



**AgEcon** SEARCH  
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

*The World's Largest Open Access Agricultural & Applied Economics Digital Library*

**This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.**

**Help ensure our sustainability.**

Give to AgEcon Search

AgEcon Search  
<http://ageconsearch.umn.edu>  
[aesearch@umn.edu](mailto:aesearch@umn.edu)

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

CANTER

9014 ✓

Department of Economics  
UNIVERSITY OF CANTERBURY

CHRISTCHURCH, NEW ZEALAND



**TWO RESULTS IN BALANCED-GROWTH  
EDUCATIONAL POLICY**

**Alan E. Woodfield**

GIANNINI FOUNDATION OF  
AGRICULTURAL ECONOMICS  
LIBRARY

MAR 20 1991

**Discussion Paper**

**No. 9014**

This paper is circulated for discussion and comments. It should not be quoted without the prior approval of the author. It reflects the views of the author who is responsible for the facts and accuracy of the data presented. Responsibility for the application of material to specific cases, however, lies with any user of the paper and no responsibility in such cases will be attributed to the author or to the University of Canterbury.

Department of Economics, University of Canterbury  
Christchurch, New Zealand

***Discussion Paper No. 9014***

**December 1990**

**TWO RESULTS IN BALANCED-GROWTH  
EDUCATIONAL POLICY**

**Alan E. Woodfield**

## 1. INTRODUCTION

This paper is written as a tribute to Richard Manning whose early death in 1989 robbed the profession of an exceptionally fine theorist. It seeks to offer some modest generalizations to, and clarifications of, two results in balanced-growth educational policy obtained by Manning [1975, 1976a]. These are core results in a series of contributions to supply-oriented models of education and growth due to Manning and his associates (Manning [1976b, 1977a, 1977b, 1978, 1979a, 1979b, 1982, 1985], Henry and Manning [1981], Manning and Dalziel [1984, 1985], Jans [1989]), and which form the basis for Leonard's assessment [1990, p.172] that Manning had "earned himself a lasting place in the economic analysis of educational policies".

The basic approach of Manning's 1975 paper was to consider a situation in which workers begin life unskilled, but some of whom are transformed into skilled workers according to a Leontief-technology education sector. Skilled and unskilled workers are combined in a neoclassical production function to produce a consumption good, and conditions are obtained for maximization of per capita consumption. These in turn yield an educational Golden Rule for the balanced growth skill level, and with which is associated an optimal wage differential involving few parameters.

Section II of the present paper considers the implications of permitting the labour force growth rate to be influenced by educational policy. Section III investigates conditions obtained by Manning [1976a] which appear to argue against the training of women, and introduces some mechanisms which may weaken this particular result.

## II. EDUCATION, GROWTH, AND ENDOGENOUS FERTILITY

Let  $S$  denote skilled labour,  $U$  unskilled labour,  $L$  total labour,  $n_u$  the constant exponential birth rate in the unskilled sector,  $n_s$  the birth rate

in the skilled sector, and  $m_u$  and  $m_s$  the respective unskilled and skilled mortality rates in each sector.

In Manning,  $n_u = n_s = n$ . Here, I allow for the birth rate in the skilled sector to be determined by educational policy, using arguments from the microeconomics of fertility behaviour. There, theory and supporting empirical evidence suggest a strong link between completed education (especially female) and the number of children ever born. This appears to work through the wage-raising effect of education leading to substitution away from time-intensive commodities such as child services, and the substitution of quality for quantity in the production of a given amount of child services if quality has a high income-elasticity of demand (holding the shadow price of quality constant).<sup>1</sup>

Now assume that no distinction is made between male and female workers or their wages, and assume that all people are born (and enter the education system) with equally augmentable basic endowed levels of competence, namely, the unskilled level.<sup>2</sup> The latter assumption requires both similarity of genetic ability endowments across skilled and unskilled groups,<sup>3</sup> and that if skilled parents devote higher absolute quantities of consumption goods to their children, nevertheless, the educability of these children is not thereby changed. Thus, skilled parents will bear unskilled children, but will be assumed to have a lower completed fertility rate than the unskilled. The overall birth rate  $\bar{n}$  will then depend on the proportion of skilled to total population, that is to say, the natural gross increment to the unskilled will be decreasing in the skill level of the population. This is expressed in equation (1) as,

$$\bar{n} = \alpha n_s + (1-\alpha)n_u, \quad n_s \leq n_u, \quad (1)$$

where  $\alpha = S/L$ , the proportion of skilled to total labour.

Denote by  $Esg$  the flow of newly trained workers, where  $E$  denotes (skilled) teachers,  $s$  is the student-staff ratio and  $g$  is the graduation rate. The stock of skilled workers will grow at the rate  $(Esg - m_s S)$ , while unskilled workers will grow at the rate  $(\bar{n}L - Esg - m_u U)$ . The time rate of change of the proportion of skilled labour in the workforce is given by,

$$\dot{\alpha} = \left( \frac{\dot{S}}{S} - \frac{\dot{L}}{L} \right) \alpha = (esg - m_s \hat{n}) \alpha, \quad (2)$$

where  $\hat{n} = \alpha(n_s - m_s) + (1-\alpha)(n_u - m_u)$  is the net rate of population growth, and where  $e = E/S$  is the ratio of teachers to total skilled workers. In steady growth,  $\dot{\alpha} = 0$ , implying that  $e^* = (\hat{n} + m_s)/sg$ . Henceforth, it is assumed that  $0 < e^* < 1$ .

Per worker output of consumption goods  $c = C/L$  is given by the linear homogeneous, quasi-concave production function,

$$c = f(X/L, Y/L) = f(\alpha - \alpha e, 1 - \alpha - \alpha es), \quad (3)$$

where  $X = S - E$  and  $Y = U - Es$  denote skilled and unskilled workers available for producing consumer goods. Substituting the balanced growth solution for  $e^*$  in (2) yields,

$$c = f\left(\alpha - \alpha(\hat{n} + m_s)/sg, 1 - \alpha - \alpha(\hat{n} + m_s)/g\right). \quad (4)$$

Golden Rule educational policy is that which maximizes consumption per worker with respect to the skill ratio. Differentiating (4) with respect to  $\alpha$ , setting the result equal to zero, and solving for the ratio of the marginal products of skilled and unskilled workers  $f_x/f_y$  yields,

$$\frac{f_x}{f_y} = \frac{1 + \{[(1-2\alpha)(n_u - m_u + m_s) + 2\alpha n_s]/g\}}{1 - \{[(1-2\alpha)(n_u - m_u + m_s) + 2\alpha n_s]/sg\}} \equiv \gamma. \quad (5)$$

Following Manning, for sufficiently large values of the student-staff ratio, (5) can be approximated as,

$$f_x/f_y \approx 1 + \{[(1-2\alpha)(n_u - m_u + m_s) + 2\alpha n_s]/g\}. \quad (6)$$

Since competitive wages are proportional to marginal products, (6) implies the following.

Proposition 1.

If birth rates and mortality rates of skilled and unskilled sectors of the labour force differ, the competitive wage differential for skill expressed as a proportion of the unskilled wage is given by,

$$(W_s/W_u) - 1 \approx [(1-2\alpha)(n_u - m_u + m_s) + 2\alpha n_s]/g .$$

To interpret this proposition in the context of Manning's results, assume first that  $n_s = n_u = n$ , in which case, (6) implies the following.

Proposition 2.

If only mortality rates of skilled and unskilled workers differ, the competitive wage differential for skill is given by,

$$(W_s/W_u) - 1 \approx [n + (1-2\alpha)(m_s - m_u)]/g .$$

Proposition 2 differs from the results obtained by Manning. Instead, Manning obtains the expression  $[n + (1-\alpha)(m_s - m_u)]/g = [\tilde{n} + m_s]/g$  for the wage differential for skill. The reason for the difference is that Manning (1975, p.105) explicitly assumes  $\tilde{n} = n - \alpha m_s - (1-\alpha)m_u$  to be constant. Such an assumption, however, implies that  $d\tilde{n}/d\alpha = 0$ , which is satisfied if, and only if,  $m_s = m_u$ , in which case  $\tilde{n} + m_s = n$ . If one wishes to distinguish between mortality rates,  $\tilde{n}$  cannot be constant as educational policy varies.<sup>4</sup>

The implications of Proposition 2, however, are to strengthen the conclusions drawn by Manning. First, if  $\tilde{n}$  is independent of  $\alpha$ , then  $m_s = m_u$ , and the optimal wage differential is simply the gross rate of population growth times the length of the training period. Since Manning concludes that the wage differential will be rather small (10-60 percent for plausible parameter values), the conclusion is unambiguously strengthened; the wage differential will be contained in an even lower range.



Secondly, if  $m_s \neq m_u$ , the wage differential will still be smaller than that given by Manning if the mortality rate of skilled workers exceeds that of unskilled workers. Since skilled workers are, on average, older than their unskilled counterparts, ceteris paribus we would expect  $m_s > m_n$ .<sup>5</sup> If so, the wage differential is monotonically increasing in the mortality differential if the optimal skill ratio is less than one-half, otherwise it is monotonically decreasing. In the latter case, the optimal wage differential will be less than  $n/g$ , a possibility which is not admitted in Manning's version. In the present version, however, a given mortality differential will now receive less weight in determining the wage differential.

As for the impact of endogenous population growth, what is required is a comparison of the wage differentials implied in Propositions 1 and 2. Substituting  $n_u$  for  $n$  in Proposition 2, and applying some routine algebra, permits the following result to be obtained.

Proposition 3.

If the birth rate in the skilled sector is less than the birth rate in the unskilled sector, the wage differential for skill will be lower than when birth rates are uniform and equal to that of the unskilled sector.

That is, if the strict inequality holds in (1), the optimal wage differential for skill will be smaller than the case where education has no impact on fertility. The corresponding optimal skill level, however, will be greater, since  $f_x/f_y$  is decreasing in  $\alpha$ . Education now buys some reduction in the unskilled workforce, and so some potential teachers who otherwise would be required to maintain the skill ratio can be released into the production of consumption goods.

### III. GROWTH AND FEMALE EDUCATION

In Manning [1976a], the basic model is generalized to permit differences among groups of persons who differ with respect to the parameters  $s$ ,  $g$  and  $m$ . Manning pays particular attention to the distinction between males and females. In a particular version of the model, it is assumed that males and females have common graduation rates and student-staff ratios, but that social pressures which force women to specialize temporarily in child-rearing activity leads to systematic male/female differences in mortality rates.

Notwithstanding the lower genetic vulnerability of females, which is manifest in lower conception and birth rates, and smaller age-specific death rates,<sup>6</sup> early workforce retirement by females is alleged to be of sufficient importance to ensure that  $m_f$  (the female mortality rate) exceeds  $m_m$  (the male mortality rate). Manning then demonstrates that it is never optimal to educate any females until all men who can be educated (that is, satisfy the conditions for a constant ratio of skilled to total males) are in fact educated. Not all males who can be educated generally need be educated in equilibrium, so that the desire for education by females may be completely thwarted. Further, Manning argues a strong case for stability of this result; since women are trained less on average and hence earn less, if either parent must remain home to rear children, the female typically has a lower opportunity cost.<sup>7</sup> Allocation of household time in this manner across families will maintain a high mortality rate for females, making it rational for a planned economy to specialize (perhaps completely) in the education of males, a case of Adam's rib brought painfully up to date. What is more, Manning shows that reductions in female mortality are necessary but not generally sufficient to lead to an increase in female education; in some

cases, reduction in female mortality leads simply to a transfer of unskilled males into the education sector, and their replacement by females.<sup>8</sup>

Of course, it is possible for many women to reduce their mortality rates to that of men, either by explicit contracts or changes in conventions involving sharing of child-rearing and other house-related activities with the fathers of their children.<sup>9</sup> Also, since women live longer, even when one compensates for the possible positive relation between labor force participation and physical mortality, women could "buy back" their child-rearing years by postponing their retirement date.<sup>10</sup> Problems arise, however, if skills are subject to rapid decay during periods of absence from the labour market, or if the objective function of instantaneous consumption maximization is replaced by the maximization of the discounted utility from consumption so that output postponements become more costly. An ameliorating factor is that the value of child consumption services is neglected in the objective function and its inclusion may well modify the conclusion relating to the "waste" of female education.

In what follows, I make the following inroads into the alleged rational basis for educational discrimination against women. First, it is argued that Manning's strong results are predicated by an assumption which requires the net rate of labour force growth for each group to be constant and uniform. Relaxation of this assumption leads to potentially different results.

Secondly, it is argued that specialized child-care services provided by skilled women raises the rate of cognitive development of their children above those of the unskilled. As a consequence, offspring of skilled mothers are characterised by a higher graduation rate/shorter training period than their counterparts, which in turn reduces the average mortality of skilled females.<sup>11</sup> It is of interest to note that the empirical

literature suggests a direct link between mother's education and (especially early) measured ability of their offspring, while father's education appears to have a direct impact on length of schooling and earnings. The latter are also functions of ability, but do not seem to be directly related to mother's education.<sup>12</sup>

The outcome of the above assumptions is that potentially trainable young people graduate quickly or slowly, and spend rather much or rather little time in the labour force, depending on whether or not their mothers are skilled. The proportion of easily trainable persons is then dependent on the ratio of skilled women to the total female workforce. In Manning, the cost of educating a female relative to a potentially trainable unskilled male is the output lost due to the female's early retirement. To keep the output of consumption goods constant, the flow of new female graduates would have to exceed the corresponding flow of male graduates, requiring additional teachers who could have been employed on skilled tasks in the consumption goods sector. In the present context, one of the costs of not educating women is that the supply of able students is no longer forthcoming at the same rate, so that the average graduation rate is lower, and the average mortality rate higher, than in the presence of a skilled female workforce.

Finally, Manning's model is modified so that male and female unskilled labour are substitutable in production, but not on a man for woman basis. A simple assumption is that the total unskilled labour force  $Y$  equals the sum of  $Y_m$  unskilled males and  $\lambda Y_f$  unskilled females, where  $\lambda \in [0,1]$ . A possible motivation for this assumption is that, on average, unskilled tasks are 'muscle-intensive' and that males have an absolute advantage in these activities. Changing output composition, and the removal of entry barriers

to women, however, make this assumption increasingly difficult to defend. Skilled males and females, however, are assumed to be perfect substitutes.

Development of the formal model closely parallels that of the aggregative model. The time rate of change of the proportion of skilled to total persons in group  $i$ ,  $i = m, f$ , is given by,

$$\dot{\alpha}_i = \left[ e_i s g(\alpha_f) - m_{si}(\alpha_f) - \hat{n}_i(\alpha_f) \right] \alpha_i, \quad i = m, f, \quad (7)$$

where  $e_i = E_i/S_i$  is the proportion of skilled persons  $i$  engaged in teaching,  $s$  and  $g$  are the student-staff ratios and average graduate rates common to females and males alike, and where  $g'(\alpha_f) \geq 0$ ,  $m_{si}$  is the mortality rate of skilled persons  $i$  with  $m'_{si}(\alpha_f) \leq 0$ , and  $\hat{n}_i$  is the net reproduction rate of the  $i$ th labour force, given by,

$$\hat{n}_i = n_i - \alpha_i m_{si}(\alpha_f) - (1 - \alpha_i) m_{ui}, \quad i = m, f, \quad (8)$$

where  $n_i$  is the birth rate of group  $i$ , and  $m_{ui}$  is the mortality rate of unskilled persons  $i$ . Our earlier assumptions imply the following restrictions:  $m_{uf} = m_{sf}(0)$ ,  $m_{uf} < m_{sf}(\alpha_f)$  for  $\alpha_f \in (0, 1]$ ;  $m_{um} < m_{sm}(0)$ ,  $m_{um} \geq m_{sm}(\alpha_f)$  for  $\alpha_f \in (0, 1]$ .

The steady-state ratio of educators to total skilled persons in the  $i$ th group is obtained by setting (8) equal to zero and solving as,

$$e_i^* = \left[ n_i + (1 - \alpha_i)(m_{sm} - m_{um}) \right] / s g, \quad i = m, f, \quad (9)$$

Skilled workers per unit of labour available for consumption goods production is  $X/L = (1 - e_m^*)\alpha_m p_m + (1 - e_f^*)\alpha_f p_f$ , where  $p_m = L_m/L$  and  $p_f = L_f/L$ , and unskilled workers per unit of labour similarly available is  $Y/L = 1 - (1 + e_m^*)\alpha_m p_m - (1 + e_f^*)\alpha_f p_f$ . The skilled workers ratio  $X/L$  will be positive if  $0 < e_i^* < 1$  ( $i = m, f$ ), which is assumed to be satisfied. The condition for  $Y/L > 0$  is,

$$\alpha_i \leq \frac{1}{1 + [n_i + (1 - \alpha_i)(m_{si} - m_{ui})]/g} \equiv \delta_i, \quad i = m, f, \quad (10)$$

which is a constraint on the maximization of consumption per worker.

The maximand now becomes,

$$c = f \left[ \sum_{i=m, f} \left( 1 - \frac{n_i + (1 - \alpha_i)(m_{si} - m_{ui})}{sg} \alpha_i p_i \right) \right. \\ \left. 1 - \sum_{i=m, f} \left( 1 + \frac{n_i + (1 - \alpha_i)(m_{si} - m_{ui})}{g} \alpha_i p_i \lambda_i \right) \right] \quad (11)$$

where  $\lambda_m = 0$  and  $\lambda_f = \lambda$ .

The Kuhn-Tucker conditions for this problem can be shown to imply that,

$$f_x/f_y \leq \gamma_i \text{ as } \alpha_i^* = \begin{cases} 0 \\ c[0, \delta_i] \\ \delta_i \end{cases}, \quad i = m, f, \quad (12)$$

where,<sup>13</sup>

$$\gamma_m \approx 1 + \frac{n_m + (1 - 2\alpha_m)(m_{sm} - m_{um})}{g}, \quad (13)$$

$$\gamma_f \approx 1 + \left\{ \frac{n_f + (1 - 2\alpha_f)(m_{sf} - m_{uf}) - \eta_f m_{sf} (1 - \alpha_f) - [n_f + (1 - \alpha_f)(m_{sf} - m_{uf})] \mu}{g} \right\} \lambda \\ - \left\{ \frac{(1 - \alpha_m) \eta_m m_{sm} + [n_m + (1 - \alpha_m)(m_{sm} - m_{um})] \mu}{\alpha_f g} \right\} \alpha_m p_m, \quad (14)$$

and where  $\mu = \alpha_f g' / g \geq 0$  is the elasticity of the graduation rate with respect to the proportion of women who are skilled, and  $\eta_i = -\alpha_f m'_{si} / m_i \geq 0$ , ( $i = m, f$ ), is the elasticity of the mortality rate of skilled workers of type  $i$  with respect to the proportion of skilled women.

Whether or not women (or men) will be educated depends on the relative magnitudes of the  $\gamma_i$  and  $f_x/f_y$ . Manning [1976a, pp.383-4] develops an ordering rule for groups which leads to the following decision rule for the education authority.

- (i) educate no persons from group  $i$  (that is, set  $\alpha_i^* = 0$ ) if  $f_x/f_y > \gamma_i$ ;
- (ii) educate some persons from group  $i$  (that is, set  $\alpha_i^* \in [0, \delta_i]$ ) if  $f_x/f_y = \gamma_i$ ;
- (iii) educate the maximum number of persons compatible with steady growth (that is, set  $\alpha_i^* = \delta_i$ ) if  $f_x/f_y < \gamma_i$ .

Recall that a necessary condition for women to be educated in Manning's analysis is that all males who can be educated, are educated. This means that  $\alpha_f^* = 0$  if  $\alpha_m^* \leq \delta_m$ , while  $\alpha_f^* > 0$  requires that  $\alpha_m^* = \delta_m$  and  $f_x/f_y \leq \gamma_f$ . Clearly, Manning's result requires that  $\gamma_m < \gamma_f$ . The following result, however, is implied by (13) and (14).

Proposition 4.

If  $\mu = \eta_m = \eta_f = 0$ , and  $\lambda = 1$ , then  $\gamma_m < \gamma_f$ , and males receive educational priority, as  $n_m + (1-2\alpha_m)(m_{sm} - m_{um}) < n_f + (1-2\alpha_f)(m_{sf} - m_{uf})$ .

Proposition 4 differs from the corresponding result obtained by Manning, whose own condition (in a slightly different notation) is,

$$\gamma_m < \gamma_f \text{ as } n + m_{sm} < n + m_{sf}, \quad (15)$$

and, since "the only difference between the groups is their mortality rates",<sup>14</sup>  $\gamma_m < \gamma_f$  and males receive educational priority. But what is  $n$ ? Manning [1976a, p.382] writes that "the natural rate of net population growth is the growth rate of the total workforce in each group". This appears to imply that  $\hat{n}_m = \hat{n}_f = n$  is the common (across-sex) net rate of labour force expansion, an assumption sufficient to permit the conclusion that it is the male/female mortality rates of skilled workers alone that determine which of the two groups gets priority. But  $\hat{n}_i$  is defined to equal  $\left( n_i - \alpha_i m_{si} - (1-\alpha_i) m_{ui} \right)$ ,  $i = m, f$ . In general,  $\hat{n}_i$  is a function of the skill ratio  $\alpha_i$ , and it is this dependence which is captured in Proposition 4. Moreover,  $\hat{n}_i$  is independent of  $\alpha_i$  if either,

(i)  $m_{si} = m_{ui}$ ,  $n_i = \text{constant}$ , or,

(ii)  $m_{si} \neq m_{ui}$ ,  $n_i = n_i(\alpha_i)$ ,  $(dn_i/d\alpha_i) + m_{ui} = m_{si}$ .

If  $m_{ui} < m_{si}$ , condition (ii) requires  $dn_i/d\alpha_i$  to be positive. This implies the very odd requirement that the birth rate of group  $i$  be increasing in the proportion of that group who are skilled. If, however,  $\hat{n}_i$  remains constant as  $\alpha_i$  varies, and  $\hat{n}$  is common to both sexes, then it is true that  $\gamma_m < \gamma_f$  as  $m_{sm} < m_{sf}$ , and Manning's strong result holds. But if  $\hat{n}_i$  is independent of  $\alpha_i$  because  $m_{si} = m_{ui}$ , then Manning's condition becomes  $\gamma_m < \gamma_f$  as  $n_m < n_f$ . The educational priority rule for males would then be contingent on a relation between gross labour force growth rates by sex, not on a relation between mortality rates of skilled labour between sexes, and Manning's result will fail to hold, even if the mortality rates of skilled workers differ by sex.

What Proposition 4 tells us instead is that if gross accretion rates to the labour force are the same for each sex and are independent of the respective skill ratios, the decision on educational priority by sex depends on the difference between the skilled and unskilled mortality rates of each sex. A comparison of the mortality rates of skilled workers alone is valid only if the mortality rates of all unskilled workers are the same. And although unskilled mortality differentials may be narrowing due to changes in conventions and contracts, evidence would suggest that  $m_{um}$  is less than  $m_{uf}$ . Thus, the case for educating women is partially reestablished.

In the more general case considered here, (13) and (14) imply the following.

Proposition 5.

If graduation and mortality elasticities with respect to the proportion of skilled females are positive, and unskilled male and female labour are imperfectly substitutable, then  $\gamma_m < \gamma_f$ , and males receive educational



priority, as  $n_m + (1-2\alpha_m)(m_{sm}-m_{um}) < (n_f+(1-2\alpha_f)(m_{sf}-m_{uf})-\mu_f m_{sf}(1-\alpha_f) - [n_f+(1-\alpha_f)(m_{sf}-m_{uf})]\mu)\lambda - \{(1-\alpha_m)\eta_m m_{sm} + [n_m+(1-\alpha_m)(m_{sm}-m_{um})]\mu\}p_m \alpha_m / \alpha_f$ .

The implications of these generalizations may be most easily seen by considering some special cases of Proposition 5. First, assuming  $\lambda < 1$ , if the graduation and mortality elasticities are zero, and  $n_m = n_f = n$ , Proposition (5) implies that men will receive educational priority as  $(1-2\alpha_m)(m_{sm}-m_{um}) < (1-2\alpha_f)(m_{sf}-m_{uf})\lambda$ . If the condition for male educational priority is met when  $\lambda = 1$ , there is no guarantee that it will be met when  $\lambda$  is less than unity, especially if it is substantially so. Since minor differences in mortality differentials between skilled and unskilled workers can be sufficient to lead to sex-specialized education, minor variations in  $\lambda$  could lead to reversals in the choice of sex-related educational priority. Estimates of  $\lambda$  in the context of a time series of cross-section Cobb-Douglas technologies are contained in Woodfield [1973], where it is shown that  $\lambda$  appears fairly stable over time, the median estimate being approximately 0.7.

Next, if  $\lambda = 1$  and  $n_m = n_f = n$ , if  $\gamma_m < \gamma_f$  in the presence of zero values for  $\mu$ ,  $\eta_m$ , and  $\eta_f$ , there is no guarantee that this condition will continue to be met when the graduation and mortality rate elasticities are positive. It is easily checked that  $\gamma_f$  is unambiguously reduced in this case. The presence of nonzero elasticities clearly improves the chances of women being educated, and of receiving educational priority, but naturally offers no guarantees. The reason is that the gains offered through the increases in workforce trainability and reductions in mortality may be insufficient to offset the losses associated with high mortality of skilled females.

An examination of (13)-(14) reveals that whereas  $\gamma_i$  generally depends on the own-skill ratio  $\alpha_i$ ,  $\gamma_f$  depends on both the own-skill ratio  $\alpha_f$  and the

male-skill ratio  $\alpha_m$ . Clearly,  $\partial \gamma_f / \partial \alpha_m > 0$  if the elasticities are assumed to be constant, which means that a change in any factor tending to reduce the equilibrium skill-intensity of males will raise (or at least not lower) the rate at which females are being skilled. For instance, if males are currently being skilled at rate  $\delta_m$  (implying  $f_x/f_y > \gamma_m$ ), and there is (say) a reduction in the male skilled-unskilled mortality differential, then males can be trained at a maximal rate of  $\tilde{\delta}_m < \delta_m$ , and the equilibrium male skill-ratio  $\alpha_m$  must fall, in turn reducing  $\gamma_f$ . Suppose no females were previously skilled, implying  $f_x/f_y < \gamma_f$ . The reduction in  $\gamma_f$  induced by the reduction in  $\alpha_m$  may be sufficient to replace this latter inequality by an equality, or even an inequality with opposite sign. This interdependence arises only when either (or both)  $\mu$  and  $\eta_m$  are positive; it does not depend on the skilled-female mortality elasticity  $\eta_f$ .

#### IV. CONCLUSION

The purpose of this paper was to clarify and extend two major results in balanced growth educational policy due to Manning. In Manning's view, the most important result of his [1975] paper was the golden-rule wage differential, which he suggested [1975, p.511] that "even on a generous interpretation of statistics, ... seems much below prevailing differentials". Manning pointed to the difficulty in expanding selected areas of training due to monopoly power of professional bodies, and while he experienced the unfortunate effects of limitation of entry due to underfunding in his own discipline, he was also a harsh critic of those who appeared instead to be imposing "limitation of exit" through lengthy courses and high failure rates, especially at advanced levels of study. The present paper, by weakening an unnecessarily restrictive assumption made by Manning, along with allowing the training process to affect the level of fertility,

strengthens Manning's results in that optimal wage differentials are further reduced.

Further, Manning concluded his [1976a] paper which contained an optimizing basis for the preferential selection of men for training, by interpreting his results as unfashionable. Nevertheless, he also recognized that they resulted from the proper accounting for the consumption costs of so-called "enlightened" educational policy. The present paper again relaxes an unnecessarily restrictive assumption of Manning. It also introduces new considerations whereby male and female education are not symmetric in their impacts on graduation and mortality rates of skilled workers, and considers the case of imperfect substitutability of unskilled male and female workers. The results go some way to restore fashion, but without introducing faddism, or arguments that education of certain groups is intrinsically worthwhile. However, the results are highly aggregative, and serve to yield insight rather than realistic decision rules. One interesting generalization is to permit fertility differences across families according to their male/female skill composition. Another is to permit differences across families in their home investments in children, permitting much greater variation in the degree of educability and workforce mortality of their offspring. Experimentation with models of these types, however, seem to complicate matters substantially without adding much insight.

## REFERENCES

- Barro, R., and G. Becker, [1989], "Fertility choice in a model of economic growth", Econometrica 57, pp.481-501.
- Becker, G.S., [1973], "An economic theory of marriage. Part I", Journal of Political Economy, 81, pp.813-46.
- , [1981], A Treatise on the Family, (Cambridge, Harvard University Press).
- , [1988], "Family economics and macro behavior", American Economic Review 78, pp.1-13.
- Benhabib, J., and K. Nishimura, [1990], "Endogenous fertility and growth", Working Papers in Economics and Econometrics No.201, Australian National University.
- Ben-Porath, Y., [1977], "Fertility and economic growth: Some microeconomic aspects", in M.D. Intriligator (ed.), Frontiers of Quantitative Economics III B, (Amsterdam, North-Holland), pp.631-56.
- Blandy, R.J., and A.E. Woodfield, [1977], "Towards a characteristics asset-portfolio theory of fertility", in L. Ruzicka (ed.), The Economic and Social Supports for High Fertility, (Canberra, ANU Press), pp.55-89.
- Eckstein, T., and K.I. Wolpin, [1982], "Endogenous fertility in an overlapping generations model", (mimeograph), (Yale University).
- Erickson, M.T., [1978], Child Psychopathology, (N.J., Prentice-Hall).
- Goldberger, A.S., [1977], "Twin methods: A skeptical view", in P. Taubman (ed.), Kinometrics: Determinants of Socioeconomic Success Within and Between Families, (Amsterdam, North-Holland), pp.299-324.
- Gronau, R., [1976], "The allocation of time of Israeli women", Journal of Political Economy 84, pp.201-20.

- Henry, K.R., and R. Manning, [1981], "International trade and the skill composition of the labour force", (mimeograph), (University of Canterbury).
- Jans, I., [1989], "Incentives in a general equilibrium education model", Chapter 6 of Extensions and Applications of Principal-Agent Problems, Ph.D. thesis, (University of Canterbury).
- Leibenstein, H., [1974], "An interpretation of the economic theory of fertility: Promising path or blind alley?", Journal of Economic Literature 12, pp.457-79.
- Leibowitz, A., [1972], "Education and the allocation of women's time", (mimeograph), (N.Y., NBER).
- , [1977], "Family background and economic success: A review of the evidence", in P. Taubman (ed.), ibid, pp.9-33.
- Leonard, D., [1990], "Richard Manning 1943-1989", Economic Record 66, pp.171-74.
- Manning, R., [1975], "Optimal aggregative development of a skilled workforce", Quarterly Journal of Economics 89, pp.504-11.
- , [1976a], "Issues in optimal educational policy in a context of balanced growth", Journal of Economic Theory 13, pp.380-95.
- , [1976b], "A descriptive model of the aggregative development of a skilled workforce", (mimeograph), (University of NSW).
- , [1977a], "A labour theory of labour value", 6th Conference of Economists, Hobart, Tasmania.
- , [1977b], "Optimal aggregative development of a skilled workforce with explicit recognition of the time-delay in education", ANZAAS, Melbourne.

- \_\_\_\_\_, [1978], "Optimal wage differentials in balanced growth in an aggregative model of education with higher degrees", Australian Economic Papers 17, pp.81-90.
- \_\_\_\_\_, [1979a], "Two theorems concerning optimal educational policy in balanced growth", Journal of Economic Theory 17, pp.465-72.
- \_\_\_\_\_, [1979b], "Optimal human and physical capital accumulation in a fixed coefficients economy", 8th Conference of Economists, La Trobe University, Melbourne.
- \_\_\_\_\_, [1982], "Trade, education, and growth", International Economic Review 23, pp.83-106.
- \_\_\_\_\_, [1985], "Some issues in human capital theory", 14th Conference of Economists, University of New South Wales, Sydney.
- Manning, R., and P.C. Dalziel, [1984], "The getting of wisdom: Optimal policies with respect to the age of acquisition of skills", (mimeograph), (University of Canterbury).
- \_\_\_\_\_, [1985], "Graduation rates and educational policy", (mimeograph), (University of Canterbury).
- Rutter, M., [1975], Helping Troubled Children, (N.Y., Plenum Press).
- Schultz, T.P. (ed.), [1974], Economics of the family: Marriage, Children, and Human Capital, (Chicago, University of Chicago Press).
- Schultz, T.P. (1976), "Determinants of fertility", in A.J. Coale (ed.), Economic Factors in Population Growth, (London, Macmillan), pp.90-124.
- Willis, R.J., [1985], "Externalities and population", National Opinion Research Corporation, (Washington, D.C., National Academic Press).
- Woodfield, A., [1973], "Two aspects of labour input specification in the aggregate production function", New Zealand Economic Papers 5, pp.77-95.

## FOOTNOTES

1. Expositions and critical surveys of this literature may be found in Ben-Porath [1977], Leibenstein [1974], Schultz [1974], Schultz [1976] and Becker [1981]. The incorporation of fertility decisions into modern growth theory is relatively recent; see, for example, Barro and Becker [1989], Becker [1988], Benhabib and Nishimura [1990], Eckstein and Wolpin [1982] and Willis [1985].
2. Manning frequently points out that the model does not permit a variable age of labor market entry, and hence is best thought of as referring to post-basic schooling.
3. I shall not cast myself into the fiery furnace of the heritability-environment debate regarding educational/earnings outcomes. It would appear, however, that little will be resolved in this area until we can measure the extent to which parents of dizygotic twins (or pairs of siblings, generally) differentiate between their children in terms of ability/earnings - related home inputs compared to parents of monozygotic twins. On this, see Goldberger [1977].
4. It might, however, be justified as an approximation.
5. In what follows in Section II, this assumption will be made explicitly.
6. For an elaboration of this point, see Erickson [1978], Chapter 5.
7. This distinction is of prime importance in the economic theory of marriage developed by Becker [1964].
8. For details of these arguments, see Manning [1976a], pp.385-9.

9. Technically, women need to be absent from the labor force for a time "necessary" to prepare, deliver, and recover from the birth of each child. This period may be very short indeed. However, many skilled women do not farm out their young children. One argument about why they do not is that they then lose much of the opportunity to develop individualistic characteristics in their children. A discussion of this issue may be found in Blandy and Woodfield [1977].
10. See the discussion in Rutter [1975], Chapter 3. Note that many professions whose members are public sector employees (and some private sector employees) are bound by maximal retirement age provisions which are generally sex-neutral in spirit and letter, but implicitly discriminate against women according to the argument in this section.
11. It could be argued that unskilled females will allocate their additional nonmarket time in favour of activities which also build skills in their children. Our assumption requires that unskilled mothers' time is very unproductive in training children, that diminishing returns set in very quickly once children reach school age (and schooling reduces the time available to teach children at home), or that these mothers do not take the opportunity. The latter is an empirical question, on which evidence is both scanty and conflicting. Gronau's [1976] study of the time allocation patterns of a sample of Israeli women in 1969 found labour force participation and female education positively related, yet women with little education bore more children and spent less total time in child-care activities than their more highly educated counterparts. Gronau also conjectures that uneducated womens' child-care time is relatively chore-intensive. Some contrary results for U.S. women are found by Leibowitz [1972].



12. For a survey and discussion of these, and related, results, see Leibowitz [1977].
13. The approximations in (14) and (15) are obtained by partially differentiating (12) with respect to  $\alpha_m$  and  $\alpha_f$ , equating to zero, solving for  $f_x/f_y$  respectively, and, as in Section II, assuming a sufficiently large value for the student-staff ratio that the denominators on the right-hand side of each resulting expression are (approximately) unity.
14. Cf. Manning [1976a], p.385.

## LIST OF DISCUSSION PAPERS\*

- No. 8601 Estimating the Error Variance in Regression After a Preliminary Test of Restrictions on the Coefficients, by David E. A. Giles, Judith A. Mikolajczyk and T. Dudley Wallace.
- No. 8602 Search While Consuming, by Richard Manning.
- No. 8603 Implementing Computable General Equilibrium Models: Data Preparation, Calibration, and Replication, by K. R. Henry, R. Manning, E. McCann and A. E. Woodfield.
- No. 8604 Credit Rationing: A Further Remark, by John G. Riley.
- No. 8605 Preliminary-Test Estimation in Mis-Specified Regressions, by David E. A. Giles.
- No. 8606 The Positive-Part Stein-Rule Estimator and Tests of Linear Hypotheses, by Aman Ullah and David E. A. Giles.
- No. 8607 Production Functions that are Consistent with an Arbitrary Production-Possibility Frontier, by Richard Manning.
- No. 8608 Preliminary-Test Estimation of the Error Variance in Linear Regression, by Judith A. Clarke, David E. A. Giles and T. Dudley Wallace.
- No. 8609 Dual Dynamic Programming for Linear Production/Inventory Systems, by E. Grant Read and John A. George.
- No. 8610 Ownership Concentration and the Efficiency of Monopoly, by R. Manning.
- No. 8701 Stochastic Simulation of the Reserve Bank's Model of the New Zealand Economy, by J. N. Lye.
- No. 8702 Urban Expenditure Patterns in New Zealand, by Peter Hampton and David E. A. Giles.
- No. 8703 Preliminary-Test Estimation of Mis-Specified Regression Models, by David E. A. Giles.
- No. 8704 Instrumental Variables Regression Without an Intercept, by David E. A. Giles and Robin W. Harrison.
- No. 8705 Household Expenditure in Sri Lanka: An Engel Curve Analysis, by Mallika Dissanayake and David E. A. Giles.
- No. 8706 Preliminary-Test Estimation of the Standard Error of Estimate in Linear Regression, by Judith A. Clarke.
- No. 8707 Invariance Results for FIML Estimation of an Integrated Model of Expenditure and Portfolio Behaviour, by P. Dorian Owen.
- No. 8708 Social Cost and Benefit as a Basis for Industry Regulation with Special Reference to the Tobacco Industry, by Alan E. Woodfield.
- No. 8709 The Estimation of Allocation Models With Autocorrelated Disturbances, by David E. A. Giles.
- No. 8710 Aggregate Demand Curves in General-Equilibrium Macroeconomic Models: Comparisons with Partial-Equilibrium Microeconomic Demand Curves, by P. Dorian Owen.
- No. 8711 Alternative Aggregate Demand Functions in Macro-economics: A Comment, by P. Dorian Owen.
- No. 8712 Evaluation of the Two-Stage Least Squares Distribution Function by Imhof's Procedure by P. Cribbitt, J. N. Lye and A. Ullah.
- No. 8713 The Size of the Underground Economy: Problems and Evidence, by Michael Carter.
- No. 8714 A Computable General Equilibrium Model of a Fisheries Method to Close the Foreign Sector, by Ewen McCann and Keith McLaren.
- No. 8715 Preliminary-Test Estimation of the Scale Parameter in a Mis-Specified Regression Model, by David E. A. Giles and Judith A. Clarke.
- No. 8716 A Simple Graphical Proof of Arrow's Impossibility Theorem, by John Fountain.
- No. 8717 Rational Choice and Implementation of Social Decision Functions, by Manimay Sen.
- No. 8718 Divisia Monetary Aggregates for New Zealand, by Ewen McCann and David E. A. Giles.
- No. 8719 Telecommunications in New Zealand: The Case for Reform, by John Fountain.
- No. 8801 Workers' Compensation Rates and the Demand for Apprentices and Non-Apprentices in Victoria, by Pasquale M. Sgro and David E. A. Giles.
- No. 8802 The Adventures of Sherlock Holmes, the 48% Solution, by Michael Carter.
- No. 8803 The Exact Distribution of a Simple Pre-Test Estimator, by David E. A. Giles.
- No. 8804 Pre-testing for Linear Restrictions in a Regression Model With Student-t Errors, by Judith A. Clarke.

*(continued on next page)*

- No. 8805 Divisia Monetary Aggregates and the Real User Cost of Money, by Ewen McCann and David Giles.
- No. 8806 The Management of New Zealand's Lobster Fishery, by Alan Woodfield and Pim Borren.
- No. 8807 Poverty Measurement: A Generalization of Sen's Result, by Prasanta K. Pattanaik and Manimay Sen.
- No. 8808 A Note on Sen's Normalization Axiom for a Poverty Measure, by Prasanta K. Pattanaik and Manimay Sen.
- No. 8809 Budget Deficits and Asset Sales, by Ewen McCann.
- No. 8810 Unorganized Money Markets and 'Unproductive' Assets in the New Structuralist Critique of Financial Liberalization, by P. Dorian Owen and Otton Solis-Fallas.
- No. 8901 Testing for Financial Buffer Stocks in Sectoral Portfolio Models, by P. Dorian Owen.
- No. 8902 Provisional Data and Unbiased Prediction of Economic Time Series by Karen Browning and David Giles.
- No. 8903 Coefficient Sign Changes When Restricting Regression Models Under Instrumental Variables Estimation, by David E. A. Giles.
- No. 8904 Economies of Scale in the New Zealand Electricity Distribution Industry, by David E. A. Giles and Nicolas S. Wyatt.
- No. 8905 Some Recent Developments in Econometrics: Lessons for Applied Economists, by David E. A. Giles.
- No. 8906 Asymptotic Properties of the Ordinary Least Squares Estimator in Simultaneous Equations Models, by V. K. Srivastava and D. E. A. Giles.
- No. 8907 Unbiased Estimation of the Mean Squared Error of the Feasible Generalised Ridge Regression Estimator, by V. K. Srivastava and D. E. A. Giles.
- No. 8908 An Unbiased Estimator of the Covariance Matrix of the Mixed Regression Estimator, by D. E. A. Giles and V. K. Srivastava.
- No. 8909 Pre-testing for Linear Restrictions in a Regression Model with Spherically Symmetric Disturbances, by Judith A. Giles.
- No. 9001 The Durbin-Watson Test for Autocorrelation in Nonlinear Models, by Kenneth J. White.
- No. 9002 Determinants of Aggregate Demand for Cigarettes in New Zealand, by Robin Harrison and Jane Chetwyd.
- No. 9003 Unemployment Duration and the Measurement of Unemployment, by Manimay Sengupta.
- No. 9004 Estimation of the Error Variance After a Preliminary-Test of Homogeneity in a Regression Model with Spherically Symmetric Disturbances, by Judith A. Giles.
- No. 9005 An Expository Note on the Composite Commodity Theorem, by Michael Carter.
- No. 9006 The Optimal Size of a Preliminary Test of Linear Restrictions in a Mis-specified Regression Model, by David E. A. Giles, Offer Lieberman, and Judith A. Giles.
- No. 9007 Inflation, Unemployment and Macroeconomic Policy in New Zealand: A Public Choice Analysis, by David J. Smyth and Alan E. Woodfield.
- No. 9008 Inflation — Unemployment Choices in New Zealand and the Median Voter Theorem, by David J. Smyth and Alan E. Woodfield.
- No. 9009 The Power of the Durbin-Watson Test when the Errors are Heteroscedastic, by David E. A. Giles and John P. Small.
- No. 9010 The Exact Distribution of a Least Squares Regression Coefficient Estimator After a Preliminary t-Test, by David E. A. Giles and Virendra K. Srivastava.
- No. 9011 Testing Linear Restrictions on Coefficients in a Linear Regression Model with Proxy variables and Spherically Symmetric Disturbances, by Kazuhiro Ohtani and Judith A. Giles.
- No. 9012 Some Consequences of Applying the Goldfeld-Quandt Test to Mis-Specified Regression Models, by David E. A. Giles and Guy N. Saxton.
- No. 9013 Pre-testing in a Mis-specified Regression Model, by Judith A. Giles.
- No. 9014 Two Results in Balanced-Growth Educational Policy, by Alan E. Woodfield.

\* Copies of these Discussion Papers may be obtained for \$4 (including postage, price changes occasionally) each by writing to the Secretary, Department of Economics, University of Canterbury, Christchurch, New Zealand. A list of the Discussion Papers prior to 1986 is available on request.