



*The World's Largest Open Access Agricultural & Applied Economics Digital Library*

**This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.**

**Help ensure our sustainability.**

Give to AgEcon Search

AgEcon Search

<http://ageconsearch.umn.edu>

[aesearch@umn.edu](mailto:aesearch@umn.edu)

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

*No endorsement of AgEcon Search or its fundraising activities by the author(s) of the following work or their employer(s) is intended or implied.*

CANTER

9111V

Department of Economics  
UNIVERSITY OF CANTERBURY

CHRISTCHURCH, NEW ZEALAND



GIANNINI FOUNDATION OF  
AGRICULTURAL ECONOMICS  
LIBRARY

NOV 1 1991

WITHDRAWN

**THE EXACT POWERS OF SOME AUTOCORRELATION  
TESTS WHEN THE DISTURBANCES  
ARE HETEROSCEDASTIC**

**John P. Small**

***Discussion Paper***

**No. 9111**

This paper is circulated for discussion and comments. It should not be quoted without the prior approval of the author. It reflects the views of the author who is responsible for the facts and accuracy of the data presented. Responsibility for the application of material to specific cases, however, lies with any user of the paper and no responsibility in such cases will be attributed to the author or to the University of Canterbury.

Department of Economics, University of Canterbury  
Christchurch, New Zealand

***Discussion Paper No. 9111***

September 1991

**THE EXACT POWERS OF SOME AUTOCORRELATION  
TESTS WHEN THE DISTURBANCES  
ARE HETEROSCEDASTIC**

John P. Small



THE EXACT POWERS OF SOME AUTOCORRELATION  
TESTS WHEN THE DISTURBANCES ARE HETEROSCEDASTIC

John P. Small

Department of Economics, University of Canterbury, Private Bag, Christchurch,  
New Zealand.

September 1991

Abstract

This paper considers the exact finite sample powers of five popular tests for AR(1) disturbances when one of several types of heteroscedasticity is also present. Severe reductions in power are found, particularly under strong positive autocorrelation. Factors influencing these power reductions are identified analytically and the limiting powers are also considered.

## 1. Introduction

In applied econometric work, testing for serial independence of the regression disturbances is a routine and necessary practice. Such testing can reveal to the researcher signs of specification error or poor explanatory performance, as well as information about the appropriate estimation technique to employ. The general agreement on the importance of detecting autocorrelation has produced a large literature on the subject<sup>1</sup> and has led to a variety of tests.

The most widely used such test for the last 40 years has been the bounds test of Durbin and Watson (1950, 1951) which has two major advantages: it is easy to calculate the test statistic and the tabulated bounds are independent of the data used. The drawbacks of the DW bounds test are the possibility of inconclusive results and sub-optimal power properties under certain conditions. Several easily applied alternatives have been suggested which overcome the inconclusive region difficulties, for example the Sign Change test (Geary (1970)).

More recently, great improvements in computer technology have avoided the problem of the inconclusive bounds test by permitting easy application of the exact DW test and this has refocussed attention on the task of developing tests which have optimal power properties. The major offerings in this field are from Berenblut and Webb (1973) and King (1981, 1985), who suggest tests which are more powerful than the DW test for particular areas of the parameter space. As well as having powerful tests at our disposal we would ideally use tests which maintain their power when their underlying assumptions are violated in some commonly occurring ways. Such problems as departures from normality, omitted regressors or superfluous regressors should not seriously weaken a test for autocorrelation (or any test), as these violations are likely to occur and will typically be unknown to the applied researcher. This paper explores one

such scenario. We use exact techniques to study the power functions of five tests for autocorrelation when they are applied to a model in which the disturbance variance is heteroscedastic in one of three different ways. This study extends work by Epps and Epps (1977) and Giles and Small (1991) (both of which studied the power of the Durbin Watson test when the errors are heteroscedastic) to include the Berenblut and Webb (1973) test, the alternative DW test (King (1981)) and two versions of King's (1985) point optimal test.

The next section outlines the structure and strengths of each test. Some theoretical results arising from these models are presented in Section 3, which is followed by a description of the data used in the numerical evaluations. Section 5 reports the main findings of the study and Section 6 offers some concluding comments.

## 2. The tests

Consider the standard linear regression model

$$y = X\beta + u \quad (1)$$

where  $y$  is a  $(T \times 1)$  vector of observations on the dependent variable,  $X$  is a  $(T \times k)$  full rank non-stochastic regressor matrix,  $\beta$  is a  $(k \times 1)$  parameter vector and  $u$  is a  $(T \times 1)$  vector of disturbances following the AR(1) process:

$$u_t = \rho u_{t-1} + \varepsilon_t \quad |\rho| < 1, t = 1, \dots, T \quad (2)$$

We consider tests of  $H_0 : \rho = 0$  against  $H_a^+ : \rho > 0$  and  $H_a^- : \rho < 0$  individually, each conducted at the 5% nominal size.

The statistics for each of the tests considered can be written as a ratio of quadratic forms in  $u$ , the general form of this ratio being

$$r = \frac{u' Qu}{u' Mu} \quad (3)$$

where  $M = I - X(X'X)^{-1}X'$  and  $Q$  is some other nonstochastic  $(T \times T)$  matrix, the form of which determines the individual test statistic.

(i) The Durbin Watson (DW) test

This has

$$Q = MAM \text{ where } A = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & & \\ 0 & -1 & 2 & & \\ \vdots & & & & \\ 0 & 0 & \dots & 0 & -1 & 1 \end{bmatrix}.$$

The DW test is an approximately Uniformly Most Powerful (UMP) test of  $H_0 : \rho = 0$  against  $H_a^+$  for all design matrices, the approximation being due to a small modification made to the density function of the stationary AR(1) error process by Durbin and Watson (1950). In addition, when the columns of  $X$  are linear combinations of  $k$  of the eigenvectors of  $A$ , the DW test is an approximately Uniformly Most Powerful Invariant (UMPI) test against  $H_a^+$ . Throughout this paper invariance is with respect to an affine transformation of the dependent variable.

(ii) The Berenblut and Webb (BW) test

Berenblut and Webb (1973) proposed two tests, the first of which used  $Q = MBM$ , where

$$B = \begin{bmatrix} 2 & -1 & 0 & \dots & 0 \\ -1 & 2 & & & \\ 0 & & & & \\ \vdots & & & & 0 \\ 0 & \dots & 0 & -1 & 1 \end{bmatrix}.$$

This statistic arose from a consideration of a modified form of the density function of non-stationary first order autoregressive errors. The second test statistic was arrived at by considering the likelihood ratio test of  $H_0 : \rho = 0$  against  $H_a : \rho \neq 0$  (again in the context of a nonstationary AR(1) process), and replacing the inverse of the covariance matrix of the AR(1) process with  $B$ . This gave rise to a statistic using:

$$Q = B - BX(X'BX)^{-1}X'B.$$



This second test will be referred to as the BW test and was shown by Berenblut and Webb to possess the optimal power qualities of both the DW test and the first Berenblut and Webb test. In an empirical evaluation Berenblut and Webb found that the BW test was more powerful than the DW test at high values of  $\rho$  for six design matrices.

(iii) The alternative Durbin Watson (ADW) test

Here

$$Q = MA_0M, \text{ where } A_0 = \begin{bmatrix} 2 & -1 & 0 & \dots & 0 \\ -1 & 2 & & & \vdots \\ 0 & & 2 & & 0 \\ \vdots & & & \ddots & -1 \\ 0 & \dots & 0 & -1 & 2 \end{bmatrix}.$$

King (1981) proposed this test and found it to be a Locally Best Invariant (LBI) test in the neighbourhood of  $\rho = 0$ . In the same paper King discussed results from an empirical comparison of the power functions of the DW and ADW tests, in which the latter generally performed better than the former against negative autocorrelation and for  $\rho < 0.5$ .

(iv) The point optimal ( $S(\rho_1)$ ) tests

This class of tests, due to King (1985), has

$$Q = \Sigma(\rho_1)^{-1} - \Sigma(\rho_1)^{-1}X[X'\Sigma(\rho_1)^{-1}X]^{-1}X'\Sigma(\rho_1)^{-1}$$

where  $\Sigma(\rho_1) = \frac{1}{1-\rho_1^2} V(\rho_1)$  and  $V(\rho_1)$  is  $V$  with  $\rho$  fixed at some chosen value,  $\rho_1$ .

King showed that this test is most powerful invariant (MPI) when the selected value for  $\rho_1$  is the true value of  $\rho$ . He studied the power of two versions of the Point Optimal test,  $\rho_1 = 0.5$  and  $\rho_1 = 0.75$ , and the DW, ADW and BW tests using a wide range of design matrices. King found that small power increases occurred when using either  $S(0.5)$  or  $S(0.75)$ , in preference to DW or ADW, with large samples of smoothly evolving regressors, and more significant differences were apparent in smaller samples. In the case of Watson's (1955) matrix, the

$S(\rho_1)$  and BW tests had power functions which approached unity for  $\rho \rightarrow 1$ , in contrast to the DW and ADW power functions which peaked at  $\rho = 0.75$  and then declined.

### 3. Theoretical discussion

The power function of each test against  $H_a^+ : \rho > 0$ , for a nominal significance level of  $100\alpha\%$  and its associated critical value,  $r^*(\alpha)$ , can be found by substituting values of  $\rho$  into the expression

$$\text{pr}\left\{r < r^*(\alpha) \mid V = V(\rho)\right\}. \quad (4)$$

The analogous expression for testing against  $H_a^- : \rho < 0$  is  $\text{Pr}(r > r^*(\alpha) \mid V = V(\rho))$  but the critical values used are different. The structure of each test statistic which is given by (3), allows the use of the well known manipulations, in the style of Koerts and Abrahamse (1969), for example, to write (4) as:

$$\begin{aligned} & \text{pr}\left\{u'(Q - r^*M)u < 0 \mid V = V(\rho)\right\} \\ &= \text{pr}\left\{\sum_{j=1}^T \lambda_j \chi_j^2 \leq 0\right\} \end{aligned} \quad (5)$$

where the  $\lambda_j$ 's are the eigenvalues of  $(Q - r^*M)V$  and the  $\chi_j^2$ 's are independent central chi square variates, each with one degree of freedom. The following lemma from Evans and King (1985) provides a useful unification.

#### Lemma 1

If  $Q = B - BX(X'BX)^{-1}X'B$   
and  $M = I - X(X'X)^{-1}X'$   
then  $QM = MQ = Q$  so that  $Q = MQM$

The proof follows immediately from the definitions of  $M$  and  $Q$ . This lemma enables us to write the BW and  $S(\rho_1)$  tests as DW-type tests with a particular  $A$

matrix and so to represent the power of all tests considered as depending on the eigenvalues of  $M(A-r*I)MV$  for some non-stochastic  $A$ . The form of (5) allows computation of the power of each test for any covariance matrix using, for example, the FQUAD routine from Koerts and Abrahamse (1969) or Davies' (1980) algorithm. The numerical evaluations described below were conducted using a Fortran version of Davies' algorithm contained in the SHAZAM (White et al. (1990)) computer package.

The first practical question which arises is whether to introduce heteroscedasticity into the  $\varepsilon_t$ 's or the  $u_t$ 's of (2). In practice either one, the other, both or neither of these variables will be heteroscedastic. For the purposes of a controlled study, however, a choice must be made. It is not difficult to show (a proof is available on request) that heteroscedastic  $\varepsilon_t$ 's imply a different degree of heteroscedasticity in the  $u_t$ 's for each value of  $\rho$ . Furthermore, the limiting case of  $\rho = 1$  results in homoscedasticity for the  $u_t$ 's. For these reasons heteroscedasticity was introduced directly into  $u$ . We assume, therefore, that  $\varepsilon_t \sim \text{NID}(0, \sigma_\varepsilon^2)$ ,  $t = 1, \dots, T$ . The covariance matrix of a homoscedastic  $u$  vector defined by (2) is

$$E(u'u) = \left( \frac{\sigma_\varepsilon^2}{1-\rho^2} \right) V \text{ where } V = \begin{bmatrix} 1 & \rho & \rho^2 & \dots & \rho^{T-1} \\ \rho & 1 & \rho & & \vdots \\ \rho^2 & \rho & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \rho \\ \rho^{T-1} & \rho^{T-2} & \dots & \rho & 1 \end{bmatrix}.$$

The following forms of heteroscedasticity are considered

$$\text{var}(u_t) = kZ_t^\alpha \quad (6)$$

$$\text{var}(u_t) = k(1 + \gamma Z_t) \quad (7)$$

$$\text{var}(u_t) \begin{cases} 1 & t \leq T_1 \\ h & t > T_1 \end{cases} \quad t = 1, 2, \dots, T_1, \dots, T \quad (8)$$

where  $Z_t$  is a suitable transformation of the value of the  $j$ th regressor at time  $t$ ,  $X_{jt}$ , and  $\alpha$ ,  $\gamma$  and  $h$  are selected constants. The proportionality constant,  $k = \sigma_e^2 / 1 - \rho^2$ , does not influence the power or size of the tests considered and shall not concern us further.

These three heteroscedastic schemes are intended to represent forms which are likely to occur in practise. Multiplicative heteroscedasticity (6) has been considered by several writers, including Harvey (1976) who concentrated on the estimation of a model with this characteristic. The process is easily generalised to include dependence on more than one regressor but this study is restricted to the simple case outlined above. The additive heteroscedasticity of (7) can represent any situation where the variance of the dependent variable is assumed to be a linear function of some transformation of the regressors. A notable special case is the random coefficient model of Hildreth and Houck (1968). The third model of  $\text{var}(u_t)$  is likely to arise when the regression parameters exhibit a structural break, and will be referred to as Chow heteroscedasticity after the well-known F tests for this phenomenon.

There is no good reason to assume homoscedasticity when a test for AR(1) errors is conducted. Heteroscedasticity of the forms (7) and (8), in particular, are likely to occur in time-series regressions, while testing for spatial autocorrelation when using cross-section data provides another motivation for this study.

We turn now to the question of the appropriate form of the covariance matrix of  $u$  when both heteroscedasticity and autocorrelation are present. Giles and Small (1991) assume that the vector of disturbance variances is substituted into the leading diagonal of  $V$ , giving

$$V^* = \begin{bmatrix} \sigma_1^2 & \rho & \dots & \dots & \rho^{T-1} \\ \rho & \sigma_2^2 & & & \rho^{T-2} \\ \rho^2 & & \ddots & & \vdots \\ \vdots & & & \ddots & \rho \\ \rho^{T-1} & \rho^{T-2} & \dots & \rho & \sigma_T^2 \end{bmatrix}$$

where  $\text{var}(u_t) = k\sigma_t^2$ .

This covariance matrix arises naturally from the Hildreth-Houck random coefficient model with AR(1) errors. It can also be derived from a model in which only the intercept is random and in this context some extra insight is gained. Suppose that

$$y_t = X_t' \beta + \mu_t + u_t$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

where  $u_t \sim N(0, \sigma_\mu^2)$  is the random intercept and  $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$  is independent of  $\mu_t$ . Then  $v_t = \mu_t + u_t$  has covariance matrix given by  $kV^*$  with  $\sigma_t^2 = (1 + \sigma_\mu^2/k)$ ,  $\forall t$ . If we let  $\sigma_t^2 = \lambda$  (which imposes homoscedasticity) then  $V^*$  reflects the following autoregressive process

$$\lambda u_t = \rho^s u_{t-s} + \varepsilon_t \quad t = 1, \dots, T, \quad (9)$$

in which the first autocorrelation is  $\rho/\lambda$ , while all subsequent ones are  $\rho$ . Since  $\lambda > 1$  the first autocorrelation is weaker than all others and this reduces the average power of all tests considered here, irrespective of the data, as the following theorem shows.

#### Theorem 1

When the autoregressive process is given by (9) rather than (2), the average value of the test statistic is increased when testing against  $H_a^+$ .

#### Proof

Let  $S = M(A - r^*I)M$  and consider the first moment of  $(r - r^*)$  which is given by  $E(u' Su) = \text{tr}(SV)$ ,

Consider  $V^* = V + \Lambda$   
 where  $\Lambda = \text{diag}(\lambda^*)$   
 and  $\lambda^* = \lambda - 1 > 0$ .

If  $V^*$  is the true covariance matrix of  $u$  then we must compare  $\text{tr}(SV)$  with  $\text{tr}(SV^*)$ .

$$\begin{aligned}\text{tr}(SV^*) &= \text{tr}(S(V+\Lambda)) \\ &= \text{tr}(SV) + \sum_{i=1}^T q_{ii} \lambda^* \\ &= \text{tr}(SV) + \lambda^* \text{tr}(S)\end{aligned}$$

but  $\text{tr}(S) = E(r-r^*) \Big|_{\rho=0}$

and we know that  $E(r) \Big|_{\rho=0} > r^*$ , when testing against  $H_a^+$ .

So  $\text{tr}(S) > 0$  and  $\text{tr}(SV^*) > \text{tr}(SV)$ . #

So, at least on average, the probability of rejecting the null hypothesis in favour of  $H_a^+$  is therefore reduced as  $\lambda$  increases. The same argument can be used to show that when testing against  $H_a^-$  the average value of the test statistic is reduced, with the same power effect, on average.

This result provides the motivation for choosing transformations used in constructing  $Z_t$  which constrain  $\sigma_t^2$  to a minimum value of unity, while still reflecting the variability of  $X_{jt}$ . For (6),  $Z_t = X_{jt}/\min X_{jt}$  while (7) has  $Z_t = (X_{jt} - \min(X_{jt})) / (\max(X_{jt}) - \min(X_{jt}))$ .

The other obvious method of introducing heteroscedasticity is to apply the back-substitution procedure associated with the AR(1) process to the heteroscedastic  $u_t$ 's. This, in a sense, assumes that the heteroscedasticity "came first", and results in the following covariance matrix, neglecting the scale factor,

$$V^{**} = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \rho^2\sigma_1\sigma_3 & \dots & \rho^{T-1}\sigma_1\sigma_T \\ \rho\sigma_1\sigma_2 & \sigma_2^2 & \rho\sigma_2\sigma_3 & & \rho^{T-2}\sigma_2\sigma_T \\ \rho^2\sigma_1\sigma_3 & \rho\sigma_2\sigma_3 & \sigma_3^2 & & \vdots \\ \vdots & & & \ddots & \vdots \\ \rho^{T-1}\sigma_1\sigma_T & \dots & \dots & \dots & \sigma_T^2 \end{bmatrix}$$

Both  $V^*$  and  $V^{**}$  have been used in this study, which evaluates all powers exactly.

### 3.1 The Limiting Power

The data dependence of the distribution of  $r$  precludes direct analytic evaluation of the powers of the tests under consideration in almost all cases. Some results are obtainable, however, at the boundaries of the parameter space by examining the limits of the eigenvalues of (5) as  $\rho \rightarrow \pm 1$ . This technique was used by Krämer (1985) to prove that, for regressions with no intercept, the limiting power of the DW test as  $\rho \rightarrow 1$  is always zero or unity. A more involved, but similar, analysis enabled Zeisel (1989) to show that when an intercept is present the power of the DW test as  $\rho \rightarrow 1$  lies strictly between these two values. More recently, Small (1991) has generalised both of these results to the ADW, BW and  $S(\rho_1)$  tests.

This section presents two further limiting power results which apply when the disturbances are heteroscedastic in particular ways.

#### *Theorem 2*

When  $\text{var}(u_t) = kZ_t^\alpha$  and  $\text{cov}(u)$  is given by  $V^{**}$ , the limiting power of all tests considered as  $\rho \rightarrow 1$  is zero or unity unless  $\alpha = 2$ , irrespective of the presence of an intercept.

#### *Proof.*

Under the conditions of Theorem 2, the covariance matrix of  $u$  is given by



$$V^{**} = \begin{bmatrix} Z_1^\alpha & (Z_2 Z_1)^{\alpha/2} & \dots & (Z_1 Z_1)^{\alpha/2} \\ (Z_1 Z_2)^{\alpha/2} & Z_2^\alpha & & (Z_1 Z_2)^{\alpha/2} \\ (Z_1 Z_2)^{\alpha/2} & (Z_2 Z_3)^{\alpha/2} & Z_3^\alpha & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ (Z_1 Z_1)^{\alpha/2} & \dots & \dots & Z_1^\alpha \end{bmatrix}$$

$$= \left( \min(X_j) \right)^{\alpha/2} \begin{bmatrix} X_{j1}^{\alpha/2} \\ \vdots \\ \vdots \\ X_{jT}^{\alpha/2} \end{bmatrix} \begin{bmatrix} X_{j1}^{\alpha/2} & \dots & X_{jT}^{\alpha/2} \end{bmatrix}.$$

Notice that  $X_j^{\alpha/2}$  is not in the column space of  $X$  unless  $\alpha = 2$ . This implies that, in general,  $MV^{**} \neq 0$ . Notice also that  $V^{**}$  has rank equal to unity so that  $M(A-r^*I)MV^{**}$  has only one non-zero eigenvalue. The sign of this eigenvalue uniquely determines the power of the test, a positive eigenvalue implying a limiting power of zero. #

The following corollaries extend this result in two directions.

#### Corollary 1

The limiting power, as  $\rho \rightarrow 1$ , of all tests considered when  $\text{var}(u_t)$  is given by (7) or (8) and  $\text{cov}(u) = V^{**}$  is also zero or unity.

#### Proof

When  $\text{var}(u_t)$  is given by (7) then, as  $\rho \rightarrow 1$ ,  $V^{**} \rightarrow \nu\nu'$ ,

where  $\nu = (\sqrt{1+\gamma Z_1} \sqrt{1+\gamma Z_2} \dots \sqrt{1+\gamma Z_3})'$  is not spanned by the column space of  $X$  in general.

When  $\text{var}(u_t)$  is given by (8) and  $\rho \rightarrow 1$ ,  $V^{**} \rightarrow ww'$  where  $w = (1,1,\dots,1,h,h,\dots,h)'$  is also not spanned by the columns of  $X$ .

In both of these cases  $V^{**}$  also has unit rank so the limiting power of all tests are zero or unity. #

### Corollary 2

When  $\text{cov}(u) = V^{**}$  and  $\text{var}(u_t)$  is given by either (6), (7) or (8), the limiting power of all tests considered is always zero or unity as  $\rho \rightarrow -1$ .

#### Proof

If  $V^{**} \rightarrow \nu\nu'$  as  $\rho \rightarrow 1$  with  $\nu = (\nu_1, \nu_2, \dots, \nu_T)'$  then as  $\rho \rightarrow 1$ ,  $V^{**} \rightarrow \nu^-\nu^-'$  with  $\nu^- = (\nu_1, -\nu_2, \nu_3, -\nu_4, \dots, (-1)^{T-1}\nu_T)$ . Now if  $\nu$  is not in the column space of  $X$ , then neither is  $\nu^-$  but  $V^{**}$  has rank approaching unity as  $\rho \rightarrow -1$  under these conditions. The power must therefore approach zero or unity as  $\rho \rightarrow -1$ . #

### Theorem 3

When  $\text{var}(u_t) = k(1+\gamma Z_t)$  and  $\text{cov}(u)$  is given by  $V^*$ , the limiting power of all tests considered as  $\rho \rightarrow 1$  is constant for all  $\gamma > 0$ , providing an intercept is present.

#### Proof

Under the conditions of Theorem 3 we can decompose the covariance matrix of  $u$  as

$$V^* = \Sigma + \gamma\Lambda,$$

where  $\Sigma = ii'$  for  $i = (1, 1, \dots, 1)'$ ,  $\Lambda = \text{diag}(Z_t)$  and  $\gamma$  is a scalar.

Let  $S = M(A - r^*I)M$  and consider the eigenvalues of  $SV^*$ , being the  $\lambda$  which satisfy

$$\lambda w = SV^*w, \quad \text{for some non-null vector } w$$

or, 
$$\lambda w = S(\Sigma + \gamma\Lambda)w.$$

Now when an intercept is present,  $M\Sigma = S\Sigma = 0$  so that

$$\lambda w = \gamma S\Lambda w.$$

Consider some other non-zero scalar,  $\gamma^*$ . We can write

$$\lambda w = \gamma^* \left( \frac{\gamma}{\gamma^*} \right) S\Lambda w,$$

so 
$$\lambda \frac{\gamma}{\gamma^*} w = \gamma^* S\Lambda w,$$

thus altering the value of  $\gamma$  scales each eigenvalue by the same factor, which does not affect the rejection probability. #

#### 4. Data

Several regressor matrices were used in an empirical study in an attempt to reveal the effects of data characteristics on the tests' powers under the mis-specifications outlined above.

The design matrices used were of two sizes,  $60 \times 3$  and  $20 \times 3$ , and are characterised as follows:

- X1: A constant and the price and income series from Durbin and Watson's (1951) consumption of spirits example.
- X2: A constant, the quarterly Australian Consumers Price Index commencing 1959(1), and the same index lagged one period.
- X3: A constant, a linear time trend and observations drawn from the Normal (30,4) distribution.
- X4: A constant, a time trend and observations drawn from the Uniform [0,10] distribution.
- X5: A constant, a time trend and observations drawn from the Lognormal (2.226, 19.58) distribution<sup>2</sup>.
- X6:  $a_1, (a_2 + a_{T-1})/\sqrt{2}, (a_3 + a_{T-2})/\sqrt{2}$  where  $a_1, \dots, a_T$  are the eigenvectors corresponding to the eigenvalues of  $A$  arranged in ascending order. Note that  $a_1$  is a constant as it corresponds to the zero root of  $A$ .

These design matrices were chosen to represent a range of characteristics. The slowly evolving X1 matrix is annual data while X2 has a weak seasonal pattern. Both X1 and X2 have been used in previous studies in the general field of autocorrelation testing (e.g. King (1985) and Evans (1991)). Several previous studies have suggested the use of artificial data of various types. The lognormal data, for example, are often used to represent cross section data which are known to be skewed and therefore particularly relevant to scenarios involving heteroscedasticity. The X6 matrix was shown by Watson (1955) to

produce the most inefficient OLS estimates within the class of orthogonal matrices.

For each X matrix, one regressor was selected to be the  $X_j$  of (6) and (7). The variables used for this purpose were: Income (X1), CPI (X2), Normal (X3), Uniform (X4), Lognormal (X5) and  $(a_2 + a_T)/\sqrt{2}$  (X6). The scalars  $\alpha$  and  $\gamma$  were chosen to give desired values of the ratio.

$$h = \frac{\text{maximum var}(u_t)}{\text{minimum var}(u_t)}.$$

The degree of heteroscedasticity introduced was controlled in this way with  $h$  being set at six values ranging from 1.0 (which implies homoscedasticity) to 2.5. In the other exact work of this type, Epps and Epps (1977) and Giles and Small (1991) used the same measurement criterion for heteroscedasticity, although other measures could be considered, such as the coefficient of variation.

No size corrections were made to the heteroscedastic power functions. The reason for this is that the purpose of this study is to determine the effect of heteroscedastic disturbances on the power of each test when based on least squares residuals. If an applied worker knew that the disturbances were heteroscedastic she would use a GLS type estimator which would account for the heteroscedasticity and simultaneously return the nominal size of the autocorrelation test to its true size. This study presumes that the researcher is ignorant of the complications due to heteroscedasticity.<sup>3</sup>

## 5. Results

It is convenient to discuss the results of the numerical evaluations in two groups, distinguished by the structure of the mis-specified covariance matrix.

### 5.1 Results using $V^*$

This section discusses the results obtained by using the first version of the mis-specified covariance matrix,  $V^*$ , introduced above. All figures and tables referred to are contained in Appendix 1.

The correctly specified ( $h = 1.0$ ) power functions (e.g., Figures 1a and b) are in accord with the findings of previous studies. In the case of X6 and  $T = 60$  the results are identical to those reported by King (1985). As expected, the degree of extra power available from selecting the best test, rather than the worst, varies considerably with the data used. This can be seen by comparing Figure 1a, where there is very little difference between the tests, with Figure 1b, which is based on different data and shows that large power gains are available by selecting the best test. When using X4 (Uniform Data) with  $T = 20$  and  $\rho = 0.8$ , there is only a 2.2% increase in power obtainable by using the best test ( $S(0.75)$ ) rather than the weakest test (ADW), as can be seen from Table 2. The correctly specified power functions for X1, X2 and X3 are almost identical to those of X4 which appear in Figure 1a. This graph also confirms that the ADW test is relatively weak for  $\rho > 0.6$ , while the  $S(0.5)$ ,  $S(0.75)$  and BW tests have very similar power functions which, as a group, dominate the DW and ADW tests over this region. These rankings are reversed, however, for tests against  $H_a^-$ .

Greater differences between the tests are evident when using X5 and X6 in a correctly specified model. Table 3 is based on a sample size of 20 and shows that for X5 with  $\rho = 0.8$  and  $h = 1.0$ , the strongest test ( $S(0.75)$ ) has a power of 0.817, which is almost 6% better than the 0.773 power of the weakest test (ADW). The same comparison using X6 reveals a massive 88% improvement from using the best ( $S(0.75)$ ) rather than the worst test (DW). Figure 1b clearly illustrates the extreme differences which can arise even in correctly specified models. As was noted by King (1985), when using Watson's matrix, the power

functions of the DW and ADW tests peak at  $\rho = 0.7$  and then decline as  $\rho \rightarrow 1$ .<sup>4</sup> This is in stark contrast to the  $S(\rho_1)$  and BW tests which have power functions which are strictly increasing in  $\rho$ , for this matrix.

When heteroscedasticity is introduced there is potential for the true size of each test to be distorted away from the nominal (here 5%) level. It is obviously more difficult to compare the power functions of two tests (or the same test in two different models) when one has a higher Type I error probability than the other. The degree of size distortion encountered in this study varied with  $h$ , the true sizes of all tests in all models falling within the following ranges: [0.049, 0.052] when  $h \leq 1.2$ , [0.049, 0.055] for  $h = 1.5$ , and [0.048, 0.058] for  $h = 2.0$ . These distortions are small, relative to the power changes that are induced by increasing  $h$  above unity, which allows valid comparisons of power functions across tests and across different degrees of heteroscedasticity.

The effect of introducing a moderate ( $h = 1.5$ ) degree of heteroscedasticity can be seen in Tables 1 to 3, in which the sample size is always 20. Figures 3 and 4 present this information, and power functions for other values of  $h$ , graphically. These figures also include powers for models with different forms of heteroscedasticity, although the scale of the variances has been standardised.

The most notable features of Figures 3 and 4 is that while all tests have homoscedastic power functions which are strictly increasing in  $\rho$ , for these design matrices, all these power functions are declining in  $\rho$  as  $\rho \rightarrow 1$  for all  $h > 1$  when  $T = 20$ . Notice also that the decline in power as  $h$  increases is consistent with the effect predicted by Theorem 1.

Figure 4b graphs the power of the  $S(0.5)$  test for various degrees of additive heteroscedasticity with  $T = 20$  and using  $X_4$ . The effect predicted by Theorem 3 is clearly evident in this diagram. This data set illustrates the

least severe power reductions encountered as a consequence of heteroscedasticity. Again, the power differences between the tests for a given  $h$  are relatively small, as can be seen from Table 2.

The effect that sample size has on the power of these tests was noted by King (1985). Other things constant, a larger sample size increases the power of each test and reduces the power differences between the tests. In addition, Figures 3a and b show that the DW test is much more robust to small degrees of heteroscedasticity when  $T = 60$  than when  $T = 20$ . This effect was found to be common to all tests and all design matrices.

It can be seen from Figures 3 and 4 that the precise form of  $\text{var}(u_i)$  is not important for the general shape of the mis-specified power functions. The crucial determinant of the serious power losses evident in these graphs is the scale of the leading diagonal elements of  $V^*$ , as suggested by Theorem 1. Further weight is given to this conclusion by Figure 4a, which plots power curves for the DW test using  $X_1$  with  $T = 20$ . In this figure  $n$  represents the number of non-unity leading diagonal elements of  $V^*$ , all such elements taking a value of 2.5. The conclusion is that as the average of the diagonal elements increases, the power of the test falls.

The ranking of the tests on the basis of their powers can change as a result of increasing  $h$  above unity. For example Table 1 shows that for  $X_1$  and  $h = 1.0$  the limiting power of the BW test is superior to all others while when  $h = 1.5$  this test has the lowest limiting power (recall that BW is LMPI as  $\rho \rightarrow 1$ ). Comparisons such as this are potentially dangerous however, as they divert attention from the major effect of this form of heteroscedasticity and can give false confidence in the strength of one particular test.

## 5.2 Results using $V^{**}$

In this section we consider the power functions when the true covariance matrix is  $V^{**}$ . For these models, Appendix 2 provides selected graphs and



tabulated power values, all of which are based on a sample size of 20. In Table 4 heteroscedasticity is of the form given by (6).

When the covariances between individual disturbances reflect their heteroscedasticity the true covariance matrix is given by  $V^{**}$ . The scale of the disturbance variances is irrelevant in this case, as it affects all elements of the covariance matrix and can therefore be factored out. Another way of looking at this is to note that there is no implied mis-specification of the AR(1) process, in contrast to the  $V^*$  case considered above.

The effect of this type of mis-specification on the sizes of the tests considered is minimal. The true sizes of all tests over almost all models were found to be identical to their nominal sizes to two decimal places. The exceptions to this were minor, with true sizes falling in the range (0.056, 0.060) for high values of the heteroscedasticity parameter when the data was X2 and X6 (recall that all tests had a nominal size of 0.050).

Disregarding, for now, extremes of the parameter space, the powers of the tests were generally not significantly altered by introducing heteroscedasticity of the form  $V^{**}$ . In cases where, for given  $\rho$ , the power of a test changed by more than  $\pm 0.01$ , it was found that the data matrix used was the important factor, rather than the particular test.

These slightly larger deviations from correctly specified power functions occurred with X1 (for mid-range  $\rho > 0$ ), X3 (strongly negative  $\rho$ ), X4 (as  $\rho \rightarrow 1$ ), X5 (as  $\rho \rightarrow 1$ ) and X6 (mid-range and strongly negative  $\rho$  and as  $\rho \rightarrow 1$ ). The most serious loss of power occurred with X6 when  $\rho$  fell in the range (0.55,1).

It is interesting to note that although the BW, S(0.5) and S(0.75) tests are not intended for use against  $H_a^-$  they have higher power against this alternative than against  $H_a^+$ , for given absolute values of  $\rho$ . As usual, there is an exception to this statement which is provided by Watson's data set, X6.

The tables of Appendices 1 and 2 show power values for  $\rho = \pm 1$ . These were calculated using the techniques suggested by Krämer and Zeisel (1990). As shown in Theorem 2, for all tests considered here, when the true covariance matrix is  $V^{**}$  and  $\text{var}(u_t)$  is given by (6), (7) or (8) the limiting power (in either direction) is always zero or unity. This fact accounts for significant deviations from correctly specified power functions as  $\rho \rightarrow 1$  for X3, X4, X5, and X6, since the limiting  $V^{**}$  power for all tests using these data sets is zero. Again, the precise form of  $\text{var}(u_t)$  is less important than the structure of the covariance matrix.

The real data used produced heteroscedastic limiting powers of unity which provokes speculation as to the reasons for the difference from the artificial data in this respect.

To summarise this section, it has been found that mis-specification of the type given by  $V^{**}$  has very little effect on the size and power of all tests studied unless the AR(1) parameter is very large in absolute value. In particular, the power of all tests approached zero as  $\rho \rightarrow 1$  when artificial data were used.

## 6. Conclusion

This paper has considered the effect that heteroscedastic errors have on the power of some tests for AR(1) errors, and has found severe reductions in power under two different covariance matrix structures and three types of heteroscedasticity.

When the underlying AR(1) process is altered by the introduction of heteroscedasticity of the  $V^*$  type, the power of each test is lower (for all  $\rho$ ) the greater is the scale of the disturbances, irrespective of the particular scheme for  $\text{var}(u_t)$ .

The second heteroscedastic covariance matrix used,  $V^{**}$ , allows the AR(1) process to dominate, with the covariance terms reflecting the heteroscedasticity. In this case the most notable effect is on the limiting power as  $\rho \rightarrow \pm 1$ . Independently of the presence of an intercept, the limiting power of each test considered is either zero or one under these conditions, when various forms of heteroscedasticity contaminate the error process. The clear conclusion arising from this paper is that there is no guarantee that the popular tests for AR(1) disturbances studied have any significant power when there is heteroscedasticity present. Furthermore, it has been shown that in many such cases the probability of detecting autocorrelation declines as the autocorrelation gets stronger, and the consequences of ignoring it get more severe.

## FOOTNOTES

- This paper reports on work undertaken as part of the author's Ph.D. research. The author is indebted to Professor D.E.A. Giles and to Dr J.A. Giles for their many helpful comments and suggestions. An earlier version of this paper was presented at the Econometric Society Meeting, Sydney, 1991.
- 1. The Box-Pierce Q statistic is one of several which have been suggested.
- 2. This is the exponential of the  $N(0,1.6)$  distribution and has a coefficient of variation in the population of  $\sqrt{19.58}/2.226 = 1.988$ .
- 3. A second, and more pragmatic, reason for not correcting for size is that the degree of size distortion encountered was negligible, as discussed below.
- 4. The phenomenon of power functions falling as  $\rho \rightarrow 1$  has been noted by several other authors. Recent work on this topic includes Krämer and Zeisel (1990) and Bartels (1990).

## REFERENCES

- Bartels, R., 1990, On the power function of the Durbin Watson test, Discussion Paper No. 90-03 Department of Econometrics, University of Sydney, Australia.
- Berenblut, I.I. and G.I. Webb, 1973, A new test for autocorrelated errors in the linear regression model, *Journal of the Royal Statistical Society B*, 35, 33-50.
- Davies, R.B., 1980, The distribution of a linear combination of chi square random variables (Algorithm AS 155), *Applied Statistics* 29, 323-33.
- Durbin, J. and G.S. Watson, 1950, Testing for serial correlation in least squares regression I, *Biometrika* 37, 409-28.
- Durbin, J. and G.S. Watson, 1951, Testing for serial correlation in least squares regression II, *Biometrika* 38, 159-78.
- Durbin, J., 1954, Errors in variables, *Review of the International Statistical Institute* 22, 23-32.
- Epps, T.W. and M.L. Epps, 1977, The robustness of some standard tests for autocorrelation and heteroscedasticity when both problems are present, *Econometrica* 45, 745-53.
- Evans, M.A., 1989, Robustness and size of tests of autocorrelation and heteroscedasticity to non-normality, Working Paper No. 10/89, Department of Econometrics, Monash University, Melbourne.
- Evans, M.A. and M.L. King, 1985, Critical value approximations for tests of linear regression disturbances, *Australian Journal of Statistics* 27, 68-83.
- Geary, R.C., 1970, Relative efficiency of count of sign changes for assessing residual autoregression in least squares regression, *Biometrika* 57, 123-7.
- Giles, D.E.A. and J.P. Small, 1991, The power of the Durbin Watson test when the errors are heteroscedastic, *Economics Letters* 36, 37-41.
- Harvey, A.C., 1976, Estimating regression models with multiplicative heteroscedasticity, *Econometrica* 44, 461-465.
- Hildreth, C. and J.P. Houck, 1968, Some estimators for a linear model with random coefficients, *Journal of the American Statistical Association* 70, 872-879.
- King, M.L., 1981, The alternative Durbin Watson test: an assessment of Durbin and Watson's choice of test statistic, *Journal of Econometrics* 17, 51-66.
- King, M.L., 1985, A point optimal test for autoregressive disturbances, *Journal of Econometrics* 27, 21-37.
- Koerts, J. and A.P.J. Abrahamse, 1969, *On the Theory and Application of the General Linear Model*, Rotterdam University Press, Rotterdam.

- Krämer, W., 1985, The power of the Durbin Watson test for regressions without an intercept, *Journal of Econometrics* 28, 363-370.
- Krämer, W. and H. Zeisel, 1990, Finite sample power of linear regression autocorrelation tests, *Journal of Econometrics* 43, 363-72.
- Small, J.P., 1991, The limiting power of point optimal autocorrelation tests, Discussion Paper 9110, Department of Economics, University of Canterbury.
- Watson, G.S., 1955, Serial correlation in regression Analysis I, *Biometrika* 42, 327-41.
- White, K.J., S.D. Wong, D. Whistler, and S.A. Haun, 1990, *SHAZAM: Econometrics Computer Program (Version 6.2), Users Reference Manual*, McGraw-Hill, New York.
- Zeisel, H., 1989, On the power of the Durbin-Watson test under high autocorrelation, *Communications in Statistics, Theory and Methods* 18, 3907-3916.

## APPENDIX 1

### RESULTS USING COVARIANCE MATRIX $V^*$



Table 1

Selected powers using V\* with multiplicative heteroscedasticity

## (a) Spirits data (X1)

rho	ADW		DW		S(0.5)		S(0.75)		BW	
	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5
-1.0	1.000	0.766	1.000	0.760	1.000	0.765	1.000	0.766	1.000	0.769
-0.8	0.943	0.835	0.922	0.808	0.932	0.821	0.926	0.815	0.919	0.809
-0.6	0.764	0.642	0.737	0.617	0.749	0.628	0.740	0.621	0.727	0.612
-0.4	0.450	0.368	0.434	0.355	0.441	0.361	0.435	0.357	0.425	0.351
-0.2	0.177	0.153	0.174	0.150	0.175	0.152	0.173	0.151	0.171	0.149
0.0	0.050	0.051	0.050	0.050	0.050	0.051	0.050	0.051	0.050	0.051
0.0	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.049
0.2	0.177	0.148	0.174	0.147	0.176	0.148	0.175	0.147	0.171	0.143
0.4	0.407	0.320	0.405	0.320	0.409	0.322	0.407	0.320	0.398	0.311
0.6	0.649	0.501	0.653	0.505	0.656	0.507	0.656	0.506	0.648	0.496
0.8	0.810	0.585	0.818	0.596	0.819	0.597	0.821	0.598	0.819	0.590
1.0	0.874	0.058	0.883	0.060	0.884	0.059	0.888	0.059	0.892	0.056

## (b) CPI data (X2)

-1.0	1.000	0.698	1.000	0.684	1.000	0.681	1.000	0.670	1.000	0.659
-0.8	0.935	0.768	0.921	0.742	0.916	0.743	0.905	0.730	0.898	0.720
-0.6	0.756	0.582	0.737	0.559	0.732	0.562	0.719	0.551	0.710	0.542
-0.4	0.448	0.336	0.437	0.323	0.435	0.326	0.427	0.321	0.422	0.317
-0.2	0.177	0.145	0.175	0.141	0.175	0.143	0.173	0.142	0.172	0.141
0.0	0.050	0.051	0.050	0.050	0.050	0.051	0.050	0.051	0.050	0.051
0.0	0.050	0.049	0.050	0.052	0.050	0.050	0.050	0.050	0.050	0.050
0.2	0.173	0.138	0.171	0.142	0.173	0.138	0.172	0.137	0.170	0.136
0.4	0.398	0.285	0.396	0.291	0.402	0.288	0.400	0.286	0.397	0.285
0.6	0.635	0.436	0.638	0.447	0.647	0.444	0.647	0.444	0.645	0.443
0.8	0.793	0.488	0.799	0.507	0.808	0.500	0.810	0.502	0.809	0.502
1.0	0.859	0.066	0.866	0.084	0.873	0.069	0.876	0.070	0.876	0.071

Table 2

## Selected powers using V\* with additive heteroscedasticity

## (a) Normal data (X3)

rho	ADW		DW		S(0.5)		S(0.75)		BW	
	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5
-1.0	1.000	0.563	1.000	0.546	1.000	0.508	1.000	0.491	1.000	0.483
-0.8	0.902	0.643	0.921	0.611	0.850	0.588	0.831	0.570	0.820	0.560
-0.6	0.709	0.476	0.737	0.452	0.662	0.443	0.646	0.431	0.637	0.424
-0.4	0.420	0.278	0.437	0.267	0.399	0.265	0.391	0.260	0.386	0.257
-0.2	0.171	0.128	0.175	0.124	0.168	0.125	0.166	0.123	0.165	0.122
0.0	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.048
0.0	0.050	0.050	0.050	0.051	0.050	0.051	0.050	0.051	0.050	0.052
0.2	0.173	0.129	0.171	0.130	0.172	0.131	0.171	0.131	0.169	0.130
0.4	0.400	0.262	0.396	0.264	0.405	0.267	0.403	0.266	0.400	0.265
0.6	0.646	0.402	0.638	0.408	0.659	0.412	0.658	0.412	0.656	0.411
0.8	0.808	0.444	0.799	0.457	0.824	0.459	0.826	0.461	0.826	0.462
1.0	0.867	0.049	0.866	0.055	0.883	0.053	0.885	0.054	0.886	0.056

## (b) Uniform data (X4)

-1.0	1.000	0.687	1.000	0.680	1.000	0.667	1.000	0.659	1.000	0.656
-0.8	0.941	0.771	0.915	0.743	0.921	0.746	0.910	0.734	0.903	0.727
-0.6	0.761	0.580	0.727	0.556	0.732	0.559	0.719	0.549	0.711	0.543
-0.4	0.449	0.332	0.430	0.321	0.433	0.322	0.425	0.318	0.421	0.315
-0.2	0.177	0.143	0.174	0.141	0.174	0.141	0.173	0.141	0.172	0.140
0.0	0.050	0.050	0.050	0.051	0.050	0.051	0.050	0.051	0.050	0.051
0.0	0.050	0.050	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.049
0.2	0.173	0.137	0.170	0.133	0.173	0.136	0.171	0.134	0.170	0.132
0.4	0.400	0.285	0.397	0.278	0.405	0.286	0.403	0.283	0.400	0.280
0.6	0.645	0.441	0.649	0.438	0.658	0.447	0.658	0.445	0.656	0.443
0.8	0.808	0.500	0.819	0.505	0.824	0.510	0.826	0.509	0.826	0.508
1.0	0.869	0.049	0.881	0.044	0.884	0.048	0.886	0.046	0.887	0.046

Table 3

Selected powers using V\* with Chow-type heteroscedasticity

## (a) Lognormal data (X5)

rho	ADW		DW		S(0.5)		S(0.75)		BW	
	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5
-1.0	1.000	0.721	1.000	0.712	1.000	0.712	1.000	0.708	1.000	0.706
-0.8	0.921	0.766	0.896	0.739	0.894	0.741	0.884	0.731	0.879	0.726
-0.6	0.736	0.577	0.707	0.554	0.705	0.555	0.695	0.547	0.689	0.543
-0.4	0.436	0.335	0.421	0.324	0.420	0.325	0.414	0.322	0.411	0.319
-0.2	0.175	0.147	0.172	0.145	0.172	0.146	0.171	0.145	0.170	0.144
0.0	0.050	0.053	0.050	0.053	0.050	0.053	0.050	0.053	0.050	0.053
0.0	0.050	0.051	0.050	0.050	0.050	0.049	0.050	0.049	0.050	0.048
0.2	0.168	0.135	0.165	0.132	0.167	0.131	0.165	0.129	0.163	0.128
0.4	0.380	0.272	0.381	0.270	0.392	0.275	0.389	0.272	0.386	0.270
0.6	0.611	0.411	0.626	0.419	0.644	0.431	0.644	0.431	0.642	0.429
0.8	0.773	0.459	0.798	0.479	0.814	0.492	0.817	0.495	0.816	0.495
1.0	0.837	0.090	0.865	0.089	0.877	0.082	0.880	0.081	0.880	0.081

## (b) Watson's data (X6)

-1.0	0.000	0.067	0.000	0.080	0.000	0.033	0.000	0.032	0.000	0.032
-0.8	0.477	0.328	0.447	0.307	0.289	0.214	0.264	0.197	0.253	0.190
-0.6	0.462	0.339	0.432	0.318	0.337	0.259	0.314	0.244	0.303	0.236
-0.4	0.311	0.238	0.295	0.227	0.260	0.205	0.248	0.197	0.242	0.193
-0.2	0.147	0.124	0.143	0.121	0.137	0.117	0.134	0.115	0.133	0.114
0.0	0.050	0.051	0.050	0.051	0.050	0.051	0.050	0.051	0.050	0.051
0.0	0.050	0.051	0.050	0.051	0.050	0.051	0.050	0.051	0.050	0.051
0.2	0.146	0.123	0.142	0.120	0.142	0.120	0.134	0.114	0.129	0.111
0.4	0.299	0.227	0.291	0.221	0.339	0.251	0.327	0.242	0.317	0.234
0.6	0.434	0.308	0.424	0.300	0.604	0.416	0.603	0.416	0.595	0.409
0.8	0.450	0.278	0.440	0.272	0.819	0.518	0.829	0.536	0.828	0.536
1.0	0.301	0.097	0.308	0.097	0.925	0.101	0.935	0.100	0.936	0.099

Figure 1a  
Uniform Data;  $T=20$   
 $h = 1.0$

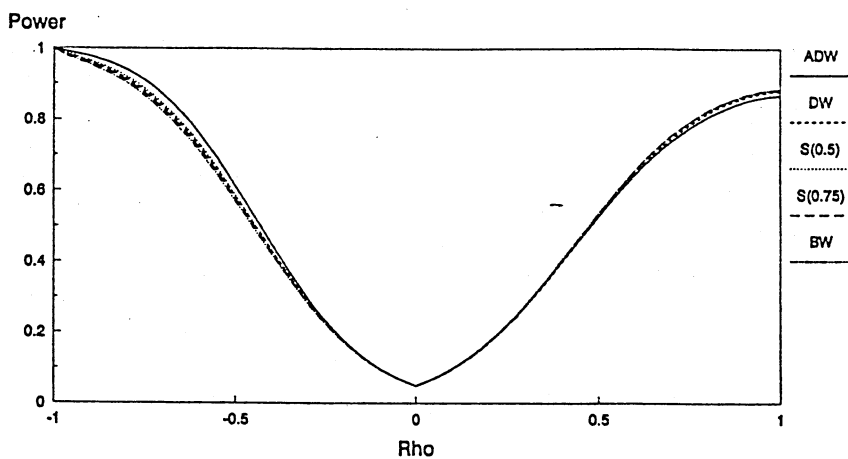


Figure 1b  
Watson's Data;  $T=20$   
 $h = 1.0$

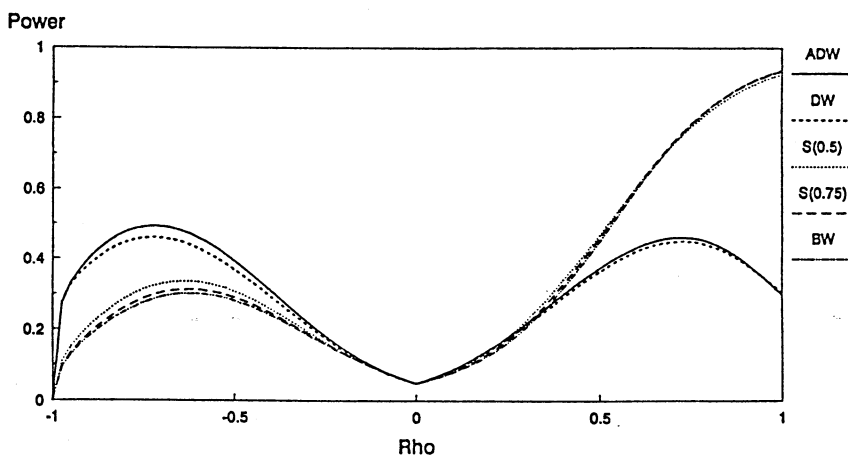


Figure 2a  
Uniform Data;  $T=20$   
 $h = 1.5$

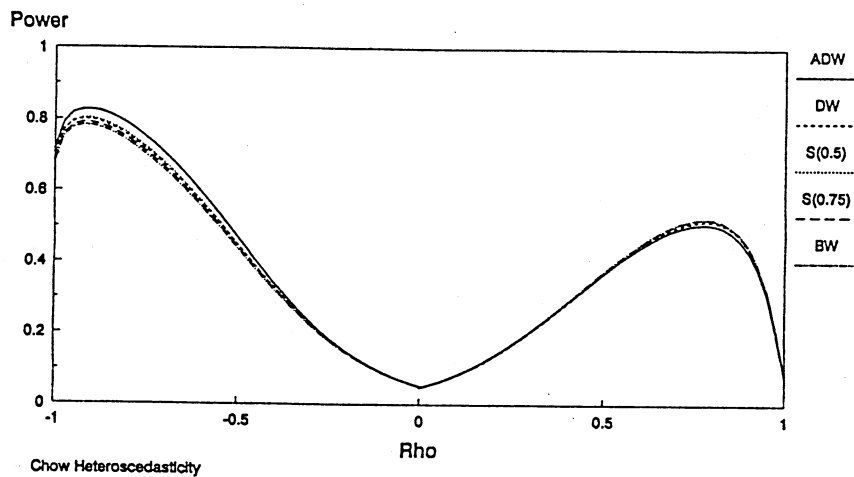


Figure 2b  
Watson's Data;  $T=20$   
 $h = 1.5$

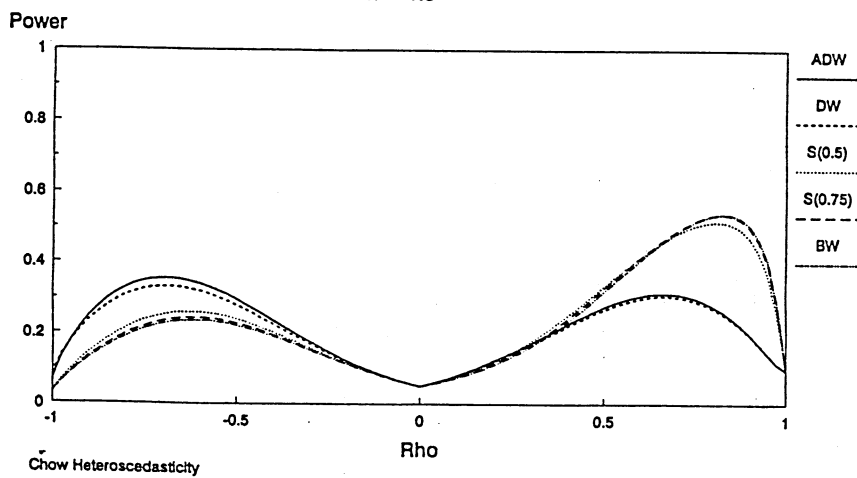


Figure 3a  
CPI Data;  $T = 60$   
DW Test

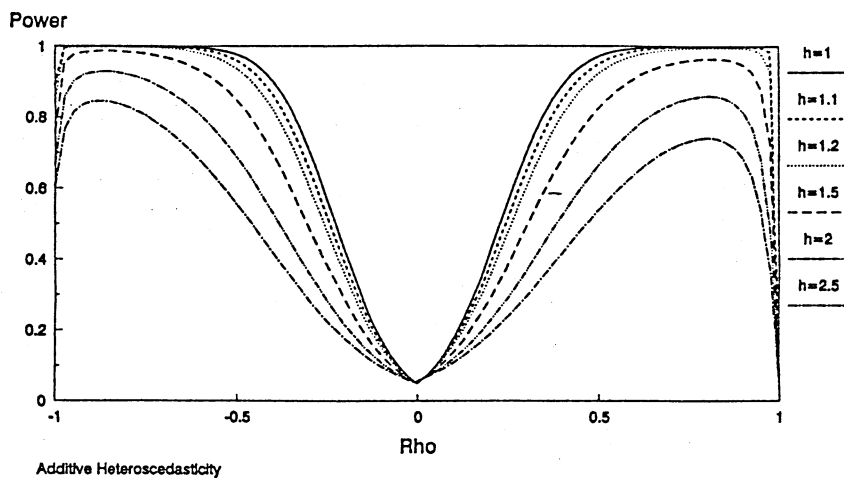


Figure 3b  
CPI Data;  $T = 20$   
DW Test

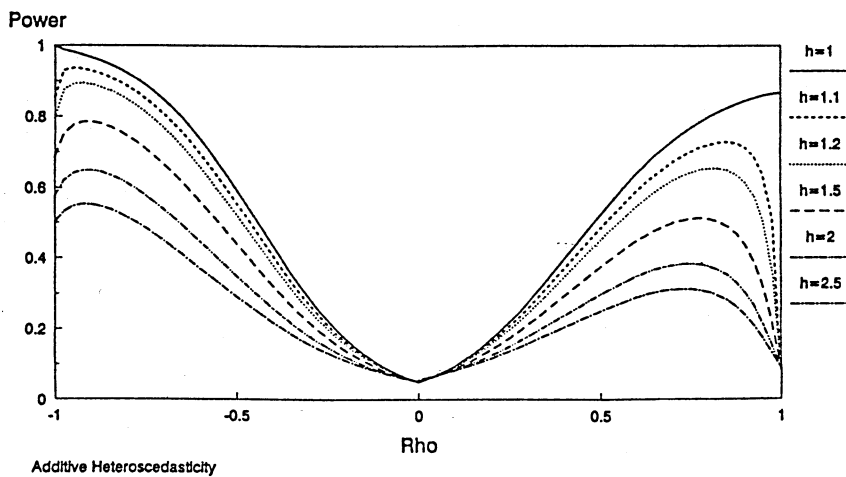


Figure 4a  
Spirits Data;  $T = 20$   
DW Test

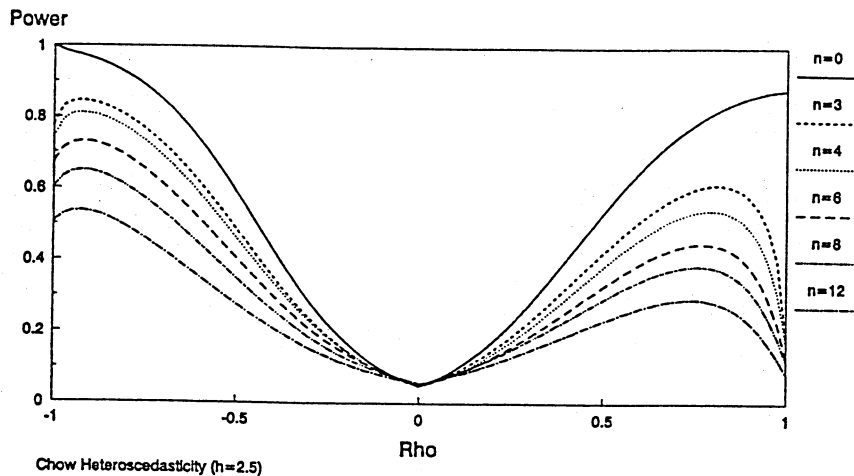
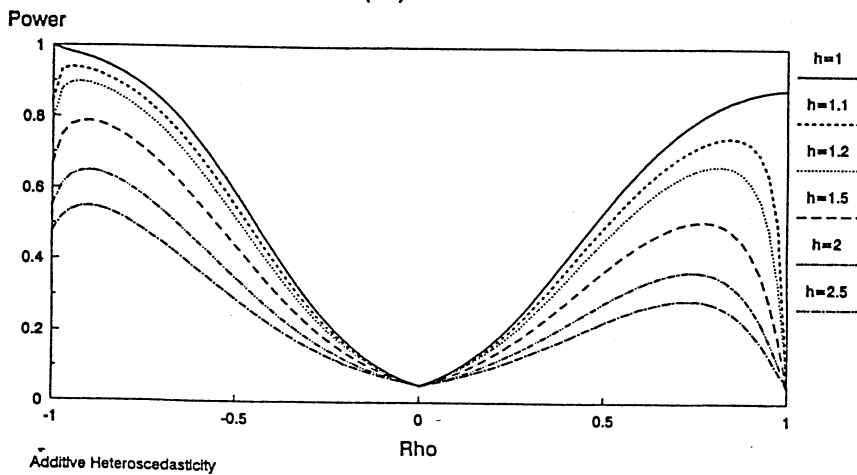


Figure 4b  
Uniform Data;  $T = 20$   
 $S(0.5)$  Test





## APPENDIX 2

RESULTS USING  
COVARIANCE MATRIX  $V^{**}$

Table 4

## Selected powers using V\*\* with multiplicative heteroscedasticity

## (a) Spirits data (X1)

Rho	ADW		DW		S(0.5)		S(0.75)		BW	
	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5
-1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.8	0.943	0.942	0.922	0.920	0.932	0.931	0.926	0.925	0.919	0.919
-0.6	0.764	0.764	0.737	0.734	0.749	0.748	0.740	0.739	0.727	0.728
-0.4	0.450	0.451	0.434	0.433	0.441	0.441	0.435	0.436	0.425	0.427
-0.2	0.177	0.178	0.174	0.174	0.175	0.176	0.173	0.175	0.171	0.173
0.0	0.050	0.051	0.050	0.050	0.050	0.051	0.050	0.051	0.050	0.051
0.0	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.049
0.2	0.177	0.173	0.174	0.171	0.176	0.173	0.175	0.172	0.171	0.167
0.4	0.407	0.401	0.405	0.400	0.409	0.403	0.407	0.401	0.398	0.391
0.6	0.649	0.643	0.653	0.647	0.656	0.650	0.656	0.650	0.648	0.641
0.8	0.810	0.805	0.818	0.813	0.819	0.814	0.821	0.816	0.819	0.813
1.0	0.874	1.000	0.883	1.000	0.884	1.000	0.888	1.000	0.892	1.000

## (b) Normal data (X3)

-1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.8	0.902	0.898	0.879	0.871	0.850	0.842	0.831	0.822	0.820	0.810
-0.6	0.709	0.702	0.680	0.670	0.662	0.653	0.646	0.635	0.637	0.626
-0.4	0.420	0.414	0.403	0.395	0.399	0.391	0.391	0.382	0.386	0.377
-0.2	0.171	0.168	0.168	0.163	0.168	0.164	0.166	0.161	0.165	0.160
0.0	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.048
0.0	0.050	0.050	0.050	0.051	0.050	0.051	0.050	0.051	0.050	0.052
0.2	0.173	0.172	0.170	0.173	0.172	0.174	0.171	0.173	0.169	0.173
0.4	0.400	0.399	0.399	0.401	0.405	0.407	0.403	0.406	0.400	0.404
0.6	0.646	0.643	0.651	0.653	0.659	0.659	0.658	0.660	0.656	0.659
0.8	0.808	0.806	0.819	0.820	0.824	0.824	0.826	0.827	0.826	0.827
1.0	0.867	0.000	0.881	0.000	0.883	0.000	0.885	0.000	0.886	0.000

Figure 5a  
Normal Data;  $T = 20$   
 $h = 1.0$

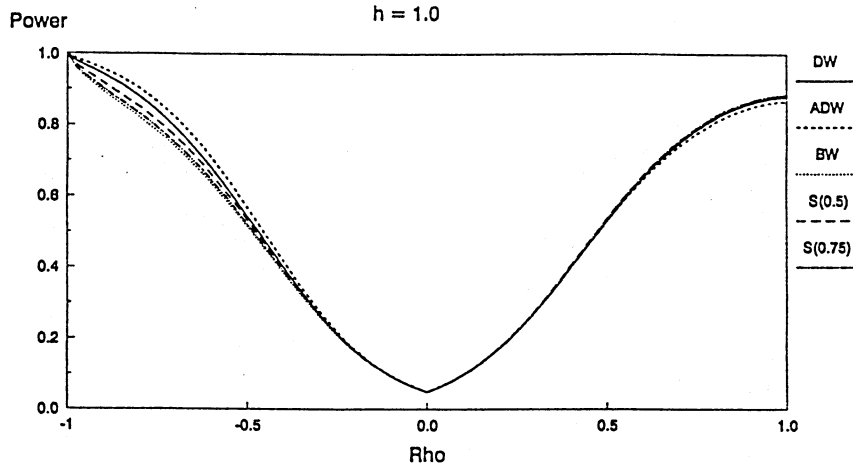


Figure 5b  
Normal Data;  $T = 20$   
 $h = 1.5$

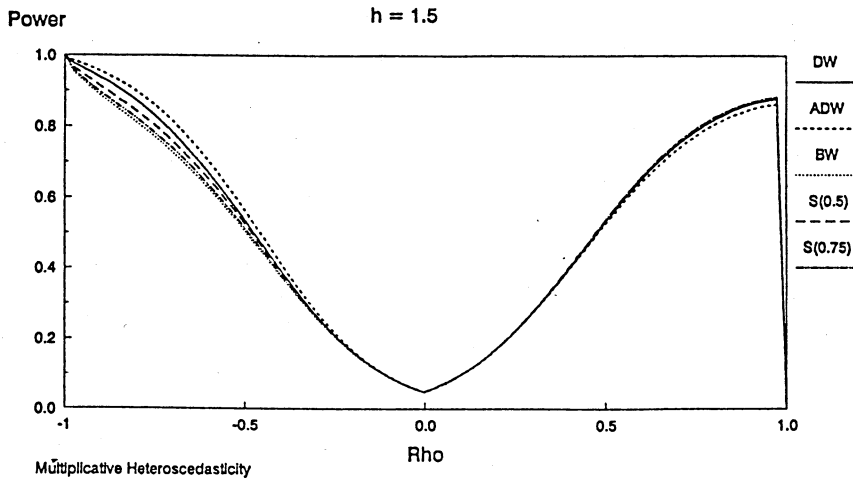


Figure 6a  
Spirits Data; T=20  
DW Test

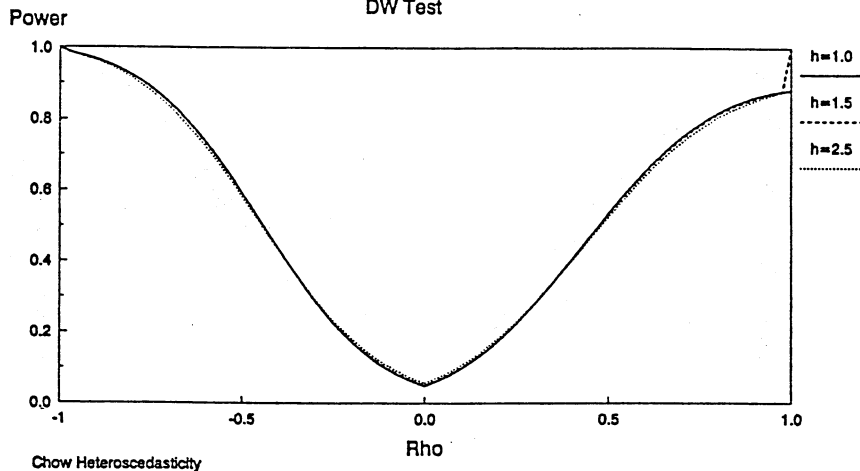
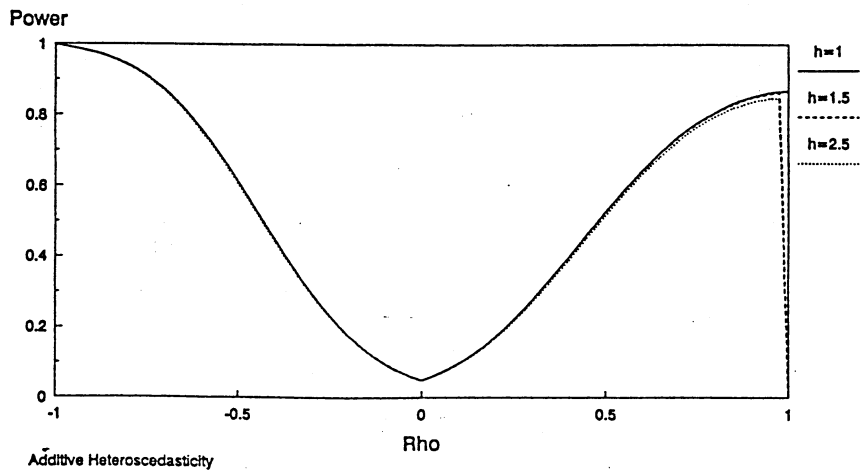


Figure 6b  
Uniform Data; T=20  
ADW Test



## LIST OF DISCUSSION PAPERS\*

- No. 8701 Stochastic Simulation of the Reserve Bank's Model of the New Zealand Economy, by J. N. Lye.
- No. 8702 Urban Expenditure Patterns in New Zealand, by Peter Hampton and David E. A. Giles.
- No. 8703 Preliminary-Test Estimation of Mis-Specified Regression Models, by David E. A. Giles.
- No. 8704 Instrumental Variables Regression Without an Intercept, by David E. A. Giles and Robin W. Harrison.
- No. 8705 Household Expenditure in Sri Lanka: An Engel Curve Analysis, by Mallika Dissanayake and David E. A. Giles.
- No. 8706 Preliminary-Test Estimation of the Standard Error of Estimate in Linear Regression, by Judith A. Clarke.
- No. 8707 Invariance Results for FIML Estimation of an Integrated Model of Expenditure and Portfolio Behaviour, by P. Dorian Owen.
- No. 8708 Social Cost and Benefit as a Basis for Industry Regulation with Special Reference to the Tobacco Industry, by Alan E. Woodfield.
- No. 8709 The Estimation of Allocation Models With Autocorrelated Disturbances, by David E. A. Giles.
- No. 8710 Aggregate Demand Curves in General-Equilibrium Macroeconomic Models: Comparisons with Partial-Equilibrium Microeconomic Demand Curves, by P. Dorian Owen.
- No. 8711 Alternative Aggregate Demand Functions in Macro-economics: A Comment, by P. Dorian Owen.
- No. 8712 Evaluation of the Two-Stage Least Squares Distribution Function by Imhof's Procedure by P. Cribbitt, J. N. Lye and A. Ullah.
- No. 8713 The Size of the Underground Economy: Problems and Evidence, by Michael Carter.
- No. 8714 A Computable General Equilibrium Model of a Fisherine Method to Close the Foreign Sector, by Ewen McCann and Keith McLaren.
- No. 8715 Preliminary-Test Estimation of the Scale Parameter in a Mis-Specified Regression Model, by David E. A. Giles and Judith A. Clarke.
- No. 8716 A Simple Graphical Proof of Arrow's Impossibility Theorem, by John Fountain.
- No. 8717 Rational Choice and Implementation of Social Decision Functions, by Manimay Sen.
- No. 8718 Divisia Monetary Aggregates for New Zealand, by Ewen McCann and David E. A. Giles.
- No. 8719 Telecommunications in New Zealand: The Case for Reform, by John Fountain.
- No. 8801 Workers' Compensation Rates and the Demand for Apprentices and Non-Apprentices in Victoria, by Pasquale M. Sgro and David E. A. Giles.
- No. 8802 The Adventures of Sherlock Holmes, the 48% Solution, by Michael Carter.
- No. 8803 The Exact Distribution of a Simple Pre-Test Estimator, by David E. A. Giles.
- No. 8804 Pre-testing for Linear Restrictions in a Regression Model With Student-t Errors, by Judith A. Clarke.
- No. 8805 Divisia Monetary Aggregates and the Real User Cost of Money, by Ewen McCann and David Giles.
- No. 8806 The Management of New Zealand's Lobster Fishery, by Alan Woodfield and Pim Borren.
- No. 8807 Poverty Measurement: A Generalization of Sen's Result, by Prasanta K. Pattanaik and Manimay Sen.
- No. 8808 A Note on Sen's Normalization Axiom for a Poverty Measure, by Prasanta K. Pattanaik and Manimay Sen.
- No. 8809 Budget Deficits and Asset Sales, by Ewen McCann.
- No. 8810 Unorganized Money Markets and 'Unproductive' Assets in the New Structuralist Critique of Financial Liberalization, by P. Dorian Owen and Otton Solis-Fallas.
- No. 8901 Testing for Financial Buffer Stocks in Sectoral Portfolio Models, by P. Dorian Owen.
- No. 8902 Provisional Data and Unbiased Prediction of Economic Time Series by Karen Browning and David Giles.
- No. 8903 Coefficient Sign Changes When Restricting Regression Models Under Instrumental Variables Estimation, by David E. A. Giles.

*(Continued on next page)*



- No. 8904 Economies of Scale in the New Zealand Electricity Distribution Industry, by David E. A. Giles and Nicolas S. Wyatt.
- No. 8905 Some Recent Developments in Econometrics: Lessons for Applied Economists, by David E. A. Giles.
- No. 8906 Asymptotic Properties of the Ordinary Least Squares Estimator in Simultaneous Equations Models, by V. K. Srivastava and D. E. A. Giles.
- No. 8907 Unbiased Estimation of the Mean Squared Error of the Feasible Generalised Ridge Regression Estimator, by V. K. Srivastava and D. E. A. Giles.
- No. 8908 An Unbiased Estimator of the Covariance Matrix of the Mixed Regression Estimator, by D. E. A. Giles and V. K. Srivastava.
- No. 8909 Pre-testing for Linear Restrictions in a Regression Model with Spherically Symmetric Disturbances, by Judith A. Giles.
- No. 9001 The Durbin-Watson Test for Autocorrelation in Nonlinear Models, by Kenneth J. White.
- No. 9002 Determinants of Aggregate Demand for Cigarettes in New Zealand, by Robin Harrison and Jane Chetwyd.
- No. 9003 Unemployment Duration and the Measurement of Unemployment, by Manimay Sengupta.
- No. 9004 Estimation of the Error Variance After a Preliminary-Test of Homogeneity in a Regression Model with Spherically Symmetric Disturbances, by Judith A. Giles.
- No. 9005 An Expository Note on the Composite Commodity Theorem, by Michael Carter.
- No. 9006 The Optimal Size of a Preliminary Test of Linear Restrictions in a Mis-specified Regression Model, by David E. A. Giles, Offer Lieberman, and Judith A. Giles.
- No. 9007 Inflation, Unemployment and Macroeconomic Policy in New Zealand: A Public Choice Analysis, by David J. Smyth and Alan E. Woodfield.
- No. 9008 Inflation — Unemployment Choices in New Zealand and the Median Voter Theorem, by David J. Smyth and Alan E. Woodfield.
- No. 9009 The Power of the Durbin-Watson Test when the Errors are Heteroscedastic, by David E. A. Giles and John P. Small.
- No. 9010 The Exact Distribution of a Least Squares Regression Coefficient Estimator After a Preliminary t-Test, by David E. A. Giles and Virendra K. Srivastava.
- No. 9011 Testing Linear Restrictions on Coefficients in a Linear Regression Model with Proxy variables and Spherically Symmetric Disturbances, by Kazuhiro Ohtani and Judith A. Giles.
- No. 9012 Some Consequences of Applying the Goldfeld-Quandt Test to Mis-Specified Regression Models, by David E. A. Giles and Guy N. Saxton.
- No. 9013 Pre-testing in a Mis-specified Regression Model, by Judith A. Giles.
- No. 9014 Two Results in Balanced-Growth Educational Policy, by Alan E. Woodfield.
- No. 9101 Bounds on the Effect of Heteroscedasticity on the Chow Test for Structural Change, by David Giles and Offer Lieberman.
- No. 9102 The Optimal Size of a Preliminary Test for Linear Restrictions when Estimating the Regression Scale Parameter, by Judith A. Giles and Offer Lieberman.
- No. 9103 Some Properties of the Durbin-Watson Test After a Preliminary t-Test, by David Giles and Offer Lieberman.
- No. 9104 Preliminary-Test Estimation of the Regression Scale Parameter when the Loss Function is Asymmetric, by Judith A. Giles and David E. A. Giles.
- No. 9105 On an Index of Poverty, by Manimay Sengupta and Prasanta K. Pattanaik.
- No. 9106 Cartels May Be Good For You, by Michael Carter and Julian Wright.
- No. 9107 Lp-Norm Consistencies of Nonparametric Estimates of Regression, Heteroskedasticity and Variance of Regression Estimate when Distribution of Regression is Known, by Radhey S. Singh.
- No. 9108 Optimal Telecommunications Tariffs and the CCITT, by Michael Carter and Julian Wright.
- No. 9109 Price Indices : Systems Estimation and Tests, by David Giles and Ewen McCann.
- No. 9110 The Limiting Power of Point Optimal Autocorrelation Tests, by John P. Small.
- No. 9111 The Exact Power of Some Autocorrelation Tests When the Disturbances are Heteroscedastic, by John P. Small.

\* Copies of these Discussion Papers may be obtained for \$4 (including postage, price changes occasionally) each by writing to the Secretary, Department of Economics, University of Canterbury, Christchurch, New Zealand. A list of the Discussion Papers prior to 1987 is available on request.