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# THE OPTIMAL SIZE OF A PRELIMINARY TEST FOR LINEAR RESTRICTIONS WHEN ESTIMATING THE REGRESSION SCALE PARAMETER

Judith A. Giles and Offer Lieberman

## Discussion Paper

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**ABSTRACT**

This paper considers the choice of critical value for a pre-test of exact linear restrictions when estimating the regression error variance. We calculate the critical value according to a mini-max risk regret criterion and compare the resulting risk functions with those generated by using the critical value which minimises the pre-test risk function. The results suggest that the latter approach is generally preferable.

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* We are grateful to David Giles and John Small for their helpful comments.

## 1. Introduction and framework.

We consider the estimation of the error variance in the classical linear regression model $y = X\beta + e$, $e \sim N(0,\sigma^2 I_T)$, after a pre-test of the hypothesis $H_0$: $R\beta=r$ vs. $H_1$: $R\beta\neq r$, where X (T×k), R (m×k), and r (m×1) are non-stochastic and X and R are of full rank. The usual test of $H_0$ is based on $\mathcal{F} = [v(e^{*\prime}e^{*}-\tilde{e}'\tilde{e})]/[m(\tilde{e}'\tilde{e})] \sim F'_{(m,v;\lambda)}$, $v=T-k$, $\tilde{e}=y-Xb$, $b=(X'X)^{-1}X'y$, $e^{*}=y-Xb^{*}$, $b^{*}=b+(X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}(r-Rb)$ and $\lambda=(R\beta-r)'[R(X'X)^{-1}R']^{-1}(R\beta-r)/2\sigma^2$.

Clarke *et al.* (1987a) derive the risk (under quadratic loss) of the pre-test estimator of $\sigma^2$ when the component estimators are the unrestricted and the restricted maximum likelihood (ML) estimators of $\sigma^2$. Their numerical evaluations show that none of the estimators considered strictly dominates, that the pre-test estimator is never preferred to either of its component estimators, and that it may have higher risk than that of both the unrestricted and the restricted estimators.

Clarke *et al.* (1987b) generalise these results to a family of estimators, which include the ML, the usual least squares (L), and the minimum mean squared error (M) estimators. Let $\tilde{\sigma}^2=\tilde{e}'\tilde{e}/(T+\delta)$ be the unrestricted estimator of $\sigma^2$ and $\sigma^{*2}=e^{*\prime}e^{*}/(T+\gamma)$ be the estimator of $\sigma^2$ which incorporates the restrictions. Then the pre-test estimator is $\hat{\sigma}^2 = \tilde{\sigma}^2 I_{(c,\infty)}(\mathcal{F}) + \sigma^{*2} I_{[0,c]}(\mathcal{F})$, where $I_{(.,.)}(\mathcal{F})$ is an indicator function with value unity if $\mathcal{F}\in(.,.)$, zero otherwise and c is the critical value of the test associated with an $\alpha$ significance level. They show that the risks under quadratic loss of $\tilde{\sigma}^2$, $\sigma^{*2}$, and $\hat{\sigma}^2$, relative to $\sigma^4$, are

$$\rho(\tilde{\sigma}^2) = \left(2v + (k+\delta)^2\right)/\left(T+\delta\right)^2 \tag{1}$$

$$\rho(\sigma^{*2}) = \left(2(m+v+4\lambda) + (m-k-\gamma+2\lambda)^2\right)/\left(T+\gamma\right)^2 \tag{2}$$

$$\rho(\hat{\sigma}^2) = 1 + \Big\{4\lambda(T+\delta)^2\left(\lambda P_{80}+(m+2)P_{60}+vP_{42}-(T+\gamma)P_{40}\right) + v(v+2)(T+\gamma)^2$$
$$- 2(T+\gamma)(T+\delta)\left(v(T+\gamma)+v(\delta-\gamma)P_{02}+m(T+\delta)P_{20}\right) + m(T+\delta)^2\left(2vP_{22}+(m+2)P_{40}\right)$$

$$+ v(v+2)(\delta-\gamma)(2T+\delta+\gamma)P_{04}\Big\}\Big/\Big[(T+\gamma)(T+\delta)\Big]^2, \tag{3}$$

where $P_{ij}=\Pr.\left[F'_{(m+i,v+j;\lambda)} \le \Big(cm(v+j)\Big)\Big/\Big(v(m+i)\Big)\right]$.

The L estimators correspond to $\delta=-k$ and $\gamma=(m-k)$, the M estimators correspond to $\delta=(2-k)$ and $\gamma=(m+2-k)$, while the ML estimators correspond to $\delta=\gamma=0$. We distinguish these three particular members with appropriate use of the subscripts L, M, and ML, respectively.

Ohtani (1988) also considers $\tilde{\sigma}^2_M$, $\sigma^{*2}_M$, and $\hat{\sigma}^2_M$. His numerical evaluations show that there exists a family of pre-test estimators which strictly dominate $\tilde{\sigma}^2_M$ and that that which uses $c=v/(v+2)$ ($c_M$ say) has the smallest risk of this family. He proves that this latter pre-test estimator is the Stein (1964) estimator. Gelfand and Dey (1988), among other things, prove the result postulated by Ohtani (see also Giles (1990)). So, the minimum risk boundary results from using $\sigma^{*2}_M$ for $\lambda\in[0,\lambda_M]$ and $\hat{\sigma}^2_M|c=c_M$ for $\lambda>\lambda_M$, where $\lambda_M$ is that value of $\lambda$ for which $\rho(\sigma^{*2}_M)=\rho(\hat{\sigma}^2_M|c=c_M)$.

Giles (1991) shows that there also exists a family of pre-test estimators which strictly dominate $\tilde{\sigma}^2_L$ and she proves that $\partial\Big(\rho(\hat{\sigma}^2_L)\Big)/\partial c=0$ when $c=0$, 1 or $\infty$, if e follows any spherically symmetric distribution of the compound normal form. Her numerical evaluations suggest that when $m\le2$ it is preferable to always pre-test using $c=1$. So, when using the L estimators with $m>2$, minimum risk is achieved by using $\sigma^{*2}_L$ for $\lambda\in[0,\lambda_L]$ and $\hat{\sigma}^2_L|c=1$ when $\lambda>\lambda_L$, where $\lambda_L$ is the value of $\lambda$ for which $\rho(\sigma^{*2}_L)=\rho(\hat{\sigma}^2_L|c=1)$.

Giles (1990) proves that $\partial\Big(\rho(\hat{\sigma}^2_{ML})\Big)/\partial c=0$ when $c=0$ or $\infty$, so that the pre-test estimator never dominates either of its component estimators when using the ML estimators. This result supports the numerical findings of Clarke *et al.* (1987a). So, the minimum risk boundary when using the ML estimators arises from using $\sigma^{*2}_{ML}$ for $\lambda\in[0,\lambda_{ML}]$ and $\tilde{\sigma}^2_{ML}$ for $\lambda>\lambda_{ML}$, where $\lambda_{ML}$ is the value of $\lambda$ for which $\rho(\sigma^{*2}_{ML})=\rho(\tilde{\sigma}^2_{ML})$.

So, given that pre-test estimators are routinely used, that $\lambda$ is usually unknown and that there exists no dominating estimator (except when $m \leq 2$ when using the L estimators), we need to ask what choice of test size will bring the pre-test risk as close as possible to the minimum risk boundary. The answer to this will depend, among other things, on the chosen optimality criterion. Two such criteria are those suggested by Brook (1976) and Toyoda and Wallace (1976). These two studies consider the "optimal" critical value for the conditional mean forecast problem after a pre-test for exact linear restrictions. Here we obtain the critical values according to the Brook (1976) mini-max regret criterion when using the ML, L, and M estimators. We then compare the pre-test risk functions that result from using the "optimal" critical value from the mini-max regret criterion and the critical value which minimises the pre-test risk.

## 2. Optimal critical values

Let $reg_{ML}=\rho(\hat{\sigma}^2_{ML})-\min\left(\rho(\sigma^{*2}_{ML}),\ \rho(\tilde{\sigma}^2_{ML})\right)$, $reg_L=\rho(\hat{\sigma}^2_L)-\min\left(\rho(\sigma^{*2}_L),\ \rho(\hat{\sigma}^2_L|c=1)\right)$, $reg_M=\rho(\hat{\sigma}^2_M)-\min\left(\rho(\sigma^{*2}_M),\ \rho(\hat{\sigma}^2_M|c=c_M)\right)$. Let $\lambda^L_i$ $(\lambda^U_i)$ be the value of $\lambda \leq \lambda^*_i$ $(>\lambda^*_i)$ such that $reg_i$ is a maximum and let $d^L_i$ $(d^U_i)$ be the corresponding value of $reg_i$, i=ML, L, M. Given that increasing c decreases $d^L_i$ but increases $d^U_i$, the mini-max regret procedure is to find the critical value $c^*_i$ such that $d^U_i = d^L_i$, and both regrets are simultaneously minimised, i=ML, L, M.

Optimal critical values, $c^*_i$, are reported in Table 1 for several values of m, v and k. We also give the significance level, $\alpha^*_i$, associated with each $c^*_i$, and the significance levels $\alpha_L$ and $\alpha_M$ which correspond to c=1 and $c=c_M$, respectively. We calculated these values using a FORTRAN program written by the authors and executed on a VAX8350. We used Davies' (1980) algorithm to evaluate the non-central F probabilities. As noted above, this analysis is irrelevant when using the L estimators and $m \leq 2$: then $\hat{\sigma}^2_L|c=1$ strictly dominates. Apart from the appropriate value of $\alpha_L$, the part of Table 1 corresponding to these cases is accordingly blank.

Regardless of which estimation procedure is used $c^*$ is not constant. This contrasts with Brook's general finding (and that of Toyoda and Wallace when $m \geq 5$) that the optimal critical value is always close to two in value. However, for a given m and k and the estimation procedure, $c^*$ is relatively constant as v varies. This implies that $\alpha^*$ decreases as v increases.

The results also illustrate that $c^*$ is not similar across the different estimation procedures, and nor is its possible range. When using the ML estimators $c^*_{ML}$ varies from 1.4 to 7.2 for the cases that we examined. This implies significance levels ranging from near 0% to over 35%, with $\alpha^*_{ML}$ decreasing dramatically with k.

The range of values for $c^*_L$, however, is much narrower. Here, $c^*_L \in [1.3,1.5]$ and $\alpha^*_L$ lies between 18% and 30% - much higher than the commonly used sizes of 1% and 5%. This concurs with the results of Brook (1976) and Toyoda and Wallace (1976), for instance. As expected, $c^*_L$ is greater that 1, because the optimality criterion will result in a pre-test which selects the restricted estimator more often than the criterion of simply minimising the pre-test risk function. So, $\alpha^*_L < \alpha_L$.

The results for the M estimators are similiar to those just discussed for the L estimators. For the cases examined, $c^*_M$ varies from 1.3 to 2.7 and $\alpha^*_M$ ranges from 8% to 35%; again higher than the commonly used levels. $\alpha^*_M$ is significantly less than $\alpha_M$, which is typically greater than 30%.

### 3. Risk comparisons.

We have calculated the optimal critical values according to the mini-max regret criterion and we have discussed the critical values which result in a minimum of the pre-test risk function. We know that the pre-test estimator based on the latter approach strictly dominates, or is equivalent to (for the

ML case), the unrestricted estimator. We used this feature in our formation of the mini-max regret criterion. The question then arises of the risk difference between these two pre-test estimators. Figures 1, 2, and 3 present typical risk results.

Figure 1 considers the ML case and shows that though there is a risk gain in using the pre-test estimator over the unrestricted estimator if $\lambda$ is in the neighbourhood of $H_0$, the risk loss from this strategy can be reasonably high if $H_0$ is sufficiently invalid. Nevertheless, given that $\lambda$ is unknown, this strategy is preferable to naively imposing the restrictions without testing their validity. However, though not illustrated in Figure 1, we find that when m and k are relatively small (for example, m=1 and k=2) then the unrestricted estimator <u>strictly dominates</u> the pre-test estimator which uses $c=c^*_{ML}$. In these situations the $\lambda$-range over which $\rho(\sigma^{*2}_{ML}) < \rho(\tilde{\sigma}^2_{ML})$ is relatively small, and so generally it is better to simply ignore the prior information and to use the unrestricted estimator (c=0).

Figure 2 considers the L case. We find that generally the mini-max regret criterion results in a pre-test estimator which is strictly dominated by the pre-test estimator which uses c=1. The exceptions are for very large values of m (m>10) and in these cases the region over which the dominance is reversed is small and the risk loss relatively minor. Consequently, the results suggest, when using the L estimators, that it is better to pre-test using c=1 rather than $c=c^*_L$.

Finally, Figure 3, considers the M case. Here, there is generally a small $\lambda$-range, in the neighbourhood of the null, for which $\rho(\hat{\sigma}^2_M|c=c^*_M) <$ $\rho(\hat{\sigma}^2_M|c=c_M)$. The risk gain, however, of using $\hat{\sigma}^2_M|c=c^*_M$ over $\hat{\sigma}^2_M|c=c_M$ in this $\lambda$-range is minor in comparison to the potential loss when $\rho(\hat{\sigma}^2_M|c=c^*_M) >$ $\rho(\hat{\sigma}^2_M|c=c_M)$. The exceptions occur for very large values of v (say, v>100). Then, $\rho(\hat{\sigma}^2_M|c=c_M) \le \rho(\hat{\sigma}^2_M|c=c^*_M)$. Accordingly, as $\lambda$ is unknown, our results suggest that it is preferable to pre-test using $c=c_M$ rather than $c=c_M^*$ when employing the M component estimators.

## 4. Conclusions

The question of the "optimal" choice of test size when estimating $\sigma^2$ arises because $\lambda$ is unobservable and because there is (typically) no strictly dominating estimator. In this paper we have calculated the optimal critical value according to a mini-max regret criterion. Our results show, for a given estimation procedure, that this varies with m, v and k. This contrasts with the criterion of using the critical value which minimises the pre-test risk function: c=0 for the ML case, c=1 for the L case, and c=v/(v+2) for the M case. Not only are the latter values simple to use but our results show that generally the risk of the pre-test estimator which uses these critical values is smaller than that which uses the critical values derived from the mini-max regret criterion.

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**Table 1**

**Optimal Critical Values and Their Significance Levels**

| m | v | k | $c^*_{ML}$ | $\alpha^*_{ML}$ | $c^*_L$ | $a^*_L$ | $\alpha_L$ | $c^*_M$ | $\alpha^*_M$ | $c_M$ | $\alpha_M$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 2 | 1.428 | 0.355 | | | 0.423 | 2.450 | 0.258 | 0.500 | 0.553 |
| 1 | 6 | 2 | 1.763 | 0.233 | | | 0.356 | 2.428 | 0.170 | 0.750 | 0.420 |
| 1 | 10 | 2 | 1.863 | 0.202 | | | 0.341 | 2.523 | 0.143 | 0.833 | 0.383 |
| 1 | 30 | 2 | 1.983 | 0.169 | | | 0.325 | 2.606 | 0.117 | 0.938 | 0.341 |
| 1 | 10 | 5 | 6.472 | 0.029 | | | 0.341 | 2.523 | 0.143 | 0.833 | 0.383 |
| 1 | 20 | 5 | 6.802 | 0.017 | | | 0.329 | 2.576 | 0.124 | 0.909 | 0.352 |
| 1 | 30 | 5 | 6.928 | 0.013 | | | 0.325 | 2.606 | 0.117 | 0.938 | 0.341 |
| 1 | 50 | 5 | 7.038 | 0.011 | | | 0.322 | 2.620 | 0.112 | 0.962 | 0.332 |
| 1 | 100 | 5 | 7.124 | 0.009 | | | 0.320 | 2.655 | 0.106 | 0.980 | 0.325 |
| 2 | 2 | 5 | 2.612 | 0.277 | | | 0.500 | 1.978 | 0.336 | 0.500 | 0.667 |
| 2 | 6 | 5 | 3.181 | 0.114 | | | 0.422 | 2.427 | 0.169 | 0.750 | 0.512 |
| 2 | 10 | 5 | 3.356 | 0.077 | | | 0.402 | 2.417 | 0.139 | 0.833 | 0.463 |
| 2 | 20 | 5 | 3.516 | 0.049 | | | 0.386 | 2.463 | 0.111 | 0.909 | 0.419 |
| 2 | 30 | 5 | 3.576 | 0.041 | | | 0.380 | 2.486 | 0.100 | 0.938 | 0.403 |
| 2 | 50 | 5 | 3.628 | 0.034 | | | 0.375 | 2.508 | 0.092 | 0.962 | 0.389 |
| 2 | 100 | 5 | 3.670 | 0.029 | | | 0.372 | 2.524 | 0.085 | 0.980 | 0.379 |
| 3 | 10 | 5 | 2.323 | 0.134 | 1.428 | 0.292 | 0.432 | 2.124 | 0.161 | 0.833 | 0.506 |
| 3 | 20 | 5 | 2.425 | 0.096 | 1.438 | 0.261 | 0.413 | 2.169 | 0.124 | 0.909 | 0.454 |
| 3 | 30 | 5 | 2.464 | 0.082 | 1.444 | 0.250 | 0.406 | 2.182 | 0.111 | 0.938 | 0.435 |
| 3 | 50 | 5 | 2.497 | 0.070 | 1.455 | 0.238 | 0.401 | 2.193 | 0.101 | 0.962 | 0.418 |
| 3 | 100 | 5 | 2.523 | 0.062 | 1.467 | 0.228 | 0.396 | 2.207 | 0.092 | 0.980 | 0.405 |
| 5 | 10 | 10 | 3.183 | 0.056 | 1.489 | 0.276 | 0.465 | 1.871 | 0.187 | 0.833 | 0.555 |
| 5 | 20 | 10 | 3.322 | 0.024 | 1.484 | 0.239 | 0.443 | 1.891 | 0.141 | 0.909 | 0.495 |
| 5 | 30 | 10 | 3.376 | 0.016 | 1.482 | 0.225 | 0.435 | 1.895 | 0.125 | 0.938 | 0.471 |
| 5 | 50 | 10 | 3.423 | 0.010 | 1.481 | 0.213 | 0.428 | 1.897 | 0.112 | 0.962 | 0.450 |
| 5 | 100 | 10 | 3.461 | 0.006 | 1.480 | 0.203 | 0.422 | 1.900 | 0.101 | 0.980 | 0.434 |
| 10 | 20 | 20 | 3.568 | 0.008 | 1.445 | 0.232 | 0.476 | 1.639 | 0.166 | 0.909 | 0.543 |
| 10 | 30 | 20 | 3.621 | 0.003 | 1.432 | 0.214 | 0.465 | 1.632 | 0.145 | 0.938 | 0.514 |
| 10 | 50 | 20 | 3.668 | 0.001 | 1.421 | 0.199 | 0.456 | 1.623 | 0.127 | 0.962 | 0.488 |
| 10 | 100 | 20 | 3.707 | 0.000 | 1.423 | 0.181 | 0.449 | 1.614 | 0.113 | 0.980 | 0.465 |
| 30 | 30 | 32 | 1.999 | 0.031 | 1.394 | 0.184 | 0.500 | 1.406 | 0.178 | 0.938 | 0.570 |
| 30 | 40 | 32 | 2.013 | 0.020 | 1.329 | 0.198 | 0.494 | 1.391 | 0.163 | 0.952 | 0.550 |
| 30 | 80 | 32 | 2.036 | 0.006 | 1.299 | 0.178 | 0.482 | 1.365 | 0.138 | 0.976 | 0.514 |

Fig. 1. ML : m=1, v=10, k=5.

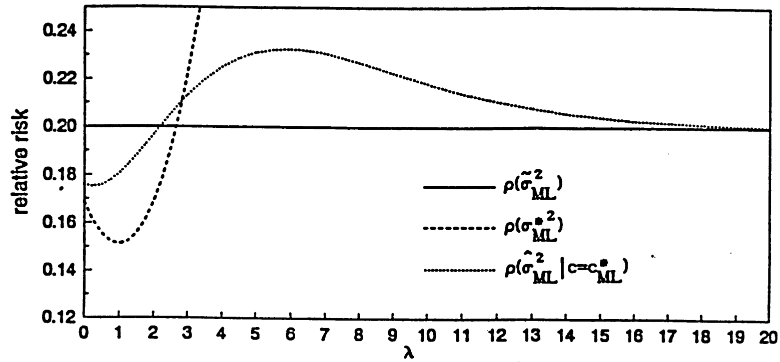



Fig. 2. L : m=3, v=10, k=5.

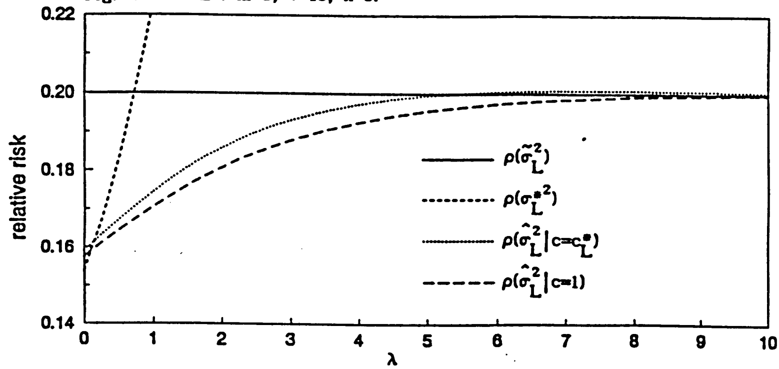
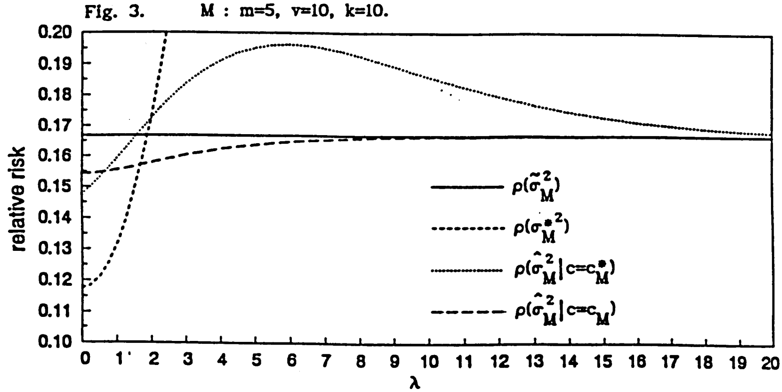



Fig. 3. M : m=5, v=10, k=10.



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