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# TESTING LINEAR RESTRICTIONS ON COEFFICIENTS IN A LINEAR REGRESSION MODEL WITH PROXY VARIABLES AND SPHERICALLY SYMMETRIC DISTURBANCES

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ABSTRACT

In this paper we consider the power function of the classical F test for linear restrictions on the coefficients in a linear regression model with spherically symmetric disturbances when proxies are used in the place of unobservable regressors. We numerically evaluate the power function assuming multivariate Student-t (Mt) regression disturbances in a simple univariate regression model. Our results show the effects on the power function of the degrees of freedom of the Mt distribution and of the correlation between the omitted and the proxy variable.

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## 1. Introduction

Frequently, in applied regression analysis using economic data, we cannot observe some of the variables suggested by our economic theory. Then, the researcher must decide whether to omit the unobservable variables or to replace them by proxies. Within the errors-in-variables framework, and assuming only one unobservable variable, McCallum (1972) and Wickens (1972) show that the use of a proxy variable leads to smaller asymptotic bias than omitting the problem variable. Barnow (1976) shows that this result may not be generally applicable if there is more than one unobservable variable; we may be better to delete the unobservable variable. In terms of mean squared error, this latter finding is supported by the studies of Aigner (1974), Frost (1979), Kinal and Lahiri (1981), Ohtani (1981), and Srivastava and Madhuliea (1990). In particular, Srivastava and Madhuliea (1990) show that we should always exclude the unobservable variables, rather than proxy them, acccording to a mean squared error matrix criterion.

These studies concentrate on the effects on the properties of the parameter estimates when proxy variables are used. Relatively little research considers the corresponding implications for hypothesis testing. Exceptions include Ohtani (1985), Kakimoto and Ohtani (1985), and Ohtani (1987). This research assumes that the disturbance term follows a normal distribution. However, there is a wide body of literature which suggests that some economic data series (in particular, financial data), may be generated by processes which exhibit more kurtosis than the normal distribution. [See, for example, Mandelbrot (1963), Fama (1965), Blattberg and Gonedes (1974), Praetz (1972), Praetz and Wilson (1978), Lau *et al.* (1990), Rainbow and Praetz (1986) and Hall *et al.* (1989).

This has obvious implications for the distribution of the regression disturbance term, and accordingly there has been increasing interest in the sampling properties of estimators and test statistics for non-normally distributed disturbances. One alternative assumption to normality is the wider one of spherical symmetry.[1] Many studies have investigated linear regression models with spherically symmetric disturbances including Thomas (1970), Zellner (1976), King (1979), Ullah and Zinde-Walsh (1984, 1985), Judge *et al.* (1985), Sutradhar and Ali (1986), Ullah and Phillips (1986), Andrews and Phillips (1987), Zinde-Walsh and Ullah (1987), Singh (1988), Sutradhar (1988), Giles (1990a,b,c), and Ohtani (1990).[2]

The extension to this broader family of distributions is easily motivated by some of the aforementioned empirical studies. Further, the probability density functions (pdf's) of a sub-class of this family of distributions can be written as a variance mixture of normal distributions (we denote this sub-class of families as $SSD_N$). This implies that each individual error term may be normally distributed but that their joint distribution is non-normal if the variance is itself a random variable. The multivariate-t (Mt) distribution is one member of this family of distributions (which includes normality as a further special case), and it arises if the variance mixing distribution is the inverted gamma distribution. Then the marginal distributions are univariate Student-t and they may have thicker tails than under normality.

In particular, assuming a correctly specified regression model Thomas (1970) derives the non-null distribution of the classical F test for linear restrictions on the regression coefficients and shows that it depends on the specifc form of the SSD. King (1979) shows that this test is a UMPI size-$\alpha$ test for the wider case of elliptically symmetric disturbances. Sutradhar (1988) (unaware of Thomas (1970) or Ullah and Phillips (1986)) examines the power of the test when the regression disturbances are Mt, and finds that it depends on the degrees of freedom of the Mt population. He computes the power for various cases and shows that if the restrictions are in the neighbourhood of being true then the power of the test is at least the same as that which would result from a normality assumption. However, for relatively large hypothesis error the power for finite degrees of freedom of the Mt distribution is lower. Giles (1990a) derives the non-null distribution of the test statistic when the regression disturbances are $SSD_N$ and we have omitted relevant regressors from the design matrix. She does not, however, examine the power function of the test.

In this paper we extend this research by considering the sampling properties of the F test statistic in a linear regression model with proxy variables and $SSD_N$ disturbances. In the next section we provide the model framework and we derive, in Section 3, the non-null distribution of the test statistic. Under the null, this distribution is not, in general, central F. The exception is if the null hypothesis is that the coefficients on the unobservable variables are zero. We show this result in Section 4. The power of the test, however, depends on the variance mixing distribution. To examine the power function of the test we consider the special case of Mt regression disturbances and a simple univariate regression model under a variety of alternative scenarios in Section 5. The paper concludes with some final remarks in Section 6.

## 2. The model and the test statistic.

Suppose that the true regression model is

$$y = X_1\beta_1 + X_2\beta_2 + u, \qquad (1)$$

where y is an $(n \times 1)$ vector of observations on the dependent variable, $X_1$ is an $(n \times k_1)$ matrix of observable independent variables, $X_2$ is an $(n \times k_2)$ matrix of unobservable independent variables, $\beta_1$ and $\beta_2$ are $(k_1 \times 1)$ and $(k_2 \times 1)$

vectors of regression coefficients, and u is an $(n\times 1)$ vector of error terms. We assume that $X_1$ and $X_2$ are non-stochastic (or independent of u).

As $X_2$ is unobservable, there is an incentive to replace it with a matrix of proxy variables. Denoting this $(n\times k_2)$ matrix as $X_2^*$, we write the model which includes the proxy variables as

$$y = X_1\beta_1^* + X_2^*\beta_2^* + u^* . \tag{2}$$

Let $X = [X_1,X_2]$, $X^* = [X_1,X_2^*]$, $\beta' = [\beta_1',\beta_2']$, $\beta^{*\prime} = [\beta_1^{*\prime},\beta_2^{*\prime}]$ so that (1) and (2) can be written as

$$y = X\beta + u \quad , \tag{3}$$

and

$$y = X^*\beta^* + u^* \quad . \tag{4}$$

As to the error terms, we assume that u has a non-normal distribution of the form $f(u) = \int_0^\infty f_N(u)f(\tau)d\tau$, where $f_N(u)$ is f(u) when $u \sim N(0,\tau^2 I_n)$, and $f(\tau)$ is supported on $[0,\infty)$. So $E(u) = 0$, and $E(uu') = \sigma_u^2 I_n$, where $\sigma_u^2 = f(\tau^2)$. This class of non-normal distributions is a subclass of the family of spherically symmetric disturbances which can be expressed as a variance mixture of normal distributions. When the disturbances are Mt we have

$$f(\tau) = \left[2/\Gamma(\nu/2)\right]\left(\nu\sigma^2/2\right)^{\nu/2} \tau^{-(\nu+1)} e^{-\nu\sigma^2/2\tau^2} \tag{5}$$

and

$$f(u) = \left[\nu^{\nu/2}\Gamma\left((\nu+n)/2\right)\right]\left[\pi^{\nu/2}\Gamma(\nu/2)\sigma^n\right]^{-1}$$

$$\times \left[\nu + u'u/\sigma^2\right]^{-(\nu+n)/2} \quad . \tag{6}$$

The mean and covariance matrix of u, for $\nu > 2$, are $E(u) = 0$ and $E(uu') = \left(\nu\sigma^2/(\nu-2)\right)I_n$ .

We suppose there are m ($\le$k) exact linear restrictions on the regression coefficients given by,

$$R\beta = r \quad ,$$

where R is an (m×k) ($k=k_1+k_2$) known non-stochastic matrix and r is a known (m×1) non-stochastic vector. The test statistic for the null hypothesis, $H_0$: $R\beta = r$, is

$$F = (p/m)(Rb^*-r)'\left[RS^{*-1}R'\right]^{-1}(Rb^*-r)/e^{*\prime}e^*, \tag{7}$$

where $b^* = S^{*-1}X^{*\prime}y$, $e^* = y-X^*b^*$, $S^* = X^{*\prime}X^*$, $p = n-k$.

## 3. The non-null distribution of F.

The pdf of F is

$$f(F) = \int_0^\infty f_N(F)f(\tau)d\tau \quad , \tag{8}$$

where $f_N(F)$ is f(F) when $u \sim N(0,\tau^2 I_n)$. Ohtani (1983) shows that $f_N(F)$ is doubly non-central F with m and p degrees of freedom and non-centrality parameters $\lambda/\tau^2$ and $\theta/\tau^2$, where

$$\lambda = \left(RS^{*-1}X^{*\prime}X\beta-r\right)'\left[RS^{*-1}R'\right]^{-1}\left(RS^{*-1}X^{*\prime}X\beta-r\right) \; , \tag{9}$$

$$\theta = \beta'X'M^*X\beta \; , \tag{10}$$

and $M^* = I_n - X^*S^{*-1}X^{*\prime}$. So, (Johnson and Kotz (1970))

$$f_N(F) = e^{-(\lambda+\theta)/2\tau^2}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}(\lambda/2)^i(\theta/2)^j(\tau^2)^{-(i+j)}$$

$$\times\ m^{\frac{m}{2}+i}\ p^{\frac{p}{2}+j}\ F^{\frac{1}{2}m+i-1}\ (p+mF)^{-\frac{1}{2}(m+p)-i-j}\left[i!j!B\left(\tfrac{1}{2}m+i;\ \tfrac{1}{2}p+j\right)\right]^{-1},$$

and

$$f(F) = \sum_{i=0}^{\infty}\ \sum_{j=0}^{\infty}\ (\lambda/2)^{i}(\theta/2)^{j}\ m^{\frac{m}{2}+i}\ p^{\frac{p}{2}+j}\ F^{\frac{1}{2}m+i-1}$$

$$\times\ (p+mF)^{-\frac{1}{2}(m+p)-i-j}\left[i!j!B\left(\tfrac{1}{2}m+i;\ \tfrac{1}{2}p+j\right)\right]^{-1}$$

$$\times \int_{0}^{\infty} e^{-(\lambda+\theta)/2\tau^{2}}(\tau^{2})^{-(i+j)}f(\tau)d\tau \quad , \tag{11}$$

where B(.;.) is the beta function.

$f(F)$ depends on $f(\tau)$ and we can regard (11) as a weighted average of the non-null distribution of F under normality with the form of the variance mixing distribution determining the weights. Assuming that $X_2$ is observable, if the linear restrictions relate only to the coefficients of $X_1$, then (11) collapses first, to the non-null distribution derived by Giles (1990a) when we omit $X_2$, that is, we exclude relevant regressors; and secondly, to the non-null distribution derived by Giles (1990b) when the design matrix is properly specified.

When $\tau$ is an inverted gamma random variable with a pdf given by (5) then $u \sim Mt\left(0,\nu\sigma^{2}/(\nu-2)I_{n}\right)$ and

$$f_{Mt}(F) = \sum_{i=0}^{\infty}\ \sum_{j=0}^{\infty}\ C_{ij}\ F^{\frac{m}{2}+i-1}\ \left(p+mF\right)^{-(m+p)/2-i-j}\ , \tag{12}$$

where $$C_{ij} = 2^{i+j}\nu^{\nu/2}\left(\lambda/(2\sigma^2)\right)^i\left(\theta/(2\sigma^2)\right)^j\left[\nu+\lambda/\sigma^2+\theta/\sigma^2\right]^{-i-j-\nu/2}$$

$$\times\, \Gamma\left(i+j+\nu/2\right)\, m^{\frac{m}{2}+i}\, p^{\frac{p}{2}+j}\left[\Gamma\left(\frac{\nu}{2}\right) i!j! B\left(\frac{m}{2}+i;\ \frac{p}{2}+j\right)\right]^{-1}.$$

(12) follows from (11) by using the change of variable $\tau^2 = 1/Z$, then $t = (\lambda+\theta+\nu\sigma^2)Z/2$, and by noting that $\int_0^\infty e^{-t}t^{f-1}dt = \Gamma(f)$.

Note that, in general, $\lambda$ and $\theta$ are non-zero even if $H_0$ is true. Accordingly, the usual procedure of testing the restrictions by using a critical value from the central F distribution is invalid for all members of the $SSD_N$ family when we include proxy variables [3]. The exception is when the null hypothesis is that $\beta_2=0$. We consider this case in the next section.

## 4. The null distribution of F when $H_0$: $\beta_2 = 0$.

In this section, we consider the null distribution of F when the linear restrictions are that $\beta_2 = 0$; that is $R = \left[0,\ I_{k_2}\right]$ and r=0. The unobservable variables are insignificant under this null hypothesis.

To simplify the problem under this null we write

$$S^{*-1} = \begin{bmatrix} S^{*11} & -S^{*12} \\ -S^{*21} & S^{*22} \end{bmatrix}$$

where

$$S^{*11} = \left(X_1'X_1\right)^{-1} + \left(X_1'X_1\right)^{-1}X_1'X_2S^{*22}X_2^{*\prime}X_1\left(X_1'X_1\right)^{-1},$$

$$S^{*12} = S^{*21\prime} = \left(X_1'X_1\right)^{-1}X_1'X_2^*S^{*22}$$

$$S^{*22} = \left[X_2^{*\prime}X_2^* - X_2^{*\prime}X_1\left(X_1'X_1\right)^{-1}X_1'X_2^*\right]^{-1},$$

and so, when $R = [0, I_{k_2}]$, $RS^{*-1}R' = S^{*22}$, $RS^{*-1}X^{*\prime}X\beta = \left(S^{*22}X_2^{*\prime} - S^{*21}X_1'\right) \times \left(X_1\beta_1 + X_2\beta_2\right)$, and as $\left(S^{*22}\right)^{-1}\left(S^{*22}X_2^{*\prime} - S^{*21}X_1'\right)X_1 = 0$, $\lambda$ is given by

$$\lambda = \beta_2'X_2'\left(S^{*22}X_2^{*\prime} - S^{*21}X_1'\right)'\left(S^{*22}\right)^{-1}\left(S^{*22}X_2^{*\prime} - S^{*21}X_1'\right)X_2\beta_2 \quad . \tag{13}$$

Further, we can write $\theta$ as

$$\theta = \left(X_1\beta_1 + X_2\beta_2\right)' M^* \left(X_1\beta_1 + X_2\beta_2\right)$$

$$= \beta_2'X_2'M^*X_2\beta_2 \; . \tag{14}$$

as $X_1'M^* = \left[I_{k_1}, \;\; 0\right] X^{*\prime}M^* = 0.$

Clearly, under the null hypothesis $H_0$: $\beta_2 = 0$, $\lambda = \theta = 0$ and so,

$$f(F) = \left(B(m/2;p/2)\right)^{-1} m^{\frac{m}{2}} p^{\frac{p}{2}} F^{\frac{m}{2}-1} (p+mF)^{-(m+p)/2} \,, \tag{15}$$

which is the pdf of a central F random variate with m and p degrees of freedom. Hence, the test based on F is robust to the departure from normality that we are considering if the null hypothesis is $H_0 : \beta_2 = 0$. However, clearly from (11) the power of the test depends on the variance mixing distribution.

## 5. The power of the test.

The distribution function of F is

$$\mathcal{F}(c) = \Pr.(F<c) = \int_0^c f(F)dF,$$

which after applying the transformation $z = mF/(p+mF)$, is given by

$$\mathcal{F}(c) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (\lambda/2)^i(\theta/2)^j\left(i!j!\right)^{-1} I_{c*}\left( \tfrac{1}{2}m+i;\ \tfrac{1}{2}p+j \right)$$

$$\times \int_0^\infty e^{-(\lambda+\theta)/2\tau^2}(\tau^2)^{-(i+j)} f(\tau)d\tau \tag{16}$$

where $c^* = mc/(p+mc)$, and $I_{c^*}(.;.)$ is the incomplete beta function ratio. (16) depends on $f(\tau)$, and so to numerically consider its effect on the size and the power of the F test we assume that $\tau$ is an inverted gamma random variable so that $u \sim Mt\left(0, \nu\sigma^2/(\nu-2)I_n\right)$. Then

$$\mathcal{F}_{Mt}(c) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} D_{ij}\, I_{c^*}\left(\tfrac{1}{2}m+i;\ \tfrac{1}{2}p+j\right), \tag{17}$$

where $$D_{ij} = 2^{i+j}\ \nu^{\nu/2}\left(\lambda/(2\sigma^2)\right)^i\left(\theta/(2\sigma^2)\right)^j\left[\nu+\lambda/\sigma^2+\theta/\sigma^2\right]^{-i-j-\nu/2}$$

$$\times\ \Gamma\left(i+j+\nu/2\right)\left[i!j!\ \Gamma(\nu/2)\right]^{-1}.$$

(17) follows from (16) in the same way that we obtained (12) from (11). We numerically evaluate (17) to examine the power performance of the F test when the regression disturbances are Mt and we have included proxy variables. To simplify this task we consider the following simple model:

$$Y = \beta_1 1_1 + \beta_2 X_2 + u, \tag{18}$$

where $1_1$ is an $(n\times1)$ vector consisting of ones, $X_2$ is an $(n\times1)$ vector of the unobservable variable, and $\beta_1$ and $\beta_2$ are scalar coefficients. The model with the proxy variable $X_2^*$ is

$$Y = \beta_1^* 1_1 + \beta_2^* X_2^* + u^*. \tag{19}$$

We assume that $X_2$ and $X_2^*$ are measured as deviations from their sample means. (This involves no loss of generality.)

### 5.1 $H_0 : \beta_2=0$

We consider first $H_0 : \beta_2=0$. From Section 4 we know that under this null F is a central F random variate with 1 and (n-2) degrees of freedom.

Let $t_{\beta_2} = \beta_2(X_2'X_2)^{\frac{1}{2}}/\sigma$ and $r_{22*} = X_2^{*\prime}X_2/\left[\left(X_2^{*\prime}X_2^*\right)\left(X_2'X_2\right)\right]^{\frac{1}{2}}$, so that $\lambda/\sigma^2 = t_{\beta_2}^2 r_{22*}^2$, and $\theta/\sigma^2 = t_{\beta_2}^2\left(1 - r_{22*}^2\right)$. $r_{22*}$ is the correlation coefficient between the unobservable variable, $X_2$, and its proxy variable, $X_2^*$. When $r_{22*}$ is very close to 1 the proxy variable is said to be "rich", while it is "poor" when $r_{22*}$ is close to 0. Clearly, if there is perfect correlation between $X_2^*$ and $X_2$ ($r_{22*}$=1) then $\theta/\sigma^2 = 0$ and $\lambda/\sigma^2 = t_{\beta_2}^2$. This latter case is the one examined by Sutradhar (1988).

We can regard $t_{\beta_2}$ as a measure of the hypothesis error. *Ceteris paribus*, if $H_0$ is true then $t_{\beta_2} = 0$ (and for the special case we consider here, $\lambda/\sigma^2 = \theta/\sigma^2 = 0$), while $t_{\beta_2}^2$ increases with $\beta_2$, as do also the non-centrality parameters.

We have calculated the power of the test, at the 5% significance level, when n = 10,20,30; $\nu$ = 5,10,20,50,100,$\infty$ (normal); $r_{22*}$ = 0.1, 0.3, 0.5, 0.7, 0.9, 0.95, and for various values of $t_{\beta_2}$ [4]. Typical results are given in Table 1 and illustrated in Figure 1. The results show that the F test is biased when the proxy variable is very poor, say when $r_{22*}$ = 0.1, regardless of the value of $\nu$. For a given value of $\nu$, the power of the test decreases for $t_{\beta_2} \in (0,6]$ indicating that the test is misleading as it fails to detect the alternative hypothesis over this range of $t_{\beta_2}$; that is, the power is less than the size of the test. Further, for $t_{\beta_2} \in (0,6]$ the power of the test is <u>higher</u> the smaller the value of $\nu$. For $t_{\beta_2} > 6$ the power increases, dramatically so, for relatively large values of $\nu$, while for relatively small values of $\nu$ the power increases relatively slowly.

If the proxy variable is a 'better substitute' for the unobservable variable then we find that the power increases with $\nu$, regardless of the value of $t_{\beta_2}$. For a 'moderate' proxy variable, say $r_{22*} = 0.5$, there is a considerable loss in power as $\nu$ decreases from $\infty$, particularly for relatively larger values of $t_{\beta_2}$ (hypothesis error). This carries over for a 'rich' proxy variable say, $r_{22*} = 0.95$, but the difference between the powers is relatively smaller. The results for this latter case are similar to those of Sutradhar (1988, p.179).

### 5.2 $H_0 : \beta_1=0$

Our aim in this section is to ascertain whether the results of Srivastava and Madhuliea (1990) extend to a comparison of power functions. That is, we address the question of whether the power of the test is higher if we omit the regressor than if we use a proxy variable. For this purpose we consider the model given by equation (18) subject to the hypothesis that $\beta_1=0$. Though this particular testing problem is usually not of interest to econometricians its examination may give some light to the question under investigation. (See also Ohtani (1985).)

Let $t_{\beta_1}=n^{1/2}\beta_1/\sigma$ so that $\lambda/\sigma^2=t^2_{\beta_1}$ and $\theta/\sigma^2=t^2_{\beta_2}(1-r^2_{22*})$. When we simply omit the unobservable variable $\theta/\sigma^2=t^2_{\beta_2}$.[5] Note first that $t_{\beta_1}$ is a measure of the hypothesis error, secondly, that $\lambda/\sigma^2=0$ under $H_0$, and finally, that $\theta/\sigma^2$, and hence, $t_{\beta_2}$ (for a given value of $r^2_{22*}$), indicates the extent of the mis-specification error whether from excluding the variable or from using an (imperfect) proxy variable.

We have calculated the true significance level and the power of the F test, for this case, when the nominal significance level is 5%[6], for n=10, 20, 30; $\nu$=5, 10, 20, 50, 100, $\infty$; $r_{22*}$=0.1, 0.5, 0.95; $t_{\beta_2}$=1, 10; and for various values of $t_{\beta_1}$. Typical results are given in Tables 2 and 3, and Figure 2.

When $t_{\beta_2}=1$ (see Table 2), that is, the degree of model mis-specification is relatively small, we find little difference between the sizes of the procedures for all $\nu$, though the size when $r_{22*}=0.95$ is marginally closer to 0.05. Nevertheless, as the power functions have different test sizes we cannot make a definite comparison of their powers. Our results suggest that for (relatively) small $t_{\beta_2}$ the ordering of powers reflects the ordering of the corresponding sizes. So, for small $t_{\beta_2}$ the power of the test is similiar whether we omit or proxy the unobservable variable, regardless of the value of $\nu$.

This result does not extend to all degrees of mis-specification, as is evident from the results given in Table 3, and depicted in Figure 2. In this case $t_{\beta_2}=10$. We now observe a wide variation in the sizes and powers of the test for the different procedures and for different values of $\nu$. Specifically, we find, *ceteris paribus*, that as $t_{\beta_2}$ increases the size and the power of the test firstly decrease, attain their minima, and then increase towards unity with the rates of convergence depending on $\nu$ and $r_{22*}$ (*ceteris paribus*, they are faster for large $\nu$, or small $r_{22*}$, or if the unobservable variable is simply omitted).[7]

So, in particular, if the model is sufficiently mis-specified then the size of the test is unity; that is, we always reject $H_0$ *even if it is true.* That an increase in mis-specification error causes the size/power to increase is easily seen from $\delta\mathcal{F}_{Mt}(c)/\delta\theta$ which is given by

$$\frac{\delta\mathcal{F}_{Mt}(c)}{\delta\theta} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} b_{ij}\left[ j\left( \nu+ \frac{\lambda}{\sigma^2} \right) - \left( i+ \frac{\nu}{2} \right) \frac{\theta}{\sigma^2} \right] \tag{20}$$

where

$$b_{ij} = 2^{i+j}\,\nu^{\nu/2}\left(\frac{\lambda}{2\sigma^2}\right)^i \frac{\Gamma(i+j+\nu/2)}{i!j!\Gamma(\nu/2)}\left(\frac{1}{2\sigma^2}\right)\left(\frac{\theta}{2\sigma^2}\right)^{j-1}\left[\nu+\frac{\lambda}{\sigma^2}+\frac{\theta}{\sigma^2}\right]^{i-j-\nu/2-1}.$$

From (20) it is clear that with $\lambda$ and $\nu$ fixed the negative term dominates and so, $\mathcal{F}_{Mt}(c)$ (the power) will (eventually) decrease (increase) as $\theta$ increases.

Given the marked differences in the test sizes, it is difficult to compare the power functions. We can only make a definite conclusion when one procedure has both a lower test size and a higher power than another procedure. For the example given here this occurs, for small $\nu$, when $r_{22*}=0.95$. Then, the test has a smaller size but higher power (for $t_{\beta_1}\geq 3$) using the "rich" proxy variable than omitting the regressor . This conclusion does not hold, however, when the proxy variable is poor. Though not illustrated in the results presented here, there is a degree of mis-specification (i.e. $t_{\beta_2}$) for which this feature is evident for all possible values of $\nu$.

These results suggest, therefore, that the performance of the test is, typically, better if we include a "good" proxy variable than if we omit the unobservable variable. However, regardless of whether we omit or proxy the unobservable variable, the test is not robust to severe mis-specification.

## 6. Final remarks.

In this paper we have investigated the power function of the classical F test for linear restrictions on the coefficients in a linear regression model with spherically symmetric disturbances when proxy variables are used in the place of unobservable regressors. We have shown that the test is invalid unless the null hypothesis is that the coefficients on the unobservable variables are zero.

To illustrate the power functions we considered a simple model with multivariate Student-t regression disturbances. The results showed that if the proxy variable is a good substitute for the unobservable variable then the power functions are similar for all $\nu$. However, if the proxy is "poor" then the power for relatively small $\nu$ is higher than for $\nu=\infty$ if the restrictions are in the neighbourhood of being true. Otherwise, it is significantly lower for small $\nu$ than for large $\nu$. Further, if the proxy is "poor", regardless of value of $\nu$, the test fails to detect the alternative hypothesis for a wide range of the hypothesis error. We also investigated whether the power is higher if we proxy the unobservable variable than if we simply omit it. Our results suggest there is little difference if the proxy is "poor" but that it is preferable to use a proxy when it is "rich". We find that the test is not robust if the degree of mis-specification is high, particularly when we omit the variable or we use a "poor" proxy.

The numerical analysis assumes that there is only one unobservable variable. It remains for future research to determine whether these results extend to the case of more than one unobservable variable. The extent to which our results are valid for other forms of non-normality (for instance, iid non-normal regression disturbances) also needs investigation.

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## Footnotes

1. A discussion of this family of distributions is beyond our scope. See, for example, Kelker (1970) and Muirhead (1982).

2. Zellner (1976), Ullah and Zinde-Walsh (1984), Sutradhar and Ali (1986), Ullah and Phillips (1986), Singh (1988), Sutradhar (1988) and Ohtani (1990), consider the special case of Mt regression disturbances.

3. Similarly, the F test statistic is invalid when we merely omit relevant regressors and the linear restrictions relate only to the coefficients on $X_1$. See, for instance, Mittelhammer (1984) and Giles (1990a).

4. The function is symmetric for positive and negative $\beta_2$, and so we only consider the former.

5. For this latter case we take R=1 and r=0 in (7), and let $X^*=[1]:n\times 1$ in (9) and (10).

6. Recall that the test is only valid under the null for $H_0:\beta_2=0$.

7. Though this feature is not evident from Tables 2 and 3 for $r_{22*}=0.95$, the more detailed numerical evaluations that we have undertaken show that it does hold for all $r_{22*}$.

**Table 1**

**Power of the F test at the 5% significance level for n=20 (c=4.414)**

| $r_{22*}$ | $t_{\beta_2}$ | $\nu$ = 5 | 10 | 20 | 50 | 100 | $\infty$ |
|---|---|---|---|---|---|---|---|
| 0.1 | 0 | .0500 | .0500 | .0500 | .0500 | .0500 | .0500 |
| | 1 | .0457 | .0457 | .0456 | .0456 | .0456 | .0456 |
| | 2 | .0358 | .0354 | .0352 | .0350 | .0350 | .0349 |
| | 3 | .0254 | .0243 | .0237 | .0233 | .0231 | .0230 |
| | 4 | .0171 | .0153 | .0143 | .0137 | .0135 | .0133 |
| | 5 | .0116 | .0092 | .0080 | .0073 | .0071 | .0069 |
| | 6 | .0137 | .0059 | .0043 | .0044 | .0042 | .0039 |
| | 7 | .0383 | .0119 | .0056 | .0038 | .0269 | 1.0000 |
| | 8 | .0985 | .0499 | .0219 | .0446 | 1.0000 | 1.0000 |
| | 9 | .1907 | .1412 | .0936 | 1.0000 | 1.0000 | 1.0000 |
| | 10 | .3002 | .2845 | .2963 | 1.0000 | 1.0000 | 1.0000 |
| 0.5 | 0 | .0500 | .0500 | .0500 | .0500 | .0500 | .0500 |
| | 1 | .0703 | .0704 | .0705 | .0706 | .0706 | .0706 |
| | 2 | .1209 | .1223 | .1231 | .1236 | .1238 | .1239 |
| | 3 | .1843 | .1884 | .1907 | .1921 | .1926 | .1931 |
| | 4 | .2497 | .2567 | .2607 | .2632 | .2640 | .2649 |
| | 5 | .3158 | .3229 | .3274 | .3306 | .3317 | .3329 |
| | 6 | .3882 | .3914 | .3929 | .3943 | .3984 | .3984 |
| | 7 | .4702 | .4931 | .4891 | .5168 | .5136 | .6602 |
| | 8 | .5814 | .5985 | .6517 | .7300 | .9534 | 1.0000 |
| | 9 | .6660 | .7017 | .7744 | .9797 | 1.0000 | 1.0000 |
| | 10 | .7382 | .8254 | .9064 | 1.0000 | 1.0000 | 1.0000 |
| 0.95 | 0 | .0500 | .0500 | .0500 | .0500 | .0500 | .0500 |
| | 1 | .1457 | .1457 | .1456 | .1456 | .1456 | .1455 |
| | 2 | .4045 | .4158 | .4217 | .4253 | .4265 | .4277 |
| | 3 | .6642 | .7037 | .7271 | .7426 | .7481 | .7538 |
| | 4 | .8268 | .8779 | .9069 | .9255 | .9319 | .9383 |
| | 5 | .9112 | .9544 | .9747 | .9854 | .9885 | .9913 |
| | 6 | .9529 | .9833 | .9940 | .9979 | .9987 | .9993 |
| | 7 | .9739 | .9937 | .9986 | .9998 | .9999 | 1.0000 |
| | 8 | .9848 | .9976 | .9997 | 1.0000 | 1.0000 | 1.0000 |
| | 9 | .9908 | .9990 | .9999 | 1.0000 | 1.0000 | 1.0000 |
| | 10 | .9942 | .9996 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

Figure 1 : Power functions for n=20 and $H_0 : \beta_2 = 0$

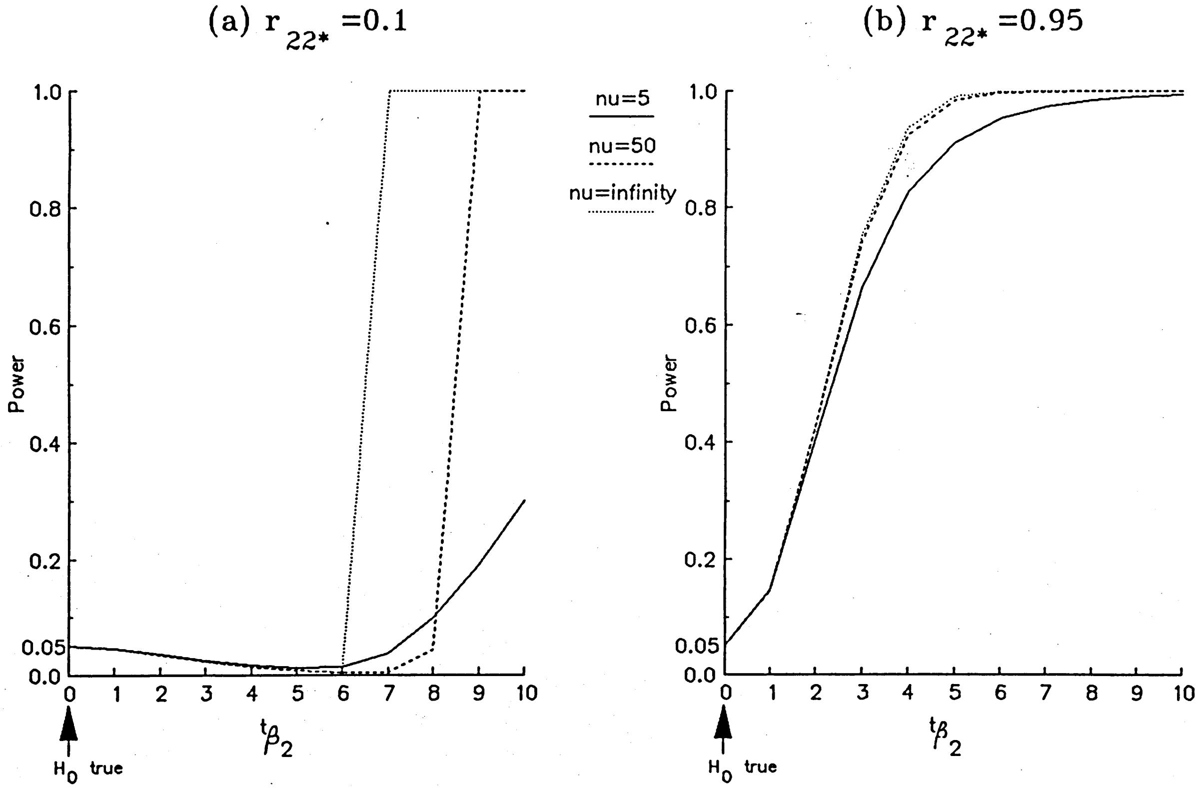


Table 2

Power of the F test at the nominal 5% significance level

for n=20 (c=4.414) when $H_0$: $\beta_1$=0 and $t_{\beta_2}$=1.0 *

| $r_{22*}$ | $t_{\beta_1}$ | $\nu$ 5 | 10 | 20 | 50 | 100 | $\infty$ |
|---|---|---|---|---|---|---|---|
| - | 0 | .0449 | .0449 | .0449 | .0449 | .0448 | .0448 |
| | 1 | .1454 | .1460 | .1463 | .1464 | .1465 | .1465 |
| | 2 | .4272 | .4408 | .4480 | .4524 | .4540 | .4555 |
| | 3 | .6988 | .7422 | .7679 | .7851 | .7912 | .7974 |
| | 4 | .8549 | .9056 | .9334 | .9506 | .9564 | .9621 |
| | 5 | .9294 | .9684 | .9851 | .9929 | .9950 | .9967 |
| 0.1 | 0 | .0448 | .0447 | .0447 | .0447 | .0447 | .0447 |
| | 1 | .1443 | .1449 | .1452 | .1454 | .1454 | .1455 |
| | 2 | .4246 | .4379 | .4450 | .4494 | .4509 | .4524 |
| | 3 | .6964 | .7394 | .7650 | .7820 | .7881 | .7943 |
| | 4 | .8534 | .9041 | .9320 | .9493 | .9551 | .9609 |
| | 5 | .9286 | .9678 | .9847 | .9926 | .9947 | .9965 |
| 0.5 | 0 | .0460 | .0459 | .0459 | .0459 | .0459 | .0459 |
| | 1 | .1474 | .1479 | .1481 | .1482 | .1483 | .1483 |
| | 2 | .4290 | .4426 | .4499 | .4544 | .4559 | .4574 |
| | 3 | .6991 | .7425 | .7683 | .7856 | .7917 | .7980 |
| | 4 | .8546 | .9053 | .9332 | .9505 | .9563 | .9620 |
| | 5 | .9290 | .9682 | .9850 | .9929 | .9949 | .9967 |
| 0.95 | 0 | .0494 | .0494 | .0494 | .0494 | .0494 | .0494 |
| | 1 | .1563 | .1563 | .1563 | .1563 | .1562 | .1562 |
| | 2 | .4411 | .4556 | .4633 | .4680 | .4696 | .4713 |
| | 3 | .7063 | .7509 | .7775 | .7953 | .8016 | .8082 |
| | 4 | .8577 | .9086 | .9365 | .9537 | .9594 | .9651 |
| | 5 | .9303 | .9692 | .9858 | .9934 | .9954 | .9970 |

* "-" in the column of $r_{22*}$ means that the unobservable variable is omitted.

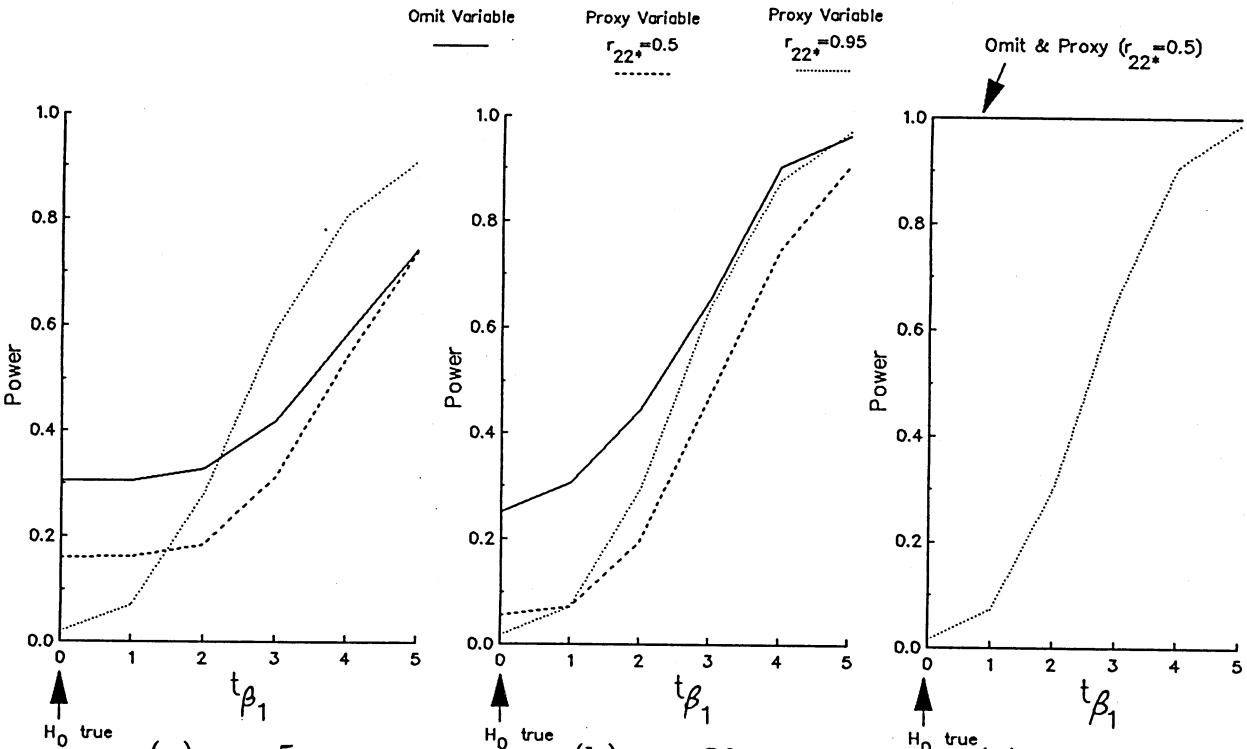
## Table 3

**Power of the F test at the nominal 5% significance level for n=20 (c=4.414) when $H_0$: $\beta_1=0$ and $t_{\beta_2}=10.0$ ***

| $r_{22*}$ | $t_{\beta_1}$ | $\nu$ = 5 | 10 | 20 | 50 | 100 | ∞ |
|---|---|---|---|---|---|---|---|
| - | 0 | .3046 | .2850 | .2497 | 1.0000 | 1.0000 | 1.0000 |
| | 1 | .3060 | .2921 | .3049 | 1.0000 | 1.0000 | 1.0000 |
| | 2 | .3268 | .3522 | .4438 | 1.0000 | 1.0000 | 1.0000 |
| | 3 | .4179 | .5073 | .6514 | 1.0000 | 1.0000 | 1.0000 |
| | 4 | .5805 | .7009 | .9041 | 1.0000 | 1.0000 | 1.0000 |
| | 5 | .7418 | .8485 | .9635 | 1.0000 | 1.0000 | 1.0000 |
| 0.1 | 0 | .2988 | .2774 | .2400 | 1.0000 | 1.0000 | 1.0000 |
| | 1 | .3002 | .2845 | .2963 | 1.0000 | 1.0000 | 1.0000 |
| | 2 | .3211 | .3459 | .4381 | 1.0000 | 1.0000 | 1.0000 |
| | 3 | .4127 | .5032 | .6488 | 1.0000 | 1.0000 | 1.0000 |
| | 4 | .5753 | .6979 | .9037 | 1.0000 | 1.0000 | 1.0000 |
| | 5 | .7362 | .8457 | .9632 | 1.0000 | 1.0000 | 1.0000 |
| 0.5 | 0 | .1592 | .1071 | .0567 | .0163 | 1.0000 | 1.0000 |
| | 1 | .1622 | .1098 | .0714 | 1.0000 | 1.0000 | 1.0000 |
| | 2 | .1847 | .1631 | .1948 | 1.0000 | 1.0000 | 1.0000 |
| | 3 | .3119 | .3610 | .4703 | 1.0000 | 1.0000 | 1.0000 |
| | 4 | .5368 | .6282 | .7461 | 1.0000 | 1.0000 | 1.0000 |
| | 5 | .7382 | .8254 | .9064 | 1.0000 | 1.0000 | 1.0000 |
| 0.95 | 0 | .0201 | .0186 | .0177 | .0171 | .0169 | .0167 |
| | 1 | .0690 | .0705 | .0714 | .0719 | .0721 | .0723 |
| | 2 | .2803 | .2879 | .2922 | .2949 | .2958 | .2968 |
| | 3 | .5902 | .6189 | .6351 | .6455 | .6491 | .6528 |
| | 4 | .8047 | .8516 | .8779 | .8947 | .9004 | .9062 |
| | 5 | .9090 | .9509 | .9707 | .9813 | .9845 | .9875 |

* "-" in the column of $r_{22*}$ means that the unobservable variable is omitted.

Figure 2 : Power functions for n=20 when $H_0\!:\!\beta_1=0$ & $t_{\beta_2} = 10$




(a) nu=5 (b) nu=20 (c) nu=infinity

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