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PRELIMINARY-TEST ESTIMATION OF THE REGRESSION SCALE PARAMETER WHEN THE LOSS FUNCTION IS ASYMMETRIC

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ABSTRACT

Various studies have considered the risk properties under quadratic loss, of estimators of the scale parameter after a preliminary test for exact linear restrictions on the regression coefficients. This loss function is symmetric though, arguably, under-estimation of the scale has greater consequences than over-estimation. In this paper we consider the LINEX loss function, which allows for an asymmetric penalty. We derive the exact risk of estimators of the error variance after a pre-test of exact restrictions and we numerically evaluate the derived expressions. The results are compared with those under quadratic loss so that the effects of the asymmetry can be ascertained.

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1. Introduction

There is a well-established literature relating to the estimation of the coefficient vector in the linear regression model after a preliminary test of the validity of exact linear restrictions on this vector. Useful summaries are given by Judge and Bock (1978, 1983), for example. More recently, there has been interest in the estimation of the scale parameter in this model after the same preliminary test (e.g., Clarke *et al.* (1987a,b), Ohtani (1988), Gelfand and Dey (1988a,b), Clarke (1990), Giles (1990, 1991), Giles and Lieberman (1991)).

The literature on pre-test estimation to date centers on risk under quadratic loss. When estimating the scale parameter of a regression model, a quadratic loss structure may be unduly restrictive. In particular, the estimation of this parameter (in the form of either the variance or the standard deviation of the regression errors) is needed for the construction of "standard errors", confidence intervals, and test statistics. Under-estimation of the scale parameter is, arguably, of greater consequence than its over-estimation. The former situation results in the reporting of standard errors which are unduly optimistic with respect to the precision of the coefficient estimates, and t-statistics which tend to be distorted in favour of "significant" results, other things equal. Accordingly, it may be preferable to consider a pre-test estimator of the regression model's scale parameter which is based on an asymmetric loss function.

It is well known that the choice of loss function can affect estimator rankings (e.g., Hirano (1973)). Recently, Srivastava and Rao (1990) have considered the use of the (asymmetric) LINEX loss function (e.g., Varian (1975), Zellner (1986)) in the context of estimating the error variance of the normal linear regression model. This paper combines their analysis with that of Clarke *et al.* (1987b) in considering the estimation of this parameter under LINEX loss, after a pre-test of exact restrictions on the model's coefficients.

Section 2 discusses the model and notation. The risk functions of the pre-test, the unrestricted, and the restricted estimators of the error variance are derived in Section 3; and Section 4 evaluates and discusses these risks in comparison with their counterparts based on a quadratic loss structure. Some concluding remarks appear in Section 5.

2. The Estimation Problem

Consider the regression model,

$$y = X\beta + u \quad ; \quad u \sim N(0,\sigma^2 I_T) \tag{1}$$

where y and u are (T×1); X is (T×k), non-stochastic and of rank k; and β is (k×1).

Also, consider m (≤k) independent linear restrictions on β, given by $R\beta = r$, where R is (m×k) of rank m; r is (m×1); and both R and r are non-stochastic. Applying Ordinary Least Squares estimation to (1) yields

$$\tilde{\beta} = (X'X)^{-1}X'y$$

with associated residual vector,

$$\tilde{u} = y - X\tilde{\beta} \ .$$

Imposing the m restrictions on β and applying Restricted Least Squares estimation to (1) yields

$$\beta^* = \tilde{\beta} + (X'X)^{-1}R'\left[R(X'X)^{-1}R'\right]^{-1}(r-R\tilde{\beta}),$$

with associated residual vector,

$$u^* = y - X\beta^* \ .$$

The uniformly most powerful invariant size - α test of

$$H_0: R\beta = r \quad \underline{\text{vs}} \quad H_A: R\beta \neq r$$

rejects H_0 when $f > c(\alpha)$, where

$$f = \left[(u^{*\prime}u^{*} - \tilde{u}'\tilde{u})/\tilde{u}'\tilde{u}\right]\left[\nu/m\right] ,$$

$$\nu = T - k,$$

and $c(\alpha)$ satisfies $\int_0^{c(\alpha)} dF_{(m,\nu)} = 1 - \alpha$, where $F_{(m,\nu)}$ denotes the central F-distribution with m and ν degrees of freedom. The statistic f follows this distribution if H_0 is true, and under H_A it is distributed as non-central F ($F'_{(m,\nu;\lambda)}$) with these degrees of freedom and non-centrality parameter

$$\lambda = (r-R\beta)'\left[R(X'X)^{-1}R'\right]^{-1}(r-R\beta)/2\sigma^2 \quad . \tag{2}$$

The pre-test estimator of β referred to in Section 1 is

$$\hat{\beta} = \begin{cases} \tilde{\beta} & ; \quad f > c(\alpha) \\ \beta^* & ; \quad f \le c(\alpha) \end{cases}$$

and the associated pre-test estimator of σ^2 is

$$\hat{\sigma}^2 = \begin{cases} \tilde{\sigma}^2 & ; \quad f > c(\alpha) \\ \sigma^{*2} & ; \quad f \le c(\alpha) \end{cases} .$$

Clarke *et al.* (1987a) consider the risk of $\hat{\sigma}^2$, under quadratic loss, when $\tilde{\sigma}^2$ and σ^{*2} are the unrestricted and restricted maximum likelihood estimators of σ^2. This analysis is generalised by Clarke *et al.* (1987b) to the case where $\hat{\sigma}^2$ is based on component estimators from the families,

$$\tilde{\sigma}^2 = \tilde{u}'\tilde{u}/(\nu+\gamma) \tag{3}$$

$$\sigma^{*2} = u^{*\prime}u^{*}/(\nu+m+\delta) . \tag{4}$$

Maximum likelihood (ML) component estimators correspond to $\gamma = k$, $\delta = k - m$; least squares (LS) component estimators correspond to $\gamma = \delta = 0$; and minimum Mean Squared Error (MSE) component estimators[1] correspond to $\gamma = \delta = 2$.

Srivastava and Rao (1990) derive the risk of $\tilde{\sigma}^2$ in (2) under LINEX loss.

This loss function is of the form[2]

$$L(\tilde{\varepsilon}) = \exp(a\tilde{\varepsilon}) - a\tilde{\varepsilon} - 1 , \tag{5}$$

where

$$\tilde{\varepsilon} = (\tilde{\sigma}^2 - \sigma^2)/\sigma^2 \tag{6}$$

is the relative estimation error, and the parameter 'a' determines the asymmetry of $L(\tilde{\varepsilon})$ about the origin. If 'a' is small enough for $a^j \approx 0$ $(j \geq 3)$, then $L(\tilde{\varepsilon})$ is approximately quadratic. The risk of $\tilde{\sigma}^2$ is $R(\tilde{\sigma}^2) = E[L(\tilde{\varepsilon})]$.

The same approach can be taken with respect to the risks of σ^{*2} and $\hat{\sigma}^2$. Defining ε^* and $\hat{\varepsilon}$ analogously to $\tilde{\varepsilon}$ in (6), and substituting into (5), we can define $R(\sigma^{*2})$ and $R(\hat{\sigma}^2)$ under LINEX loss. The derivation of these risk expressions is considered next.

3. Risk Under LINEX Loss

The risk functions of $\tilde{\sigma}^2$, σ^{*2} and $\hat{\sigma}^2$ are stated in the following result:

Theorem 1

Under the assumptions of Section 2,

$$R(\tilde{\sigma}^2) = e^{-a}\left(\frac{\nu+\gamma}{\nu+\gamma-2a}\right)^{\nu/2} + \left(\frac{a\gamma}{\nu+\gamma}\right) - 1 \tag{7}$$

$$R(\sigma^{*2}) = \sum_{i=0}^{\infty} e^{-(\lambda+a)}\left(\frac{\lambda^i}{i!}\right)\left(\frac{\nu+m+\delta}{\nu+m+\delta-2a}\right)^{(\nu+m+2i)/2}$$

$$+ \left(\frac{a(\delta-2\lambda)}{\nu+m+\delta}\right) - 1 \tag{8}$$

$$R(\hat{\sigma}^2) = R(\tilde{\sigma}^2) + e^{-a}\left\{ - \sum^{\infty} \frac{\left(2a/(\nu+\gamma)\right)^i}{i!} \frac{\Gamma\left(\frac{\nu+2i}{2}\right)}{\Gamma(\nu/2)} P_{m,\nu+2i;\lambda}\right.$$

$$\left. + \sum_{r=0}^{\infty} \sum_{i=1}^{\infty} \frac{e^{-\lambda}\lambda^r}{r!} \frac{\left(2a/(\nu+m+\delta)\right)^i}{i!} \left(L_{1i}'L_{2i}\right)\right\}$$

$$- a \left[m(\nu+\gamma)P_{m+2;\nu;\lambda} - \nu(m+\delta-\gamma)P_{m,\nu+2;\lambda} + 2\lambda(\nu+\gamma)P_{m+4,\nu;\lambda}\right] /$$

$$\left((\nu+\gamma)(\nu+m+\delta)\right), \qquad (9)$$

where

$$P_{m+\phi,\nu+\varphi;\lambda} = \Pr.\left[F'_{(m+\phi,\nu+\varphi;\lambda)} \leq \left(cm(\nu+\varphi)\right)/\left(\nu(m+\phi)\right)\right];$$

$$\phi,\ \varphi = 0,\ 1,\ 2,\ \ldots$$

L_{1i}' is a $(1\times(i+1))$ vector equal to the $(i+1)$'th row of Pascal's Triangle; $i = 1, 2, \ldots$

and L_{2i} is an $((i+1)\times1)$ vector with elements

$$M_j = \frac{\Gamma\left(\frac{\nu+2(i-j)}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{\Gamma\left(\frac{m+2j+2r}{2}\right)}{\Gamma\left(\frac{m+2r}{2}\right)} P_{m+2j+2r,\nu+2(i-j);0}\ ;$$

$$j = 0, 1, \ldots, i\ .$$

The expression in (7) is stated by Srivastava and Rao (1990, p.5). A proof of Theorem 1 appears in the Appendix. With some tedious manipulations involving repeated use of the Binomial Theorem, it can be verified that (9) collapses to (7) and (8) respectively, if $\alpha \to 1$ ($c \to 0$) or $\alpha \to 0$ ($c \to \infty$). In the same way, and using the infinite series expansion of the exponential function, it can be shown that (7) - (9) collapse to their quadratic loss counterparts if 'a' is sufficiently small that third-order and higher terms are negligible.

Exact numerical evaluations of these risks are discussed in the next section. Prior to this we derive critical values which result in an extremum of the pre-test risk function. In particular, we are interested in whether the results of Ohtani (1988), Gelfand and Dey (1988a), and Giles (1990, 1991) extend to the LINEX risks. These authors show that under quadratic loss the pre-test risk function is minimised when[3] $c=0$, ∞, or $c=c^*$ where

$$c^* = \Big((m+d-\gamma)\nu\Big)\Big/\Big(m(\nu+\gamma)\Big) \ . \qquad (10)$$

So, $c^*=1$ for the LS components, $\nu/(\nu+2)$ for the MSE components, and zero for the ML components. These studies also show that the pre-test estimator which uses $c=c^*$ strictly dominates the unrestricted estimator. Theorem 2 gives the critical values which can result in an extremum of the LINEX pre-test risk.

Theorem 2

Under the assumptions of Section 2,

$$\partial R(\hat{\sigma}^2)/\partial c = 0 \text{ when } c=0,\ c=\infty, \text{ or } c=c^* \ ,$$

where c^* is defined by (10).

A proof of this theorem is given in the Appendix. So, the values of c which result in turning points under quadratic loss also result in turning points under LINEX loss. However, depending on the value of a, c^* may maximise the pre-test risk as well as minimise it. The numerical evaluations, which we now consider, illustrate this result.

4. Numerical Evaluations

Examples of the LINEX loss function are given in Figure 1 for $a=-0.5$, -2.0, and -5.0. Values of $a<0$ penalise under-estimation of the scale parameter more heavily then over-estimation. The quadratic loss functions shown in Figure 1 for comparison are scaled to match the limiting form of LINEX loss. That is, $L(\tilde{\varepsilon}) = a^2\tilde{\varepsilon}/2$.

These values of a are also used when evaluating the risks in (7) - (9). The risks are evaluated numerically on a VAX 6340 using FORTRAN code which incorporates Davies' (1980) algorithm to evaluate the non-central and central F probabilities and various other algorithms from Press *et al.* (1986). The infinite series in (8) and (9) converge rapidly with a convergence tolerance of 10^{-9}. Corresponding results for the risks of $\tilde{\sigma}^2$, σ^{*2}, and $\hat{\sigma}^2$ under quadratic loss are computed in the same way. In this case the formulae of Clarke *et al.* (1987b) are used after scaling by $a^2/2$ for comparability with the LINEX results. The LINEX risk results are illustrated in Figures 2 - 4, for maximum likelihood, least squares and minimum MSE component estimators of σ^2. Table 1 provides a comparison of the quadratic and LINEX results. In each case we consider $\nu=30$, k=5, and m=3. These results are typical of those we have examined, full details of which are available on request.

In each case two critical values are considered for the preliminary test of restrictions - one associated with a 5% significance level, and one which is "optimal" in some sense. For the LS and MSE components we use c=1 and $c=\nu/(\nu+2)$ respectively for the latter. These are the values of c which result in an extremum of the pre-test risk function as shown in Theorem 2 above. In the case of the ML components $c^*=0$, implying that $\hat{\sigma}^2 = \tilde{\sigma}^2$. Consequently, for this estimator we use the "optimal" critical value reported by Giles and Lieberman (1991) according to the mini-max regret criterion under quadratic loss. For the case presented in Figure 2 this "optimal" critical value is 2.464, which corresponds to an 8.2% significance level.

In Figure 2, which illustrates the LINEX risk functions for ML components, we see that when a = -0.5 the risks are <u>qualitatively</u> the same as those reported by Clarke *et al.* (1987a). In particular, the pre-test estimator can have higher risk than either of its components, the risk of σ^{*2} is always smaller than that of $\tilde{\sigma}^2$ and $\hat{\sigma}^2$ when H_0 is true, and it is preferable to ignore the prior information when $\lambda > \lambda^*$, where λ^* is that value of λ for which $R(\tilde{\sigma}^2) = R(\sigma^{*2})$.

These results continue to hold qualitatively as the loss asymmetry increases. Quantitatively though, λ^* increases with an increase in asymmetry, as does the range over which $\hat{\sigma}^2$ has higher risk than either of its components. Further, the λ value for which $\partial R(\hat{\sigma}^2)/\partial\lambda = 0$ need no longer be greater than λ^* if the loss function is asymmetric in the way investigated here. Finally, the mini-max regret criterion "optimal" critical value varies with a. A detailed investigation of this issue is beyond the scope of this paper, though the results suggest that the "optimal" critical value would increase (α decrease) with increases in the loss asymmetry.

The features discussed for the ML components are also observed for the LS components in Figure 3. Further, we see that when using the LS components $\tilde{\sigma}^2$ may strictly dominate $\hat{\sigma}^2$ - even if c=1 and if H_0 is true. In addition, though λ^* increases with higher loss asymmetry, the potential risk gain of σ^{*2} over $\tilde{\sigma}^2$ is relatively small. Then the results suggest, given that λ is unknown, that it may be preferable to always ignore the prior information.

The MSE component risks behave in a very similar fashion to the LS component risks. In particular, Figure 4b illustrates that using $c=\nu/(\nu+2)$ can both maximise and minimise the pre-test risk. As with the LS components, the unrestricted estimator can strictly dominate the pre-test estimator and the relative risk gain of σ^{*2} over $\tilde{\sigma}^2$ decreases with higher loss asymmetry.

Finally, Table 1 presents the LINEX risks relative to the corresponding quadratic risks. In each case we have scaled the results so that effectively $R(\tilde{\sigma}^2) = R(\sigma^{*2}) = R(\hat{\sigma}^2) = 1$ under quadratic loss. The results illustrate that we cannot generalise on whether the risk under LINEX loss is higher or lower than under quadratic loss.

5. Conclusions

In this paper we have relaxed the conventional assumption, in the preliminary-test estimation literature, that the loss structure is quadratic. By adopting a LINEX loss function we are able to see how asymmetric departures from quadratic loss may affect certain known results. Such asymmetry may be very relevant in the estimation of the regression scale parameter. We find that the risk functions for the pre-test, unrestricted, and restricted estimators of this parameter are robust to mild departures from quadratic loss, at least qualitatively. However, as the degree of asymmetry increases, these results change in several important ways.

Accordingly, it is clear that other existing results in the pre-test literature are unlikely to be robust to major departures from the assumed quadratic loss structure. Work in progress investigates this matter further.

Appendix

Proof of Theorem 1

(i) $$R(\tilde{\sigma}^2) = E\left[\exp\left(a(\tilde{\sigma}^2-\sigma^2)/\sigma^2\right) - a(\tilde{\sigma}^2 - \sigma^2)/\sigma^2 - 1\right], \tag{A.1}$$

where $\tilde{\sigma}^2 = \tilde{u}'\tilde{u}/(\nu+\gamma)$.

Let $z = \tilde{u}'\tilde{u}/\sigma^2$, which is χ^2_ν. Then,

$$E\left[a(\tilde{\sigma}^2-\sigma^2)/\sigma^2\right] = aE(z)/(\nu+\gamma) - a$$

$$= -a\gamma/(\nu+\gamma) \tag{A.2}$$

$$E\left[\exp\left(a(\tilde{\sigma}^2-\sigma^2)/\sigma^2\right)\right] = \int_0^\infty e^{az/(\nu+\gamma)-a} f(z)dz,$$

where $f(z) = \left(2^{\nu/2}/\Gamma(\nu/2)\right) z^{\nu/2-1} e^{-z/2}$.

Let $t = z(\nu+\gamma-2a)/(2(\nu+\gamma))$; then

$$E\left[\exp\left(a(\tilde{\sigma}^2-\sigma^2)/\sigma^2\right)\right] = \frac{e^{-a}(\nu+\gamma)^{\nu/2}}{\Gamma\left(\frac{\nu}{2}\right)(\nu+\gamma-2a)^{\nu/2}} \int_0^\infty e^{-t}\, t^{\nu/2-1}dt$$

$$= e^{-a}\left(\frac{\nu+\gamma}{\nu+\gamma-2a}\right)^{\nu/2}. \tag{A.3}$$

Substituting (A.2) and (A.3) into (A.1) yields (7).

(ii) $$R(\sigma^{*2}) = E\left[\exp\left(a(\sigma^{*2}-\sigma^2)/\sigma^2\right) - a(\sigma^{*2}-\sigma^2)/\sigma^2-1\right], \tag{A.4}$$

where $\sigma^{*2} = u^{*\prime}u^*/(\nu+m+\delta)$.

Let $w = u^{*\prime}u^*/\sigma^2$, which is $\chi^{2\prime}_{\nu+m;\lambda}$, where the non-centrality parameter (λ) is defined in (2). Then,

$$E\left[a(\sigma^{*2}-\sigma^2)/\sigma^2\right] = aE(w)/(\nu+m+\delta) - a$$

$$= a(\nu+m+2\lambda)/(\nu+m+\delta) - a$$

$$= a(2\lambda-\delta)/(\nu+m+\delta)\ . \tag{A.5}$$

$$E\left[\exp\left(a(\sigma_*^2-\sigma^2)/\sigma^2\right)\right] = \int_0^\infty e^{aw/(\nu+m+\delta)-a} f(w)dw\ ,$$

where

$$f(w) = \sum_{i=0}^{\infty} \left(\frac{e^{-\lambda}\lambda^i}{i!}\right) \frac{w^{(\nu+m+2i)/2-1}\ e^{-w/2}}{2^{(\nu+m+2i)/2}\ \Gamma\left((\nu+m+2i)/2\right)}\ .$$

Let $s = w(\nu+m+\delta-2a)/(2(\nu+m+\delta))$; then

$$E\left[\exp\left(a(\sigma_*^2-\sigma^2)/\sigma^2\right)\right] = \sum_{i=0}^{\infty} \frac{e^{-(\lambda+a)}\lambda^i}{i!\Gamma\left((\nu+m+2i)/2\right)2^{(\nu+m+2i)/2}}$$

$$\times \int_0^\infty e^{-s} \left[\frac{2(\nu+m+\delta)}{\nu+m+\delta-2a}\right]^{(\nu+m+2i)/2} s^{(\nu+m+2i)/2-1} ds$$

$$= \sum_{i=0}^{\infty} \left[\frac{e^{-(\lambda+a)}\lambda^i}{i!}\right]\left[\frac{\nu+m+\delta}{\nu+m+\delta-2a}\right]^{(\nu+m+2i)/2}\ . \tag{A.6}$$

Substituting (A.5) and (A.6) into (A.4) yields (8).

(iii) $$R(\hat{\sigma}^2) = E\left[\exp\left(a(\hat{\sigma}^2-\sigma^2)/\sigma^2\right) - a(\hat{\sigma}^2-\sigma^2)/\sigma^2 - 1\right]$$

$$= E(Q_1) - E(Q_2) - 1 \tag{A.7}$$

where $Q_1 = \exp\left(a(\hat{\sigma}^2-\sigma^2)/\sigma^2\right)$ and $Q_2 = a(\hat{\sigma}^2-\sigma^2)/\sigma^2$.

Now, $\hat{\sigma}^2 = \tilde{\sigma}^2 + (\sigma_*^2 - \tilde{\sigma}^2)I_{[0,c]}(f)$ where $I_{(.,.)}(f)$ is an indicator function which is unity if f lies within the subscripted range, zero otherwise, and as $w = z + x$ where $x \sim \chi'^2_{m;\lambda}$ we have,

$$\hat{\sigma}^2 = \sigma^2\left\{(\nu+m+\delta)z + \left[(\nu+\gamma)x - (m+\delta-\gamma)z\right]I_{[0,c]}(\nu x/mz)\right\}/$$

$$\Big((\nu+m+\delta)(\nu+\gamma)\Big). \tag{A.8}$$

z and x are independent and so using Lemma 1 of Clarke *et al.* (1987a) we have

$$E\Big(zI_{[0,c]}(\nu x/mz)\Big) = \nu P_{m,\nu+2;\lambda} ,$$

$$E\Big(xI_{[0,c]}(\nu x/mz)\Big) = mP_{m+2,\nu;\lambda} + 2\lambda P_{m+4,\nu;\lambda} .$$

Substituting these expressions into (A.8) gives

$$E(Q_2) = - a\gamma/ (\nu+\gamma) + a\Big[m(\nu+\gamma)P_{m+2,\nu;\lambda}$$

$$- \nu(m+\delta-\gamma) P_{m,\nu+2;\lambda} + 2\lambda(\nu+\gamma)P_{m+4,\nu;\lambda}\Big]\Big/\Big((\nu+\gamma)(\nu+m+\delta)\Big). \tag{A.9}$$

Turning now to derive $E(Q_1)$ we write

$$\frac{a\hat{\sigma}^2}{\sigma^2} = b_0 z + (b_1 x-b_2 z)I_{[0,c]}(\nu x/mz)$$

$$= Q_3, \quad \text{say,}$$

where $b_0 = a/(\nu+\gamma)$, $b_1 = a/(\nu+m+\delta)$, $b_2 = a(m+\delta-\gamma)/\Big((\nu+\gamma)(\nu+m+\delta)\Big)$ and note that $b_1 = b_0 - b_2$. We desire $E(\exp(Q_3))$, which we can write, as

$$E\Big[\exp(Q_3)\Big] = 1 + E(Q_3) + \frac{E(Q_3)^2}{2!} + \frac{E(Q_3)^3}{3!} + \dots .$$

Now, using Lemma 1 of Clarke (1990) we have

$$E\Big(Z^{n_1}I_{[0,c]}(\nu x/mz)\Big) = 2^{n_1} \frac{\Gamma\left(\frac{\nu+2n_1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} P_{m,\nu+2n_1;\lambda}$$

$$= 2^{n_1} \frac{\Gamma\left(\frac{\nu+2n_1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \sum_{r=0}^{\infty} \frac{e^{-\lambda}\lambda^r}{r!} P_{m+2r,\nu+2n_1;0} \qquad \text{(A.10)}$$

and

$$E\left(x^{n_2} I_{[0,c]}(\nu x/mz)\right) = 2^{n_2} \sum_{r=0}^{\infty} \frac{e^{-\lambda}\lambda^r}{r!} \frac{\Gamma\left(\frac{m+2n_2+2r}{2}\right)}{\Gamma\left(\frac{m+2r}{2}\right)} P_{m+2r+2n_2,r;0} \qquad \text{(A.11)}$$

where n_1 and n_2 are any real values such that $n_1 > (-\nu/2)$ and $n_2 > (-m/2)$. Using (A.10) and (A.11) we have

$$E\left[\exp\ (Q_3)\right] = 1 + 2\left\{ b_0 \frac{\Gamma\left(\frac{\nu+2}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} + \sum_{r=0}^{\infty} \frac{e^{-\lambda}\lambda^r}{r!} \left[\left(b_1 - b_0\right) \frac{\Gamma\left(\frac{\nu+2}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\right.\right.$$

$$\left.\left.\times P_{m+2r,\nu+2;0} + b_1 \frac{\Gamma\left(\frac{m+2+2r}{2}\right)}{\Gamma\left(\frac{m+2r}{2}\right)} P_{m+2+2r,\nu;0}\right]\right\}$$

$$+ 2^2\left\{ b_0^2 \frac{\Gamma\left(\frac{\nu+4}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} + \sum_{r=0}^{\infty} \frac{e^{-\lambda}\lambda^r}{r!} \left[\left(b_1^2 - b_0^2\right) \frac{\Gamma\left(\frac{\nu+4}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} P_{m+2r,\nu+4;0}\right.\right.$$

$$\left.\left.+ b_1^2 \frac{\Gamma\left(\frac{m+4+2r}{2}\right)}{\Gamma\left(\frac{m+2r}{2}\right)} P_{m+4+4r,\nu;0} + 2b_1^2 \frac{\Gamma\left(\frac{\nu+2}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{\Gamma\left(\frac{m+2+2r}{2}\right)}{\Gamma\left(\frac{m+2r}{2}\right)} P_{m+2+2r,\nu+2;0}\right]\right\}$$

$$+2^3 \left\{ b_0^3 \frac{\Gamma\left(\frac{\nu+6}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} + \sum_{r=0}^{\infty} \frac{e^{-\lambda}\lambda^r}{r!} \left[\left(b_1^3 - b_0^3\right) \frac{\Gamma\left(\frac{\nu+6}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} P_{m+2r,\nu+6;0}\right.\right.$$

$$+ b_1^3 \frac{\Gamma\left(\frac{m+6+2r}{2}\right)}{\Gamma\left(\frac{m+2r}{2}\right)} P_{m+6+2r,\nu;0} + 3b_1^3 \frac{\Gamma\left(\frac{\nu+2}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{\Gamma\left(\frac{m+4+2r}{2}\right)}{\Gamma\left(\frac{m+2r}{2}\right)} P_{m+4+2r,\nu+2;0}$$

$$+3b_1^3 \frac{\Gamma\left(\frac{\nu+4}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{\Gamma\left(\frac{m+2+2r}{2}\right)}{\Gamma\left(\frac{m+2r}{2}\right)} P_{m+2+2r,\nu+4;0}\Bigg]\Bigg\} + \ldots$$

$$= \sum_{i=0}^{\infty} \frac{\left(2b_0\right)^i}{i!} \frac{\Gamma\left(\frac{\nu+2i}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} - \sum_{i=1}^{\infty} \frac{\left(2b_0\right)^i}{i!} \frac{\Gamma\left(\frac{\nu+2i}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} P_{m,\nu+2i;\lambda}$$

$$+ \sum_{i=0}^{\infty} \frac{\left(2b_1\right)^i}{i!} \sum_{r=0}^{\infty} \frac{e^{-\lambda}\lambda^r}{r!} \left(L_{1i}' L_{2i}\right) .$$

Finally, if $|\, 2a/(\nu+\gamma)| < 1$, which is not restrictive in practice, then

$$\sum_{i=0}^{\infty} \frac{\left(2b_0\right)^i}{i!} \frac{\Gamma\left(\frac{\nu+2i}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} = \left[\frac{\nu+\gamma}{\nu+\gamma-2a} \right]^{\nu/2}$$

and so

$$E(Q_1) = e^{-a}\left\{\left[\frac{\nu+\gamma}{\nu+\gamma-2a} \right]^{\nu/2} - \sum_{i=1}^{\infty} \frac{\left(2b_0\right)^i}{i!} \frac{\Gamma\left(\frac{\nu+2i}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} P_{m,\nu+2i;\lambda}\right.$$

$$\left. + \sum_{i=1}^{\infty} \frac{\left(2b_1\right)^i}{i!} \sum_{r=0}^{\infty} \frac{e^{-\lambda}\lambda^r}{r!} \left(L_{1i}' L_{2i}\right)\right\} . \qquad (A.12)$$

Substituting (A.9) and (A.12) into (A.7) yields (9).

Proof of Theorem 2

Using the infinite series expansion of the exponential function we write

$$R(\hat{\sigma}^2) = E\left[\frac{a^2}{2!} \left((\hat{\sigma}^2-\sigma^2)/\sigma^2\right)^2 + \frac{a^3}{3!} \left((\hat{\sigma}^2-\sigma^2)/\sigma^2\right)^3 + \dots \right]$$

$$= E\left[\frac{a^2}{2!} \left[\left((\nu+m+\delta)z - (\nu+m+\delta)(\nu+\gamma)\right)\right.\right.$$

$$\left. + \left((\nu+\gamma)x - (m+\delta-\gamma)z\right) I_{[0,c]}(\nu x/mz)\right]^2 \Big/ \left((\nu+m+\delta)(\nu+\gamma)\right)^2$$

$$+ \frac{a^3}{3!} \left[\left((\nu+m+\delta)z-(\nu+m+\delta)(\nu+\gamma)\right)\right.$$

$$\left.\left. + \left((\nu+\gamma)x-(m+\delta-\gamma)z\right) I_{[0,c]}(\nu x/mz)\right]^3 \Big/ \left((\nu+m+\delta)(\nu+\gamma)\right)^3 + \dots \right]$$

$$= E\left[\Delta^* + \left((\nu+\gamma)x - (m+\delta-\gamma)z\right) \Phi \; I_{[0,c]}(\nu x/mz)\right], \tag{A.13}$$

where

$$\Delta^* = \frac{a^2}{2!} \Delta^2 + \frac{a^3}{3!} \Delta^3 + \dots$$

$$\Delta = (\nu+m+\delta)z - (\nu+m+\delta)(\nu+\gamma)$$

$$\Phi = \frac{a^2}{2!} \left[2\Delta + \left((\nu+\gamma)x-(m+\delta-\gamma)z\right)\right]$$

$$+ \frac{a^3}{3!} \left[\left((\nu+\gamma)x-(m+\delta-\gamma)z\right)^2 + 3\Delta^2\right.$$

$$\left. + 3\Delta\left((\nu+\gamma)x - (m+\delta-\gamma)z\right)\right] + \dots \quad .$$

Now $I_{[0,c]}(\nu x/mz) = I_{[0,u]}(x)$ where $u = mcz/\nu$, so (A.13) is

$$R(\hat{\sigma}^2) = E_z\left[\Delta^* + E_x\left[\left((\nu+\gamma)x-(m+\delta-\gamma)z\right)\Phi I_{[0,u]}(x)\right]\right]$$

$$= E_z\left[\Delta^* + \int_0^u \left((\nu+\gamma)x-(m+\delta-\gamma)z\right)\Phi f(x)dx\right]$$

where f(.) is the density function of a $\chi^{2'}_{m;\lambda}$ variate.

So,

$$\frac{\partial R(\hat{\sigma}^2)}{\partial c} = E_z\left[\frac{\partial u}{\partial c} \cdot \frac{\partial}{\partial u} \int_0^u \left((\nu+\gamma)x-(m+\delta-\gamma)z\right)\Phi f(x)dx\right]$$

$$= E_z\left[zf\left(\frac{mcz}{\nu}\right)\left((\nu+\gamma)mc/\nu - (m+\delta-\gamma)\right)\Phi^*\right] \qquad \text{(A.14)}$$

where $\Phi^* = \Phi$ when $x = mcz/\nu$.

(A.14) will be zero when $c = 0, \infty$ and when

$$(\nu+\gamma)mc/\nu - (m+\delta-\gamma) = 0$$

i.e.

$$c^* = \left((m+\delta-\gamma)\nu\right)/\left(m(\nu+\gamma)\right).$$

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Footnotes

* We are grateful to Viren Srivastava for allowing us to refer to Srivastava and Rao (1990).

1. In this case, σ^{*2} has minimum MSE in the family (4) if $R\beta = r$.

2. Strictly, this is a simplified form of the general LINEX function, $L_G(\tilde{\varepsilon}) = b\left(\exp(a\tilde{\varepsilon}) - (C\tilde{\varepsilon}/b) - 1\right)$. We require $C = ab$ for $L_G(\tilde{\varepsilon})$ to be a minimum when $\tilde{\varepsilon} = 0$. We also set the proportionality factor, b, to unity.

3. $\hat{\sigma}^2 = \tilde{\sigma}^2$ if $c = 0$, and $\hat{\sigma}^2 = \sigma^{*2}$ if $c = \infty$.

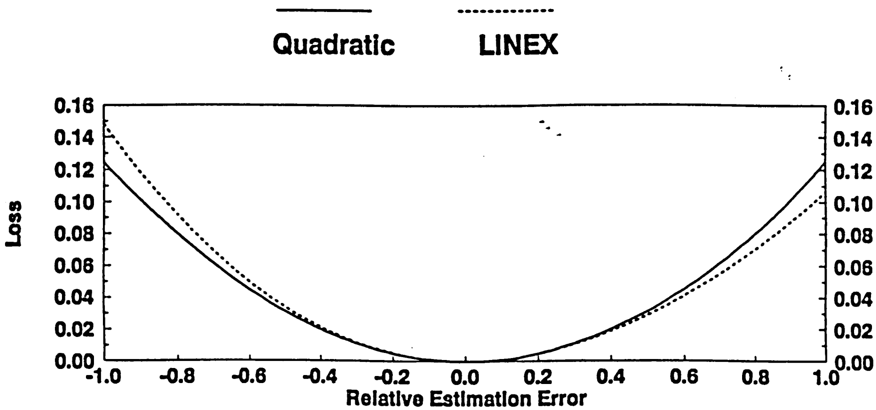

Figure 1a. Alternative loss functions when a=-0.5

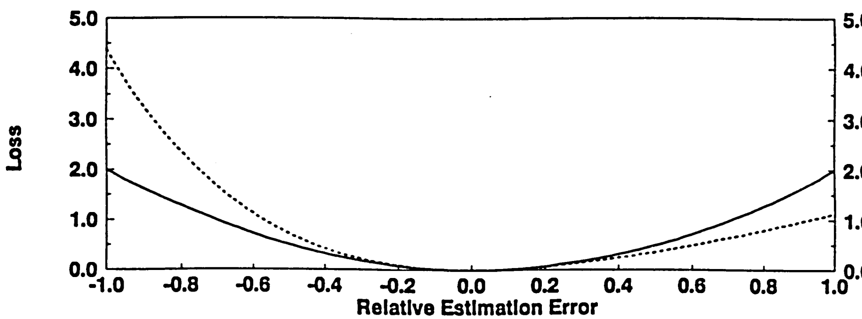

Figure 1b. Alternative loss functions when a=-2.0

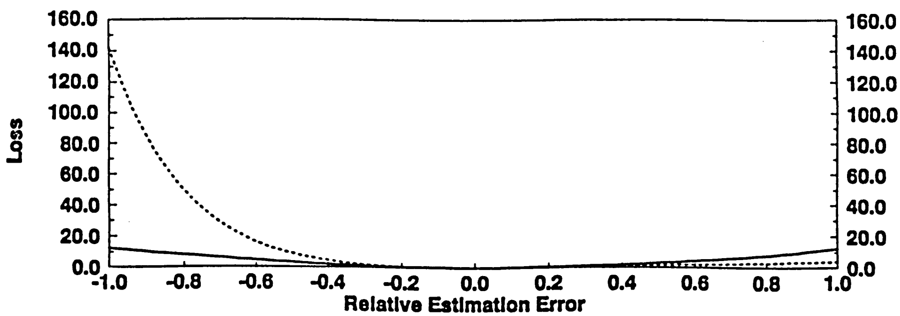

Figure 1c. Alternative loss functions when a=-5.0

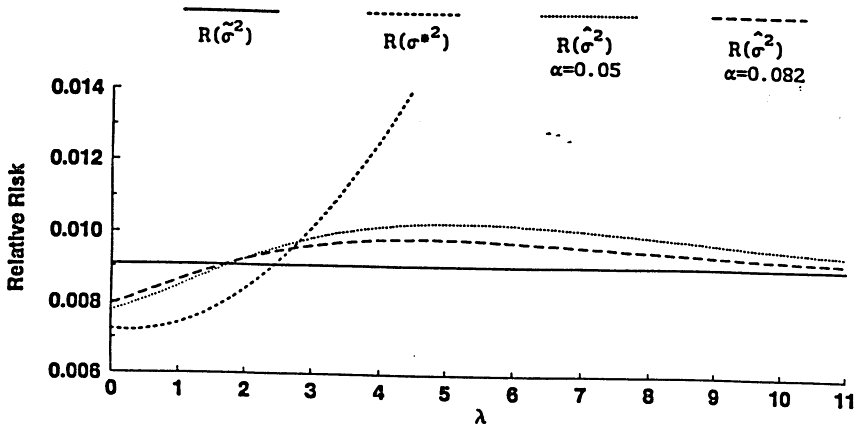

Figure 2a. LINEX risk functions for ML components when a=-0.5

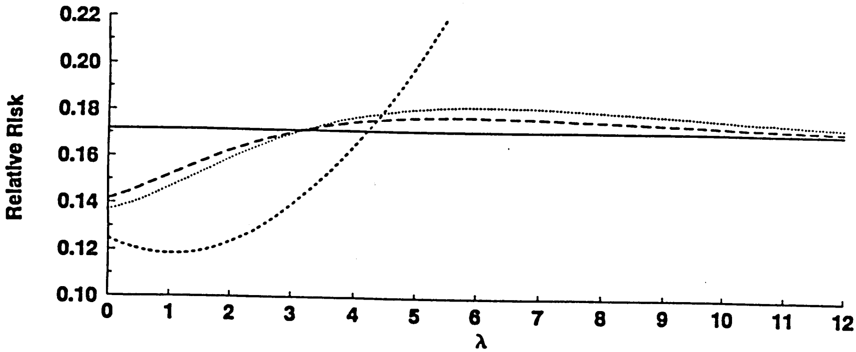

Figure 2b. LINEX risk functions for ML components when a=-2.0

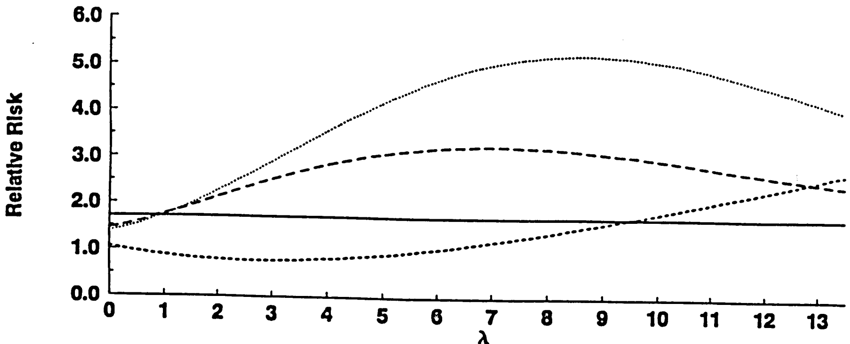

Figure 2c. LINEX risk functions for ML components when a=-5.0

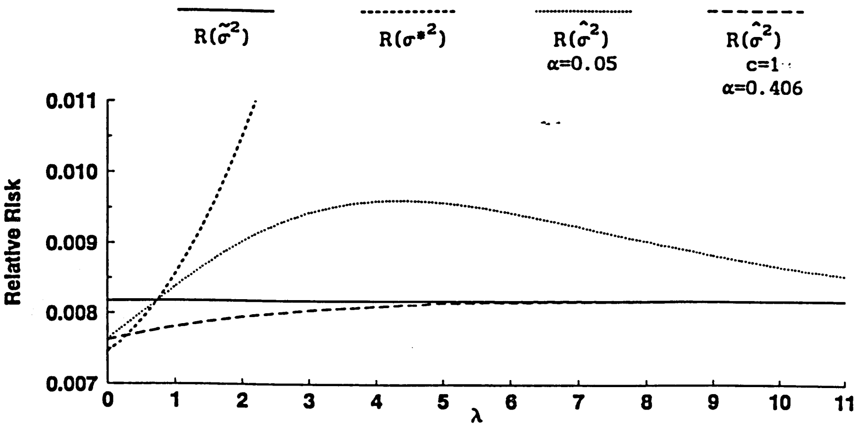

Figure 3a LINEX risk functions for LS components when a=-0.5

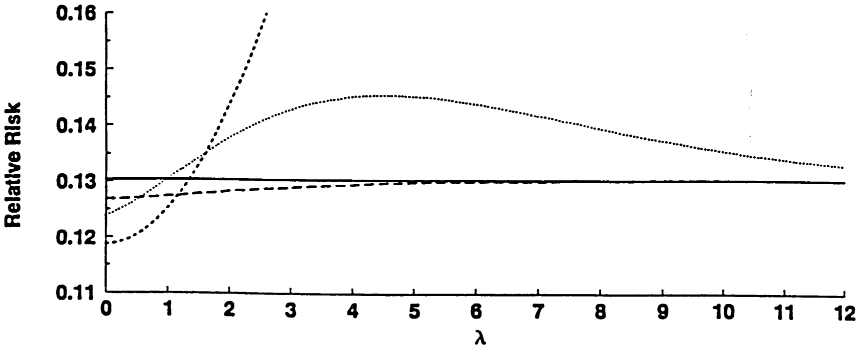

Figure 3b LINEX risk functions for LS components when a=-2.0

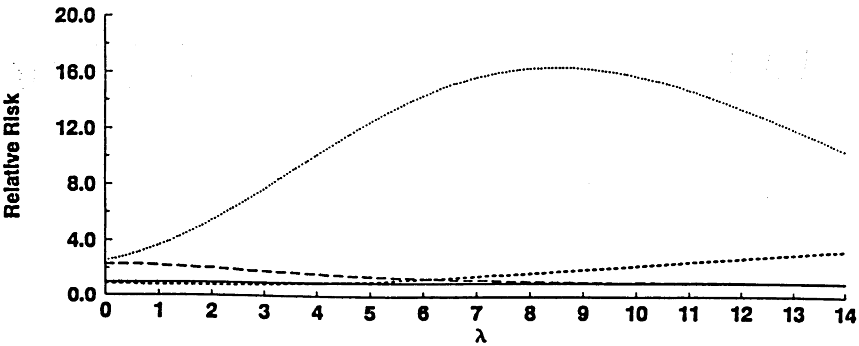

Figure 3c LINEX risk functions for LS components when a=-5.0

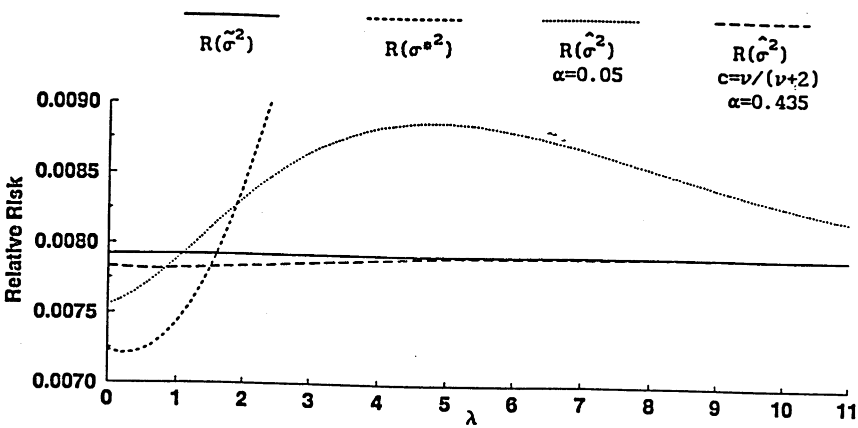

Figure 4a LINEX risk functions for MSE components when a=-0.5

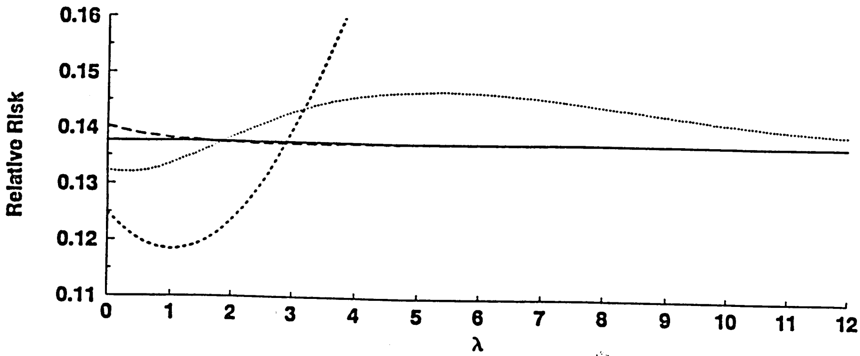

Figure 4b LINEX risk functions for MSE components when a=-2.0

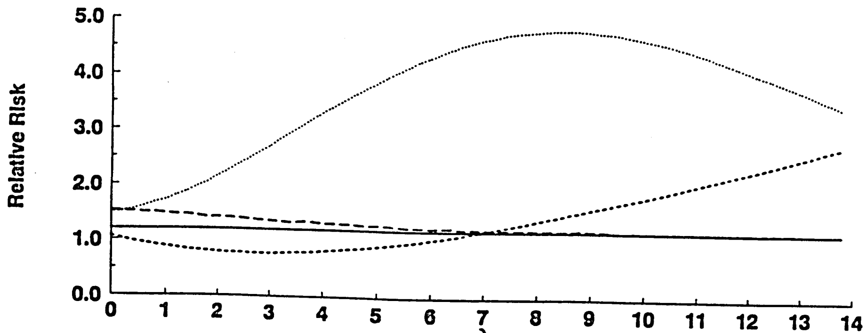

Figure 4c LINEX risk functions for MSE components when a=-5.0

TABLE 1 LINEX risks relative to quadratic risks*

λ	$R(\tilde{\sigma}^2)$		$R(\sigma^{*2})$		$R(\hat{\sigma}^2)$ a=-2.0		$R(\hat{\sigma}^2)$ a=-5.0	
	a=-2.0	a=-5.0	a=-2.0	a=-5.0	α=0.05	$\alpha=\alpha^*$	α=0.05	$\alpha=\alpha^*$
				ML Components				
0	1.2385	1.9693	1.0927	1.4868	1.1248	1.1375	1.8337	1.8591
1	1.2385	1.9693	0.9801	1.1650	1.0947	1.1185	2.0611	2.0688
2	1.2385	1.9693	0.8811	0.9011	1.0858	1.1188	2.4676	2.3376
3	1.2385	1.9693	0.8065	0.7177	1.0896	1.1281	2.9656	2.6539
4	1.2385	1.9693	0.7541	0.6015	1.1005	1.1417	3.5362	2.9603
5	1.2385	1.9693	0.7176	0.5277	1.1158	1.1570	4.1074	3.2083
6	1.2385	1.9693	0.6900	0.4794	1.1329	1.1726	4.6312	3.3910
7	1.2385	1.9693	0.6680	0.4458	1.1512	1.1874	5.0708	3.4872
8	1.2385	1.9693	0.6492	0.4199	1.1674	1.1992	5.3693	3.5041
9	1.2385	1.9693	0.6321	0.3988	1.1831	1.2105	5.5108	3.4364
10	1.2385	1.9693	0.6163	0.3807	1.1962	1.2186	5.4974	3.3270
				LS Components				
0	0.9782	1.1800	0.9802	1.1647	1.0024	1.0385	3.2699	2.9899
1	0.9782	1.1800	0.8744	0.8879	0.9498	1.0111	4.2634	2.8060
2	0.9782	1.1800	0.7983	0.7008	0.9236	0.9961	5.8596	2.5552
3	0.9782	1.1800	0.7455	0.5851	0.9125	0.9878	7.8780	2.2137
4	0.9782	1.1800	0.7089	0.5133	0.9110	0.9826	10.2016	1.9961
5	0.9782	1.1800	0.6812	0.4668	0.9138	0.9811	12.5679	1.7544
6	0.9782	1.1800	0.6587	0.4340	0.9207	0.9789	14.7212	1.5966
7	0.9782	1.1800	0.6394	0.4085	0.9287	0.9782	16.4851	1.4487
8	0.9782	1.1800	0.6218	0.3875	0.9375	0.9782	17.6193	1.3464
9	0.9782	1.1800	0.6055	0.3694	0.9457	0.9775	18.0608	1.2859
10	0.9782	1.1800	0.5898	0.3533	0.9529	0.9782	17.8058	1.2640
				MSE Components				
0	1.1016	1.5346	1.0927	1.4868	1.1154	1.1499	2.0081	1.9948
1	1.1016	1.5346	0.9801	1.1650	1.0645	1.1282	2.1874	1.9222
2	1.1016	1.5346	0.8811	0.9011	1.0390	1.1152	2.5837	1.8391
3	1.1016	1.5346	0.8065	0.7177	1.0280	1.1090	3.0991	1.7510
4	1.1016	1.5346	0.7541	0.6015	1.0267	1.1062	3.7150	1.6935
5	1.1016	1.5346	0.7176	0.5277	1.0309	1.1043	4.3303	1.6575
6	1.1016	1.5346	0.6900	0.4794	1.0389	1.1026	4.8789	1.6015
7	1.1016	1.5346	0.6680	0.4458	1.0489	1.1017	5.3363	1.5798
8	1.1016	1.5346	0.6492	0.4199	1.0578	1.1025	5.6226	1.5713
9	1.1016	1.5346	0.6321	0.3988	1.0671	1.1016	5.7296	1.5645
10	1.1016	1.5346	0.6163	0.3807	1.0751	1.1016	5.6678	1.5412

* α^*=0.082 for the ML components, 0.406 (c=1) for the LS components, and 0.435 ($c=\nu/(\nu+2)$) for the MSE components.

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