



The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search

<http://ageconsearch.umn.edu>

aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

No endorsement of AgEcon Search or its fundraising activities by the author(s) of the following work or their employer(s) is intended or implied.

CANTER

9011✓

Department of Economics
UNIVERSITY OF CANTERBURY

CHRISTCHURCH, NEW ZEALAND



**TESTING LINEAR RESTRICTIONS ON COEFFICIENTS IN A
LINEAR REGRESSION MODEL WITH PROXY VARIABLES
AND SPHERICALLY SYMMETRIC DISTURBANCES**

Kazuhiro Ohtani and Judith A. Giles

GIANNINI FOUNDATION OF
AGRICULTURAL ECONOMICS
LIBRARY

JAN 14 1991

Discussion Paper

No. 9011

This paper is circulated for discussion and comments. It should not be quoted without the prior approval of the author. It reflects the views of the author who is responsible for the facts and accuracy of the data presented. Responsibility for the application of material to specific cases, however, lies with any user of the paper and no responsibility in such cases will be attributed to the author or to the University of Canterbury.

Department of Economics, University of Canterbury
Christchurch, New Zealand

Discussion Paper No. 9011

October 1990

**TESTING LINEAR RESTRICTIONS ON COEFFICIENTS IN A
LINEAR REGRESSION MODEL WITH PROXY VARIABLES
AND SPHERICALLY SYMMETRIC DISTURBANCES**

Kazuhiro Ohtani
(Kobe University)

and

Judith A. Giles
(University of Canterbury)

TESTING LINEAR RESTRICTIONS ON COEFFICIENTS IN A
LINEAR REGRESSION MODEL WITH PROXY VARIABLES
AND SPHERICALLY SYMMETRIC DISTURBANCES*

Kazuhiro Ohtani
Kobe University

and

Judith A. Giles
University of Canterbury

October 1990

ABSTRACT

In this paper we consider the power function of the classical F test for linear restrictions on the coefficients in a linear regression model with spherically symmetric disturbances when proxies are used in the place of unobservable regressors. We numerically evaluate the power function assuming multivariate Student-t (Mt) regression disturbances in a simple univariate regression model. Our results show the effects on the power function of the degrees of freedom of the Mt distribution and of the correlation between the omitted and the proxy variable.

* We are grateful to David Giles for several helpful suggestions and comments.

Correspondence:

Kazuhiro Ohtani
Faculty of Economics
Kobe University
Rokko, Nada, Kobe
JAPAN

1. Introduction

Frequently, in applied regression analysis using economic data, we cannot observe some of the variables suggested by our economic theory. Then, the researcher must decide whether to omit the unobservable variables or to replace them by proxies. Within the errors-in-variables framework, and assuming only one unobservable variable, McCallum (1972) and Wickens (1972) show that the use of a proxy variable leads to smaller asymptotic bias than omitting the problem variable. Barnow (1976) shows that this result may not be generally applicable if there is more than one unobservable variable; we may be better to delete the unobservable variable. In terms of mean squared error, this latter finding is supported by the studies of Aigner (1974), Frost (1979), Kinal and Lahiri (1981), Ohtani (1981), and Srivastava and Madhuliea (1990). In particular, Srivastava and Madhuliea (1990) show that we should always exclude the unobservable variables, rather than proxy them, according to a mean squared error matrix criterion.

These studies concentrate on the effects on the properties of the parameter estimates when proxy variables are used. Relatively little research considers the corresponding implications for hypothesis testing. Exceptions include Ohtani (1985), Kakimoto and Ohtani (1985), and Ohtani (1987). This research assumes that the disturbance term follows a normal distribution. However, there is a wide body of literature which suggests that some economic data series (in particular, financial data), may be generated by processes which exhibit more kurtosis than the normal distribution. [See, for example, Mandelbrot (1963), Fama (1965), Blattberg and Gonedes (1974), Praetz (1972), Praetz and Wilson (1978), Lau *et al.* (1990), Rainbow and Praetz (1986) and Hall *et al.* (1989).

This has obvious implications for the distribution of the regression disturbance term, and accordingly there has been increasing interest in the

sampling properties of estimators and test statistics for non-normally distributed disturbances. One alternative assumption to normality is the wider one of spherical symmetry.¹ Many studies have investigated linear regression models with spherically symmetric disturbances including Thomas (1970), Zellner (1976), King (1979), Ullah and Zinde-Walsh (1984, 1985), Judge *et al.* (1985), Sutradhar and Ali (1986), Ullah and Phillips (1986), Andrews and Phillips (1987), Zinde-Walsh and Ullah (1987), Singh (1988), Sutradhar (1988), Giles (1990a,b,c), and Ohtani (1990).²

The extension to this broader family of distributions is easily motivated by some of the aforementioned empirical studies. Further, the probability density functions (pdf's) of a sub-class of this family of distributions can be written as a variance mixture of normal distributions (we denote this sub-class of families as SSD_N). This implies that each individual error term may be normally distributed but that their joint distribution is non-normal if the variance is itself a random variable. The multivariate-t (Mt) distribution is one member of this family of distributions (which includes normality as a further special case), and it arises if the variance mixing distribution is the inverted gamma distribution. Then the marginal distributions are univariate Student-t and they may have thicker tails than under normality.

In particular, assuming a correctly specified regression model Thomas (1970) derives the non-null distribution of the classical F test for linear restrictions on the regression coefficients and shows that it depends on the specific form of the SSD. King (1979) shows that this test is a UMPI size- α test for the wider case of elliptically symmetric disturbances. Sutradhar (1988) (unaware of Thomas (1970) or Ullah and Phillips (1986)) examines the power of the test when the regression disturbances are Mt, and finds that it depends on the degrees of freedom of the Mt population. He computes the

power for various cases and shows that if the restrictions are in the neighbourhood of being true then the power of the test is at least the same as that which would result from a normality assumption. However, for relatively large hypothesis error the power for finite degrees of freedom of the M_t distribution is lower. Giles (1990a) derives the non-null distribution of the test statistic when the regression disturbances are SSD_N and we have omitted relevant regressors from the design matrix. She does not, however, examine the power function of the test.

In this paper we extend this research by considering the sampling properties of the F test statistic in a linear regression model with proxy variables and SSD_N disturbances. In the next section we provide the model framework and we derive, in Section 3, the non-null distribution of the test statistic. Under the null, this distribution is not, in general, central F. The exception is if the null hypothesis is that the coefficients on the unobservable variables are zero. We show this result in Section 4. The power of the test, however, depends on the variance mixing distribution. To examine the power function of the test we consider the special case of M_t regression disturbances and a simple univariate regression model under a variety of alternative scenarios in Section 5. The paper concludes with some final remarks in Section 6.

2. The model and the test statistic.

Suppose that the true regression model is

$$y = X_1\beta_1 + X_2\beta_2 + u, \quad (1)$$

where y is an $(n \times 1)$ vector of observations on the dependent variable, X_1 is an $(n \times k_1)$ matrix of observable independent variables, X_2 is an $(n \times k_2)$ matrix of unobservable independent variables, β_1 and β_2 are $(k_1 \times 1)$ and $(k_2 \times 1)$

vectors of regression coefficients, and u is an $(n \times 1)$ vector of error terms. We assume that X_1 and X_2 are non-stochastic (or independent of u).

As X_2 is unobservable, there is an incentive to replace it with a matrix of proxy variables. Denoting this $(n \times k_2)$ matrix as X_2^* , we write the model which includes the proxy variables as

$$y = X_1 \beta_1^* + X_2^* \beta_2^* + u^* \quad (2)$$

Let $X = [X_1, X_2]$, $X^* = [X_1, X_2^*]$, $\beta' = [\beta_1', \beta_2']$, $\beta^{*'} = [\beta_1^{*'}, \beta_2^{*'}]$ so that (1) and (2) can be written as

$$y = X\beta + u \quad (3)$$

and

$$y = X^* \beta^* + u^* \quad (4)$$

As to the error terms, we assume that u has a non-normal distribution of the form $f(u) = \int_0^\infty f_N(u)f(\tau)d\tau$, where $f_N(u)$ is $f(u)$ when $u \sim N(0, \tau^2 I_n)$, and $f(\tau)$ is supported on $[0, \infty)$. So $E(u) = 0$, and $E(uu') = \sigma_u^2 I_n$, where $\sigma_u^2 = f(\tau^2)$. This class of non-normal distributions is a subclass of the family of spherically symmetric disturbances which can be expressed as a variance mixture of normal distributions. When the disturbances are Mt we have

$$f(\tau) = \left[\frac{2}{\Gamma(\nu/2)} \right] \left(\nu \sigma^2 / 2 \right)^{\nu/2} \tau^{-(\nu+1)} e^{-\nu \sigma^2 / 2 \tau^2} \quad (5)$$

and

$$\begin{aligned} f(u) &= \left[\nu^{v/2} \Gamma((\nu+n)/2) \right] \left[\pi^{v/2} \Gamma(\nu/2) \sigma^n \right]^{-1} \\ &\times \left[\nu + u' u / \sigma^2 \right]^{-(\nu+n)/2} \end{aligned} \quad (6)$$

The mean and covariance matrix of u , for $\nu > 2$, are $E(u) = 0$ and $E(uu') = \left(\nu \sigma^2 / (\nu - 2) \right) I_n$.

We suppose there are m ($\leq k$) exact linear restrictions on the regression coefficients given by,

$$R\beta = r,$$

where R is an $(m \times k)$ ($k = k_1 + k_2$) known non-stochastic matrix and r is a known $(m \times 1)$ non-stochastic vector. The test statistic for the null hypothesis, $H_0: R\beta = r$, is

$$F = (p/m)(Rb^* - r)' \left[RS^{*-1}R' \right]^{-1} (Rb^* - r) / e^{*'} e^*, \quad (7)$$

where $b^* = S^{*-1}X^{*'}y$, $e^* = y - X^*b^*$, $S^* = X^{*'}X^*$, $p = n - k$.

3. The non-null distribution of F .

The pdf of F is

$$f(F) = \int_0^\infty f_N(F)f(\tau)d\tau, \quad (8)$$

where $f_N(F)$ is $f(F)$ when $u \sim N(0, \tau^2 I_n)$. Ohtani (1983) shows that $f_N(F)$ is doubly non-central F with m and p degrees of freedom and non-centrality parameters λ/τ^2 and θ/τ^2 , where

$$\lambda = \left(RS^{*-1}X^{*'}X\beta - r \right)' \left[RS^{*-1}R' \right]^{-1} \left(RS^{*-1}X^{*'}X\beta - r \right), \quad (9)$$

$$\theta = \beta'X'M^*X\beta, \quad (10)$$

and $M^* = I_n - X^*S^{*-1}X^{*'}$. So, (Johnson and Kotz (1970))

$$f_N(F) = e^{-(\lambda+\theta)/2\tau^2} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (\lambda/2)^i (\theta/2)^j (\tau^2)^{-(i+j)}$$

$$\times m^{\frac{m}{2}+i} p^{\frac{p}{2}+j} F^{\frac{1}{2}m+i-1} (p+mF)^{-\frac{1}{2}(m+p)-i-j} \left[i!j!B\left(\frac{1}{2}m+i; \frac{1}{2}p+j\right) \right]^{-1},$$

and

$$\begin{aligned} f(F) &= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (\lambda/2)^i (\theta/2)^j m^{\frac{m}{2}+i} p^{\frac{p}{2}+j} F^{\frac{1}{2}m+i-1} \\ &\times (p+mF)^{-\frac{1}{2}(m+p)-i-j} \left[i!j!B\left(\frac{1}{2}m+i; \frac{1}{2}p+j\right) \right]^{-1} \\ &\times \int_0^{\infty} e^{-(\lambda+\theta)/2\tau^2} (\tau^2)^{-(i+j)} f(\tau) d\tau, \end{aligned} \quad (11)$$

where $B(.,.)$ is the beta function.

$f(F)$ depends on $f(\tau)$ and we can regard (11) as a weighted average of the non-null distribution of F under normality with the form of the variance mixing distribution determining the weights. Assuming that X_2 is observable, if the linear restrictions relate only to the coefficients of X_1 , then (11) collapses first, to the non-null distribution derived by Giles (1990a) when we omit X_2 , that is, we exclude relevant regressors; and secondly, to the non-null distribution derived by Giles (1990b) when the design matrix is properly specified.

When τ is an inverted gamma random variable with a pdf given by (5) then $u \sim \text{Mt}\left(0, \nu\sigma^2/(\nu-2)I_n\right)$ and

$$f_{\text{Mt}}(F) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} C_{ij} F^{\frac{m}{2}+i-1} (p+mF)^{-(m+p)/2-i-j}, \quad (12)$$

where

$$C_{ij} = 2^{i+j} \nu^{\nu/2} \left(\lambda / (2\sigma^2) \right)^i \left(\theta / (2\sigma^2) \right)^j \left[\nu + \lambda / \sigma^2 + \theta / \sigma^2 \right]^{-i-j-\nu/2} \\ \times \Gamma(i+j + \nu/2) m^{\frac{m}{2}+i} p^{\frac{p}{2}+j} \left[\Gamma\left(\frac{\nu}{2}\right) {}_2F_1\left(\frac{m}{2}+i; \frac{p}{2}+j\right) \right]^{-1}.$$

(12) follows from (11) by using the change of variable $\tau^2 = 1/Z$, then $t = (\lambda + \theta + \nu\sigma^2)Z/2$, and by noting that $\int_0^\infty e^{-t} t^{f-1} dt = \Gamma(f)$.

Note that, in general, λ and θ are non-zero even if H_0 is true. Accordingly, the usual procedure of testing the restrictions by using a critical value from the central F distribution is invalid for all members of the SSD_N family when we include proxy variables³. The exception is when the null hypothesis is that $\beta_2 = 0$. We consider this case in the next section.

4. The null distribution of F when $H_0: \beta_2 = 0$.

In this section, we consider the null distribution of F when the linear restrictions are that $\beta_2 = 0$; that is $R = \begin{bmatrix} 0, I_{k_2} \end{bmatrix}$ and $r=0$. The unobservable variables are insignificant under this null hypothesis.

To simplify the problem under this null we write

$$S^{*-1} = \begin{bmatrix} S^{*11} & -S^{*12} \\ -S^{*21} & S^{*22} \end{bmatrix}$$

where

$$S^{*11} = (X_1' X_1)^{-1} + (X_1' X_1)^{-1} X_1' X_2 S^{*22} X_2' X_1 (X_1' X_1)^{-1},$$

$$S^{*12} = S^{*21} = (X_1' X_1)^{-1} X_1' X_2 S^{*22}$$

$$S^{*22} = \left[X_2' X_2 - X_2' X_1 (X_1' X_1)^{-1} X_1' X_2 \right]^{-1},$$

and so, when $R = [0, I_{k_2}]$, $RS^{*-1}R' = S^{*22}$, $RS^{*-1}X^{*'}X\beta = \left(S^{*22}X_2^{*'} - S^{*21}X_1' \right) \times (X_1\beta_1 + X_2\beta_2)$, and as $\left(S^{*22} \right)^{-1} \left(S^{*22}X_2^{*'} - S^{*21}X_1' \right) X_1 = 0$, λ is given by

$$\lambda = \beta_2' X_2' \left(S^{*22}X_2^{*'} - S^{*21}X_1' \right)' \left(S^{*22} \right)^{-1} \left(S^{*22}X_2^{*'} - S^{*21}X_1' \right) X_2 \beta_2 \quad (13)$$

Further, we can write θ as

$$\begin{aligned} \theta &= \left(X_1\beta_1 + X_2\beta_2 \right)' M^* \left(X_1\beta_1 + X_2\beta_2 \right) \\ &= \beta_2' X_2' M^* X_2 \beta_2 \end{aligned} \quad (14)$$

$$\text{as } X_1' M^* = \begin{bmatrix} I_{k_1} & 0 \end{bmatrix} X^{*'} M^* = 0.$$

Clearly, under the null hypothesis $H_0: \beta_2 = 0$, $\lambda = \theta = 0$ and so,

$$f(F) = \left(B(m/2; p/2) \right)^{-1} m^{\frac{m}{2}} p^{\frac{p}{2}} F^{\frac{m}{2}-1} (p+mF)^{-(m+p)/2}, \quad (15)$$

which is the pdf of a central F random variate with m and p degrees of freedom. Hence, the test based on F is robust to the departure from normality that we are considering if the null hypothesis is $H_0: \beta_2 = 0$. However, clearly from (11) the power of the test depends on the variance mixing distribution.

5. The power of the test.

The distribution function of F is

$$\mathcal{F}(c) = \text{Pr.}(F < c) = \int_0^c f(F) dF,$$

which after applying the transformation $z = mF/(p+mF)$, is given by

$$\mathcal{F}(c) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (\lambda/2)^i (\theta/2)^j \left(i! j! \right)^{-1} I_{c^*} \left(\frac{1}{2} m + i; \frac{1}{2} p + j \right)$$

$$\times \int_0^{\infty} e^{-(\lambda+\theta)/2\tau^2} (\tau^2)^{-(i+j)} f(\tau) d\tau \quad (16)$$

where $c^* = mc/(p+mc)$, and $I_{c^*}(\cdot; \cdot)$ is the incomplete beta function ratio. (16) depends on $f(\tau)$, and so to numerically consider its effect on the size and the power of the F test we assume that τ is an inverted gamma random variable so that $u \sim Mt\left(0, \nu\sigma^2/(\nu-2)I_n\right)$. Then

$$\mathcal{F}_{Mt}(c) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} D_{ij} I_{c^*} \left(\frac{1}{2}m+i; \frac{1}{2}p+j \right), \quad (17)$$

where
$$D_{ij} = 2^{i+j} \nu^{\nu/2} \left(\lambda/(2\sigma^2) \right)^i \left(\theta/(2\sigma^2) \right)^j \left[\nu + \lambda/\sigma^2 + \theta/\sigma^2 \right]^{-i-j-\nu/2} \\ \times \Gamma \left(i+j+\nu/2 \right) \left[i!j! \Gamma(\nu/2) \right]^{-1}.$$

(17) follows from (16) in the same way that we obtained (12) from (11). We numerically evaluate (17) to examine the power performance of the F test when the regression disturbances are Mt and we have included proxy variables. To simplify this task we consider the following simple model:

$$Y = \beta_1 1_1 + \beta_2 X_2 + u, \quad (18)$$

where 1_1 is an $(n \times 1)$ vector consisting of ones, X_2 is an $(n \times 1)$ vector of the unobservable variable, and β_1 and β_2 are scalar coefficients. The model with the proxy variable X_2^* is

$$Y = \beta_1^* 1_1 + \beta_2^* X_2^* + u^*. \quad (19)$$

We assume that X_2 and X_2^* are measured as deviations from their sample means. (This involves no loss of generality.)

5.1 $H_0 : \beta_2 = 0$

We consider first $H_0 : \beta_2 = 0$. From Section 4 we know that under this null F

is a central F random variate with 1 and (n-2) degrees of freedom.

Let $t_{\beta_2} = \beta_2(X_2'X_2)^{\frac{1}{2}}/\sigma$ and $r_{22*} = X_2^{*'}X_2/\left[\left(X_2^{*'}X_2^*\right)\left(X_2'X_2\right)\right]^{\frac{1}{2}}$, so that $\lambda/\sigma^2 = t_{\beta_2}^2 r_{22*}^2$, and $\theta/\sigma^2 = t_{\beta_2}^2 \left(1 - r_{22*}^2\right)$. r_{22*} is the correlation coefficient between the unobservable variable, X_2 , and its proxy variable, X_2^* . When r_{22*} is very close to 1 the proxy variable is said to be "rich", while it is "poor" when r_{22*} is close to 0. Clearly, if there is perfect correlation between X_2^* and X_2 ($r_{22*}=1$) then $\theta/\sigma^2 = 0$ and $\lambda/\sigma^2 = t_{\beta_2}^2$. This latter case is the one examined by Sutradhar (1988).

We can regard t_{β_2} as a measure of the hypothesis error. *Ceteris paribus*, if H_0 is true then $t_{\beta_2} = 0$ (and for the special case we consider here, $\lambda/\sigma^2 = \theta/\sigma^2 = 0$), while $t_{\beta_2}^2$ increases with β_2 , as do also the non-centrality parameters.

We have calculated the power of the test, at the 5% significance level, when $n = 10, 20, 30$; $\nu = 5, 10, 20, 50, 100, \infty$ (normal); $r_{22*} = 0.1, 0.3, 0.5, 0.7, 0.9, 0.95$, and for various values of $t_{\beta_2}^4$. Typical results are given in Table 1 and illustrated in Figure 1. The results show that the F test is biased when the proxy variable is very poor, say when $r_{22*} = 0.1$, regardless of the value of ν . For a given value of ν , the power of the test decreases for $t_{\beta_2} \in (0, 6]$ indicating that the test is misleading as it fails to detect the alternative hypothesis over this range of t_{β_2} ; that is, the power is less than the size of the test. Further, for $t_{\beta_2} \in (0, 6]$ the power of the test is higher the smaller the value of ν . For $t_{\beta_2} > 6$ the power increases, dramatically so, for relatively large values of ν , while for relatively small values of ν the power increases relatively slowly.

If the proxy variable is a 'better substitute' for the unobservable

variable then we find that the power increases with ν , regardless of the value of t_{β_2} . For a 'moderate' proxy variable, say $r_{22*} = 0.5$, there is a considerable loss in power as ν decreases from ∞ , particularly for relatively larger values of t_{β_2} (hypothesis error). This carries over for a 'rich' proxy variable say, $r_{22*} = 0.95$, but the difference between the powers is relatively smaller. The results for this latter case are similar to those of Sutradhar (1988, p.179).

5.2 $H_0 : \beta_1 = 0$

Our aim in this section is to ascertain whether the results of Srivastava and Madhuliea (1990) extend to a comparison of power functions. That is, we address the question of whether the power of the test is higher if we omit the regressor than if we use a proxy variable. For this purpose we consider the model given by equation (18) subject to the hypothesis that $\beta_1 = 0$. Though this particular testing problem is usually not of interest to econometricians its examination may give some light to the question under investigation. (See also Ohtani (1985).)

Let $t_{\beta_1} = n^{1/2}\beta_1/\sigma$ so that $\lambda/\sigma^2 = t_{\beta_1}^2$ and $\theta/\sigma^2 = t_{\beta_2}^2(1-r_{22*}^2)$. When we simply omit the unobservable variable $\theta/\sigma^2 = t_{\beta_2}^2$.⁵ Note first that t_{β_1} is a measure of the hypothesis error, secondly, that $\lambda/\sigma^2 = 0$ under H_0 , and finally, that θ/σ^2 , and hence, t_{β_2} (for a given value of r_{22*}^2), indicates the extent of the mis-specification error whether from excluding the variable or from using an (imperfect) proxy variable.

We have calculated the true significance level and the power of the F test, for this case, when the nominal significance level is 5%⁶, for $n=10, 20, 30$; $\nu=5, 10, 20, 50, 100, \infty$; $r_{22*}=0.1, 0.5, 0.95$; $t_{\beta_2}=1, 10$; and for

various values of t_{β_1} . Typical results are given in Tables 2 and 3, and Figure 2.

When $t_{\beta_2}=1$ (see Table 2), that is, the degree of model mis-specification is relatively small, we find little difference between the sizes of the procedures for all ν , though the size when $r_{22^*}=0.95$ is marginally closer to 0.05. Nevertheless, as the power functions have different test sizes we cannot make a definite comparison of their powers. Our results suggest that for (relatively) small t_{β_2} the ordering of powers reflects the ordering of the corresponding sizes. So, for small t_{β_2} the power of the test is similar whether we omit or proxy the unobservable variable, regardless of the value of ν .

This result does not extend to all degrees of mis-specification, as is evident from the results given in Table 3, and depicted in Figure 2. In this case $t_{\beta_2}=10$. We now observe a wide variation in the sizes and powers of the test for the different procedures and for different values of ν . Specifically, we find, *ceteris paribus*, that as t_{β_2} increases the size and the power of the test firstly decrease, attain their minima, and then increase towards unity with the rates of convergence depending on ν and r_{22^*} (*ceteris paribus*, they are faster for large ν , or small r_{22^*} , or if the unobservable variable is simply omitted).⁷

So, in particular, if the model is sufficiently mis-specified then the size of the test is unity; that is, we always reject H_0 even if it is true. That an increase in mis-specification error causes the size/power to increase is easily seen from $\partial \mathcal{F}_{Mt}(c)/\partial \theta$ which is given by

$$\frac{\partial \mathcal{F}_{Mt}(c)}{\partial \theta} = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} b_{ij} \left[j \left(\nu + \frac{\lambda}{\sigma^2} \right) - \left(i + \frac{\nu}{2} \right) \frac{\theta}{\sigma^2} \right] \quad (20)$$

where

$$b_{ij} = 2^{i+j} \nu^{\nu/2} \left(\frac{\lambda}{2\sigma^2} \right)^i \frac{\Gamma(i+j+\nu/2)}{i!j!\Gamma(\nu/2)} \left(\frac{1}{2\sigma^2} \right) \left(\frac{\theta}{2\sigma^2} \right)^{j-1} \left[\nu + \frac{\lambda}{\sigma^2} + \frac{\theta}{\sigma^2} \right]^{i-j-\nu/2-1}.$$

From (20) it is clear that with λ and ν fixed the negative term dominates and so, $\mathcal{F}_{Mt}(c)$ (the power) will (eventually) decrease (increase) as θ increases.

Given the marked differences in the test sizes, it is difficult to compare the power functions. We can only make a definite conclusion when one procedure has both a lower test size and a higher power than another procedure. For the example given here this occurs, for small ν , when $r_{22^*} = 0.95$. Then, the test has a smaller size but higher power (for $t_{\beta_1} \geq 3$) using the "rich" proxy variable than omitting the regressor. This conclusion does not hold, however, when the proxy variable is poor. Though not illustrated in the results presented here, there is a degree of mis-specification (i.e. t_{β_2}) for which this feature is evident for all possible values of ν .

These results suggest, therefore, that the performance of the test is, typically, better if we include a "good" proxy variable than if we omit the unobservable variable. However, regardless of whether we omit or proxy the unobservable variable, the test is not robust to severe mis-specification.

6. Final remarks.

In this paper we have investigated the power function of the classical F test for linear restrictions on the coefficients in a linear regression model with spherically symmetric disturbances when proxy variables are used in the place of unobservable regressors. We have shown that the test is invalid unless the null hypothesis is that the coefficients on the unobservable variables are zero.

To illustrate the power functions we considered a simple model with

multivariate Student-t regression disturbances. The results showed that if the proxy variable is a good substitute for the unobservable variable then the power functions are similar for all ν . However, if the proxy is "poor" then the power for relatively small ν is higher than for $\nu=\infty$ if the restrictions are in the neighbourhood of being true. Otherwise, it is significantly lower for small ν than for large ν . Further, if the proxy is "poor", regardless of value of ν , the test fails to detect the alternative hypothesis for a wide range of the hypothesis error. We also investigated whether the power is higher if we proxy the unobservable variable than if we simply omit it. Our results suggest there is little difference if the proxy is "poor" but that it is preferable to use a proxy when it is "rich". We find that the test is not robust if the degree of mis-specification is high, particularly when we omit the variable or we use a "poor" proxy.

The numerical analysis assumes that there is only one unobservable variable. It remains for future research to determine whether these results extend to the case of more than one unobservable variable. The extent to which our results are valid for other forms of non-normality (for instance, iid non-normal regression disturbances) also needs investigation.

References

- Aigner, D.J., 1974, MSE dominance of least squares with errors-of-observation, *Journal of Econometrics* 2, 365-372.
- Andrews, D.K. and P.C.B. Phillips, 1987, Best median-unbiased estimation in linear regression with bounded asymmetric loss functions, *Journal of the American Statistical Association* 82, 886-893.
- Barnow, B.S., 1976, The use of proxy variables when one or two independent variables are measured with error, *The American Statistician* 30, 119-121.
- Blattberg, R.C. and N.J. Gonedes, 1974, A comparison of the stable and Student distributions as statistical models for stock prices, *Journal of Business* 47, 244-280.
- Fama, E.F., 1965, The behaviour of stock market prices, *Journal of Business* 38, 34-105.
- Frost, P.A., 1979, Proxy variables and specification bias, *Review of Economics and Statistics* 61, 323-325.
- Giles, J.A., 1990a, Preliminary-test estimation of a mis-specified linear model with spherically symmetric disturbances, Ph.D. thesis, University of Canterbury.
- Giles, J.A., 1990b, Pre-testing for linear restrictions in a regression model with spherically symmetric disturbances, *Journal of Econometrics*, forthcoming.
- Giles, J.A., 1990c, Estimation of the error variance after a preliminary-test of homogeneity in a regression model with spherically symmetric disturbances, mimeo., University of Canterbury.
- Hall, J.A., B.W. Brorsen and S.H. Irwin, 1989, The distribution of futures prices: A test of the stable Paretian and mixture of normals hypotheses, *Journal of Financial and Quantitative Analysis*, 24, 105-116.
- Johnson, N.L. and S. Kotz, 1970, *Distributions in statistics : Continuous univariate distributions-2* (McGraw-Hill, New York).
- Judge, G.G., S. Miyazaki and T.A. Yancey, 1985, Minimax estimators for the location vectors of spherically symmetric densities, *Econometric Theory* 1, 509-417.
- Kakimoto, S. and K. Ohtani, 1985, On the use of a proxy variable in the test for homoscedasticity, *Economics Letters* 18, 153-156.
- Kelker, D., 1970, Distribution theory of spherical distributions and a location-scale parameter generalization, *Sankhya A* 32, 419-430.
- Kinal, T. and K. Lahiri, 1981, Exact sampling distribution of the omitted variable estimator, *Economics Letters* 8, 121-127.
- King, M.L., 1979, Some aspects of statistical inference in the linear regression model, Ph.D. thesis, University of Canterbury.

- Lau, A. H-L., H-S. Lau and J.R. Wingender, 1990, The distribution of stock returns: New evidence against the stable model, *Journal of Business and Economic Statistics* 8, 217-223.
- Mandelbrot, B.B., 1963, The variation of certain speculative prices, *Journal of Business* 36, 394-419.
- McCallum, B.T., 1972, Relative asymptotic bias from errors of omission and measurement, *Econometrica* 40, 757-758.
- Mittelhammer, R.C., 1984, Restricted least squares, pre-test, OLS and Stein rule estimators: Risk comparisons under model misspecification, *Journal of Econometrics* 25, 151-164.
- Muirhead, R.J., 1982, *Aspects of multivariate statistical theory* (Addison-Wesley, Reading, Mass.).
- Ohtani, K., 1981, On the use of a proxy variable in prediction: An MSE comparison, *Review of Economics and Statistics* 63, 627-628.
- Ohtani, K., 1983, Preliminary test predictor in the linear regression model including a proxy variable, *Journal of the Japan Statistical Society* 13, 11-19.
- Ohtani, K., 1985, A note on the use of a proxy variable in testing hypothesis, *Economics Letters* 17, 107-110.
- Ohtani, K., 1987, Some sampling properties of the two-stage test in a linear regression with a proxy variable, *Communications in Statistics - Theory and Methods* 16, 717-729.
- Ohtani, K., 1990, Some sampling properties of the least squares estimator over an interval constraint when error terms have a multivariate t distribution, mimeo., Kobe University.
- Praetz, P.D., 1972, The distribution of share price changes, *Journal of Business* 45, 49-55.
- Praetz, P.D. and E.J.G. Wilson, 1978, The distribution of stock market returns: 1858-1973, *Australian Journal of Management* 3, 79-90.
- Rainbow, K. and P.D. Praetz, 1986, The distribution of returns in Sydney wool futures, in B.A. Goss (ed.), *Futures markets: Their establishment and performance* (Croom Helm, London).
- Singh, R.S., 1988, Estimation of error variance in linear regression models with errors having multivariate Student- t distribution with unknown degrees of freedom, *Economics Letters* 27, 47-53.
- Srivastava, V.K. and Madhuliea, 1990, Use of proxy variables in regression analysis, *Journal of Quantitative Economics* 6, 71-74.
- Sutradhar, B.C., 1988, Testing linear hypothesis with t -error variable, *Sankhya B* 50, 175-180.

- Sutradhar, B.C. and M.M. Ali, 1986, Estimation of the parameters of a regression model with a multivariate t error variable, *Communications in Statistics - Theory and Methods* 15, 429-450.
- Thomas, D.H., 1970, Some contributions to radial probability distributions, statistics, and the operational calculi, Ph.D. thesis, Wayne State University.
- Ullah, A. and P.C.B. Phillips, 1986, Distribution of the F -ratio, *Econometric Theory* 2, 449-452.
- Ullah, A. and V. Zinde-Walsh, 1984, On the robustness of LM, LR, and W tests in regression models, *Econometrica* 52, 1055-1066.
- Ullah, A. and V. Zinde-Walsh, 1985, Estimation and testing in a regression model with spherically symmetric errors, *Economics Letters* 17, 127-132.
- Wickens, M.R., 1972, A note on the use of proxy variables, *Econometrica* 40, 759-761.
- Zellner, A., 1976, Bayesian and non-Bayesian analysis of the regression model with multivariate Student- t error terms, *Journal of the American Statistical Association* 71, 400-405.
- Zinde-Walsh, V. and A. Ullah, 1987, On robustness of tests of linear restrictions in regression models with elliptical error distributions, in I.B. MacNeill and G.J. Umphrey (eds), *Time series and econometric modelling* (D. Reidel, Boston).

Footnotes

1. A discussion of this family of distributions is beyond our scope. See, for example, Kelker (1970) and Muirhead (1982).
2. Zellner (1976), Ullah and Zinde-Walsh (1984), Sutradhar and Ali (1986), Ullah and Phillips (1986), Singh (1988), Sutradhar (1988) and Ohtani (1990), consider the special case of Mt regression disturbances.
3. Similarly, the F test statistic is invalid when we merely omit relevant regressors and the linear restrictions relate only to the coefficients on X_1 . See, for instance, Mittelhammer (1984) and Giles (1990a).
4. The function is symmetric for positive and negative β_2 , and so we only consider the former.
5. For this latter case we take $R=1$ and $r=0$ in (7), and let $X^*=[1]:n \times 1$ in (9) and (10).
6. Recall that the test is only valid under the null for $H_0:\beta_2=0$.
7. Though this feature is not evident from Tables 2 and 3 for $r_{22^*}=0.95$, the more detailed numerical evaluations that we have undertaken show that it does hold for all r_{22^*} .

Table 1

Power of the F test at the 5% significance level for $n=20$ ($c=4.414$)

r_{22}^*	t_{β_2}	ν					
		5	10	20	50	100	∞
0.1	0	.0500	.0500	.0500	.0500	.0500	.0500
	1	.0457	.0457	.0456	.0456	.0456	.0456
	2	.0358	.0354	.0352	.0350	.0350	.0349
	3	.0254	.0243	.0237	.0233	.0231	.0230
	4	.0171	.0153	.0143	.0137	.0135	.0133
	5	.0116	.0092	.0080	.0073	.0071	.0069
	6	.0137	.0059	.0043	.0044	.0042	.0039
	7	.0383	.0119	.0056	.0038	.0269	1.0000
	8	.0985	.0499	.0219	.0446	1.0000	1.0000
	9	.1907	.1412	.0936	1.0000	1.0000	1.0000
	10	.3002	.2845	.2963	1.0000	1.0000	1.0000
0.5	0	.0500	.0500	.0500	.0500	.0500	.0500
	1	.0703	.0704	.0705	.0706	.0706	.0706
	2	.1209	.1223	.1231	.1236	.1238	.1239
	3	.1843	.1884	.1907	.1921	.1926	.1931
	4	.2497	.2567	.2607	.2632	.2640	.2649
	5	.3158	.3229	.3274	.3306	.3317	.3329
	6	.3882	.3914	.3929	.3943	.3984	.3984
	7	.4702	.4931	.4891	.5168	.5136	.6602
	8	.5814	.5985	.6517	.7300	.9534	1.0000
	9	.6660	.7017	.7744	.9797	1.0000	1.0000
	10	.7382	.8254	.9064	1.0000	1.0000	1.0000
0.95	0	.0500	.0500	.0500	.0500	.0500	.0500
	1	.1457	.1457	.1456	.1456	.1456	.1455
	2	.4045	.4158	.4217	.4253	.4265	.4277
	3	.6642	.7037	.7271	.7426	.7481	.7538
	4	.8268	.8779	.9069	.9255	.9319	.9383
	5	.9112	.9544	.9747	.9854	.9885	.9913
	6	.9529	.9833	.9940	.9979	.9987	.9993
	7	.9739	.9937	.9986	.9998	.9999	1.0000
	8	.9848	.9976	.9997	1.0000	1.0000	1.0000
	9	.9908	.9990	.9999	1.0000	1.0000	1.0000
	10	.9942	.9996	1.0000	1.0000	1.0000	1.0000

Figure 1 : Power functions for $n=20$ and $H_0 : \beta_2 = 0$

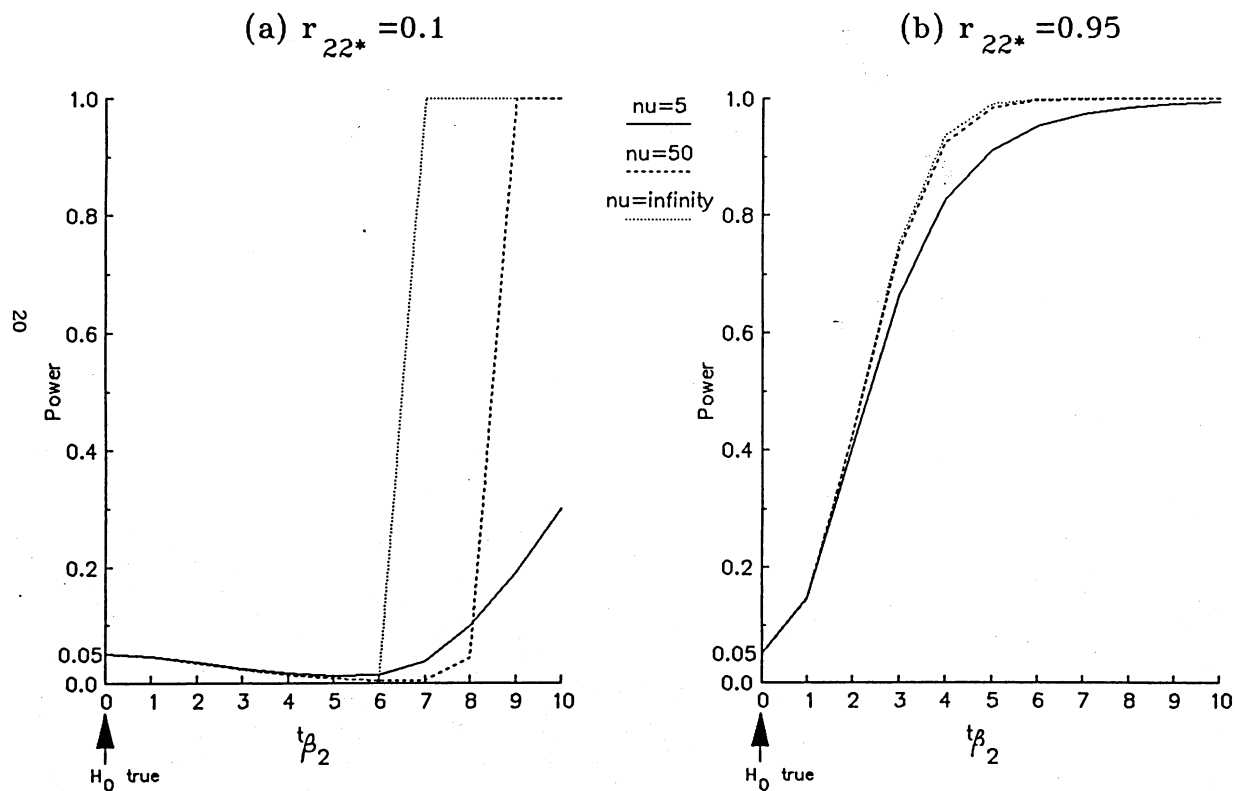


Table 2

Power of the F test at the nominal 5% significance level

for $n=20$ ($c=4.414$) when $H_0: \beta_1=0$ and $t_{\beta_2}=1.0$ *

r_{22}^*	t_{β_1}	ν					
		5	10	20	50	100	∞
-	0	.0449	.0449	.0449	.0449	.0448	.0448
	1	.1454	.1460	.1463	.1464	.1465	.1465
	2	.4272	.4408	.4480	.4524	.4540	.4555
	3	.6988	.7422	.7679	.7851	.7912	.7974
	4	.8549	.9056	.9334	.9506	.9564	.9621
	5	.9294	.9684	.9851	.9929	.9950	.9967
0.1	0	.0448	.0447	.0447	.0447	.0447	.0447
	1	.1443	.1449	.1452	.1454	.1454	.1455
	2	.4246	.4379	.4450	.4494	.4509	.4524
	3	.6964	.7394	.7650	.7820	.7881	.7943
	4	.8534	.9041	.9320	.9493	.9551	.9609
	5	.9286	.9678	.9847	.9926	.9947	.9965
0.5	0	.0460	.0459	.0459	.0459	.0459	.0459
	1	.1474	.1479	.1481	.1482	.1483	.1483
	2	.4290	.4426	.4499	.4544	.4559	.4574
	3	.6991	.7425	.7683	.7856	.7917	.7980
	4	.8546	.9053	.9332	.9505	.9563	.9620
	5	.9290	.9682	.9850	.9929	.9949	.9967
0.95	0	.0494	.0494	.0494	.0494	.0494	.0494
	1	.1563	.1563	.1563	.1563	.1562	.1562
	2	.4411	.4556	.4633	.4680	.4696	.4713
	3	.7063	.7509	.7775	.7953	.8016	.8082
	4	.8577	.9086	.9365	.9537	.9594	.9651
	5	.9303	.9692	.9858	.9934	.9954	.9970

* "-" in the column of r_{22}^* means that the unobservable variable is omitted.

Table 3

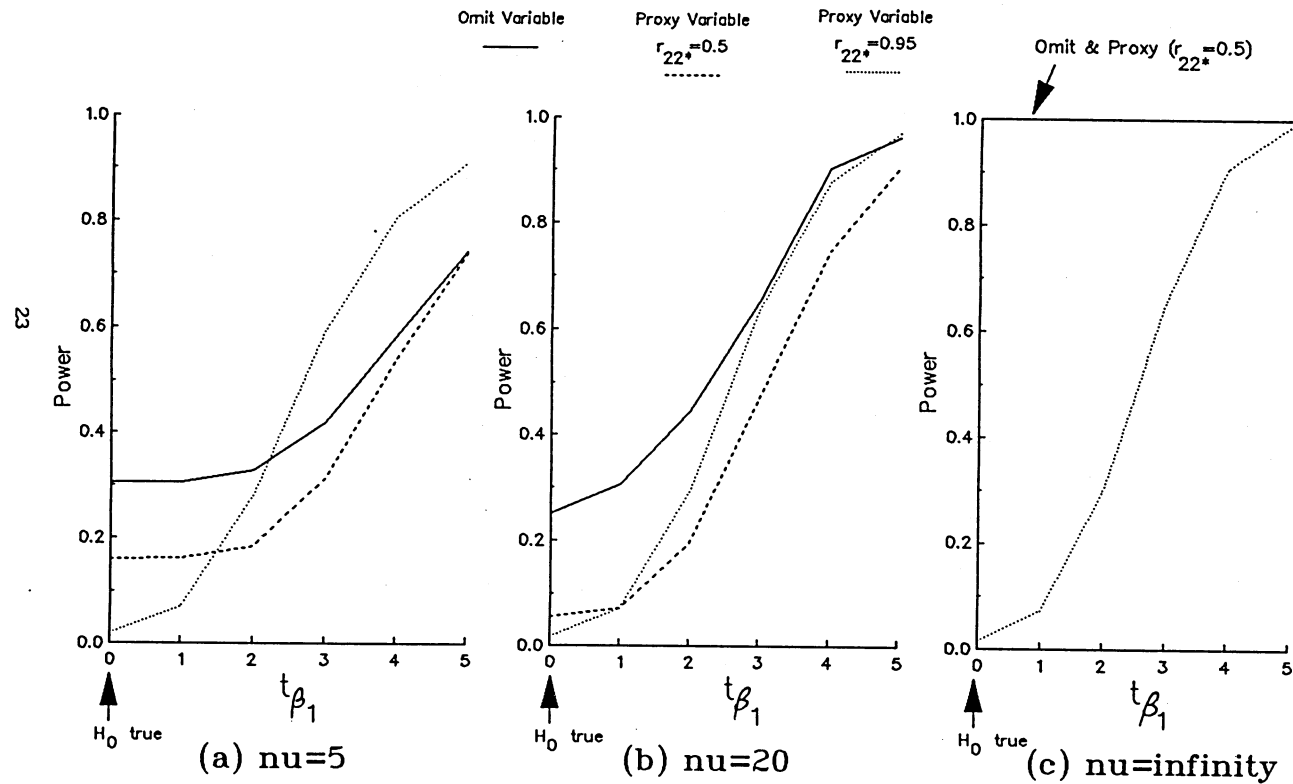
Power of the F test at the nominal 5% significance level

for $n=20$ ($c=4.414$) when $H_0: \beta_1=0$ and $t_{\beta_2}=10.0$ *

r_{22}^*	t_{β_1}	ν					
		5	10	20	50	100	∞
-	0	.3046	.2850	.2497	1.0000	1.0000	1.0000
	1	.3060	.2921	.3049	1.0000	1.0000	1.0000
	2	.3268	.3522	.4438	1.0000	1.0000	1.0000
	3	.4179	.5073	.6514	1.0000	1.0000	1.0000
	4	.5805	.7009	.9041	1.0000	1.0000	1.0000
	5	.7418	.8485	.9635	1.0000	1.0000	1.0000
0.1	0	.2988	.2774	.2400	1.0000	1.0000	1.0000
	1	.3002	.2845	.2963	1.0000	1.0000	1.0000
	2	.3211	.3459	.4381	1.0000	1.0000	1.0000
	3	.4127	.5032	.6488	1.0000	1.0000	1.0000
	4	.5753	.6979	.9037	1.0000	1.0000	1.0000
	5	.7362	.8457	.9632	1.0000	1.0000	1.0000
0.5	0	.1592	.1071	.0567	.0163	1.0000	1.0000
	1	.1622	.1098	.0714	1.0000	1.0000	1.0000
	2	.1847	.1631	.1948	1.0000	1.0000	1.0000
	3	.3119	.3610	.4703	1.0000	1.0000	1.0000
	4	.5368	.6282	.7461	1.0000	1.0000	1.0000
	5	.7382	.8254	.9064	1.0000	1.0000	1.0000
0.95	0	.0201	.0186	.0177	.0171	.0169	.0167
	1	.0690	.0705	.0714	.0719	.0721	.0723
	2	.2803	.2879	.2922	.2949	.2958	.2968
	3	.5902	.6189	.6351	.6455	.6491	.6528
	4	.8047	.8516	.8779	.8947	.9004	.9062
	5	.9090	.9509	.9707	.9813	.9845	.9875

* "-" in the column of r_{22}^* means that the unobservable variable is omitted.

Figure 2 : Power functions for $n=20$ when $H_0: \beta_1=0$ & $t_{\beta_2} = 10$



LIST OF DISCUSSION PAPERS*

- No. 8501 Perfectly Discriminatory Policies in International Trade, by Richard Manning and Koon-Lam Shea.
- No. 8502 Perfectly Discriminatory Policy Towards International Capital Movements in a Dynamic World, by Richard Manning and Koon-Lam Shea.
- No. 8503 A Regional Consumer Demand Model for New Zealand, by David E. A. Giles and Peter Hampton.
- No. 8504 Optimal Human and Physical Capital Accumulation in a Fixed-Coefficients Economy, by R. Manning.
- No. 8601 Estimating the Error Variance in Regression After a Preliminary Test of Restrictions on the Coefficients, by David E. A. Giles, Judith A. Mikołajczyk and T. Dudley Wallace.
- No. 8602 Search While Consuming, by Richard Manning.
- No. 8603 Implementing Computable General Equilibrium Models: Data Preparation, Calibration, and Replication, by K. R. Henry, R. Manning, E. McCann and A. E. Woodfield.
- No. 8604 Credit Rationing: A Further Remark, by John G. Riley.
- No. 8605 Preliminary-Test Estimation in Mis-Specified Regressions, by David E. A. Giles.
- No. 8606 The Positive-Part Stein-Rule Estimator and Tests of Linear Hypotheses, by Aman Ullah and David E. A. Giles.
- No. 8607 Production Functions that are Consistent with an Arbitrary Production-Possibility Frontier, by Richard Manning.
- No. 8608 Preliminary-Test Estimation of the Error Variance in Linear Regression, by Judith A. Clarke, David E. A. Giles and T. Dudley Wallace.
- No. 8609 Dual Dynamic Programming for Linear Production/Inventory Systems, by E. Grant Read and John A. George.
- No. 8610 Ownership Concentration and the Efficiency of Monopoly, by R. Manning.
- No. 8701 Stochastic Simulation of the Reserve Bank's Model of the New Zealand Economy, by J. N. Lye.
- No. 8702 Urban Expenditure Patterns in New Zealand, by Peter Hampton and David E. A. Giles.
- No. 8703 Preliminary-Test Estimation of Mis-Specified Regression Models, by David E. A. Giles.
- No. 8704 Instrumental Variables Regression Without an Intercept, by David E. A. Giles and Robin W. Harrison.
- No. 8705 Household Expenditure in Sri Lanka: An Engel Curve Analysis, by Mallika Dissanayake and David E. A. Giles.
- No. 8706 Preliminary-Test Estimation of the Standard Error of Estimate in Linear Regression, by Judith A. Clarke.
- No. 8707 Invariance Results for FIML Estimation of an Integrated Model of Expenditure and Portfolio Behaviour, by P. Dorian Owen.
- No. 8708 Social Cost and Benefit as a Basis for Industry Regulation with Special Reference to the Tobacco Industry, by Alan E. Woodfield.
- No. 8709 The Estimation of Allocation Models With Autocorrelated Disturbances, by David E. A. Giles.
- No. 8710 Aggregate Demand Curves in General-Equilibrium Macroeconomic Models: Comparisons with Partial-Equilibrium Microeconomic Demand Curves, by P. Dorian Owen.
- No. 8711 Alternative Aggregate Demand Functions in Macro-economics: A Comment, by P. Dorian Owen.
- No. 8712 Evaluation of the Two-Stage Least Squares Distribution Function by Imhof's Procedure by P. Cribbitt, J. N. Lye and A. Ullah.
- No. 8713 The Size of the Underground Economy: Problems and Evidence, by Michael Carter.
- No. 8714 A Computable General Equilibrium Model of a Fishery Method to Close the Foreign Sector, by Ewen McCann and Keith McLaren.
- No. 8715 Preliminary-Test Estimation of the Scale Parameter in a Mis-Specified Regression Model, by David E. A. Giles and Judith A. Clarke.
- No. 8716 A Simple Graphical Proof of Arrow's Impossibility Theorem, by John Fountain.
- No. 8717 Rational Choice and Implementation of Social Decision Functions, by Manimay Sen.
- No. 8718 Divisia Monetary Aggregates for New Zealand, by Ewen McCann and David E. A. Giles.
- No. 8719 Telecommunications in New Zealand: The Case for Reform, by John Fountain.

(Continued on back cover)

- No. 8801 Workers' Compensation Rates and the Demand for Apprentices and Non-Apprentices in Victoria, by Pasquale M. Sgro and David E. A. Giles.
- No. 8802 The Adventures of Sherlock Holmes, the 48% Solution, by Michael Carter.
- No. 8803 The Exact Distribution of a Simple Pre-Test Estimator, by David E. A. Giles.
- No. 8804 Pre-testing for Linear Restrictions in a Regression Model With Student-t Errors, by Judith A. Clarke.
- No. 8805 Divisia Monetary Aggregates and the Real User Cost of Money, by Ewen McCann and David Giles.
- No. 8806 The Management of New Zealand's Lobster Fishery, by Alan Woodfield and Pim Borren.
- No. 8807 Poverty Measurement: A Generalization of Sen's Result, by Prasanta K. Pattanaik and Manimay Sen.
- No. 8808 A Note on Sen's Normalization Axiom for a Poverty Measure, by Prasanta K. Pattanaik and Manimay Sen.
- No. 8809 Budget Deficits and Asset Sales, by Ewen McCann.
- No. 8810 Unorganized Money Markets and 'Unproductive' Assets in the New Structuralist Critique of Financial Liberalization, by P. Dorian Owen and Otton Solis-Fallas.
- No. 8901 Testing for Financial Buffer Stocks in Sectoral Portfolio Models, by P. Dorian Owen.
- No. 8902 Provisional Data and Unbiased Prediction of Economic Time Series by Karen Browning and David Giles.
- No. 8903 Coefficient Sign Changes When Restricting Regression Models Under Instrumental Variables Estimation, by David E. A. Giles.
- No. 8904 Economies of Scale in the New Zealand Electricity Distribution Industry, by David E. A. Giles and Nicolas S. Wyatt.
- No. 8905 Some Recent Developments in Econometrics: Lessons for Applied Economists, by David E. A. Giles.
- No. 8906 Asymptotic Properties of the Ordinary Least Squares Estimator in Simultaneous Equations Models, by V. K. Srivastava and D. E. A. Giles.
- No. 8907 Unbiased Estimation of the Mean Squared Error of the Feasible Generalised Ridge Regression Estimator, by V. K. Srivastava and D. E. A. Giles.
- No. 8908 An Unbiased Estimator of the Covariance Matrix of the Mixed Regression Estimator, by D. E. A. Giles and V. K. Srivastava.
- No. 8909 Pre-testing for Linear Restrictions in a Regression Model with Spherically Symmetric Disturbances, by Judith A. Giles.
- No. 9001 The Durbin-Watson Test for Autocorrelation in Nonlinear Models, by Kenneth J. White.
- No. 9002 Determinants of Aggregate Demand for Cigarettes in New Zealand, by Robin Harrison and Jane Chetwyd.
- No. 9003 Unemployment Duration and the Measurement of Unemployment, by Manimay Sengupta.
- No. 9004 Estimation of the Error Variance After a Preliminary-Test of Homogeneity in a Regression Model with Spherically Symmetric Disturbances, by Judith A. Giles.
- No. 9005 An Expository Note on the Composite Commodity Theorem, by Michael Carter.
- No. 9006 The Optimal Size of a Preliminary Test of Linear Restrictions in a Mis-specified Regression Model, by David E. A. Giles, Offer Lieberman, and Judith A. Giles.
- No. 9007 Inflation, Unemployment and Macroeconomic Policy in New Zealand: A Public Choice Analysis, by David J. Smyth and Alan E. Woodfield.
- No. 9008 Inflation — Unemployment Choices in New Zealand and the Median Voter Theorem, by David J. Smyth and Alan E. Woodfield.
- No. 9009 The Power of the Durbin-Watson Test when the Errors are Heteroscedastic, by David E. A. Giles and John P. Small.
- No. 9010 The Exact Distribution of a Least Squares Regression Coefficient Estimator After a Preliminary t-Test, by David E. A. Giles and Virendra K. Srivastava.
- No. 9011 Testing Linear Restrictions on Coefficients in a Linear Regression Model with Proxy Variables and Spherically Symmetric Disturbances by Kazuhiro Ohtani and Judith A. Giles.

* Copies of these Discussion Papers may be obtained for \$4 (including postage, price changes occasionally) each by writing to the Secretary, Department of Economics, University of Canterbury, Christchurch, New Zealand.