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ON AN INDEX OF POVERTY

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by

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Abstract. We propose a poverty measure that satisfies a number of properties that make it sensitive to the level of absolute deprivation of the poor. These properties are often violated by several poverty measures discussed in the literature. The measure corresponds to a Cobb-Douglas social welfare function which has a number of egalitarian features.

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1. Introduction

In his pioneering work, Sen (1976) suggested a poverty measure that introduced the inequality in the distribution of income among the poor as one of the basic ingredients in the measurement of poverty. Sen motivated this distributional consideration in a poverty measure from considerations of interpersonal equity: the poorer an individual, the more should the individual's poverty count in the aggregate measure. In Sen's measure, the notion of interpersonal equity is introduced via the procedure of rank order weighting of the shortfall of a person's income from the poverty line. This leads him to a poverty measure whose underlying measure of inequality corresponds to the Gini index of the income distribution of the poor.

While distributional considerations take note of the "relative deprivation" of the poor in a poverty measure, Sen (1979, 1983) emphasized elsewhere the need to conceptualise poverty also in terms of "absolute deprivation". Thus, in arguing that poverty cannot be viewed only as an issue of inequality, he writes:

> Relative deprivation cannot, however, be the only basis of judging poverty. A famine, for example, will be readily accepted as a case of acute poverty no matter what the relative standards are. Indeed, there is an irreducible core of "absolute deprivation" in the notion of poverty which translates reports of starvation, severe malnutrition and visible hardship into a diagnosis of poverty without waiting to ascertain first the relative picture. Thus the approach of relative deprivation supplements rather than supplants the analysis of poverty in terms of absolute dispossession. (Sen (1979, p.289))

In the recent literature on the aggregate measures of poverty, considerable attention has been paid to the relative deprivation aspect of the measurement of poverty both directly, as in the contributions of Takayama (1979) and

Clark, Hemming and Ulph (1981), and indirectly by considering the transfer-sensitivity properties of a poverty measure, as in the contributions of Thon (1979), Kakwani (1980) and Hagenaars (1987). However, in comparative terms, the absolute deprivation aspect of poverty has received much less attention in the literature.

In this paper, we focus attention on the absolute deprivation aspect of poverty in the measurement of poverty. We suggest a number of properties that a measure of poverty may be required to satisfy to capture this aspect of poverty. We then propose a poverty measure that is motivated by these requirements.

In the context of poverty measurement, it is natural to relate the absolute deprivation of a poor individual to the extent of his income shortfall from the poverty line. Thus, a preliminary requirement for a poverty measure to be responsive to the level of absolute deprivation is that the measure is sensitive to the income levels of the poor. Sen (1976) introduces this requirement in the form of the monotonicity axiom: given other things, an increase in the income shortfall of a person below the poverty line must increase the poverty measure. Taken together, the properties of a poverty measure that we suggest may be regarded as strengthening the monotonicity axiom, to introduce greater sensitivity of the measure to the income levels of the poor.[1]

The central notion that we wish to capture in measuring poverty is that absolute deprivation increases more than proportionately to the reduction in the income of the poor. We specify three properties of a poverty measure to capture this requirement. The first is that, given other things, the rate at which the poverty measure registers an increase as a result of a decrease in

the income of an individual below the poverty line increases as the individual becomes poorer. A related, though distinct, property of a poverty measure for the purpose at hand is the following. Let i and j be two poor individuals. Then, we require that, other things being the same, for a given reduction in i's income, the increase in j's income required to keep the level of poverty unchanged should increase with decreases in i's income. Finally, we require that, everything else being the same, given any two poor individuals, the elasticity of the poverty measure with respect to a specified change in the income should be greater for the relatively poorer individual.

If we relate these properties to those of a social welfare function underlying the poverty measure, then they correspond respectively to the properties of positive and decreasing marginal weights with respect to incomes, diminishing marginal rate of substitution between incomes, and greater elasticity with respect to lower incomes of the poor. The last property, in particular, implies that the social welfare function underlying the poverty measure is S-concave, and therefore, incorporates an equality-preferring notion of welfare.[2,3] This in turn implies that the social welfare function, and given the corresponding properties, the poverty measure itself will be sensitive to the distribution of income among the poor. Thus, while the properties we noted above are motivated by considerations of the absolute deprivation of the poor, they will also be responsive to the relative deprivation aspect of poverty, which has come to be recognized as one of the central concepts in measuring poverty following Sen (1976).[4]

As well as taking sharper note of the absolute deprivation of the poor, these properties, as general properties of a poverty index, seem to us to be

clearly reasonable. The first of our three properties corresponds directly to the notion that the adverse welfare impact of a reduction in the income of an individual increases more than proportionately as the individual becomes poorer.[5] The second property requires that, given any two poor individuals, the way the poverty measure compensates for the loss of income of one in terms of increased income for the other places progressively greater weight on the income level of the individual whose poverty is increasing. In a similar vein, the third property introduces a clear emphasis on the responsiveness of the poverty measure to the lower levels of income. If social welfare is taken to be utilitarian or additively separable, then, given cardinally measurable, interpersonally comparable individual welfare functions, these properties could be motivated by the notion of diminishing marginal utility. However, in the context of the measurement of poverty, if one intuitively accepts that the intensity of poverty increases disproportionately as the income shortfall of an individual gets larger, these properties seem to be clearly desirable properties of a poverty measure even without a cardinal framework with diminishing marginal utilities.

As Sen (1976) has noted, the ordinal weighting of incomes in his poverty measure via the rank order weights has the limitation that the marginal weights are not sensitive to the income levels as long as the income ranks of the individuals do not change. On the other hand, the foregoing properties require that these weights are sensitive to the income levels of the individuals. Essentially, this requires that the poverty measure corresponds to stronger strict concavity conditions than the property of strict S-concavity underlying the Sen index. In addition, both the poverty measure as well as a social welfare function underlying the measure should satisfy the appropriate measurability-comparability conditions associated with these

properties. The poverty measure we propose is an attempt to meet these requirements.

In the following section, we introduce the formal framework of our analysis, and show that a specific version of the last of our three properties, namely, greater elasticity with respect to the income of a relatively poorer person, together with some technical restrictions, leads to a unique poverty measure. In Section 3, we show that this measure satisfies the three "responsiveness" properties discussed above. We also show in this section that most of the poverty measures suggested in the literature do not satisfy one or more of these responsiveness properties. In Section 4, we show that our measure corresponds to a class of poverty measures suggested by Blackorby and Donaldson (1980). Drawing on their important work, we clarify the nature of the social welfare function implied by our measure. We conclude in Section 5.

2. Derivation of the Poverty Measure

Consider an economy $S = \{1,\ldots,n\}$ of n individuals. Let $y = (y_1,\ldots,y_n)$ be the vector of incomes of the individuals. For technical reasons, we assume that $y_i > 0$, $i = 1,\ldots,n$; since y_i can be arbitrarily small, this assumption does not entail any loss of generality.

Let z be the <u>poverty line</u>. We assume throughout that the set of individuals and the poverty line are fixed. Let $Q = \{i \in S | y_i < z\}$ be the set of the poor. Without loss of generality, we let $Q = \{1,\ldots,q\}$, $q \leq n$. Given y, $y_Q = (y_1,\ldots,y_q)$ denotes the income vector of the poor.[6]

In this paper, we are concerned with a class of poverty measures which Sen (1981) has called 'focused' poverty indices, whose values are independent of the incomes of the non-poor. In order to define this class, we use the notion of an aggregation function for the poor, or, simply, an aggregation function, which we define to be a real-valued function F on the set of income vectors of the poor. We shall assume that F is increasing in its arguments. A natural interpretation of F is that it corresponds to a social welfare function for the poor, but, in general, F can be any aggregate measure defined on the income vectors y_Q. We shall call $F(y_Q)$ an aggregate income index of the poor.

Given an aggregation function F, a poverty measure P for an income vector y_Q of the poor is a weighted shortfall of the aggregate income index of the poor from the poverty line:

$$P = A(n,z,q)\left[z - F(y_Q)\right] . \tag{1}$$

The intuition underlying the definition of a poverty index in (1) is similar to Sen's (1976) definition of poverty as a weighted aggregate shortfall of the income of the poor from the poverty line with the following difference however: in (1), we aggregate the incomes of the poor and then measure the shortfall of this aggregate from the poverty line, whereas in Sen (1976) the income shortfalls of the poor are measured first, and then aggregated. Note that, aside from defining poverty as an aggregate shortfall, very little is imposed on the form of a measure in (1). Indeed, since the form of F is unrestricted, (1) defines a very large class of poverty measures.

We now introduce a set of properties in the form of axioms for a poverty measure that will specify F and A(n,z,q) in (1), and lead to the poverty measure proposed in this paper.

Given y_Q, let r_i be the number associated with $i \in Q$ if y_i is the r_i-highest income in y_Q, ties broken arbitrarily. r_i is called the <u>rank of</u> $i \in Q$ <u>in</u> y_Q.

Axiom 1. Given other things, the elasticity of the aggregation function F with respect to a change in the level of income y_i, $i \in Q$, is proportional to the rank of i in y_Q. The proportionality factor, while identical for each $i \in Q$, is independent of all y_j, $j \in Q$.

Axiom 1 is intended to capture the notion that the elasticity of the poverty measure is greater with respect to a change in the income of a relatively poorer individual. Since, given n, z and q, the poverty measure is a monotone transform of the aggregation function, specifying the aggregation function to capture the requirement builds it into the poverty measure. The way the requirement is specified for F - that the elasticity of F with respect to a poor individual's income is proportional to the corresponding income rank - is, of course, arbitrary. The justification for it must lie in the fact that the requirement of greater proportional response of the poverty measure with respect to lower incomes is an ordinal one, and any set of numbers that decrease with income will suffice to capture the requirement. We follow Sen (1976) in using the ordinal ranks since they capture the normative aspect of the relative positions of the poor, and reflect the fact that these are, essentially, welfare weights. As well, they can be defined in a straightforward way given the income vector of the poor.[7]

Our next axiom is a normalization axiom.

Axiom 2. If for all $i \in Q$, $y_i \to z$, then $P \to 0$; if for all $i \in Q$, $y_i \to 0$, then $P \to q/n$.

Axiom 2 requires that when the incomes of all the poor individuals approach z, the poverty measure approaches the value 0; and when they all approach 0, the measure approaches the "head-count ratio" q/n. Note that the second part of the axiom implies that if all the individuals in the economy are poor, and if their incomes all approach 0, then the poverty measure approaches the value 1. Thus Axiom 2 essentially introduces a zero-one normalization of the poverty measure.[8]

Our last two axioms are a homogeneity and a continuity restriction on the poverty measure.

Axiom 3. P is homogeneous of degree zero in z and y_Q.

Axiom 4. P is continuous.

Axiom 3 is a convenient property of a poverty measure, since it makes the measure independent of the dimensions of income. Axiom 4 requires that the poverty measure varies continuously with the incomes of the poor, and is clearly an unexceptionable requirement.[9]

The following theorem shows that Axioms 1, 2, 3 and 4 uniquely characterize a poverty measure.[10]

Theorem 1. Axioms 1, 2, 3 and 4 characterize the following poverty measure:

$$P = \frac{q}{nz}\left(z - \prod_{i \in Q} y_i^{k_i}\right) \tag{2}$$

where $k_i = r_i / \sum_i r_i$, $i \in Q$.

Proof. It is clear that the poverty measure P given by (2) satisfies Axioms 1, 2, 3 and 4. We have only to show that if a poverty measure in (1) satisfies these axioms, then it has the form given by (2).

Let $\overline{P}$ be a poverty measure in (1) satisfying Axioms 1-4. It is sufficient to show that $\overline{P} = P$ in the domain $D = \{y_Q | y_i \neq y_j$ for all $i,j \in Q$, $i \neq j\}$. For, the complement of D in the domain of $\overline{P}$ is the set $D' = \{y_Q | y_i = y_j$ for some $i,j \in Q$, $i \neq j\}$, and $\overline{D} = D \cup D'$ is the closure of D. Since $\overline{P}$ and P are both continuous functions by Axiom 4, if $\overline{P} = P$ in D, it will follow from a standard argument for continuous functions that $\overline{P} = P$ in $\overline{D}$. In the remainder of the proof, we therefore consider the set D in the domain of $\overline{P}$.

Let $y_Q = (y_1,\ldots,y_q) \in D$. It is clear that, for each $i \in Q$, for some real number $\varepsilon > 0$, given any change in income from y_i to $y_i + h$, $|h| \leq \varepsilon$, the ranks r_i will remain unchanged for each i. Then, by Axiom 1, we have:

$$\frac{\Delta \ell n F_i}{\Delta \ell n y_i} = \alpha r_i, \quad i = 1,\ldots,q, \tag{3}$$

where $\Delta \ell n F_i = \ell n F(y_1,\ldots,y_i + h,\ldots,y_q) - \ell n F(y_Q)$,

and $\Delta \ell n y_i = \ell n(y_i + h) - \ell n y_i$,

and where α is independent of y_i, $i \in Q$. Letting $h \to 0$, and noting that the right-hand side of (3) is constant for each i in the process of taking this limit, it is clear from (3) that $\ell n F$ is differentiable with respect to $\ell n y_Q$ at y_Q, where $\ell n y_Q = (\ell n y_1,\ldots,\ell n y_q)$. Hence, restricting attention to an appropriate neighbourhood of y_Q, we have:

$$\frac{\partial \ell n F}{\partial \ell n y_i} = \alpha r_i \quad \text{for all } i \in Q\ , \tag{4}$$

where α is independent of y_i, $i \in Q$. (4) implies that $\ell n F$ is linear in $\ell n y_i$, $i \in Q$, and has the form:

$$\ell nF = \sum_{i \in Q} \alpha r_i \ell n y_i + \gamma \ , \tag{5}$$

where γ is independent of y_i, $i \in Q$. By exponentiation of both sides of (5) and by letting $\beta = e^{\gamma}$ we get:

$$F = \beta \prod_{i \in Q} y_i^{\alpha r_i} \ . \tag{6}$$

In view of Axiom 2, letting $y_i \to 0$, $i \in Q$, we get from (1) and (6):

$$A(n,z,q) = q/nz \ . \tag{7}$$

By Axiom 3, P is homogeneous of degree zero in z and y_Q. Given (1) and (7), this implies that F is homogeneous of degree one in y_Q. Hence from (6) we get:

$$\alpha = 1/ \sum_{i \in Q} r_i \ . \tag{8}$$

Now, letting $y_i \to z$, $i \in Q$, and using Axiom 2, we have from (1), (6), (7) and (8),

$$\beta = 1 \ . \tag{9}$$

It follows from (1), (6), (7), (8) and (9) that $\overline{P} = P$ in D. This completes the proof. □

3. Some Properties of Poverty Measures

In this section, we verify that the poverty measure given by (2) satisfies the properties we discussed in Section 1. We also note some of the poverty indices which have been proposed in the literature, and examine how far they satisfy these properties.

Two of the basic requirements for a poverty measure are the axioms of Monotonicity and Transfer:

Monotonicity Axiom (M): Given other things, a reduction in the income of a poor individual must increase poverty.

Transfer Axiom (T): Given other things, a transfer of income from a poor individual to a relatively less poor individual must increase poverty, provided the number of the poor individuals in the economy does not change as a result of the transfer.[11]

Suppose a poverty measure satisfies axioms M and T. For such a poverty measure, we now formulate a set of "responsiveness" conditions along the lines suggested in Section 1.

Responsiveness Axiom 1 (R_1): Given everything else, the rate at which the poverty measure registers an increase with respect to a reduction in the income of a poor individual increases with a decrease in the income of the individual.

Responsiveness Axiom 2 (R_2): Given other things, for any two poor individuals i and j, for a given reduction in the income of i, the increase in j's income required to keep the poverty level unchanged increases with decreases in i's income.

Responsiveness Axiom 3 (R_3): Given other things, for any two poor individuals i and j, if i is poorer than j, then the elasticity of the poverty measure for a specified change in the income is greater with respect to i's income than that with respect to j's income.

We now verify that our poverty index satisfies M, T and R_1-R_3. Since the index is decreasing in the income of each poor individual, M holds. Since the index is symmetric and strictly quasi-convex in these incomes, T also holds.[12] Strict convexity of the measure in each y_i ensures that R_1 is satisfied. Strict quasi-convexity of the poverty measure in the incomes of the poor, together with the fact that the measure is decreasing in each y_i, ensures the fulfillment of R_2. Finally, given Axiom 1, R_3 is satisfied since the poverty measure is a monotone transform of its associated aggregation function.

We now recall several poverty measures, including that of Sen (1976), which have been proposed in the literature. We shall show that each of these measures violates at least one of our three responsiveness axioms.

In stating these measures, it will be convenient to index the individuals in a decreasing order of the incomes:

$$y_1 \leq y_2 \leq \ldots \leq y_q \leq \ldots . \leq y_n .$$

Consider the following measures of poverty.

P^1 Sen (1976): $$\frac{2}{(q+1)nz} \sum_{i=1}^{q} (z - y_i)(q + 1 - i)$$

P^2 Thon (1979): $$\frac{2}{(n+1)nz} \sum_{i=1}^{q} (z - y_i)(n + 1 - i)$$

P^3 Takayama (1979): $$1 + \frac{1}{n} - \frac{2}{n^2\mu} \sum_{i=1}^{n} y_i^*(n + 1 - i)$$

where $\mu = (1/n) \sum_{i=1}^{n} y_i^*$, and y_i^* is the "censored" income of individual i,

i.e., $y_i^* = z$ if $y_i \geq z$ and $y_i^* = y_i$ if $y_i < z$.

P^4 Kakwani (1980): $$\frac{q}{\left(\sum_{i=1}^{q} i^k\right) nz} \sum_{i=1}^{q} (z - y_i)(q + 1 - i)^k, \; k \geq 1.$$

P^5 Clark, Hemming and Ulph (1981):

$$\frac{q}{nz}\left[\frac{1}{q}\sum_{i=1}^{q} (z - y_i)^\alpha\right]^{1/\alpha}, \; \alpha \geq 1.$$

P^6 Foster, Greer and Thorbecke (1984):

$$\frac{1}{n}\sum_{i=1}^{q}\left(\frac{z - y_i}{z}\right)^\alpha, \; \alpha \geq 0.$$

P^7 Hagenaars (1987): $$1 - \frac{\prod_{i=1}^{q} y_i^{1/n}}{z^{q/n}}.$$

Barring two exceptional cases, all these measures satisfy M and T. The first exceptional case is the violation of M by the Takayama (1979) measure P^3. The second arises in the context of P^5 (the Clark-Hemming-Ulph measures) where $\alpha = 1$, and P^6 (The Foster-Greer-Thorbecke measures) where $\alpha \leq 1$. For these respective values of α, P^5 and P^6 will violate the transfer axiom. In what follows, when discussing P^5 and P^6, we restrict our attention to the cases where $\alpha > 1$.

It is clear that P^1, P^2, P^3 and P^4, being linear in each y_i, will violate R_1. Also, the marginal rate of substitution between any two incomes is independent of the incomes in these measures, and thus each of these measures will violate R_2. Finally, it can be verified that they will all violate R_3. For example, at any point where $y_i \neq y_j$ for all $i,j \in Q$, the elasticity of the Sen measure P^1 with respect to y_i is given by

$$\frac{2y_i(q + 1 - i)}{(q + 1)nzP^1}.$$

Since if $y_i < y_j$, $(q + 1 - i) > (q + 1 - j)$, and hence it is clear that P^1 may violate R_3.

For $\alpha > 1$, the Clark-Hemming-Ulph measures satisfy R_1 and R_2. This follows since, for $\alpha > 1$, P^5 is strictly convex and decreasing in each y_i (thus satisfying R_1); also, it is an increasing transformation of a function which is strictly convex and decreasing in the incomes of the poor, being given by a sum of functions which are strictly convex and decreasing in the corresponding y_i (hence R_2 holds). However, these measures will violate R_3. To see this, note that the elasticity of P^5 with respect to y_i is given by

$$\frac{y_i(z - y_i)^{\alpha-1}}{\sum_{j=1}^{q} (z - y_j)^{\alpha}}.$$

Clearly, P^5 may violate R_3.[13]

Finally, for $\alpha > 1$, the class of Foster-Greer-Thorbecke measures P^6 and the Hagenaars measure P^7 fare exactly as the class of Clark-Hemming-Ulph measures. These measures are strictly convex and decreasing in the incomes of the poor, as well as being strictly convex and decreasing in each y_i, and will satisfy R_1 and R_2. However, the elasticities of these measures are given respectively by

$$\frac{\alpha y_i(z - y_i)^{\alpha-1}}{nz^{\alpha}P^6} \quad \text{and} \quad \frac{\prod_{i=1}^{q} y_i^{1/n}}{nz^{q/n}P^7}. \tag{10}$$

It is clear that P^6 and P^7 will violate R_3.

4. Welfare Implications of the Poverty Measure

The poverty measure we derived in Section 2 can be rewritten as:

$$P = \frac{q}{n}\left(\frac{z - \prod_{i \in Q} y_i^{k_i}}{z}\right) \tag{10}$$

Thus, the poverty measure can be simply interpreted as the percentage of the shortfall of the aggregate income index of the poor from the poverty line income, weighted by the head-count ratio. As discussed below, the term in the square brackets in (10) can be related to the family of inequality indices due to Atkinson (1970), Kolm (1976) and Sen (1973).

Though we started with a different view of the problem, it turns out that our measure is a member of a general class of poverty measures proposed by Blackorby and Donaldson (1980). This class of poverty measures is given by:

$$\mathcal{P} = f\left(\frac{q}{n}, \frac{z - \xi(y_Q)}{z}\right), \tag{11}$$

where $\xi(y_Q)$ is the Atkinson (1970)-Kolm (1969)-Sen (1973) "equally distributed equivalent" (ede) income restricted to the incomes of the poor, as measured by a homothetic social welfare function. If we require $\mathcal{P}$ to be homogeneous of degree one in each of its arguments and normalize so that $f(1,1) = 1$, then the class of measures given by (11) becomes:

$$\mathcal{P} = \frac{q}{n}\left(\frac{z - \xi(y_Q)}{z}\right). \tag{12}$$

Blackorby and Donaldson (1980) show that Sen's (1976) poverty measure is a member of the class defined by (12), where the social welfare function for

evaluating $\xi(y_Q)$ is the Gini social welfare function. It is clear that the class of measures defined by (12) is a subclass of that defined by (1): with $F = \xi$, every measure in (12) is also a measure in (1), but the converse is, of course, not necessarily true.

Since $F(y_Q)$ in (2) can be interpreted as the ede income of the poor for y_Q given by F,[14] one sees that the poverty measure (2) is a member of the Blackorby-Donaldson class of measures (12), and that the social welfare function underlying (2) is a Cobb-Douglas welfare function. The welfare properties of this class of functions have been closely investigated by Blackorby and Donaldson (1978). They find that the functions have some very attractive ethical features, a particularly appealing property being that "if the distribution of income is very skewed, then improving the distribution among those who are not poor has little impact on social welfare" (1978, p.75). Furthermore, a welfare function in this class has "the advantage of being "Rawlsian" when any income is small and approach[es] the arithmetic mean when incomes are close to equality" (1978, p.79).

Note also that the Cobb-Douglas function in (2) belongs to a non-additive-separable class of social welfare functions. Among the poverty measures we have discussed in Section 3, the Clark-Hemming-Ulph measures P^5, the Foster-Greer-Thorbecke measures P^6, and the Hagenaars measure P^7 are based on an underlying social welfare function which is additive separable in the individual welfares. Sen (1973), in particular, has argued against additive separability as a basis for welfare judgements, for it makes the social valuation of the welfare of an individual independent of the welfare levels of others. Consequently, social welfare judgements based on a notion

of additive separability cannot properly reflect such concerns as the relative deprivation aspect of the individual welfares.

We turn now to the measurability/comparability aspects of the measure. It is clear that the poverty measure (2), and, more generally, a measure in the class given by (12), can be interpreted as a cardinal index: Since $F(y_Q)$ gives the ede income of the poor with respect to y_Q, the poverty index in (2) measures the percentage shortfall of the ede income of the poor from the poverty line income. Furthermore, as has been shown by Blackorby and Donaldson (1982) and Roberts (1980), the Cobb-Douglas function in (2) corresponds to the requirement of ratio-scale comparability on the individual welfare functions. Given the cardinal significance of the measure, it becomes possible to compare, e.g., the percentage changes in the index; similarly, with the ratio-scale comparability of the individual welfare functions, interpersonal comparisons of the percentage changes in utility can be made. These are precisely the requirements that correspond to the type of comparisons involved in the responsiveness properties of our poverty measure.

We finally note that, while the poverty measure (2) is not "decomposable", its associated aggregation function, i.e., the Cobb-Douglas function, has some very attractive aggregation-consistency properties, as shown by Blackorby and Donaldson (1978). In particular, they have shown that it is one of the two functions that satisfies the following aggregation-consistency property: for any arbitrary partition of a population group into m subgroups, the ede income computed from the original income distribution in terms of the function equals the ede income computed from the vector of ede incomes for the m subgroups, in terms of a function having the same functional form. Thus, if we construct a set of subindices

of poverty for any arbitrary m sub- groups of the poor, then we can use the ede incomes computed for these indices to define the aggregate ede incomes for the group, and the corresponding aggregate index of poverty, using an appropriate Cobb-Douglas welfare function to compute the ede incomes. For empirical applications of the poverty index, this aggregation-consistency property is clearly of importance.

5. Concluding Remarks

In this paper, we have put forward a measure of poverty. The measure satisfies several plausible "responsiveness" properties that introduce greater sensitivity to the level of absolute deprivation of the poor. A responsiveness axiom together with some technical restrictions was used to characterize the measure. The measure corresponds to a Cobb-Douglas welfare function which possesses a strong equality-preferring bias, and lends it a considerable amount of intuitive appeal.

NOTES

1. This may be contrasted with the approach taken in a number of contributions cited above, which attempt to strengthen the "transfer axiom" suggested by Sen (1976).

2. Given symmetry of the social welfare function, which is implied by the statement of these properties, S-concavity of the function follows from a result in Berge (1963, pp.184-87), since the elasticity property requires that the marginal weights are greater for lower incomes, and this implies that a progressive transfer of income must increase welfare. For a discussion and a general statement of the result in Berge - due to Hardy, Littlewood and Polya - see Dasgupta, Sen and Starrett (1973).

3. An S-concave social welfare function (defined on individual incomes) is equality-preferring in the sense that, for any distribution, a distribution obtained by a convex combination of the income levels of the given distribution is ranked no worse. This is equivalent to the requirement that unambiguous increases in the inequality in the distributions - as judged by Lorenz rankings - do not increase welfare. See Dasgupta, Sen and Starrett (1973), and Sen (1973).

4. It should be noted, however, that transfer-related properties can also be motivated by considerations of absolute deprivation. However, in so far as these properties, in general, do not take note of the variation in the income of an individual except in relation to that of another individual, it is clear that their motivation is rooted primarily in a notion of relative, rather than that of absolute, deprivation.

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3. An S-concave social welfare function (defined on individual incomes) is equality-preferring in the sense that, for any distribution, a distribution obtained by a convex combination of the income levels of the given distribution is ranked no worse. This is equivalent to the requirement that unambiguous increases in the inequality in the distributions - as judged by Lorenz rankings - do not increase welfare. See Dasgupta, Sen and Starrett (1973), and Sen (1973).

4. It should be noted, however, that transfer-related properties can also be motivated by considerations of absolute deprivation. However, in so far as these properties, in general, do not take note of the variation in the income of an individual except in relation to that of another individual, it is clear that their motivation is rooted primarily in a notion of relative, rather than that of absolute, deprivation.

5. Atkinson (1987) quotes Watts (1968) who noted that "poverty becomes more severe at an increasing rate", essentially the same requirement as the present one.

6. Note that, in defining the poor, we have used the weak definition (a poor individual is one with an income less than the poverty line income), whereas Sen (1976) uses the strong definition (individuals having incomes no greater than the poverty line income are considered poor). For various implications of these two definitions for poverty measurement, see Donaldson and Weymark (1986).

7. For the normative considerations underlying the use of rank orders to construct weights - one that goes back to Borda (1781) - see Sen (1974, 1976). See also Young (1974).

8. Note that Axiom 2 is implied by, but does not imply, Sen's (1976) normalization axiom.

9. Note that, if a focused poverty measure is defined as a real-valued function on the set of income vectors y, then it may fail to be continuous in its entire domain. In particular, it may be discontinuous at any point where some individual has exactly the poverty line income. However, in this framework, the appeal of continuity for the poverty measure at all points where no individual has the poverty line income is clear enough. The poverty measure we characterize below can be derived in this framework using this weaker continuity assumption, along with Axioms 1-3. Similarly, Axioms 1-3, together with a somewhat more specific continuity requirement - that the poverty measure is continuous at all points where the income of each individual is no greater than the poverty line income - will characterize the poverty measure when the strong definition of the poor is used.

10. In proving the theorem, we assume that the aggregation function $F > 0$, given $y_i > 0$, $i = 1,\ldots,q$. Given that F is increasing, this normalization of F can be shown to be implied by Axioms 1 and 4.

11. Note that our statement of this axiom corresponds to a weaker version of the transfer axiom in Sen (1976): "Given other things, a pure transfer of income from a person below the poverty line to anyone who is richer must increase the poverty measure". Moreover, although formulated in terms of a transfer from a relatively poorer to a relatively richer individual, one can state the axiom equivalently in terms of a transfer from a relatively richer to a relatively poorer person. For a discussion of these and other related axioms, see Donaldson and Weymark (1986).

12. The poverty measure is strictly quasi-convex, since the Cobb-Douglas function in (2) is strictly quasi-concave. In its differentiable domain, strict quasi-concavity of the function can be verified by using, e.g., the Arrow-Enthoven conditions. Extension of strict quasi-concavity to the entire domain follows from the continuity of the function given by Axiom 4. Symmetry of the poverty measure is implied by the specification of Axiom 1. Given symmetry and strict quasi-convexity of the index, satisfaction of the transfer axiom follows from the theorems of Dasgupta, Sen and Starrett (1973, Theorem 1) and Rothschild and Stiglitz (1973, Theorem 1).

13. We do not discuss here a second measure of poverty proposed by Clark, Hemming and Ulph (1981), which can be also shown not to satisfy all three of our responsiveness properties.

14. This follows from the fact that F is homogeneous of degree one and normalized so that $F(1,\ldots,1) = 1$. See Blackorby and Donaldson (1980).

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