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Abstract

This paper considers the effect of heteroscedastic regression errors on the size of the Chow test for structural stability. We show that bounds can be placed on the true size of this test in the light of such misspecification, and on the true critical value needed to achieve any desired significance level when using the test under various degrees of heteroscedasticity. These bounds are data-independent, and some cases are tabulated. An example is given to illustrate the practical application of the critical value bounds.

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I. Introduction

The Chow (1960) test for the constancy of the regression coefficient vector over the sample is one of the most widely used diagnostic tests in applied econometrics. In its various forms, this test amounts to one of the validity of particular exact restrictions on the regression coefficients (e.g., Fisher (1970)).

It is well known that the statistics associated with the various forms of the Chow test are F-distributed under the null hypothesis of parameter stability (and non-central F under the alternative hypothesis), provided that certain conditions are satisfied. The usual assumption of normal errors can be relaxed to one of spherical symmetry (provided that the errors are homoscedastic) without affecting the null distributions of these statistics, but their distributions under the alternative (and hence their power) are sensitive to this relaxation (e.g., Ullah and Phillips (1986), Giles (1991)). The assumption of homoscedastic disturbances over the full sample cannot be relaxed without distorting both the null and alternative distributions of the test statistics even with normal errors. In the face of heteroscedasticity, we have a form of the Behrens-Fisher problem.

Several studies have considered the effect of this misspecification of the model on the Chow test. For example, Toyoda (1974) approximates the distribution of the test statistic in this case, and Schmidt and Sickles (1977) provide exact evidence. Other authors have proposed alternative tests which might be robust to heteroscedasticity or which allow for its presence in some way. For example Jayatissa (1977) suggests a finite-sample test which has been criticised by Honda (1982) and others. Watt (1979) proposes an asymptotic Wald test whose exact distribution is discussed by Ohtani and Toyoda (1985), and finite-sample bounds for which are described by Ohtani and Kobayashi (1986). A further test is suggested by Weerahandi (1987).

MacKinnon (1989) derives heteroscedasticity-robust variants of the Chow test which have asymptotic validity, but whose finite-sample properties are rather mixed.

Given its ease of construction, the Chow test continues to be used widely in favour of the proposed alternatives, even in situations where the homoscedasticity assumption is unreasonable. Following Schmidt and Sickles (1977) it is quite straightforward to determine the true (as oppposed to nominal) size of the Chow test for any specific data matrix and known actual level of heteroscedasticity, by using the techniques of Imhof (1961) or Davies (1980). This is somewhat analogous to computing an exact Durbin-Watson test rather than using the tabulated bounds on the critical values, and can be undertaken with the SHAZAM package (White et al. (1990)). However, the size distortion is data-specific, and most applied researchers (who may not have easy access to software for computing the distribution of ratios of quadratic forms in normal random vectors) are unlikely to proceed in this way, even given an estimate of the degree of heteroscedasticity.

Instead, it is common for the Chow test to be applied without allowance for possible heteroscedasticity despite its well known inadequacy in this case. Accordingly, for different degrees of heteroscedasticity, it would be helpful to have bounds on the true critical values for the test (or, equivalently, bounds on its true size) which are independent of the data values in the sample. In this paper we use the results of Kiviet (1980) to construct such bounds. The problem and notation are formalised in the next section. Section III details the construction of the bounds, and Section IV reports our results. Some concluding comments appear in Section V.

II. Model and Notation

Consider a sample of $T = T_1 + T_2$ observations and the model

$$y_i = X_i\beta_i + u_i \quad ; \qquad i = 1,2 \tag{1}$$

where y_i is $(T_i\times 1)$; X_i is $(T_i\times K)$, non-stochastic and of rank K $(< T_i)$; and β_i is $(K\times 1)$; $i = 1,2$. The same variables enter the model in each sub-sample but (typically) with different values.

The most common form of the "Chow test" for parameter stability considers H_0: $\beta_1 = \beta_2$ vs. H_A: $\beta_1 \neq \beta_2$. This may be expressed as a standard test of linear restrictions by writing (1) as

$$y \equiv \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix}\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\equiv X\beta + u,$$

and the null hypothesis as H_0:$R\beta = r$, where $R = (I_K,-I_K)$ and $r = 0$. The Chow test statistic is

$$f = \left(\frac{T-2K}{K}\right)(R\hat{\beta}-r)'[R(X'X)^{-1}R']^{-1}(R\hat{\beta}-r)/e'e,$$

where $\hat{\beta} = (X'X)^{-1}X'y$ and $e = y - X\hat{\beta}$.

If $u \sim N(0,\sigma^2 I_T)$ then f is F-distributed with K and $(T-2K)$ degrees of freedom if H_o is true, and it is non-central F with these degrees of freedom and non-centrality parameter $\phi = (R\beta-r)'[R(X'X)^{-1}R']^{-1}(R\beta-r)/2\sigma^2$ under H_A. It is readily verified that <u>under H_0</u>,

$$f = \left(\frac{T-2K}{K}\right) u'Au/u'Mu \ , \tag{2}$$

where
$$M = I_T - X(X'X)^{-1}X'$$

$$A = X(X'X)^{-1}R'\left[R(X'X)^{-1}R'\right]^{-1}R(X'X)^{-1}X'$$

and both M and A are idempotent. If $u \sim N(0,\sigma^2\Omega)$, for arbitrary positive definite symmetric $\Omega \neq I_T$, then the above distributional results no longer hold, though (2) is still valid.

As the Chow test is a special case of the usual test of h linear restrictions on β, the results of Kiviet (1980) can be used to derive bounds on f when $u \sim N(0,\sigma^2\Omega)$. The bounds on f can be used to construct bounds on its critical value for any chosen significance level, or on the true significance level of the test constructed by rejecting H_0 if $f > F^c(\alpha)$, where $F^c(\alpha)$ is the $100\alpha\%$ critical value based on the (wrongly) assumed $F_{K,T-2K}$ null distribution. All of these bounds depend on T, K and Ω, but they are independent of X.

As noted in the Introduction, the assumption that $u \sim N(0,\sigma^2 I_T)$ is often unreasonable when applying the Chow test. A more realistic assumption is that $u_i \sim N\left(0,\sigma_i^2 I_{T_i}\right)$, or $u \sim N(0,\sigma_1^2\Omega)$ where $\Omega = \text{diag.}(\omega_i)$ and

$$\omega_i = \begin{cases} 1 & ; \quad i = 1,\ldots,T_1 \\ \theta = \sigma_2^2/\sigma_1^2 & ; \quad i = T_1 + 1, \ldots, T \end{cases}$$

As Ω depends on T_1 and T_2 here, these separate values partially determine the various bounds.

III. Calculation of the Bounds

Recalling the form of f in (2) and applying the principal theorem of Kiviet (1980, p.354), it follows that <u>under the null hypothesis</u> $f_L \leq f \leq f_u$, where

$$f_L = \left(\frac{T-2K}{K}\right)\left(\sum_{i=1}^{K} \lambda_i \chi_i^2 \Big/ \sum_{i=2K+1}^{T} \lambda_i \chi_i^2\right) \tag{3}$$

$$f_u = \left(\frac{T-2K}{K}\right)\left(\sum_{i=1}^{K} \lambda_{T-K+i} \chi_i^2 \Big/ \sum_{i=2K+1}^{T} \lambda_{i-2K} \chi_i^2\right) . \tag{4}$$

The χ_i^2 are independent central Chi square variates, each with one degree of freedom and $\lambda_1 \le \lambda_2 \le \ldots \le \lambda_T$ are the eigenvalues of Ω. As Ω is diagonal here, the λ_i's are its (appropriately ordered) diagonal elements.

Because (3) and (4) hold only under H_0, it follows that the true $100\alpha\%$ upper critical value, $C(\alpha)$, of f satisfies $C_L(\alpha) \le C(\alpha) \le C_u(\alpha)$, where

$$Pr.\left(f_L \ge C_L(\alpha)\right) = \alpha \tag{5}$$

$$Pr.\left(f_u \ge C_u(\alpha)\right) = \alpha \quad . \tag{6}$$

Using (3) and (4), $C_L(\alpha)$ and $C_u(\alpha)$ may be computed by noting they are values satisfying

$$Pr.\left(\sum_{i=1}^{T} w_i\chi_i^2 \le 0\right) = 1 - \alpha \tag{7}$$

and

$$Pr.\left(\sum_{i=1}^{T} z_i\chi_i^2 \le 0\right) = 1 - \alpha, \tag{8}$$

where

$$w_i = \begin{cases} \lambda_1(T-2K)/K & i=1,\ldots,K \\ -\lambda_1 \; C_L(\alpha) & i=2K+1,\ldots,T \\ 0 & \text{otherwise} \end{cases}$$

$$z_i = \begin{cases} \lambda_{T-K+1}(T-2K)/K & i=1,\ldots,K \\ -\lambda_{1-2K} \, C_u(\alpha) & i=2K+1,\ldots,T \\ 0 & \text{otherwise} \end{cases} \quad .$$

The calculation of (7) and (8) is a standard problem. We use a FORTRAN version of Davies' (1980) algorithm to calculate these probabilities, searching over $C_L(\alpha)$ and $C_u(\alpha)$ to satisfy (7) and (8).

Similarly, the <u>true</u> size (α_0) of the test which is constructed by rejecting H_0 if $f > F^c(\alpha)$, satisfies $\alpha_L \le \alpha_0 \le \alpha_u$, where

$$\alpha_L = \Pr.\left(f_L \geq F^c(\alpha)\right) \tag{9}$$

$$\alpha_u = \Pr.\left(f_u \geq F^c(\alpha)\right). \tag{10}$$

Using (3) and (4),

$$\alpha_L = 1 - \Pr.\left(\sum_{i=1}^{T} w_i^* \chi_i^2 \leq 0\right) \tag{11}$$

$$\alpha_u = 1 - \Pr.\left(\sum_{i=1}^{T} z_i^* \chi_i^2 \leq 0\right) \tag{12}$$

where the w_i^*'s and z_i^*'s equal the w_i's and z_i's respectively, but with $F^c(\alpha)$ replacing both $C_L(\alpha)$ and $C_u(\alpha)$. Davies' routine facilitates the direct calculation of α_L and α_u.

The usefulness of the above bounds is that they are independent of the data - they depend only[1] on α, K, T_1, T_2 and θ . Finally, if the errors are homoscedastic, $\lambda_i = 1$ for all i. Then recalling the additivity properties of independent Chi square variates, it follows that $f_L = f = f_u$, $C_L(\alpha) = F^c(\alpha) = C_u(\alpha)$, and $\alpha_L = \alpha = \alpha_u$.

IV. RESULTS

Given the form of Ω, the bounds will obviously exhibit certain symmetries. For example, if $T_1 = T_2$, the results for $\theta = 0.1$ are identical to those for $\theta = 10$, etc. Similarly, those for $T_1 = 10$ and $T_2 = 20$ when $\theta = 0.1$ are identical to those for $T_2 = 10$ and $T_1 = 20$ when $\theta = 10$, etc.

Table 1 illustrates the bounds on $C(\alpha)$ and α_0 for $\alpha = 0.05$ and the values of T_1, T_2, K and θ considered in Table I of Schmidt and Sickles (1977, p. 1295). Their results provide exact values of α_0 for certain specific X matrices. It can be verified that α_L and α_u in Table 1 bound all of their reported α_0's (and the nominal $\alpha = 0.05$). Also, for fixed T_1, T_2 and K, as θ departs from unity (and H_0 becomes increasingly false), the values of $(C_u(\alpha) -$

TABLE 1.-BOUNDS ON TRUE CRITICAL VALUES AND SIZES OF CHOW TEST WHEN THE NOMINAL SIGNIFICANCE LEVEL IS 5%

T_1	T_2	k	θ	C_L	C_u	α_L	α_u
10	10	2	0.01	0.065	13.134	0.000	0.318
			0.1	0.595	9.884	0.000	0.260
			1	3.634	3.634	0.050	0.050
25	25	2	0.01	0.062	7.476	0.000	0.250
			0.1	0.564	6.583	0.000	0.214
			1	3.200	3.200	0.050	0.050
20	30	2	0.01	0.079	10.171	0.000	0.345
			0.1	0.687	8.297	0.000	0.287
			1	3.200	3.200	0.050	0.050
			10	0.478	5.444	0.000	0.160
			100	0.050	5.904	0.000	0.181
40	10	2	0.01	0.037	4.151	0.000	0.095
			0.1	0.365	4.034	0.000	0.089
			1	3.200	3.200	0.050	0.050
			10	1.191	16.422	0.001	0.513
			100	0.177	33.667	0.000	0.658

$C_L(\alpha)$) and $(\alpha_u - \alpha_L)$ increase, as expected. This is consistent with the results in Tables 4-6 of Kiviet (1980, pp.356-7) for the general F-test in the case of AR(1) or MA(1) errors[2].

The effects on the bounds of varying T_1, T_2 and K are best seen by considering Table 1 in conjunction with the Appendix tables. The latter provide bounds on C(0.05) for a range of situations likely to be encountered in the applied work. In practice, having computed the Chow test statistic, we require a critical value in order to implement the test. In the face of possible heteroscedasticity, the Appendix provides bounds on such critical values.[3]

For $\theta < 1$, increasing T_1 (ceteris paribus) leads to decreases in $C_L(0.05)$, $C_u(0.05)$ and their difference. The converse result emerges if $\theta > 1$. For increasing T_2, these results are reversed in general. Exceptions[4] can be seen for $C_u(0.05)$ for $\theta = 10$ or 100 in Table A4. Changing the value of K, ceteris paribus, produces less clear patterns in the results. This is to be expected as K determines both the numerator and denominator degrees of freedom of f, and so its effect on $F^c(0.05)$ and the bounds depends on T_1 and T_2.

The usefulness of Tables A1 - A4 is best seen with an example. Gujarati (1972) discusses[5] a structural shift in the relationship between unemployment (UN) and vacancies (VAC) in Great Britain in 1966. Using the quarterly data given by Gujarati (1988, p.465) we have fitted the model $UN_i = \beta_1 + \beta_2 VAC_i + u_i$ over the periods 1959(2) - 1970(2), 1959(2) - 1966(3) and 1966(4) - 1970(2). The corresponding OLS sums of squared residuals are 2.72600, 0.22581 and 0.35241. As $T_1 = 30$, $T_2 = 15$ and $K = 2$, the Chow test statistic takes the value 76.147. The corresponding 5% tabulated F-value (for 2 and 4 degrees of freedom) is 3.226, so we would reject the null hypothesis of structural stability. However, dividing the sub-sample sums of squared residuals by their respective degrees of freedom, and taking their ratio, gives an estimate of $\hat{\theta} = 0.297$. This suggests that the regression errors are moderately heteroscedastic. From Table A1, when $\theta = 0.1$, $C_L(0.05) = 0.434$ and $C_u(0.05) = 4.959$. As $76.147 > 4.959$ we can reject the null hypothesis when $0.1 < \theta < 1$, given the patterns in the bounds. In this case, the outcome of the test is not affected by the heteroscedasticity.

V. CONCLUSIONS

While it is widely recognised that the Chow test for structural stability is invalid in the face of heteroscedastic regression errors, it continues to be used widely. To compensate for this, our tabulated critical value bounds should help applied researchers. However, they also illustrate that the appropriate choice of critical value in this case can be dramatically different from the assumed one. This highlights the extent to which a conventional application of the Chow test can be distortive, regardless of the data matrix, when the errors are heteroscedastic.

These bounds apply only to that form of the test which allows for a structural shift in the full coefficient vector, and where there are positive degrees of freedom in each sub-sample. The methods we have described can also be used if these requirements are relaxed, but this would necessitate a very extensive set of tables.

The same approach is not fruitful as a means of bounding the power of the test. It is easily shown that under the alternative hypothesis, the bounds are no longer independent of the data, so they are of little value. However, Kiviet's approach can be used on a wide range of other testing problems of importance to applied econometricians, and work in progress considers some other such cases for various forms of model misspecification.

FOOTNOTES

* We are grateful to Judy Giles and John Small for their helpful comments, and to Robert Davies for supplying the FORTRAN code for his algorithm.

1. As f in (2) is invariant to scale, the bounds are independent of the separate values of σ_1^2 and σ_2^2.

2. A glimpse of some bounds for the critical value of the form of the Chow test considered here, when the errors are AR(1) can be obtained from the entries for (his) k = 4, h = 2 in Kiviet's Table 5.

3. Corresponding tables of bounds on α_0 are available from the authors on request. The α_0 values reported in Table II of Schmidt and Sickles (1977, p.1296) lie within the appropriate bounds in our table.

4. Similar exceptions arise for critical value upper bounds reported for the F-test with ARMA (1,1) errors in Table 7 of Kiviet (1980, p.357).

5. See, also, Gujarati (1988, pp.449-450).

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APPENDIX

Bounds on Chow Test Critical Values When Errors Are Heteroscedastic

TABLE A1.-NOMINAL SIGNIFICANCE LEVEL = 5%; k = 2

		θ								
		1	0.01		0.1		10		100	
T_1	T_2		C_L	C_u	C_L	C_u	C_L	C_u	C_L	C_u
10	10	3.634	0.065	13.134	0.595	9.884	0.595	9.884	0.065	13.134
15	10	3.467	0.051	7.486	0.487	6.624	0.726	11.486	0.084	16.887
20	10	3.369	0.045	5.852	0.436	5.428	0.841	12.802	0.104	20.494
25	10	3.305	0.042	5.085	0.407	4.819	0.943	13.910	0.123	23.967
30	10	3.259	0.040	4.642	0.388	4.453	1.034	14.861	0.141	27.314
35	10	3.226	0.039	4.354	0.375	4.208	1.117	15.690	0.159	30.545
40	10	3.200	0.037	4.151	0.365	4.034	1.191	16.422	0.177	33.667
15	15	3.369	0.063	9.198	0.578	7.718	0.577	7.718	0.063	9.198
20	15	3.305	0.053	6.946	0.505	6.224	0.660	8.698	0.075	10.885
25	15	3.259	0.048	5.887	0.462	5.439	0.737	9.582	0.087	12.547
30	15	3.226	0.045	5.274	0.434	4.959	0.808	10.386	0.098	14.185
35	15	3.200	0.043	4.875	0.414	4.636	0.875	11.121	0.110	15.799
40	15	3.179	0.041	4.595	0.399	4.403	0.937	11.797	0.121	17.390
20	20	3.259	0.061	8.031	0.569	6.963	0.569	6.963	0.061	8.031
25	20	3.226	0.055	6.684	0.514	6.026	0.630	7.652	0.071	9.106
30	20	3.200	0.050	5.904	0.478	5.444	0.687	8.297	0.079	10.171
35	20	3.179	0.047	5.395	0.452	5.048	0.742	8.901	0.087	11.228
40	20	3.162	0.045	5.037	0.433	4.762	0.790	9.469	0.095	12.275
25	25	3.200	0.061	7.476	0.564	6.583	0.564	6.583	0.061	7.476
30	25	3.179	0.056	6.530	0.520	5.909	0.612	7.112	0.068	8.263
35	25	3.162	0.052	5.912	0.488	5.446	0.657	7.616	0.075	9.045
40	25	3.148	0.049	5.478	0.465	5.110	0.701	8.096	0.081	9.822
30	30	3.162	0.061	7.153	0.561	6.355	0.561	6.355	0.061	7.153
35	30	3.148	0.056	6.428	0.524	5.831	0.600	6.784	0.067	7.773
40	30	3.136	0.053	5.918	0.496	5.447	0.638	7.196	0.072	8.390
35	35	3.136	0.061	6.941	0.558	6.203	0.558	6.203	0.061	6.941
40	35	3.126	0.057	6.356	0.527	5.776	0.592	6.563	0.065	7.453
40	40	3.117	0.061	6.792	0.557	6.095	0.557	6.095	0.061	6.792

TABLE A2.-NOMINAL SIGNIFICANCE LEVEL = 5%; k = 3

		θ								
		1	0.01		0.1		10		100	
T_1	T_2		C_L	C_u	C_L	C_u	C_L	C_u	C_L	C_u
10	10	3.344	0.052	20.713	0.484	12.131	0.484	12.131	0.052	20.713
15	10	3.127	0.041	7.973	0.401	6.736	0.607	13.591	0.069	26.795
20	10	3.009	0.037	5.668	0.362	5.167	0.713	14.705	0.087	32.367
25	10	2.934	0.035	4.738	0.340	4.450	0.807	15.596	0.104	37.506
30	10	2.883	0.033	4.239	0.326	4.044	0.890	16.334	0.121	42.273
35	10	2.845	0.032	3.929	0.316	3.784	0.964	16.958	0.137	46.718
40	10	2.816	0.031	3.718	0.309	3.603	1.032	17.496	0.153	50.879
15	15	3.009	0.052	9.962	0.483	7.853	0.483	7.853	0.052	9.962
20	15	2.934	0.045	6.810	0.423	5.954	0.558	8.823	0.063	11.909
25	15	2.883	0.041	5.535	0.389	5.049	0.628	9.677	0.073	13.815
30	15	2.845	0.038	4.849	0.366	4.524	0.692	10.437	0.083	15.681
35	15	2.816	0.036	4.423	0.351	4.184	0.751	11.119	0.094	17.511
40	15	2.794	0.035	4.132	0.339	3.945	0.807	11.735	0.104	19.304
20	20	2.883	0.052	7.940	0.481	6.676	0.481	6.676	0.052	7.940
25	20	2.845	0.046	6.325	0.435	5.611	0.536	7.340	0.060	9.058
30	20	2.816	0.043	5.456	0.405	4.982	0.587	7.954	0.067	10.164
35	20	2.794	0.040	4.915	0.384	4.568	0.635	8.524	0.074	11.257
40	20	2.776	0.038	4.546	0.368	4.277	0.681	9.055	0.082	12.339
25	25	2.816	0.053	7.110	0.480	6.140	0.480	6.140	0.052	7.110
30	25	2.794	0.047	6.060	0.443	5.405	0.522	6.639	0.058	7.889
35	25	2.776	0.044	5.405	0.416	4.938	0.563	7.111	0.064	8.663
40	25	2.761	0.042	4.958	0.397	4.597	0.602	7.559	0.069	9.431
30	30	2.775	0.052	6.660	0.479	5.835	0.479	5.835	0.052	6.660
35	30	2.761	0.048	5.893	0.448	5.295	0.514	6.233	0.057	7.257
40	30	2.748	0.045	5.368	0.424	4.908	0.548	6.615	0.061	7.851
35	35	2.748	0.052	6.378	0.478	5.638	0.478	5.638	0.052	6.378
40	35	2.737	0.049	5.778	0.451	5.209	0.508	5.970	0.056	6.862
40	40	2.728	0.052	6.185	0.478	5.501	0.478	5.501	0.052	6.185

TABLE A3.-NOMINAL SIGNIFICANCE LEVEL = 5%; k = 4

		θ								
		1	0.01		0.1		10		100	
T_1	T_2		C_L	C_u	C_L	C_u	C_L	C_u	C_L	C_u
10	10	3.259	0.041	62.560	0.402	18.127	0.402	18.127	0.041	62.560
15	10	2.965	0.035	9.652	0.339	7.518	0.523	18.610	0.058	73.853
20	10	2.817	0.031	5.888	0.311	5.237	0.626	19.020	0.071	82.573
25	10	2.728	0.030	4.667	0.295	4.335	0.717	19.367	0.091	89.669
30	10	2.668	0.029	4.071	0.285	3.860	0.796	19.662	0.107	95.646
35	10	2.626	0.028	3.720	0.277	3.569	0.867	19.916	0.122	100.840
40	10	2.594	0.027	3.489	0.272	3.372	0.931	20.138	0.138	105.339
15	15	2.817	0.044	12.276	0.418	8.707	0.418	8.707	0.044	12.276
20	15	2.728	0.039	7.174	0.369	6.062	0.490	9.695	0.054	14.812
25	15	2.668	0.035	5.507	0.341	4.944	0.556	10.535	0.064	17.266
30	15	2.626	0.033	4.693	0.322	4.338	0.617	11.262	0.074	19.644
35	15	2.594	0.032	4.213	0.309	3.962	0.673	11.900	0.083	21.948
40	15	2.570	0.031	3.896	0.300	3.705	0.725	12.465	0.093	24.183
20	20	2.668	0.045	8.442	0.423	6.804	0.423	6.804	0.045	8.442
25	20	2.626	0.041	6.340	0.384	5.510	0.474	7.476	0.052	9.692
30	20	2.594	0.037	5.311	0.359	4.792	0.523	8.088	0.059	10.924
35	20	2.570	0.035	4.703	0.341	4.338	0.568	8.649	0.066	12.140
40	20	2.550	0.034	4.303	0.327	4.026	0.611	9.165	0.073	13.339
25	25	2.594	0.046	7.165	0.426	6.039	0.426	6.039	0.046	7.165
30	25	2.570	0.042	5.924	0.394	5.222	0.465	6.534	0.051	7.984
35	25	2.550	0.039	5.191	0.371	4.699	0.503	6.998	0.056	8.795
40	25	2.534	0.037	4.707	0.354	4.336	0.540	7.435	0.062	9.600
30	30	2.550	0.046	6.535	0.427	5.631	0.427	5.631	0.046	6.535
35	30	2.534	0.043	5.676	0.400	5.045	0.460	6.020	0.051	7.141
40	30	2.520	0.040	5.110	0.379	4.636	0.491	6.391	0.055	7.444
35	35	2.520	0.046	6.160	0.428	5.378	0.428	5.378	0.046	6.160
40	35	2.509	0.043	5.512	0.404	4.925	0.455	5.698	0.050	6.641
40	40	2.499	0.046	5.911	0.429	5.206	0.429	5.206	0.046	5.911

TABLE A4.-NOMINAL SIGNIFICANCE LEVEL = 5%; k = 5

		θ									
		1	0.01		0.1		10		100		
T_1	T_2		C_L	C_u	C_L	C_u	C_L	C_u	C_L	C_u	
10	10	3.326	0.033	332.582	0.333	33.258	0.333	33.258	0.033	332.582	
15	10	2.901	0.029	14.108	0.290	9.292	0.455	29.013	0.049	290.130	
20	10	2.711	0.027	6.519	0.271	5.587	0.559	27.109	0.065	271.090	
25	10	2.603	0.026	4.783	0.260	4.377	0.648	26.030	0.081	260.299	
30	10	2.534	0.025	4.037	0.253	3.797	0.727	25.336	0.096	253.356	
35	10	2.485	0.025	3.625	0.249	3.460	0.796	24.851	0.111	248.515	
40	10	2.449	0.025	3.364	0.245	3.241	0.858	24.495	0.125	244.947	
15	15	2.711	0.038	18.196	0.367	10.516	0.367	10.516	0.038	18.196	
20	15	2.603	0.034	8.069	0.327	6.482	0.438	11.470	0.048	22.034	
25	15	2.534	0.031	5.710	0.304	5.019	0.502	12.244	0.057	25.647	
30	15	2.485	0.030	4.693	0.289	4.290	0.561	12.891	0.066	29.059	
35	15	2.449	0.028	4.131	0.279	3.858	0.615	13.443	0.075	32.289	
40	15	2.422	0.028	3.776	0.272	3.574	0.665	13.922	0.084	35.354	
20	20	2.534	0.040	9.589	0.380	7.266	0.380	7.266	0.040	9.589	
25	20	2.485	0.036	6.626	0.346	5.608	0.429	7.960	0.046	11.080	
30	20	2.449	0.034	5.344	0.324	4.753	0.475	8.581	0.053	12.544	
35	20	2.422	0.032	4.635	0.309	4.237	0.519	9.141	0.060	13.980	
40	20	2.400	0.031	4.187	0.297	3.894	0.560	9.649	0.066	15.391	
25	25	2.449	0.041	7.532	0.386	6.151	0.386	6.152	0.041	7.532	
30	25	2.422	0.038	5.991	0.357	5.190	0.424	6.656	0.046	8.430	
35	25	2.401	0.035	5.136	0.337	4.599	0.460	7.125	0.051	9.318	
40	25	2.383	0.034	4.595	0.322	4.202	0.495	7.562	0.056	10.197	
30	30	2.400	0.042	6.633	0.389	5.602	0.389	5.602	0.042	6.633	
35	30	2.383	0.039	5.635	0.365	4.946	0.420	5.993	0.046	7.270	
40	30	2.363	0.037	5.003	0.347	4.499	0.450	6.363	0.050	7.903	
35	35	2.368	0.042	6.131	0.392	5.277	0.392	5.277	0.042	6.131	
40	35	2.356	0.040	5.408	0.370	4.786	0.418	5.594	0.046	6.624	
40	40	2.346	0.042	5.812	0.393	5.062	0.393	5.062	0.042	5.812	

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