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TWO RESULTS IN BALANCED-GROWTH EDUCATIONAL POLICY

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I. INTRODUCTION

This paper is written as a tribute to Richard Manning whose early death in 1989 robbed the profession of an exceptionally fine theorist. It seeks to offer some modest generalizations to, and clarifications of, two results in balanced-growth educational policy obtained by Manning [1975, 1976a]. These are core results in a series of contributions to supply-oriented models of education and growth due to Manning and his associates (Manning [1976b, 1977a, 1977b, 1978, 1979a, 1979b, 1982, 1985], Henry and Manning [1981], Manning and Dalziel [1984, 1985], Jans [1989]), and which form the basis for Leonard's assessment [1990, p.172] that Manning had "earned himself a lasting place in the economic analysis of educational policies".

The basic approach of Manning's 1975 paper was to consider a situation in which workers begin life unskilled, but some of whom are transformed into skilled workers according to a Leontief-technology education sector. Skilled and unskilled workers are combined in a neoclassical production function to produce a consumption good, and conditions are obtained for maximization of per capita consumption. These in turn yield an educational Golden Rule for the balanced growth skill level, and with which is associated an optimal wage differential involving few parameters.

Section II of the present paper considers the implications of permitting the labour force growth rate to be influenced by educational policy. Section III investigates conditions obtained by Manning [1976a] which appear to argue against the training of women, and introduces some mechanisms which may weaken this particular result.

II. EDUCATION, GROWTH, AND ENDOGENOUS FERTILITY

Let S denote skilled labour, U unskilled labour, L total labour, n_u the constant exponential birth rate in the unskilled sector, n_s the birth rate in the skilled sector, and m_u and m_s the respective unskilled and skilled mortality rates in each sector.

In Manning, $n_u = n_s = n$. Here, I allow for the birth rate in the skilled sector to be determined by educational policy, using arguments from the microeconomics of fertility behaviour. There, theory and supporting empirical evidence suggest a strong link between completed education (especially female) and the number of children ever born. This appears to work through the wage-raising effect of education leading to substitution away from time-intensive commodities such as child services, and the substitution of quality for quantity in the production of a given amount of child services if quality has a high income-elasticity of demand (holding the shadow price of quality constant).[1]

Now assume that no distinction is made between male and female workers or their wages, and assume that all people are born (and enter the education system) with equally augmentable basic endowed levels of competence, namely, the unskilled level.[2] The latter assumption requires both similarity of genetic ability endowments across skilled and unskilled groups,[3] and that if skilled parents devote higher absolute quantities of consumption goods to their children, nevertheless, the educability of these children is not thereby changed. Thus, skilled parents will bear unskilled children, but will be assumed to have a lower completed fertility rate than the unskilled. The overall birth rate $\bar{n}$ will then depend on the proportion of skilled to total population, that is to say, the natural gross increment to the unskilled will be decreasing in the skill level of the population. This is expressed in equation (1) as,

$$\bar{n} = \alpha n_s + (1-\alpha)n_u \quad , \qquad n_s \leq n_u \, , \tag{1}$$

where $\alpha = S/L$, the proportion of skilled to total labour.

Denote by Esg the flow of newly trained workers, where E denotes (skilled) teachers, s is the student-staff ratio and g is the graduation rate. The stock of skilled workers will grow at the rate $(Esg - m_s S)$, while unskilled workers will grow at the rate $(\bar{n}L - Esg - m_u U)$. The time rate of change of the proportion of skilled labour in the workforce is given by,

$$\dot{\alpha} = \left(\frac{\dot{S}}{S} - \frac{\dot{L}}{L}\right)\alpha = (esg - m_s - \hat{n})\alpha \ , \tag{2}$$

where $\hat{n} = \alpha(n_s - m_s) + (1-\alpha)(n_u - m_u)$ is the net rate of population growth, and where $e = E/S$ is the ratio of teachers to total skilled workers. In steady growth, $\dot{\alpha} = 0$, implying that $e^* = (\hat{n} + m_s)/sg$. Henceforth, it is assumed that $0 < e^* < 1$.

Per worker output of consumption goods $c = C/L$ is given by the linear homogeneous, quasi-concave production function,

$$c = f(X/L, Y/L) = f(\alpha - \alpha e, 1 - \alpha - \alpha es) \ , \tag{3}$$

where $X = S-E$ and $Y = U-Es$ denote skilled and unskilled workers available for producing consumer goods. Substituting the balanced growth solution for e^* in (2) yields,

$$c = f\left(\alpha - \alpha(\hat{n} + m_s)/sg, 1 - \alpha - \alpha(\hat{n} + m_s)/g\right). \tag{4}$$

Golden Rule educational policy is that which maximizes consumption per worker with respect to the skill ratio. Differentiating (4) with respect to α, setting the result equal to zero, and solving for the ratio of the marginal products of skilled and unskilled workers f_x/f_y yields,

$$\frac{f_x}{f_y} = \frac{1 + \{[(1-2\alpha)(n_u - m_u + m_s) + 2\alpha n_s]/g\}}{1 - \{[(1-2\alpha)(n_u - m_u + m_s) + 2\alpha n_s]/sg\}} \equiv \gamma \ . \tag{5}$$

Following Manning, for sufficiently large values of the student-staff ratio, (5) can be approximated as,

$$f_x/f_y \approx 1 + \{[(1-2\alpha)(n_u - m_u + m_s) + 2\alpha n_s]/g\} \ . \tag{6}$$

Since competitive wages are proportional to marginal products, (6) implies the following.

Proposition 1.

If birth rates and mortality rates of skilled and unskilled sectors of the labour force differ, the competitive wage differential for skill expressed as a proportion of the unskilled wage is given by,

$$(W_s/W_u) - 1 \approx [(1-2\alpha)(n_u-m_u+m_s) + 2\alpha n_s]/g \; .$$

To interpret this proposition in the context of Manning's results, assume first that $n_s = n_u = n$, in which case, (6) implies the following.

Proposition 2.

If only mortality rates of skilled and unskilled workers differ, the competitive wage differential for skill is given by,

$$(W_s/W_u) - 1 \approx [n + (1-2\alpha)(m_s-m_u)]/g \; .$$

Proposition 2 differs from the results obtained by Manning. Instead, Manning obtains the expression $[n + (1-\alpha)(m_s-m_u)]/g = [\tilde{n} + m_s]/g$ for the wage differential for skill. The reason for the difference is that Manning (1975, p.105) explicitly assumes $\tilde{n} = n - \alpha m_s - (1-\alpha)m_u$ to be constant. Such an assumption, however, implies that $d\tilde{n}/d\alpha = 0$, which is satisfied if, and only if, $m_s = m_u$, in which case $\tilde{n} + m_s = n$. If one wishes to distinguish between morality rates, $\tilde{n}$ cannot be constant as educational policy varies.[4]

The implications of Proposition 2, however, are to strengthen the conclusions drawn by Manning. First, if $\tilde{n}$ is independent of α, then $m_s = m_u$, and the optimal wage differential is simply the gross rate of population growth times the length of the training period. Since Manning concludes that the wage differential will be rather small (10-60 percent for plausible parameter values), the conclusion is unambiguously strengthened; the wage differential will be contained in an even lower range.

Secondly, if $m_s \neq m_u$, the wage differential will still be smaller than that given by Manning if the mortality rate of skilled workers exceeds that of unskilled workers. Since skilled workers are, on average, older than their unskilled counterparts, <u>ceteris paribus</u> we would expect $m_s > m_n$.[5] If so, the wage differential is monotonically increasing in the mortality differential if the optimal skill ratio is less than one-half, otherwise it is monotonically decreasing. In the latter case, the optimal wage differential will be less than n/g, a possibility which is not admitted in Manning's version. In the present version, however, a given mortality differential will now receive less weight in determining the wage differential.

As for the impact of endogenous population growth, what is required is a comparison of the wage differentials implied in Propositions 1 and 2. Substituting n_u for n in Proposition 2, and applying some routine algebra, permits the following result to be obtained.

<u>Proposition 3</u>.

If the birth rate in the skilled sector is less than the birth rate in the unskilled sector, the wage differential for skill will be lower than when birth rates are uniform and equal to that of the unskilled sector.

That is, if the strict inequality holds in (1), the optimal wage differential for skill will be smaller than the case where education has no impact on fertility. The corresponding optimal skill level, however, will be greater, since f_x/f_y is decreasing in α. Education now buys some reduction in the unskilled workforce, and so some potential teachers who otherwise would be required to maintain the skill ratio can be released into the production of consumption goods.

III. GROWTH AND FEMALE EDUCATION

In Manning [1976a], the basic model is generalized to permit differences among groups of persons who differ with respect to the parameters s, g and m. Manning pays particular attention to the distinction between males and females. In a particular version of the model, it is assumed that males and females have common graduation rates and student-staff ratios, but that social pressures which force women to specialize temporarily in child-rearing activity leads to systematic male/female differences in mortality rates.

Notwithstanding the lower genetic vulnerability of females, which is manifest in lower conception and birth rates, and smaller age-specific death rates,[6] early workforce retirement by females is alleged to be of sufficient importance to ensure that m_f (the female mortality rate) exceeds m_m (the male mortality rate). Manning then demonstrates that it is never optimal to educate any females until all men who can be educated (that is, satisfy the conditions for a constant ratio of skilled to total males) are in fact educated. Not all males who can be educated generally need be educated in equilibrium, so that the desire for education by females may be completely thwarted. Further, Manning argues a strong case for stability of this result; since women are trained less on average and hence earn less, if either parent must remain home to rear children, the female typically has a lower opportunity cost.[7] Allocation of household time in this manner across families will maintain a high mortality rate for females, making it rational for a planned economy to specialize (perhaps completely) in the education of males, a case of Adam's rib brought painfully up to date. What is more, Manning shows that reductions in female mortality are necessary but not generally sufficient to lead to an increase in female education; in some cases, reduction in female mortality leads simply to a transfer of unskilled males into the education sector, and their replacement by females.[8]

Of course, it is possible for many women to reduce their mortality rates to that of men, either by explicit contracts or changes in conventions involving sharing of child-rearing and other house-related activities with the fathers of their children.[9] Also, since women live longer, even when one compensates for the possible positive relation between labor force participation and physical mortality, women could "buy back" their child-rearing years by postponing their retirement date.[10] Problems arise, however, if skills are subject to rapid decay during periods of absence from the labour market, or if the objective function of instantaneous consumption maximization is replaced by the maximization of the discounted utility from consumption so that output postponements become more costly. An ameliorating factor is that the value of child consumption services is neglected in the objective function and its inclusion may well modify the conclusion relating to the "waste" of female education.

In what follows, I make the following inroads into the alleged rational basis for educational discrimination against women. First, it is argued that Manning's strong results are predicated by an assumption which requires the net rate of labour force growth for each group to be constant and uniform. Relaxation of this assumption leads to potentially different results.

Secondly, it is argued that specialized child-care services provided by <u>skilled</u> women raises the rate of cognitive development of their children above those of the unskilled. As a consequence, offspring of skilled mothers are characterised by a higher graduation rate/shorter training period than their counterparts, which in turn reduces the average mortality of skilled females.[11] It is of interest to note that the empirical literature suggests a direct link between mother's education and (especially early) measured ability of their offspring, while father's education appears to have a direct impact on length of schooling and earnings. The latter are also functions of ability, but do not seem to be directly related to mother's education.[12]

The outcome of the above assumptions is that potentially trainable young people graduate quickly or slowly, and spend rather much or rather little time in the labour force, depending on whether or not their mothers are skilled. The proportion of easily trainable persons is then dependent on the ratio of skilled women to the total female workforce. In Manning, the cost of educating a female relative to a potentially trainable unskilled male is the output lost due to the female's early retirement. To keep the output of consumption goods constant, the flow of new female graduates would have to exceed the corresponding flow of male graduates, requiring additional teachers who could have been employed on skilled tasks in the consumption goods sector. In the present context, one of the costs of <u>not</u> educating women is that the supply of able students is no longer forthcoming at the same rate, so that the average graduation rate is lower, and the average mortality rate higher, than in the presence of a skilled female workforce.

Finally, Manning's model is modified so that male and female unskilled labour are substitutable in production, but not on a man for woman basis. A simple assumption is that the total unskilled labour force Y equals the sum of Y_m unskilled males and λY_f unskilled females, where $\lambda \in [0,1]$. A possible motivation for this assumption is that, on average, unskilled tasks are 'muscle-intensive' and that males have an absolute advantage in these activities. Changing output composition, and the removal of entry barriers to women, however, make this assumption increasingly difficult to defend. Skilled males and females, however, are assumed to be perfect substitutes.

Development of the formal model closely parallels that of the aggregative model. The time rate of change of the proportion of skilled to total persons in group i, i = m,f, is given by,

$$\dot{\alpha}_i = \left[e_i sg(\alpha_f) - m_{si}(\alpha_f) - \hat{n}_i(\alpha_f)\right]\alpha_i, \qquad i = m,f, \qquad (7)$$

where $e_i = E_i/S_i$ is the proportion of skilled persons i engaged in teaching, s and g are the student-staff ratios and average graduate rates common to females and males alike, and where $g'(\alpha_f) \geq 0$, m_{si} is the mortality rate of skilled persons i with $m'_{si}(\alpha_f) \leq 0$, and $\hat{n}_i$ is the net reproduction rate of the ith labour force, given by,

$$\hat{n}_i = n_i - \alpha_i m_{si}(\alpha_f) - (1-\alpha_i)m_{ui}, \qquad i = m,f, \qquad (8)$$

where n_i is the birth rate of group i, and m_{ui} is the mortality rate of unskilled persons i. Our earlier assumptions imply the following restrictions: $m_{uf} = m_{sf}(0)$, $m_{uf} < m_{sf}(\alpha_f)$ for $\alpha_f \in (0,1]$; $m_{um} < m_{sm}(0)$, $m_{um} \gtreqless m_{sm}(\alpha_f)$ for $\alpha_f \in (0,1]$.

The steady-state ratio of educators to total skilled persons in the ith group is obtained by setting (8) equal to zero and solving as,

$$e_i^* = \left[n_i + (1-\alpha_i)(m_{sm}-m_{um})\right]/sg, \qquad i = m,f, \qquad (9)$$

Skilled workers per unit of labour available for consumption goods production is $X/L = (1-e_m^*)\alpha_m p_m + (1-e_f^*)\alpha_f p_f$, where $p_m = L_m/L$ and $p_f = L_f/L$, and unskilled workers per unit of labour similarly available is $Y/L = 1 - (1+e_m^*)\alpha_m p_m - (1+e_f^* s)\alpha_f p_f$. The skilled workers ratio X/L will be positive if $0 < e_i^* < 1$ (i = m,f), which is assumed to be satisfied. The condition for Y/L > 0 is,

$$\alpha_i \leq \frac{1}{1+[n_i+(1-\alpha_i)(m_{si}-m_{ui})]/g} \equiv \delta_i, \qquad i = m,f, \tag{10}$$

which is a constraint on the maximization of consumption per worker.

The maximand now becomes,

$$c = f\Big[\sum_{i=m,f}\Big(1 - \frac{n_i+(1-\alpha_i)(m_{si}-m_{ui})}{sg}\alpha_i p_i,$$

$$1 - \sum_{i=m,f}\Big(1 + \frac{n_i+(1-\alpha_i)(m_{si}-m_{ui})}{g}\alpha_i p_i \lambda_i\Big] \tag{11}$$

where $\lambda_m = 0$ and $\lambda_f = \lambda$.

The Kuhn-Tucker conditions for this problem can be shown to imply that,

$$f_x/f_y \lesseqgtr \gamma_i \text{ as } \alpha_i^* = \begin{cases} 0 \\ \varepsilon[0,\delta_i] \\ \delta_i \end{cases}, \qquad i = m,f, \tag{12}$$

where,[13]

$$\gamma_m \approx 1 + \frac{n_m + (1-2\alpha_m)(m_{sm}-m_{um})}{g}, \tag{13}$$

$$\gamma_f \approx 1 + \left\{\frac{n_f+(1-2\alpha_f)(m_{sf}-m_{uf})-\eta_f m_{sf}(1-\alpha_f)-[n_f+(1-\alpha_f)(m_{sf}-m_{uf})]\mu}{g}\right\}\lambda$$

$$- \left\{\frac{(1-\alpha_m)\eta_m m_{sm}+[n_m+(1-\alpha_m)(m_{sm}-m_{um})]\mu}{\alpha_f g}\right\}\alpha_m p_m, \tag{14}$$

and where $\mu = \alpha_f g'/g \geq 0$ is the elasticity of the graduation rate with respect to the proportion of women who are skilled, and $\eta_i = -\alpha_f m'_{si}/m_i \geq 0$, $(i = m,f)$, is the elasticity of the mortality rate of skilled workers of type i with respect to the proportion of skilled women.

Whether or not women (or men) will be educated depends on the relative magnitudes of the γ_i and f_x/f_y. Manning [1976a, pp.383-4] develops an ordering rule for groups which leads to the following decision rule for the education authority.

(i) educate no persons from group i (that is, set $\alpha_i^* = 0$) if $f_x/f_y > \gamma_i$;

(ii) educate some persons from group i (that is, set $\alpha_i^* \in [0,\delta_i]$) if $f_x/f_y = \gamma_i$.

(iii) educate the maximum number of persons compatible with steady growth (that is, set $\alpha_i^* = \delta_i$) if $f_x/f_y < \gamma_i$.

Recall that a necessary condition for women to be educated in Manning's analysis is that all males who can be educated, are educated. This means that $\alpha_f^* = 0$ if $\alpha_m^* \leq \delta_m$, while $\alpha_f^* > 0$ requires that $\alpha_m^* = \delta_m$ and $f_x/f_y \leq \gamma_f$. Clearly, Manning's result requires that $\gamma_m < \gamma_f$. The following result, however, is implied by (13) and (14).

<u>Proposition 4</u>.

If $\mu = \eta_m = \eta_f = 0$, and $\lambda = 1$, then $\gamma_m < \gamma_f$, and males receive educational priority, as $n_m + (1-2\alpha_m)(m_{sm}-m_{um}) < n_f + (1-2\alpha_f)(m_{sf}-m_{uf})$.

Proposition 4 differs from the corresponding result obtained by Manning, whose own condition (in a slightly different notation) is,

$$\gamma_m < \gamma_f \text{ as } n + m_{sm} < n + m_{sf}, \tag{15}$$

and, since "the only difference between the groups is their mortality rates",[14] $\gamma_m < \gamma_f$ and males receive educational priority. But what is n? Manning [1976a, p.382] writes that "the natural rate of net population growth is the growth rate of the total workforce in each group". This appears to imply that $\hat{n}_m = \hat{n}_f = n$ is the common (across-sex) net rate of labour force expansion, an assumption sufficient to permit the conclusion that it is the male/female mortality rates of skilled workers <u>alone</u> that determine which of the two groups gets priority. But $\hat{n}_i$ is defined to equal $\left(n_i-\alpha_i m_{si}-(1-\alpha_i)m_{ui}\right)$, i = m,f. In general, $\hat{n}_i$ is a function of the skill ratio α_i, and it is this dependence which is captured in Proposition 4. Moreover, $\hat{n}_i$ is independent of α_i if either,

(i) $m_{si} = m_{ui}$, n_i = constant, or,

(ii) $m_{si} \neq m_{ui}$, $n_i = n_i(\alpha_i)$, $(dn_i/d\alpha_i) + m_{ui} = m_{si}$.

If $m_{ui} < m_{si}$, condition (ii) requires $dn_i/d\alpha_i$ to be positive. This implies the very odd requirement that the birth rate of group i be increasing in the proportion of that group who are skilled. If, however, $\hat{n}_i$ remains constant as α_i varies, <u>and</u> $\hat{n}$ is common to both sexes, then it is true that $\gamma_m < \gamma_f$ as $m_{sm} < m_{sf}$, and Manning's strong result holds. But if $\hat{n}_i$ is independent of α_i because $m_{si} = m_{ui}$, then Manning's condition becomes $\gamma_m < \gamma_f$ as $n_m < n_f$. The educational priority rule for males would then be contingent on a relation between gross labour force growth rates by sex, not on a relation between mortality rates of skilled labour between sexes, and Manning's result will fail to hold, <u>even if the mortality rates of skilled workers differ by sex</u>.

What Proposition 4 tells us instead is that if gross accretion rates to the labour force are the same for each sex and are independent of the respective skill ratios, the decision on educational priority by sex depends on the <u>difference</u> between the skilled and unskilled mortality rates of each sex. A comparison of the mortality rates of skilled workers alone is valid only if the mortality rates of <u>all</u> unskilled workers are the same. And although unskilled mortality differentials may be narrowing due to changes in conventions and contracts, evidence would suggest that m_{um} is less than m_{uf}. Thus, the case for educating women is partially reestablished.

In the more general case considered here, (13) and (14) imply the following.

<u>Proposition 5</u>.

If graduation and mortality elasticities with respect to the proportion of skilled females are positive, and unskilled male and female labour are imperfectly substitutable, then $\gamma_m < \gamma_f$, and males receive educational priority, as $n_m + (1-2\alpha_m)(m_{sm}-m_{um}) < \{n_f+(1-2\alpha_f)(m_{sf}-m_{uf})-\mu_f m_{sf}(1-\alpha_f) - [n_f+(1-\alpha_f)(m_{sf}-m_{uf})]\mu\}\lambda - \{(1-\alpha_m)\eta_m m_{sm}+[n_m+(1-\alpha_m)(m_{sm}-m_{um})]\mu\}p_m\alpha_m/\alpha_f$.

The implications of these generalizations may be most easily seen by considering some special cases of Proposition 5. First, assuming $\lambda < 1$, if the graduation and mortality elasticities are zero, and $n_m = n_f = n$, Proposition (5) implies that men will receive educational priority as $(1-2\alpha_m)(m_{sm}-m_{um}) < (1-2\alpha_f)(m_{sf}-m_{uf})\lambda$. If the condition for male educational priority is met when $\lambda = 1$, there is no guarantee that it will be met when λ is less than unity, especially if it is substantially so. Since minor differences in mortality differentials between skilled and unskilled workers can be sufficient to lead to sex-specialized education, minor variations in λ could lead to reversals in the choice of sex-related educational priority. Estimates of λ in the context of a time series of cross-section Cobb-Douglas technologies are contained in Woodfield [1973], where it is shown that λ appears fairly stable over time, the median estimate being approximately 0.7.

Next, if $\lambda = 1$ and $n_m = n_f = n$, if $\gamma_m < \gamma_f$ in the presence of zero values for μ, η_m, and η_f, there is no guarantee that this condition will continue to be met when the graduation and mortality rate elasticities are positive. It is easily checked that γ_f is unambiguously reduced in this case. The presence of nonzero elasticities clearly improves the chances of women being educated, and of receiving educational priority, but naturally offers no guarantees. The reason is that the gains offered through the increases in workforce trainability and reductions in mortality may be insufficient to offset the losses associated with high mortality of skilled females.

An examination of (13)-(14) reveals that whereas γ_i generally depends on the <u>own</u>-skill ratio α_i, γ_f depends on both the own-skill ratio α_f and the male-skill ratio α_m. Clearly, $\partial\gamma_f/\partial\alpha_m > 0$ if the elasticities are assumed to be constant, which means that a change in any factor tending to reduce the equilibrium skill-intensity of males will raise (or at least not lower) the rate at which females are being skilled. For instance, if males are currently being skilled at rate δ_m (implying $f_x/f_y > \gamma_m$), and there is (say) a reduction in the male skilled-unskilled mortality differential, then males can be trained at a maximal rate of $\tilde{\delta}_m < \delta_m$, and the equilibrium male skill-ratio α_m must fall, in turn reducing γ_f. Suppose no females were previously skilled, implying $f_x/f_y < \gamma_f$. The reduction in γ_f induced by the reduction in α_m may be sufficient to replace this latter inequality by an equality, or even an inequality with opposite sign. This interdependence arises only when either (or both) μ and η_m are positive; it does not depend on the skilled-female mortality elasticity η_f.

IV. CONCLUSION

The purpose of this paper was to clarify and extend two major results in balanced growth educational policy due to Manning. In Manning's view, the most important result of his [1975] paper was the golden-rule wage differential, which he suggested [1975, p.511] that "even on a generous interpretation of statistics, ... seems much below prevailing differentials". Manning pointed to the difficulty in expanding selected areas of training due to monopoly power of professional bodies, and while he experienced the unfortunate effects of limitation of entry due to underfunding in his own discipline, he was also a harsh critic of those who appeared instead to be imposing "limitation of exit" through lengthy courses and high failure rates, especially at advanced levels of study. The present paper, by weakening an unnecessarily restrictive assumption made by Manning, along with allowing the training process to affect the level of fertility, strengthens Manning's results in that optimal wage differentials are further reduced.

Further, Manning concluded his [1976a] paper which contained an optimizing basis for the preferential selection of men for training, by interpreting his results as unfashionable. Nevertheless, he also recognized that they resulted from the proper accounting for the consumption costs of so-called "enlightened" educational policy. The present paper again relaxes an unnecessarily restrictive assumption of Manning. It also introduces new considerations whereby male and female education are not symmetric in their impacts on graduation and mortality rates of skilled workers, and considers the case of imperfect substitutability of unskilled male and female workers. The results go some way to restore fashion, but without introducing faddism, or arguments that education of certain groups is intrinsically worthwhile. However, the results are highly aggregative, and serve to yield insight rather than realistic decision rules. One interesting generalization is to permit fertility differences across families according to their male/female skill composition. Another is to permit differences across families in their home investments in children, permitting much greater variation in the degree of educability and workforce mortality of their offspring. Experimentation with models of these types, however, seem to complicate matters substantially without adding much insight.

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FOOTNOTES

1. Expositions and critical surveys of this literature may be found in Ben-Porath [1977], Leibenstein [1974], Schultz [1974], Schultz [1976] and Becker [1981]. The incorporation of fertility decisions into modern growth theory is relatively recent; see, for example, Barro and Becker [1989], Becker [1988], Benhabib and Nishimura [1990], Eckstein and Wolpin [1982] and Willis [1985].

2. Manning frequently points out that the model does not permit a variable age of labor market entry, and hence is best thought of as referring to post-basic schooling.

3. I shall not cast myself into the fiery furnace of the heritability-environment debate regarding educational/earnings outcomes. It would appear, however, that little will be resolved in this area until we can measure the extent to which parents of dizygotic twins (or pairs of siblings, generally) differentiate between their children in terms of ability/earnings - related home inputs compared to parents of mono zygotic twins. On this, see Goldberger [1977].

4 It might, however, be justified as an approximation.

5. In what follows in Section II, this assumption will be made explicitly.

6. For an elaboration of this point, see Erickson [1978], Chapter 5.

7. This distinction is of prime importance in the economic theory of marriage developed by Becker [1964].

8. For details of these arguments, see Manning [1976a], pp.385-9.

9. Technically, women need to be absent from the labor force for a time "necessary" to prepare, deliver, and recover from the birth of each child. This period may be very short indeed. However, many skilled women do not farm out their young children. One argument about why they do not is that they then lose much of the opportunity to develop individualistic characteristics in their children. A discussion of this issue may be found in Blandy and Woodfield [1977].

10. See the discussion in Rutter [1975], Chapter 3. Note that many professions whose members are public sector employees (and some private sector employees) are bound by maximal retirement age provisions which are generally sex-neutral in spirit and letter, but implicitly discriminate against women according to the argument in this section.

11. It could be argued that unskilled females will allocate their additional nonmarket time in favour of activities which also build skills in their children. Our assumption requires that unskilled mothers' time is very unproductive in training children, that diminishing returns set in very quickly once children reach school age (and schooling reduces the time available to teach children at home), or that these mothers do not take the opportunity. The latter is an empirical question, on which evidence is both scanty and conflicting. Gronau's [1976] study of the time allocation patterns of a sample of Israeli women in 1969 found labour force participation and female education positively related, yet women with little education bore more children and spent less total time in child-care activities than their more highly educated counterparts. Gronau also conjectures that uneducated womens' child-care time is relatively chore-intensive. Some contrary results for U.S. women are found by Leibowitz [1972].

12. For a survey and discussion of these, and related, results, see Leibowitz [1977].

13. The approximations in (14) and (15) are obtained by partially differentiating (12) with respect to α_m and α_f, equating to zero, solving for f_x/f_y respectively, and, as in Section II, assuming a sufficiently large value for the student-staff ratio that the denominators on the right-hand side of each resulting expression are (approximately) unity.

14. Cf. Manning [1976a], p.385.

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