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PRICE INDICES : SYSTEMS ESTIMATION AND TESTS

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Abstract

A deterministic view of price indices offers no measure of index reliability, a shortcoming that can be overcome by using regression analysis. Recognising that Seemingly Unrelated Regressions estimation (rather than simple regression analysis) may be appropriate, more precise interval estimates of price indices can be computed. We show that the regression approach to price index construction has benefits beyond those previously discussed in the literature. Within this framework one can determine the appropriate estimation technique and test for significant changes in the price level. The statistical sensitivity of an index to the composition of the underlying "basket" of goods can also be tested.

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1. Introduction

The stochastic approach to index construction has been discussed by Allen (1975), Bannerjee (1975), Clements and Izan (1987) and Selvanathan (1989, 1991), among others. It has the merit of allowing the measurement of index value variability - standard errors for indices can be computed by regression methods. Interpreting a price index value as a regression coefficient also provides the basis for testing several interesting hypotheses. This is not possible if the index is viewed deterministically. We consider such issues and illustrate the potential of constructing price indices by regression methods.

2. Laspeyres and Paasche Indices

Let p_{it} and q_{it} be the price and quantity of the i'th commodity (group) at time t, with t = 0 denoting the base period. It can be shown (e.g. Selvanathan (1991)) that the Laspeyres price index at time t is the Weighted Least Squares (WLS) estimator of γ_t in

$$p_{it}q_{i0} = \gamma_t p_{i0}q_{i0} + \varepsilon_t \; ; \quad i = 1,\ldots,n \tag{1}$$

after allowing for heteroscedasticity of the form

$$\text{var.}(\varepsilon_{it}) = \sigma_t^2 p_{i0}q_{i0} \, . \tag{2}$$

That is, Ordinary Least Squares (OLS) is applied to

$$(p_{it}q_{i0}/\sqrt{p_{i0}q_{i0}}) = \gamma_t\sqrt{p_{i0}q_{i0}} + u_{it} \tag{3}$$

and

$$\hat{\gamma}_t = \left(\sum_{i=1}^{n} p_{it}q_{i0}\right)\Big/\left(\sum_{i=1}^{n} p_{i0}q_{i0}\right) . \tag{4}$$

The Paasche price index in period t is the WLS estimator of β_t in the model

$$p_{it}q_{it} = \beta_t p_{i0}q_{it} + \nu_{it} \; ; \qquad i = 1,\ldots,n \tag{5}$$

where $$\text{var.}(v_{it}) = \omega_t^2 p_{i0} q_{it} \quad . \tag{6}$$

That is, OLS is applied to the model

$$\left(p_{it}q_{it}/\sqrt{p_{i0}q_{it}}\right) = \beta_t\sqrt{p_{i0}q_{it}} + e_{it} \quad , \tag{7}$$

yielding $$\hat{\beta}_t = \left(\sum_{i=1}^{n} p_{it}q_{it}\right) \Big/ \left(\sum_{i=1}^{n} p_{i0}q_{it}\right) \quad . \tag{8}$$

This approach yields standard errors for each price index at time t, so that confidence intervals and tests are readily constructed. In practice $\hat{\gamma}_t$ and $\hat{\beta}_t$ can be computed if nominal and real expenditure data are available for each commodity (group).[1] This is illustrated in Table 1 with Australian data.[2]

3: Testing the Models' Formulation

The WLS interpretation of these indices facilitates tests of some interesting hypotheses. First, the specification of the underlying models themselves can be tested: Park's (1966) procedure[3] can be used to test if the errors in (1) and (5) satisfy (2) and (6) respectively. Table 1 reports illustrative results with the Australian data which support this form of heteroscedasticity, and hence the models underpinning the Laspeyres and Paasche indices, for these data. The standard errors reported in that Table also support the models - the price indices (regression coefficients) are significantly different from zero in all cases. The R^2 values are similar for both indices, though the quality of the models varies over time.

4. Systems Estimation

The likely covariance between index values over time suggests treating groups of equations of the form (3) (or (7)) as a Seemingly Unrelated Regressions (SUR) model, and jointly estimating a full price index series. In the Laspeyres case each equation has the same regressor (x_{i0}) so the SUR and single equation results coincide.[4] SUR estimation is still advantageous here as it permits tests of cross-equation restrictions. In the Paasche case the SUR and single equation results differ and there are efficiency gains with SUR if the joint error covariance matrix is non-diagonal.

Table 2 reports SUR results relating to nine years[5] of the Australian data. The Bruesch-Pagan LM statistics, which are Chi Square distributed with 36 degrees of freedom, are 258.28 (257.32) for the Laspeyres (Paasche) cases. This rejection of a diagonal covariance matrix supports the SUR framework, within which other interesting hypotheses can be tested. For example, the Wald tests of H_0: $\gamma_t = \gamma_{t-1}(\beta_t=\beta_{t-1})$ vs. H_A: $\gamma_t \neq \gamma_{t-1}(\beta_t \neq \beta_{t-1})$ are reported in Table 2, where we see that generally the observed annual price movement is statistically significant. The official Australian CPI is also reported[6] in Table 2. The Laspeyres and Paasche point estimates are all within one standard error of the corresponding CPI values.

5. The Composition of the Basket

The regressions underlying the price indices use sample observations across commodity groups, suggesting a simple way to test the sensitivity of the indices to the composition of the "basket" of goods. Any test for structural change in a regression model can be used to test if the indices change significantly (at any point in time) with the deletion of commodities from the "basket". Table 3 reports the results of applying the Wald test for such structural change, under SUR estimation, when one commodity group is deleted from the basket at a time. We see, for example, that the absence of one of the Food, Housing, Durables or Miscellaneous groups (groups 1,4,5,10) from the basket would have significantly changed the index values in certain of the years reported. Conversely, omitting any one of the other groups would not have had a significant effect on the indices.

The same analysis can be used to test sensitivity to the deletion of more than one commodity group. For example, although not reported in Table 2, if both of groups 3 and 7 (Clothing/Footwear, Transport) are deleted from the basket the Wald statistics[7] for structural change in the Laspeyres (Paasche) index range from 0.08 to 2.1 (0.08 to 2.2), so this deletion has no significant impact on the indices. The numerical impact is also slight. For example, the Laspeyres index based on the remaining eight groups ranges from 1.0055 in 1960 to 1.8682 in 1969, as compared with the first column of results in Table 2.

6. Conclusions

The benefits of a regression-based approach to price index construction have been explored in this paper, with an emphasis on the appropriate choice of estimator and tests of economic importance. Our results demonstrate the richness of this approach and suggest that there are substantial advantages in viewing price indices in a stochastic, rather than deterministic, framework.

Table 1

OLS Price Index Estimates and Related Statistics

(Base-Value = 1 in 1960)

	Laspeyres				Paasche			
Year	$\hat{\gamma}$	s.e.	Park	R^2	$\hat{\beta}$	s.e.	Park	R^2
1961	1.0040	0.0089	1.45	0.99	1.0039	0.0090	1.35	0.99
1962	1.0216	0.0167	1.01	0.98	1.0216	0.0168	0.95	0.98
1963	1.0273	0.0174	0.31	0.97	1.0264	0.0174	0.27	0.97
1964	1.0600	0.0193	0.20	0.97	1.0588	0.0195	-0.01	0.97
1965	1.0960	0.0241	0.98	0.96	1.0948	0.0243	0.62	0.95
1966	1.1283	0.0277	0.79	0.94	1.1282	0.0279	0.49	0.94
1967	1.1681	0.0324	0.63	0.93	1.1676	0.0328	0.30	0.92
1968	1.2052	0.0375	0.69	0.91	1.2043	0.0382	0.41	0.89
1969	1.2438	0.0407	-0.26	0.89	1.2432	0.0411	-0.34	0.88
1970	1.3178	0.0512	-0.23	0.85	1.3170	0.0509	-0.17	0.84
1971	1.4038	0.0605	-0.50	0.80	1.4059	0.0605	-0.40	0.79
1972	1.4932	0.0672	-0.56	0.77	1.4934	0.0679	-0.42	0.76
1973	1.6846	0.0791	-0.95	0.76	1.6725	0.0814	-0.74	0.72
1974	1.9731	0.1089	-0.85	0.63	1.9594	0.1069	-0.40	0.65
1975	2.2712	0.1339	-0.50	0.59	2.2526	0.1328	-0.07	0.62
1976	2.5387	0.1552	-0.61	0.59	2.5135	0.1554	-0.39	0.61
1977	2.7804	0.1720	-0.44	0.60	2.7563	0.1735	-0.03	0.63
1978	3.0349	0.1808	-0.26	0.65	3.0111	0.1872	0.19	0.66
1979	3.3455	0.1954	-0.26	0.68	3.3176	0.2032	0.30	0.69
1980	3.6680	0.2266	-0.30	0.65	3.6217	0.2335	0.23	0.66
1981	4.0051	0.2652	-0.32	0.60	3.9571	0.2747	0.31	0.62

Notes: Park's test statistic is t with 8 degrees of freedom (Two-sided 5% (1%) critical values are ± 2.306 (± 3.355).) Here, Park's test involves regressing the logarithm of the squared OLS residuals from (1) or (5) on the logarithm of the regressor and testing if the slope parameter is unity.

Table 2

SUR Price Index Estimates and Related Statistics

		Laspeyres			Paasche		
Year	CPI	$\hat{\gamma}$	s.e.	W	$\hat{\beta}$	s.e.	W
1961	1.0070	1.0040	0.0089	0.1987	1.0018	0.0052	0.1161
1962	1.0106	1.0216	0.0167	2.7189	1.0170	0.0093	5.7841
1963	1.0141	0.0273	0.0174	0.2904	1.0218	0.0073	0.3604
1964	1.0530	1.0600	0.0193	32.3227	1.0532	0.0064	57.3518
1965	1.0954	1.0960	0.0241	18.5553	1.0875	0.0071	34.5844
1966	1.1201	1.1283	0.0277	48.5713	1.1191	0.0072	201.4375
1967	1.1590	1.1681	0.0324	43.5692	1.1569	0.0085	200.0351
1968	1.1873	1.2052	0.0375	29.2602	1.1919	0.0091	170.9905
1969	1.2226	1.2438	0.0407	12.7364	1.2296	0.0127	22.7748

Notes: W = Wald Statistic for testing $\gamma_t = \gamma_{t-1}$ (or $\beta_t = \beta_{t-1}$). It is χ^2 distributed with 1 degree of freedom. (5% (1%) critical values are 3.84 (6.63).)

Table 3

Wald Statistics for Structural Change

Laspeyres

Commodity Group Omitted

	1	2	3	4	5	6	7	8	9	10
1961	10.89	0.06	0.07	5.53	0.00	1.28	0.39	0.00	0.03	0.14
1962	4.78	0.01	0.02	3.26	0.27	0.18	0.31	0.01	0.04	5.58
1963	1.55	0.01	0.01	13.71	0.97	0.83	0.88	0.00	0.28	0.20
1964	1.54	0.01	0.17	7.95	2.74	0.57	1.07	0.06	0.29	0.53
1965	0.46	0.85	0.50	4.22	3.28	0.77	1.14	0.08	0.15	0.29
1966	0.63	0.51	0.55	4.05	3.57	0.80	0.76	0.18	0.20	0.47
1967	0.60	0.42	0.66	2.59	4.03	1.05	0.62	0.30	0.18	0.82
1968	0.85	0.18	0.71	2.36	3.47	1.11	0.44	0.28	0.16	1.53
1969	0.83	0.15	0.68	0.52	3.40	2.18	0.28	0.52	0.26	2.22

Paasche

Commodity Group Omitted

	1	2	3	4	5	6	7	8	9	10
1961	14.07	0.72	0.85	6.08	0.23	1.29	0.39	0.74	0.39	0.15
1962	5.86	0.16	0.13	3.72	0.27	1.52	0.41	0.70	0.51	6.15
1963	1.82	0.59	0.59	15.47	0.99	0.73	1.11	0.17	0.32	0.22
1964	0.61	0.51	0.14	9.03	2.97	0.50	1.40	0.88	0.33	0.58
1965	0.54	0.83	0.44	5.12	3.52	0.71	1.48	0.12	0.19	0.33
1966	0.77	0.48	0.51	4.87	3.83	0.73	1.02	0.28	0.23	0.50
1967	0.70	0.39	0.59	3.14	4.48	0.92	0.89	0.44	0.22	0.87
1968	0.91	0.16	0.62	2.86	4.27	0.95	0.65	0.41	0.19	1.57
1969	0.87	0.12	0.58	0.71	4.32	1.78	0.42	0.75	0.29	2.33

Notes: The Wald Statistics are χ^2 distributed with 1 degree of freedom. (5% (1%) critical values are 3.84 (6.63).) Commodity groups are numbered according to the ordering in footnote 2.

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Footnotes

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1. From (4) and (8), these price indices can also be interpreted as Instrumental Variables regression coefficient estimators, obtained by regressing p_{it} on p_{i0}, with q_{i0} or q_{it} as the instrument. This interpretation is only useful if individual price and quantity data are available.
2. These data were also used by Selvanathan (1991). They relate to ten commodity groups - Food, Beverages/Tobacco , Clothing/Footwear, Housing, Durables, Medical, Transport/Communication, Recreation, Education, Miscellaneous. The SHAZAM package (White et al. (1990)) was used for all of our computations.
3. See, also, Gujarati (1988, 329-330).
4. For example, see Srivastava and Giles (1987, 17-18).
5. The number of years (equations) considered at once, is limited by the number of commodity groups (i.e., observations per equation). In our cases, n = 10. We have also considered other nine-year groups of equations and the results are available on request.
6. These figures are those published by the Australian Bureau of Statistics, re-based for comparison with the other index estimates.
7. In this case the Wald Statistics are still distributed as $\chi^2(1)$.

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