOONOTES

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1. This probability is being calculated under the null, rather than the alternative, hypothesis, as the same error vector is assumed in (2) and (3).
2. The transformations needed to establish (5) are well known. See Koerts and Abrahamse (1971) for details, and Consigliere (1981) for their application in this context.
3. The size of the test is independent of the scale of the errors. If the error covariance matrix is scalar then the size is also independent of the regressors.
4. In each case, an intercept was included, so k=3. Similar data sets have been used in related studies by various authors.
5. A is an (n x n) tri-diagonal matrix with unit (1,1) and (n,n) elements, 2 elsewhere on the leading diagonal, and -1 as the leading off-diagonal elements. The eigenvectors of A correspond to the eignevectors ordered in terms of increasing size. The first such vector has constant elements.
6. X6 is Watson's (1955) matrix for k=3, and it produces the least efficient least squares parameter estimates in the class of orthogonal regressor matrices. X6 is known to include extreme results in the distribution of the Durbin-Watson statistic, and in other testing situations.

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