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THE EXACT POWERS OF SOME AUTOCORRELATION TESTS WHEN THE DISTURBANCES ARE HETEROSCEDASTIC

John P. Small

Discussion Paper

No. 9111

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Department of Economics, University of Canterbury
Christchurch, New Zealand

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Department of Economics, University of Canterbury, Private Bag, Christchurch, New Zealand.

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Abstract

This paper considers the exact finite sample powers of five popular tests for AR(1) disturbances when one of several types of heteroscedasticity is also present. Severe reductions in power are found, particularly under strong positive autocorrelation. Factors influencing these power reductions are identified analytically and the limiting powers are also considered.

1. Introduction

In applied econometric work, testing for serial independence of the regression disturbances is a routine and necessary practice. Such testing can reveal to the researcher signs of specification error or poor explanatory performance, as well as information about the appropriate estimation technique to employ. The general agreement on the importance of detecting autocorrelation has produced a large literature on the subject[1] and has led to a variety of tests.

The most widely used such test for the last 40 years has been the bounds test of Durbin and Watson (1950, 1951) which has two major advantages: it is easy to calculate the test statistic and the tabulated bounds are independent of the data used. The drawbacks of the DW bounds test are the possibility of inconclusive results and sub-optimal power properties under certain conditions. Several easily applied alternatives have been suggested which overcome the inconclusive region difficulties, for example the Sign Change test (Geary (1970)).

More recently, great improvements in computer technology have avoided the problem of the inconclusive bounds test by permitting easy application of the exact DW test and this has refocussed attention on the task of developing tests which have optimal power properties. The major offerings in this field are from Berenblut and Webb (1973) and King (1981, 1985), who suggest tests which are more powerful than the DW test for particular areas of the parameter space. As well as having powerful tests at our disposal we would ideally use tests which maintain their power when their underlying assumptions are violated in some commonly occurring ways. Such problems as departures from normality, omitted regressors or superfluous regressors should not seriously weaken a test for autocorrelation (or any test), as these violations are likely to occur and will typically be unknown to the applied researcher. This paper explores one such scenario. We use exact techniques to study the power functions of five tests for autocorrelation when they are applied to a model in which the disturbance variance is heteroscedastic in one of three different ways. This study extends work by Epps and Epps (1977) and Giles and Small (1991) (both of which studied the power of the Durbin Watson test when the errors are heteroscedastic) to include the Berenblut and Webb (1973) test, the alternative DW test (King (1981)) and two versions of King's (1985) point optimal test.

The next section outlines the structure and strengths of each test. Some theoretical results arising from these models are presented in Section 3, which is followed by a description of the data used in the numerical evaluations. Section 5 reports the main findings of the study and Section 6 offers some concluding comments.

2. The tests

Consider the standard linear regression model

$$y = X\beta + u \tag{1}$$

where y is a (T×1) vector of observations on the dependent variable, X is a (T×k) full rank non-stochastic regressor matrix, β is a (k×1) parameter vector and u is a (T×1) vector of disturbances following the AR(1) process:

$$u_t = \rho u_{t-1} + \varepsilon_t . \qquad |\rho| < 1,\ t = 1,\dots,T \tag{2}$$

We consider tests of $H_0 : \rho = 0$ against $H_a^+ : \rho > 0$ and $H_a^- : \rho < 0$ individually, each conducted at the 5% nominal size.

The statistics for each of the tests considered can be written as a ratio of quadratic forms in u, the general form of this ratio being

$$r = \frac{u'Qu}{u'Mu} \tag{3}$$

where $M = I - X(X'X)^{-1}X'$ and Q is some other nonstochastic (T×T) matrix, the form of which determines the individual test statistic.

(i) The Durbin Watson (DW) test

This has

$$Q = MAM \text{ where } A = \begin{bmatrix} 1 & -1 & 0 & \cdots & & 0 \\ -1 & 2 & -1 & & & \\ 0 & -1 & 2 & & & \vdots \\ \vdots & & & & & \\ & & & \ddots & 2 & -1 \\ 0 & 0 & \cdots & 0 & -1 & 1 \end{bmatrix}.$$

The DW test is an approximately Uniformly Most Powerful (UMP) test of $H_0 : \rho = 0$ against H_a^+ for all design matrices, the approximation being due to a small modification made to the density function of the stationary AR(1) error process by Durbin and Watson (1950). In addition, when the columns of X are linear combinations of k of the eigenvectors of A, the DW test is an approximately Uniformly Most Powerful Invariant (UMPI) test against H_a^+. Throughout this paper invariance is with respect to an affine transformation of the dependent variable.

(ii) The Berenblut and Webb (BW) test

Berenblut and Webb (1973) proposed two tests, the first of which used $Q = MBM$, where

$$B = \begin{bmatrix} 2 & -1 & 0 & \cdots & 0 \\ -1 & 2 & & & \\ 0 & & & & \vdots \\ & & & & 0 \\ \vdots & & & 2 & -1 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}.$$

This statistic arose from a consideration of a modified form of the density function of non-stationary first order autoregressive errors. The second test statistic was arrived at by considering the likelihood ratio test of $H_0 : \rho = 0$ against $H_a : \rho \neq 0$ (again in the context of a nonstationary AR(1) process), and replacing the inverse of the covariance matrix of the AR(1) process with B. This gave rise to a statistic using:

$$Q = B - BX(X'BX)^{-1}X'B.$$

This second test will be referred to as the BW test and was shown by Berenblut and Webb to possess the optimal power qualities of both the DW test and the first Berenblut and Webb test. In an empirical evaluation Berenblut and Webb found that the BW test was more powerful than the DW test at high values of ρ for six design matrices.

(iii) The alternative Durbin Watson (ADW) test

Here

$$Q = MA_0M, \quad \text{where } A_0 = \begin{bmatrix} 2 & -1 & 0 & \cdots & 0 \\ -1 & 2 & & & \vdots \\ 0 & & 2 & & 0 \\ \vdots & & & \ddots & -1 \\ 0 & \cdots & 0 & -1 & 2 \end{bmatrix}.$$

King (1981) proposed this test and found it to be a Locally Best Invariant (LBI) test in the neighbourhood of $\rho = 0$. In the same paper King discussed results from an empirical comparison of the power functions of the DW and ADW tests, in which the latter generally performed better than the former against negative autocorrelation and for $\rho < 0.5$.

(iv) The point optimal ($S(\rho_1)$) tests

This class of tests, due to King (1985), has

$$Q = \Sigma(\rho_1)^{-1} - \Sigma(\rho_1)^{-1}X\left[X'\Sigma(\rho_1)^{-1}X\right]^{-1}X'\Sigma(\rho_1)^{-1}$$

where $\Sigma(\rho_1) = \frac{1}{1-\rho_1^2} V(\rho_1)$ and $V(\rho_1)$ is V with ρ fixed at some chosen value, ρ_1. King showed that this test is most powerful invariant (MPI) when the selected value for ρ_1 is the true value of ρ. He studied the power of two versions of the Point Optimal test, $\rho_1 = 0.5$ and $\rho_1 = 0.75$, and the DW, ADW and BW tests using a wide range of design matrices. King found that small power increases occurred when using either S(0.5) or S(0.75), in preference to DW or ADW, with large samples of smoothly evolving regressors, and more significant differences were apparent in smaller samples. In the case of Watson's (1955) matrix, the

$S(\rho_1)$ and BW tests had power functions which approached unity for $\rho \to 1$, in contrast to the DW and ADW power functions which peaked at $\rho = 0.75$ and then declined.

3. Theoretical discussion

The power function of each test against $H_a^+ : \rho > 0$, for a nominal significance level of 100α% and its associated critical value, $r^*(\alpha)$, can be found by substituting values of ρ into the expression

$$\mathrm{pr}\left\{ r < r^*(\alpha) \mid V = V(\rho) \right\} . \tag{4}$$

The analogous expression for testing against $H_a^- : \rho < 0$ is $\Pr\{r > r^*(\alpha) | V = V(\rho)\}$ but the critical values used are different. The structure of each test statistic which is given by (3), allows the use of the well known manipulations, in the style of Koerts and Abrahamse (1969), for example, to write (4) as:

$$\mathrm{pr}\left\{ u'(Q - r^*M)u < 0 \mid V = V(\rho) \right\}$$

$$= \mathrm{pr}\left\{ \sum_{j=1}^{T} \lambda_j \chi_j^2 \le 0 \right\} \tag{5}$$

where the λ_j's are the eigenvalues of $(Q - r^*M)V$ and the χ_j^2's are independent central chi square variates, each with one degree of freedom. The following lemma from Evans and King (1985) provides a useful unification.

Lemma 1

If $\quad Q = B - BX(X'BX)^{-1}X'B$

and $\quad M = I - X(X'X)^{-1}X'$

then $\quad QM = MQ = Q$ so that $Q = MQM$

The proof follows immediately from the definitions of M and Q. This lemma enables us to write the BW and $S(\rho_1)$ tests as DW-type tests with a particular A matrix and so to represent the power of all tests considered as depending on the eigenvalues of M(A-r*I)MV for some non-stochastic A. The form of (5) allows computation of the power of each test for any covariance matrix using, for example, the FQUAD routine from Koerts and Abrahamse (1969) or Davies' (1980) algorithm. The numerical evaluations described below were conducted using a Fortran version of Davies' algorithm contained in the SHAZAM (White et al. (1990)) computer package.

The first practical question which arises is whether to introduce heteroscedasticity into the ε_t's or the u_t's of (2). In practice either one, the other, both or neither of these variables will be heteroscedastic. For the purposes of a controlled study, however, a choice must be made. It is not difficult to show (a proof is available on request) that heteroscedastic ε_t's imply a different degree of heteroscedasticity in the u_t's for each value of ρ. Furthermore, the limiting case of $\rho = 1$ results in homoscedasticity for the u_t's. For these reasons heteroscedasticity was introduced directly into u. We assume, therefore, that $\varepsilon_t \sim NID(0,\sigma_\varepsilon^2)$, $t = 1,\ldots,T$. The covariance matrix of a homoscedastic u vector defined by (2) is

$$E(u'u) = \left(\frac{\sigma_\varepsilon^2}{1-\rho^2}\right)V \text{ where } V = \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{T-1} \\ \rho & 1 & \rho & & \vdots \\ \rho^2 & \rho & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \rho \\ \rho^{T-1} & \rho^{T-2} & \cdots & \rho & 1 \end{bmatrix}.$$

The following forms of heteroscedasticity are considered

$$var(u_t) = kZ_t^\alpha \tag{6}$$

$$var(u_t) = k(1 + \gamma Z_t) \tag{7}$$

$$\mathrm{var}(u_t) \begin{cases} 1 & t \le T_1 \\ h & t > T_1 \end{cases} \qquad t = 1,2,\ldots,T_1,\ldots,T \tag{8}$$

where Z_t is a suitable transformation of the value of the jth regressor at time t, X_{jt}, and α, γ and h are selected constants. The proportionality constant, $k = \sigma_\varepsilon^2/1-\rho^2$, does not influence the power or size of the tests considered and shall not concern us further.

These three heteroscedastic schemes are intended to represent forms which are likely to occur in practise. Multiplicative heteroscedasticity (6) has been considered by several writers, including Harvey (1976) who concentrated on the estimation of a model with this characteristic. The process is easily generalised to include dependence on more than one regressor but this study is restricted to the simple case outlined above. The additive heteroscedasticity of (7) can represent any situation where the variance of the dependent variable is assumed to be a linear function of some transformation of the regressors. A notable special case is the random coefficient model of Hildreth and Houck (1968). The third model of $\mathrm{var}(u_t)$ is likely to arise when the regression parameters exhibit a structural break, and will be referred to as Chow heteroscedasticity after the well-known F tests for this phenomenon.

There is no good reason to assume homoscedasticity when a test for AR(1) errors is conducted. Heteroscedasticity of the forms (7) and (8), in particular, are likely to occur in time-series regressions, while testing for spatial autocorrelation when using cross-section data provides another motivation for this study.

We turn now to the question of the appropriate form of the covariance matrix of u when both heteroscedasticity and autocorrelation are present. Giles and Small (1991) assume that the vector of disturbance variances is substituted into the leading diagonal of V, giving

$$
V^* = \begin{bmatrix} \sigma_1^2 & \rho & \cdot & \cdot & \cdot & \cdot & \rho^{T-1} \\ \rho & \sigma_2^2 & & & & & \rho^{T-2} \\ \rho^2 & \cdot & \cdot & & & & \cdot \\ \cdot & & \cdot & \cdot & & & \cdot \\ \cdot & & & \cdot & \cdot & & \rho \\ \rho^{T-1} & \rho^{T-2} & \cdot & \cdot & \cdot & \rho & \sigma_T^2 \end{bmatrix} .
$$

where $\text{var}(u_t) = k\sigma_t^2$.

This covariance matrix arises naturally from the Hildreth-Houck random coefficient model with AR(1) errors. It can also be derived from a model in which only the intercept is random and in this context some extra insight is gained. Suppose that

$$y_t = X_t'\beta + \mu_t + u_t$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

where $u_t \sim N(0,\sigma_\mu^2)$is the random intercept and $\varepsilon_t \sim N(0,\sigma_\varepsilon^2)$ is independent of μ_t. Then $\nu_t = \mu_t + u_t$ has covariance matrix given by kV^* with $\sigma_t^2 = (1 + \sigma_\mu^2/k)$, $\forall$ t. If we let $\sigma_t^2 = \lambda$ (which imposes homoscedasticity) then V^* reflects the following autoregressive process

$$\lambda u_t = \rho^s u_{t-s} + \varepsilon_t \qquad t = 1,\ldots,T, \tag{9}$$

in which the first autocorrelation is ρ/λ, while all subsequent ones are ρ. Since $\lambda > 1$ the first autocorrelation is weaker than all others and this reduces the average power of all tests considered here, irrespective of the data, as the following theorem shows.

Theorem 1

When the autoregressive process is given by (9) rather than (2), the average value of the test statistic is increased when testing against H_a^+.

Proof

Let $S = M(A-r^*I)M$ and consider the first moment of $(r-r^*)$ which is given by $E(u'Su) = tr(SV)$, Consider $V^* = V + \Lambda$

where $\Lambda = \text{diag}(\lambda^*)$

and $\lambda^* = \lambda - 1 > 0.$

If V^* is the true covariance matrix of u then we must compare tr(SV) with tr(SV*).

$$\text{tr}(SV^*) = \text{tr}(S(V+\Lambda))$$

$$= \text{tr}(SV) + \sum_{i=1}^{T} q_{ii}\lambda^*$$

$$= \text{tr}(SV) + \lambda^*\text{tr}(S)$$

but $\text{tr}(S) = E(r-r^*)\Big|_{\rho=0}$

and we know that $E(r)\Big|_{\rho=0} > r^*$, when testing against H_a^+.

So tr(S) > 0 and tr(SV*) > tr(SV). #

So, at least on average, the probability of rejecting the null hypothesis in favour of H_a^+ is therefore reduced as λ increases. The same argument can be used to show that when testing against H_a^- the average value of the test statistic is reduced, with the same power effect, on average.

This result provides the motivation for choosing transformations used in constructing Z_t which constrain σ_t^2 to a minimum value of unity, while still reflecting the variability of X_j. For (6), $Z_t = X_{jt}/\min X_{jt}$ while (7) has $Z_t = (X_{jt}-\min(X_{jt}))/(\max(X_{jt})-\min(X_{jt}))$.

The other obvious method of introducing heteroscedasticity is to apply the back-substitution procedure associated with the AR(1) process to the heteroscedastic u_t's. This, in a sense, assumes that the heteroscedasticity "came first", and results in the following covariance matrix, neglecting the scale factor,

$$V^{**} = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \rho^2\sigma_1\sigma_3 & \cdots & \rho^{T-1}\sigma_1\sigma_T \\ \rho\sigma_1\sigma_2 & \sigma_2^2 & \rho\sigma_2\sigma_3 & & \rho^{T-2}\sigma_2\sigma_T \\ \rho^2\sigma_1\sigma_3 & \rho\sigma_2\sigma_3 & \sigma_3^2 & & \vdots \\ \vdots & & & \ddots & \vdots \\ \rho^{T-1}\sigma_1\sigma_T & \cdots & & & \sigma_T^2 \end{bmatrix}.$$

Both V* and V** have been used in this study, which evaluates all powers exactly.

3.1 The Limiting Power

The data dependence of the distribution of r precludes direct analytic evaluation of the powers of the tests under consideration in almost all cases. Some results are obtainable, however, at the boundaries of the parameter space by examining the limits of the eigenvalues of (5) as $\rho \to \pm 1$. This technique was used by Krämer (1985) to prove that, for regressions with no intercept, the limiting power of the DW test as $\rho \to 1$ is always zero or unity. A more involved, but similar, analysis enabled Zeisel (1989) to show that when an intercept is present the power of the DW test as $\rho \to 1$ lies strictly between these two values. More recently, Small (1991) has generalised both of these results to the ADW, BW and $S(\rho_1)$ tests.

This section presents two further limiting power results which apply when the disturbances are heteroscedastic in particular ways.

Theorem 2

When $var(u_t) = kZ_t^{\alpha}$ and cov(u) is given by V**, the limiting power of all tests considered as $\rho \to 1$ is zero or unity unless $\alpha = 2$, irrespective of the presence of an intercept.

Proof

Under the conditions of Theorem 2, the covariance matrix of u is given by

$$V^{**} = \begin{bmatrix} Z_1^{\alpha} & (Z_2Z_1)^{\alpha/2} & \cdots & (Z_TZ_1)^{\alpha/2} \\ (Z_1Z_2)^{\alpha/2} & Z_2^{\alpha} & & (Z_TZ_2)^{\alpha/2} \\ (Z_1Z_2)^{\alpha/2} & (Z_2Z_3)^{\alpha/2} & Z_3^{\alpha} & \vdots \\ \vdots & & \ddots & \vdots \\ (Z_1Z_T)^{\alpha/2} & \cdots & & Z_T^{\alpha} \end{bmatrix}$$

$$= \left(\min(X_j)\right)^{\alpha/2} \begin{bmatrix} X_{j1}^{\alpha/2} \\ \vdots \\ X_{jT}^{\alpha/2} \end{bmatrix} \begin{bmatrix} X_{j1}^{\alpha/2} & \cdots & X_{jT}^{\alpha/2} \end{bmatrix}.$$

Notice that $X_j^{\alpha/2}$ is not in the column space of X unless $\alpha = 2$. This implies that, in general, $MV^{**} \neq 0$. Notice also that V^{**} has rank equal to unity so that $M(A-r^*I)MV^{**}$ has only one non-zero eigenvalue. The sign of this eigenvalue uniquely determines the power of the test, a positive eigenvalue implying a limiting power of zero. #

The following corollaries extend this result in two directions.

Corollary 1

The limiting power, as $\rho \to 1$, of all tests considered when $var(u_t)$ is given by (7) or (8) and $cov(u) = V^{**}$ is also zero or unity.

Proof

When $var(u_t)$ is given by (7) then, as $\rho \to 1$, $V^{**} \to \nu\nu'$,

where $\nu = (\sqrt{1+\gamma Z_1}\ \sqrt{1+\gamma Z_2}\ \ldots\ \sqrt{1+\gamma Z_3})'$ is not spanned by the column space of X in general.

When $var(u_t)$ is given by (8) and $\rho \to 1$, $V^{**} \to ww'$ where $w = (1,1,\ldots,1,h,h,\ldots,h)'$ is also not spanned by the columns of X.

In both of these cases V^{**} also has unit rank so the limiting power of all tests are zero or unity. #

Corollary 2

When $\text{cov}(u) = V^{**}$ and $\text{var}(u_t)$ is given by either (6), (7) or (8), the limiting power of all tests considered is always zero or unity as $\rho \to -1$.

Proof

If $V^{**} \to \nu\nu'$ as $\rho \to 1$ with $\nu = (\nu_1,\nu_2,\ldots,\nu_T)'$ then as $\rho \to 1$, $V^{**} \to \nu^- \nu^{-\prime}$ with $\nu^- = (\nu_1,-\nu_2,\nu_3-\nu_4,\ldots,(-1)^{T-1}\nu_T)$. Now if ν is not in the column space of X, then neither is ν^- but V^{**} has rank approaching unity as $\rho \to -1$ under these conditions. The power must therefore approach zero or unity as $\rho \to -1$. #

Theorem 3

When $\text{var}(u_t) = k(1+\gamma Z_t)$ and $\text{cov}(u)$ is given by V^*, the limiting power of all tests considered as $\rho \to 1$ is constant for all $\gamma > 0$, providing an intercept is present.

Proof

Under the conditions of Theorem 3 we can decompose the covariance matrix of u as

$$V^* = \Sigma + \gamma\Lambda \ ,$$

where $\Sigma = ii'$ for $i = (1,1,\ldots,1)'$, $\Lambda = \text{diag}(Z_t)$ and γ is a scalar.

Let $S = M(A-r^*I)M$ and consider the eigenvalues of SV^*, being the λ which satisfy

$$\lambda w = SV^*w \ , \qquad \text{for some non-null vector } w$$

or,
$$\lambda w = S(\Sigma+\gamma\Lambda)w.$$

Now when an intercept is present, $M\Sigma = S\Sigma = 0$ so that

$$\lambda w = \gamma S\Lambda w.$$

Consider some other non-zero scalar, γ^*. We can write

$$\lambda w = \gamma^*\left(\frac{\gamma}{\gamma^*}\right)S\Lambda w \ ,$$

so
$$\lambda\frac{\gamma}{\gamma^*}w = \gamma^* S\Lambda w \ ,$$

thus altering the value of γ scales each eigenvalue by the same factor, which does not affect the rejection probability. #

4. **Data**

Several regressor matrices were used in an empirical study in an attempt to reveal the effects of data characteristics on the tests' powers under the mis-specifications outlined above.

The design matrices used were of two sizes, 60x3 and 20x3,and are characterised as follows:

X1: A constant and the price and income series from Durbin and Watson's (1951) consumption of spirits example.

X2: A constant, the quarterly Australian Consumers Price Index commencing 1959(1), and the same index lagged one period.

X3: A constant, a linear time trend and observations drawn from the Normal (30,4) distribution.

X4: A constant, a time trend and observations drawn from the Uniform [0,10] distribution.

X5: A constant, a time trend and observations drawn from the Lognormal (2.226, 19.58) distribution[2].

X6: a_1, $(a_2 + a_T)/\sqrt{2}$, $(a_3+a_{T-1})/\sqrt{2}$ where $a_1, \ldots, a_T$ are the eigenvectors corresponding to the eigenvalues of A arranged in ascending order. Note that a_1 is a constant as it corresponds to the zero root of A.

These design matrices were chosen to represent a range of characteristics. The slowly evolving X1 matrix is annual data while X2 has a weak seasonal pattern. Both X1 and X2 have been used in previous studies in the general field of autocorrelation testing (*e.g.* King (1985) and Evans (1991)). Several previous studies have suggested the use of artificial data of various types. The lognormal data, for example, are often used to represent cross section data which are known to be skewed and therefore particularly relevant to scenarios involving heteroscedasticity. The X6 matrix was shown by Watson (1955) to produce the most inefficient OLS estimates within the class of orthogonal matrices.

For each X matrix, one regressor was selected to be the X_j of (6) and (7). The variables used for this purpose were: Income (XI), CPI (X2), Normal (X3), Uniform (X4), Lognormal (X5) and $(a_2 + a_T)/\sqrt{2}$ (X6). The scalars α and γ were chosen to give desired values of the ratio.

$$h = \frac{\text{maximum var}(u_t)}{\text{minimum var }(u_t)} \ .$$

The degree of heteroscedasticity introduced was controlled in this way with h being set at six values ranging from 1.0 (which implies homoscedasticity) to 2.5. In the other exact work of this type, Epps and Epps (1977) and Giles and Small (1991) used the same measurement criterion for heteroscedasticity, although other measures could be considered, such as the coefficient of variation.

No size corrections were made to the heteroscedastic power functions. The reason for this is that the purpose of this study is to determine the effect of heteroscedastic disturbances on the power of each test when based on least squares residuals. If an applied worker knew that the disturbances were heteroscedastic she would use a GLS type estimator which would account for the heteroscedasticity and simultaneously return the nominal size of the autocorrelation test to its true size. This study presumes that the researcher is ignorant of the complications due to heteroscedasticity.[3]

5. Results

It is convenient to discuss the results of the numerical evaluations in two groups, distinguished by the structure of the mis-specified covariance matrix.

5.1 Results using V*

This section discusses the results obtained by using the first version of the mis-specified covariance matrix, V*, introduced above. All figures and tables referred to are contained in Appendix 1.

The correctly specified (h = 1.0) power functions (*e.g.*, Figures 1a and b) are in accord with the findings of previous studies. In the case of X6 and T = 60 the results are identical to those reported by King (1985). As expected, the degree of extra power available from selecting the best test, rather than the worst, varies considerably with the data used. This can be seen by comparing Figure 1a, where there is very little difference between the tests, with Figure 1b, which is based on different data and shows that large power gains are available by selecting the best test. When using X4 (Uniform Data) with T = 20 and ρ = 0.8, there is only a 2.2% increase in power obtainable by using the best test (S(0.75)) rather than the weakest test (ADW), as can be seen from Table 2. The correctly specified power functions for X1, X2 and X3 are almost identical to those of X4 which appear in Figure 1a. This graph also confirms that the ADW test is relatively weak for ρ > 0.6, while the S(0.5), S(0.75) and BW tests have very similar power functions which, as a group, dominate the DW and ADW tests over this region. These rankings are reversed, however, for tests against H_a^-.

Greater differences between the tests are evident when using X5 and X6 in a correctly specified model. Table 3 is based on a sample size of 20 and shows that for X5 with ρ = 0.8 and h = 1.0, the strongest test (S(0.75)) has a power of 0.817, which is almost 6% better than the 0.773 power of the weakest test (ADW). The same comparison using X6 reveals a massive 88% improvement from using the best (S(0.75)) rather than the worst test (DW). Figure 1b clearly illustrates the extreme differences which can arise even in correctly specified models. As was noted by King (1985), when using Watson's matrix, the power functions of the DW and ADW tests peak at $\rho = 0.7$ and then decline as $\rho \to 1$.[4] This is in stark contrast to the $S(\rho_1)$ and BW tests which have power functions which are strictly increasing in ρ, for this matrix.

When heteroscedasticity is introduced there is potential for the true size of each test to be distorted away from the nominal (here 5%) level. It is obviously more difficult to compare the power functions of two tests (or the same test in two different models) when one has a higher Type I error probability than the other. The degree of size distortion encountered in this study varied with h, the true sizes of all tests in all models falling within the following ranges: [0.049, 0.052] when $h \leq 1.2$, [0.049, 0.055] for $h = 1.5$, and [0.048, 0.058] for $h = 2.0$. These distortions are small, relative to the power changes that are induced by increasing h above unity, which allows valid comparisons of power functions across tests and across different degrees of heteroscedasticity.

The effect of introducing a moderate ($h = 1.5$) degree of heteroscedasticity can be seen in Tables 1 to 3, in which the sample size is always 20. Figures 3 and 4 present this information, and power functions for other values of h, graphically. These figures also include powers for models with different forms of heteroscedasticity, although the scale of the variances has been standardised.

The most notable features of Figures 3 and 4 is that while all tests have homoscedastic power functions which are strictly increasing in ρ, for these design matrices, all these power functions are declining in ρ as $\rho \to 1$ for all $h > 1$ when $T = 20$. Notice also that the decline in power as h increases is consistent with the effect predicted by Theorem 1.

Figure 4b graphs the power of the S(0.5) test for various degrees of additive heteroscedasticity with $T = 20$ and using X4. The effect predicted by Theorem 3 is clearly evident in this diagram. This data set illustrates the least severe power reductions encountered as a consequence of heteroscedasticity. Again, the power differences between the tests for a given h are relatively small, as can be seen from Table 2.

The effect that sample size has on the power of these tests was noted by King (1985). Other things constant, a larger sample size increases the power of each test and reduces the power differences between the tests. In addition, Figures 3a and b show that the DW test is much more robust to small degrees of heteroscedasticity when T = 60 than when T = 20. This effect was found to be common to all tests and all design matrices.

It can be seen from Figures 3 and 4 that the precise form of $var(u_t)$ is not important for the general shape of the mis-specified power functions. The crucial determinant of the serious power losses evident in these graphs is the scale of the leading diagonal elements of V*, as suggested by Theorem 1. Further weight is given to this conclusion by Figure 4a, which plots power curves for the DW test using X1 with T = 20. In this figure n represents the number of non-unity leading diagonal elements of V*, all such elements taking a value of 2.5. The conclusion is that as the average of the diagonal elements increases, the power of the test falls.

The ranking of the tests on the basis of their powers can change as a result of increasing h above unity. For example Table 1 shows that for X1 and h = 1.0 the limiting power of the BW test is superior to all others while when h = 1.5 this test has the lowest limiting power (recall that BW is LMPI as $\rho \to 1$). Comparisons such as this are potentially dangerous however, as they divert attention from the major effect of this form of heteroscedasticity and can give false confidence in the strength of one particular test.

5.2 Results using V**

In this section we consider the power functions when the true covariance matrix is V**. For these models, Appendix 2 provides selected graphs and tabulated power values, all of which are based on a sample size of 20. In Table 4 heteroscedasticity is of the form given by (6).

When the covariances between individual disturbances reflect their heteroscedasticity the true covariance matrix is given by V^{**}. The scale of the disturbance variances is irrelevant in this case, as it affects all elements of the covariance matrix and can therefore be factored out. Another way of looking at this is to note that there is no implied mis-specification of the AR(1) process, in contrast to the V^{*} case considered above.

The effect of this type of mis-specification on the sizes of the tests considered is minimal. The true sizes of all tests over almost all models were found to be identical to their nominal sizes to two decimal places. The exceptions to this were minor, with true sizes falling in the range (0.056, 0.060) for high values of the heteroscedasticity parameter when the data was X2 and X6 (recall that all tests had a nominal size of 0.050).

Disregarding, for now, extremes of the parameter space, the powers of the tests were generally not significantly altered by introducing heteroscedasticity of the form V^{**}. In cases where, for given ρ, the power of a test changed by more than $\pm$ 0.01, it was found that the data matrix used was the important factor, rather than the particular test.

These slightly larger deviations from correctly specified power functions occurred with X1 (for mid-range $\rho > 0$), X3 (strongly negative ρ), X4 (as $p \rightarrow 1$), X5 (as $\rho \rightarrow 1$) and X6 (mid-range and strongly negative ρ and as $\rho \rightarrow 1$). The most serious loss of power occurred with X6 when ρ fell in the range (0.55,1).

It is interesting to note that although the BW, S(0.5) and S(0.75) tests are not intended for use against H_a^- they have higher power against this alternative than against H_a^+, for given absolute values of ρ. As usual, there is an exception to this statement which is provided by Watson's data set, X6.

The tables of Appendices 1 and 2 show power values for $\rho = \pm 1$. These were calculated using the techniques suggested by Krämer and Zeisel (1990). As shown in Theorem 2, for all tests considered here, when the true covariance matrix is V^{**} and $var(u_t)$ is given by (6), (7) or (8) the limiting power (in either direction) is always zero or unity. This fact accounts for significant deviations from correctly specified power functions as $\rho \to 1$ for X3, X4, X5, and X6, since the limiting V^{**} power for all tests using these data sets is zero. Again, the precise form of $var(u_t)$ is less important than the structure of the covariance matrix.

The real data used produced heteroscedastic limiting powers of unity which provokes speculation as to the reasons for the difference from the artificial data in this respect.

To summarise this section, it has been found that mis-specification of the type given by V^{**} has very little effect on the size and power of all tests studied unless the AR(1) parameter is very large in absolute value. In particular, the power of all tests approached zero as $\rho \to 1$ when artificial data were used.

6. Conclusion

This paper has considered the effect that heteroscedastic errors have on the power of some tests for AR(1) errors, and has found severe reductions in power under two different covariance matrix structures and three types of heteroscedasticity.

When the underlying AR(1) process is altered by the introduction of heteroscedasticity of the V^* type, the power of each test is lower (for all ρ) the greater is the scale of the disturbances, irrespective of the particular scheme for $var(u_t)$.

The second heteroscedastic covariance matrix used, V^{**}, allows the AR(1) process to dominate, with the covariance terms reflecting the heteroscedasticity. In this case the most notable effect is on the limiting power as $\rho \to \pm 1$. Independently of the presence of an intercept, the limiting power of each test considered is either zero or one under these conditions, when various forms of heteroscedasticity contaminate the error process. The clear conclusion arising from this paper is that there is no guarantee that the popular tests for AR(1) disturbances studied have any significant power when there is heteroscedasticity present. Furthermore, it has been shown that in many such cases the probability of detecting autocorrelation declines as the autocorrelation gets stronger, and the consequences of ignoring it get more severe.

FOOTNOTES

* This paper reports on work undertaken as part of the author's Ph.D. research. The author is indebted to Professor D.E.A. Giles and to Dr J.A. Giles for their many helpful comments and suggestions. An earlier version of this paper was presented at the Econometric Society Meeting, Sydney, 1991.

1. The Box-Pierce Q statistic is one of several which have been suggested.
2. This is the exponential of the N(0,1.6) distribution and has a coefficient of variation in the population of $\sqrt{19.58}/2.226 = 1.988$.
3. A second, and more pragmatic, reason for not correcting for size is that the degree of size distortion encountered was negligible, as discussed below.
4. The phenomenon of power functions falling as $\rho \rightarrow 1$ has been noted by several other authors. Recent work on this topic includes Krämer and Zeisel (1990) and Bartels (1990).

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APPENDIX 1

RESULTS USING

COVARIANCE MATRIX V^*

Table 1

Selected powers using V* with multiplicative heteroscedasticity

(a) Spirits data (X1)

	ADW		DW		S(0.5)		S(0.75)		BW	
rho	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5
-1.0	1.000	0.766	1.000	0.760	1.000	0.765	1.000	0.766	1.000	0.769
-0.8	0.943	0.835	0.922	0.808	0.932	0.821	0.926	0.815	0.919	0.809
-0.6	0.764	0.642	0.737	0.617	0.749	0.628	0.740	0.621	0.727	0.612
-0.4	0.450	0.368	0.434	0.355	0.441	0.361	0.435	0.357	0.425	0.351
-0.2	0.177	0.153	0.174	0.150	0.175	0.152	0.173	0.151	0.171	0.149
0.0	0.050	0.051	0.050	0.050	0.050	0.051	0.050	0.051	0.050	0.051
0.0	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.049
0.2	0.177	0.148	0.174	0.147	0.176	0.148	0.175	0.147	0.171	0.143
0.4	0.407	0.320	0.405	0.320	0.409	0.322	0.407	0.320	0.398	0.311
0.6	0.649	0.501	0.653	0.505	0.656	0.507	0.656	0.506	0.648	0.496
0.8	0.810	0.585	0.818	0.596	0.819	0.597	0.821	0.598	0.819	0.590
1.0	0.874	0.058	0.883	0.060	0.884	0.059	0.888	0.059	0.892	0.056

(b) CPI data (X2)

	ADW		DW		S(0.5)		S(0.75)		BW	
rho	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5
-1.0	1.000	0.698	1.000	0.684	1.000	0.681	1.000	0.670	1.000	0.659
-0.8	0.935	0.768	0.921	0.742	0.916	0.743	0.905	0.730	0.898	0.720
-0.6	0.756	0.582	0.737	0.559	0.732	0.562	0.719	0.551	0.710	0.542
-0.4	0.448	0.336	0.437	0.323	0.435	0.326	0.427	0.321	0.422	0.317
-0.2	0.177	0.145	0.175	0.141	0.175	0.143	0.173	0.142	0.172	0.141
0.0	0.050	0.051	0.050	0.050	0.050	0.051	0.050	0.051	0.050	0.051
0.0	0.050	0.049	0.050	0.052	0.050	0.050	0.050	0.050	0.050	0.050
0.2	0.173	0.138	0.171	0.142	0.173	0.138	0.172	0.137	0.170	0.136
0.4	0.398	0.285	0.396	0.291	0.402	0.288	0.400	0.286	0.397	0.285
0.6	0.635	0.436	0.638	0.447	0.647	0.444	0.647	0.444	0.645	0.443
0.8	0.793	0.488	0.799	0.507	0.808	0.500	0.810	0.502	0.809	0.502
1.0	0.859	0.066	0.866	0.084	0.873	0.069	0.876	0.070	0.876	0.071

Table 2

Selected powers using V* with additive heteroscedasticity

(a) Normal data (X3)

rho	ADW h=1	ADW h=1.5	DW h=1	DW h=1.5	S(0.5) h=1	S(0.5) h=1.5	S(0.75) h=1	S(0.75) h=1.5	BW h=1	BW h=1.5
-1.0	1.000	0.563	1.000	0.546	1.000	0.508	1.000	0.491	1.000	0.483
-0.8	0.902	0.643	0.921	0.611	0.850	0.588	0.831	0.570	0.820	0.560
-0.6	0.709	0.476	0.737	0.452	0.662	0.443	0.646	0.431	0.637	0.424
-0.4	0.420	0.278	0.437	0.267	0.399	0.265	0.391	0.260	0.386	0.257
-0.2	0.171	0.128	0.175	0.124	0.168	0.125	0.166	0.123	0.165	0.122
0.0	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.048
0.0	0.050	0.050	0.050	0.051	0.050	0.051	0.050	0.051	0.050	0.052
0.2	0.173	0.129	0.171	0.130	0.172	0.131	0.171	0.131	0.169	0.130
0.4	0.400	0.262	0.396	0.264	0.405	0.267	0.403	0.266	0.400	0.265
0.6	0.646	0.402	0.638	0.408	0.659	0.412	0.658	0.412	0.656	0.411
0.8	0.808	0.444	0.799	0.457	0.824	0.459	0.826	0.461	0.826	0.462
1.0	0.867	0.049	0.866	0.055	0.883	0.053	0.885	0.054	0.886	0.056
(b) Uniform data (X4)										
-1.0	1.000	0.687	1.000	0.680	1.000	0.667	1.000	0.659	1.000	0.656
-0.8	0.941	0.771	0.915	0.743	0.921	0.746	0.910	0.734	0.903	0.727
-0.6	0.761	0.580	0.727	0.556	0.732	0.559	0.719	0.549	0.711	0.543
-0.4	0.449	0.332	0.430	0.321	0.433	0.322	0.425	0.318	0.421	0.315
-0.2	0.177	0.143	0.174	0.141	0.174	0.141	0.173	0.141	0.172	0.140
0.0	0.050	0.050	0.050	0.051	0.050	0.051	0.050	0.051	0.050	0.051
0.0	0.050	0.050	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.049
0.2	0.173	0.137	0.170	0.133	0.173	0.136	0.171	0.134	0.170	0.132
0.4	0.400	0.285	0.397	0.278	0.405	0.286	0.403	0.283	0.400	0.280
0.6	0.645	0.441	0.649	0.438	0.658	0.447	0.658	0.445	0.656	0.443
0.8	0.808	0.500	0.819	0.505	0.824	0.510	0.826	0.509	0.826	0.508
1.0	0.869	0.049	0.881	0.044	0.884	0.048	0.886	0.046	0.887	0.046

Table 3

Selected powers using V* with Chow-type heteroscedasticity

(a) Lognormal data (X5)

rho	ADW h=1	ADW h=1.5	DW h=1	DW h=1.5	S(0.5) h=1	S(0.5) h=1.5	S(0.75) h=1	S(0.75) h=1.5	BW h=1	BW h=1.5
-1.0	1.000	0.721	1.000	0.712	1.000	0.712	1.000	0.708	1.000	0.706
-0.8	0.921	0.766	0.896	0.739	0.894	0.741	0.884	0.731	0.879	0.726
-0.6	0.736	0.577	0.707	0.554	0.705	0.555	0.695	0.547	0.689	0.543
-0.4	0.436	0.335	0.421	0.324	0.420	0.325	0.414	0.322	0.411	0.319
-0.2	0.175	0.147	0.172	0.145	0.172	0.146	0.171	0.145	0.170	0.144
0.0	0.050	0.053	0.050	0.053	0.050	0.053	0.050	0.053	0.050	0.053
0.0	0.050	0.051	0.050	0.050	0.050	0.049	0.050	0.049	0.050	0.048
0.2	0.168	0.135	0.165	0.132	0.167	0.131	0.165	0.129	0.163	0.128
0.4	0.380	0.272	0.381	0.270	0.392	0.275	0.389	0.272	0.386	0.270
0.6	0.611	0.411	0.626	0.419	0.644	0.431	0.644	0.431	0.642	0.429
0.8	0.773	0.459	0.798	0.479	0.814	0.492	0.817	0.495	0.816	0.495
1.0	0.837	0.090	0.865	0.089	0.877	0.082	0.880	0.081	0.880	0.081

(b) Watson's data (X6)

rho	ADW h=1	ADW h=1.5	DW h=1	DW h=1.5	S(0.5) h=1	S(0.5) h=1.5	S(0.75) h=1	S(0.75) h=1.5	BW h=1	BW h=1.5
-1.0	0.000	0.067	0.000	0.080	0.000	0.033	0.000	0.032	0.000	0.032
-0.8	0.477	0.328	0.447	0.307	0.289	0.214	0.264	0.197	0.253	0.190
-0.6	0.462	0.339	0.432	0.318	0.337	0.259	0.314	0.244	0.303	0.236
-0.4	0.311	0.238	0.295	0.227	0.260	0.205	0.248	0.197	0.242	0.193
-0.2	0.147	0.124	0.143	0.121	0.137	0.117	0.134	0.115	0.133	0.114
0.0	0.050	0.051	0.050	0.051	0.050	0.051	0.050	0.051	0.050	0.051
0.0	0.050	0.051	0.050	0.051	0.050	0.051	0.050	0.051	0.050	0.051
0.2	0.146	0.123	0.142	0.120	0.142	0.120	0.134	0.114	0.129	0.111
0.4	0.299	0.227	0.291	0.221	0.339	0.251	0.327	0.242	0.317	0.234
0.6	0.434	0.308	0.424	0.300	0.604	0.416	0.603	0.416	0.595	0.409
0.8	0.450	0.278	0.440	0.272	0.819	0.518	0.829	0.536	0.828	0.536
1.0	0.301	0.097	0.308	0.097	0.925	0.101	0.935	0.100	0.936	0.099

Figure 1a

Uniform Data; T=20

h = 1.0

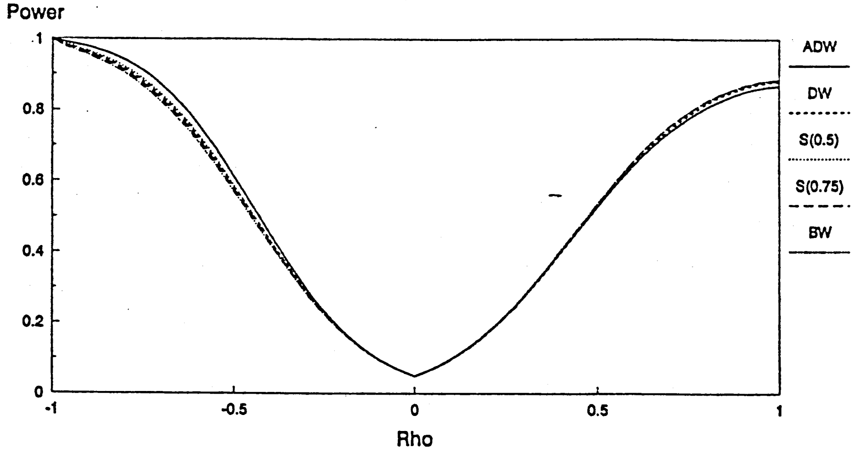


Figure 1b

Watson's Data; T=20

h = 1.0

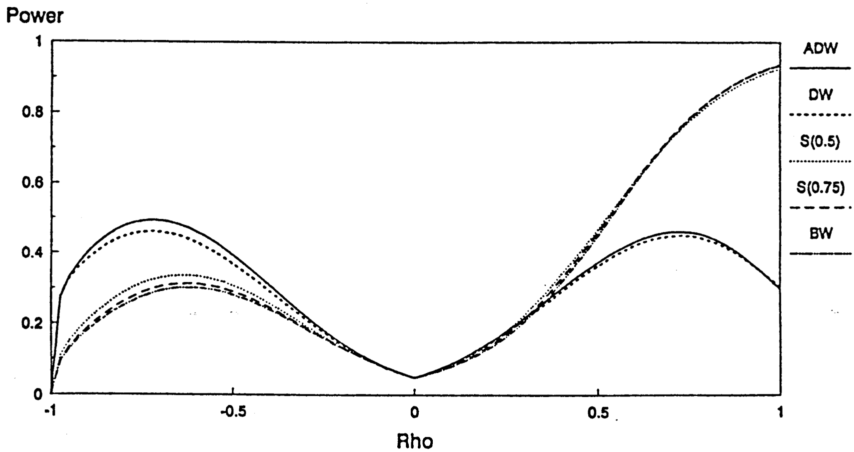
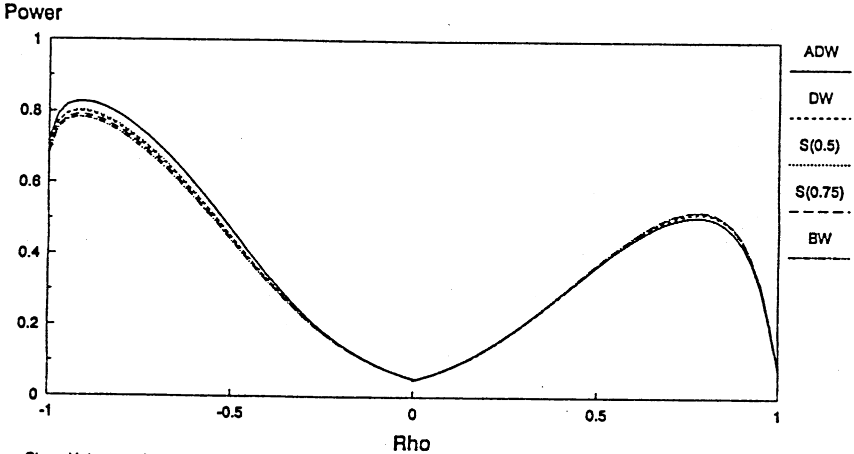
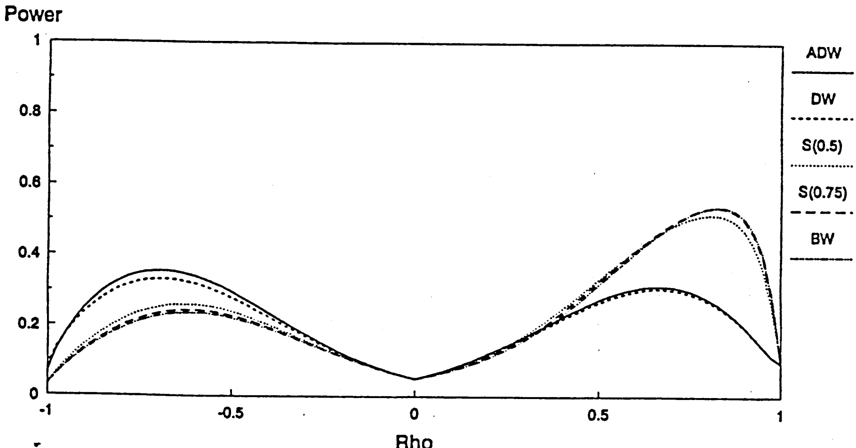


Figure 2a
Uniform Data; T=20
h = 1.5



Chow Heteroscedasticity

Figure 2b
Watson's Data; T=20
h = 1.5

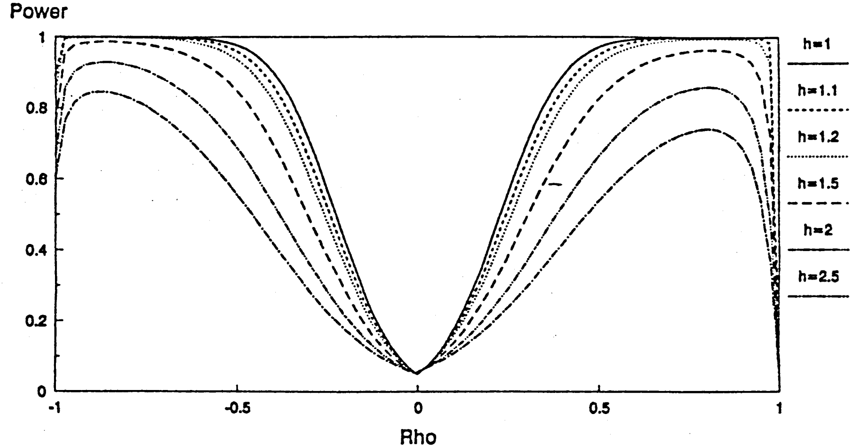


Chow Heteroscedasticity

Figure 3a

CPI Data; T = 60

DW Test

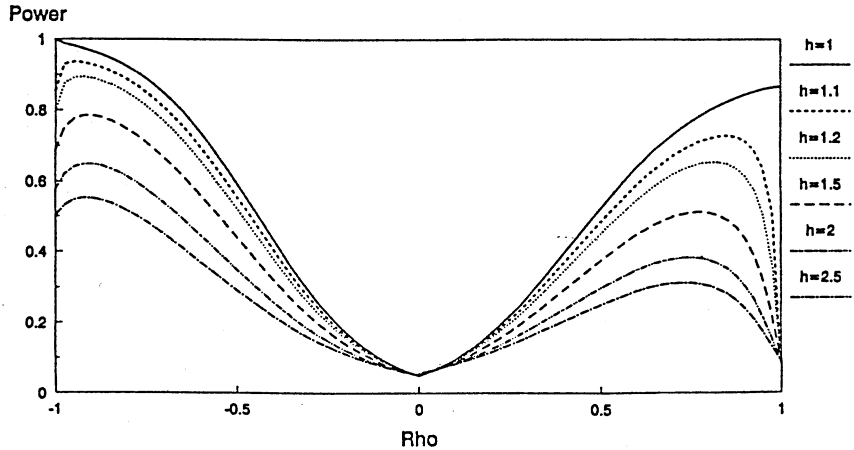


Additive Heteroscedasticity

Figure 3b

CPI Data; T = 20

DW Test

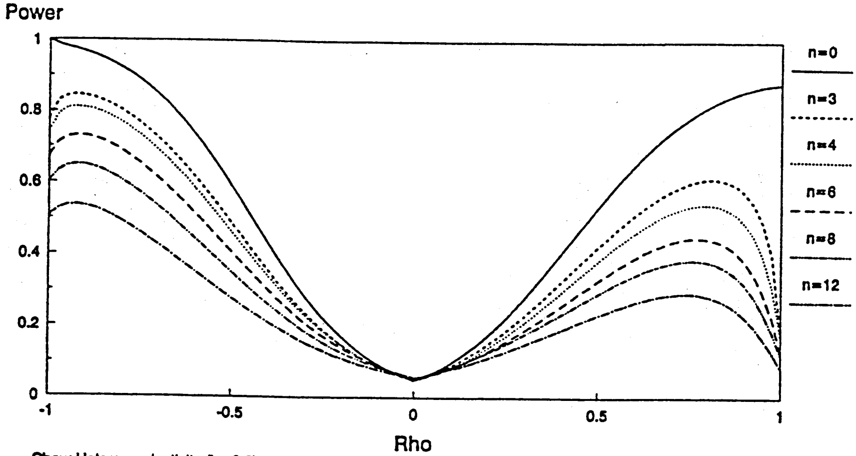


Additive Heteroscedasticity

Figure 4a

Spirits Data; T = 20

DW Test

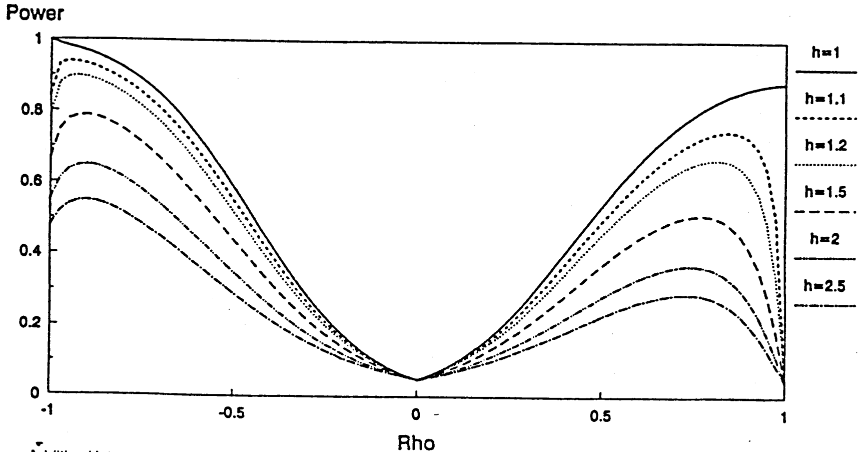


Chow Heteroscedasticity (h=2.5)

Figure 4b

Uniform Data; T = 20

S(0.5) Test



Additive Heteroscedasticity

APPENDIX 2

RESULTS USING
COVARIANCE MATRIX V^{**}

Table 4

Selected powers using V** with multiplicative heteroscedasticity

(a) Spirits data (X1)

	ADW		DW		S(0.5)		S(0.75)		BW	
Rho	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5
-1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.8	0.943	0.942	0.922	0.920	0.932	0.931	0.926	0.925	0.919	0.919
-0.6	0.764	0.764	0.737	0.734	0.749	0.748	0.740	0.739	0.727	0.728
-0.4	0.450	0.451	0.434	0.433	0.441	0.441	0.435	0.436	0.425	0.427
-0.2	0.177	0.178	0.174	0.174	0.175	0.176	0.173	0.175	0.171	0.173
0.0	0.050	0.051	0.050	0.050	0.050	0.051	0.050	0.051	0.050	0.051
0.0	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.049
0.2	0.177	0.173	0.174	0.171	0.176	0.173	0.175	0.172	0.171	0.167
0.4	0.407	0.401	0.405	0.400	0.409	0.403	0.407	0.401	0.398	0.391
0.6	0.649	0.643	0.653	0.647	0.656	0.650	0.656	0.650	0.648	0.641
0.8	0.810	0.805	0.818	0.813	0.819	0.814	0.821	0.816	0.819	0.813
1.0	0.874	1.000	0.883	1.000	0.884	1.000	0.888	1.000	0.892	1.000

(b) Normal data (X3)

	ADW		DW		S(0.5)		S(0.75)		BW	
Rho	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5	h=1	h=1.5
-1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.8	0.902	0.898	0.879	0.871	0.850	0.842	0.831	0.822	0.820	0.810
-0.6	0.709	0.702	0.680	0.670	0.662	0.653	0.646	0.635	0.637	0.626
-0.4	0.420	0.414	0.403	0.395	0.399	0.391	0.391	0.382	0.386	0.377
-0.2	0.171	0.168	0.168	0.163	0.168	0.164	0.166	0.161	0.165	0.160
0.0	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.049	0.050	0.048
0.0	0.050	0.050	0.050	0.051	0.050	0.051	0.050	0.051	0.050	0.052
0.2	0.173	0.172	0.170	0.173	0.172	0.174	0.171	0.173	0.169	0.173
0.4	0.400	0.399	0.399	0.401	0.405	0.407	0.403	0.406	0.400	0.404
0.6	0.646	0.643	0.651	0.653	0.659	0.659	0.658	0.660	0.656	0.659
0.8	0.808	0.806	0.819	0.820	0.824	0.824	0.826	0.827	0.826	0.827
1.0	0.867	0.000	0.881	0.000	0.883	0.000	0.885	0.000	0.886	0.000

Figure 5a

Normal Data; T = 20

h = 1.0

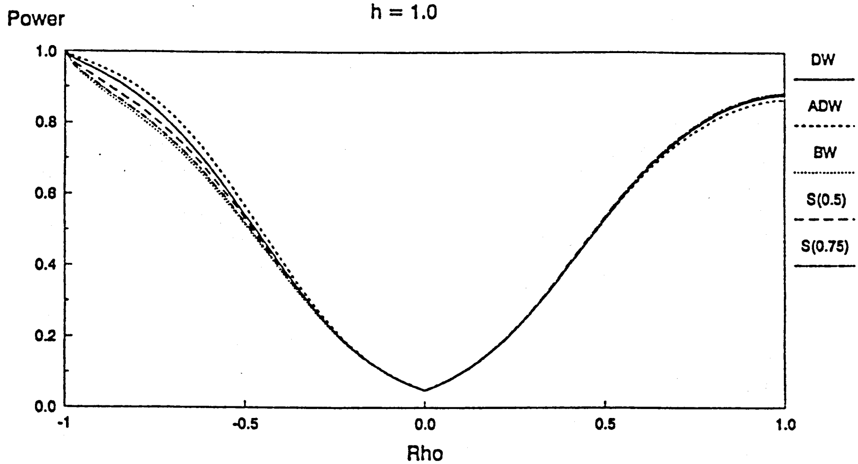
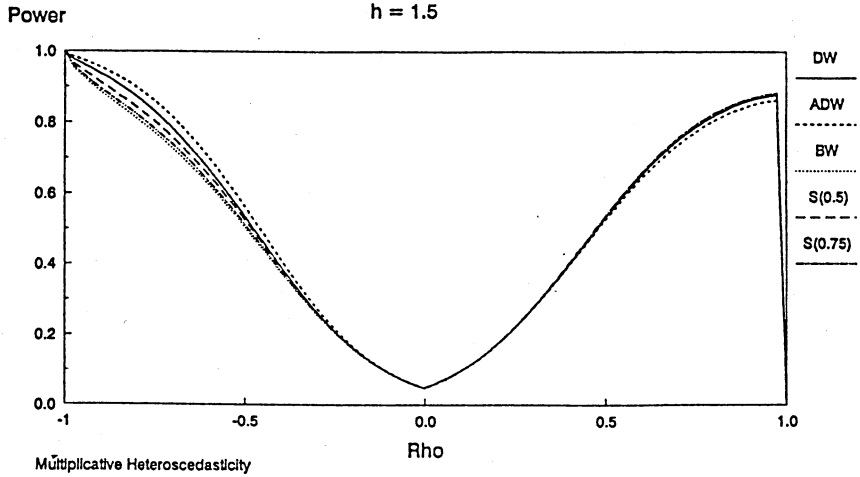


Figure 5b

Normal Data; T = 20

h = 1.5



Multiplicative Heteroscedasticity

Figure 6a
Spirits Data; T=20
DW Test

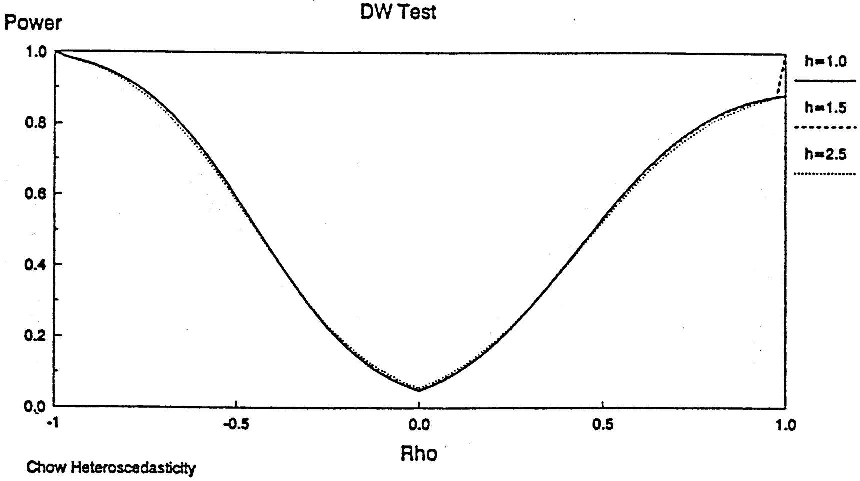
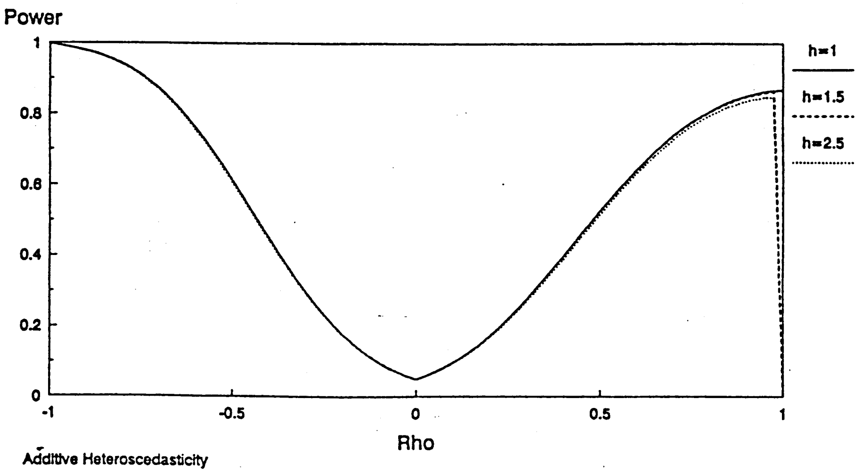


Figure 6b
Uniform Data; T=20
ADW Test



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