Department of Economics
UNIVERSITY OF CANTERBURY
CHRISTCHURCH, NEW ZEALAND

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David E. A. Giles and Virendra K. Srivastava

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David E. A. Giles
(University of Canterbury)
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Abstract

The specification of a regression model is often determined in part by testing the significance of one or more potential regressors, and then including them in the model only if they are apparently significant. This paper considers such a preliminary-test strategy in a two-regressor model when a single t-test is conducted. The exact sampling distribution of the preliminary test estimator of a regression coefficient is derived and illustrated, and some implications for confidence levels are explored.

Key Words: Preliminary Test; Linear Regression; t-Test; Confidence Intervals.

*David Giles is Professor, Department of Economics, University of Canterbury, Christchurch, 8001, New Zealand; Virendra Srivastava is Professor, Department of Statistics, Lucknow University, Lucknow, India 226007. We are grateful to Judith Giles and Offer Lieberman for their suggestions and comments, and to Robert Davies for supplying FORTRAN code for his Algorithm AS 155.

1. INTRODUCTION

"Preliminary Test" estimators are encountered frequently in econometrics and other areas of applied statistics. Such estimators arise when the choice of estimator is effectively randomised by making it dependent upon the outcome of a prior statistical test. Often, this test relates to the parameters being estimated and employs the same sample of data, though this need not be the case. The basic pre-test literature is documented by Bancroft and Han (1977) and by Judge and Bock (1978, 1983). In the context of the linear regression model, examples of preliminary test estimators abound. The particular estimator analysed here is one considered by Bancroft (1944) in his seminal paper, and relates to the least squares estimation of a regression coefficient after a t-test of the significance of a second regressor variable. This is an important problem in the context of model specification.

Pre-test estimators have been discussed almost exclusively in terms of point estimation. The literature emphasises the first two moments of such estimators and their risk under quadratic loss. To analyse the implications of pre-testing for interval estimation we require the full sampling distribution of the pre-test estimator in question. This information is also essential for other purposes, such as comparing estimators on the basis of concentration probabilities. On this, the literature is relatively silent. Bennett (1952) and Kitagawa (1967) discuss the distribution of a pre-test estimator of the mean under Normal sampling, and Bennett (1956) discusses some aspects of confidence intervals for the mean and variance in a two-sample Normal problem. The only complete analysis of the exact distribution of a pre-test estimator, together with a specific discussion of the implications for confidence interval construction, is that of Giles (1990). That study deals with the estimation of the scale parameter in a Normal population after a pre-test of variance homogeneity.

This paper derives the distribution of the estimator of a regression coefficient after a preliminary t-test. Section 2 sets out the problem and the notation used and Section 3 discusses the exact cdf of the estimator. The features of the cdf and pdf are illustrated in Section 4, and Section 5 deals with some implications for confidence levels associated with interval estimation. Some concluding remarks appear in Section 6.

2. THE PROBLEM AND NOTATION

Consider the bivariate regression model

$$y_t = \beta_1 x_{1t} + \beta_2 x_{2t} + u_t \quad ; \qquad t=1,\ldots,T \tag{2.1}$$

where the u_t's are iid $N(0,\sigma^2)$ and the absence of an intercept reflects the assumption that the variables are measured as deviations from sample means. Then $\rho^2 = \left(\sum_t x_{1t}x_{2t}\right)^2 \Big/ \left[\left(\sum_t x_{1t}^2\right)\left(\sum_t x_{2t}^2\right)\right]$ is the squared sample correlation between x_1 and x_2.

Consider the hypotheses H_0: $\beta_2 = 0$ vs. H_A: $\beta_2 \neq 0$, with associated t-statistic $t = (b_{2.1}/\sigma_2)(\nu/v)^{1/2}$, where $b_{2.1}$ is the OLS estimator of β_2 in (2.1), σ_2^2 is the variance of $b_{2.1}$, and ν denotes degrees of freedom. Let $b_{1.2}$ be the corresponding OLS estimator of β_1, and $v = \sum_t(y_t - b_{1.2}x_{1t} - b_{2.1}x_{2t})^2/\sigma^2$. Note that $v \sim \chi^2_\nu$, independently of $b_{1.2}$ and $b_{2.1}$. Let σ_1^2 be the variance of $b_{1.2}$.

Now, if b_1 is the OLS estimator of β_1 in (2.1) when the restriction $\beta_2 = 0$ is imposed, we shall call $b_{1.2}$ and b_1 the "unrestricted" and "restricted" estimators of β_1. The pre-test estimator of interest is

$$\hat{\beta}_1 = \begin{cases} b_1 & ; \quad |t| < c(\alpha) \\ b_{1.2} & ; \quad |t| \geq c(\alpha) \end{cases}$$

where $c(\alpha)$ is the tabulated t-value for a size-α two-sided test. It is readily shown that

$$b_1 = b_{1.2} + \rho(\sigma_1/\sigma_2)b_{2.1}$$

and

$$\begin{pmatrix} b_{1.2} \\ b_{2.1} \end{pmatrix} \sim N \left[\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} , \begin{pmatrix} \sigma_1^2 & -\rho\sigma_1\sigma_2 \\ -\rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} \right] .$$

The correlation between $b_{1.2}$ and $b_{2.1}$ is $-\rho$, and

$$\sigma_i^2 = \sigma^2 / \left[(\sum_t x_{it}^2)(1-\rho^2) \right] \quad ; \qquad i = 1,2.$$

We wish to determine the cdf of $\hat{\beta}_1$. That is, we require $\Pr.(\hat{\beta}_1 < \tau)$; $-\infty < \tau < \infty$. Having obtained an expression for this cdf, the corresponding pdf can be obtained by differentiation.

2. THE DISTRIBUTION FUNCTION

To derive the exact cdf of $\hat{\beta}_1$, note that

$$\begin{aligned} \Pr.(\hat{\beta}_1 < \tau) &= \Pr.\left(b_1 < \tau \ \underline{\text{and}} \ |t| < c(\alpha)\right) \\ &+ \Pr.\left(b_{1.2} < \tau \ \underline{\text{and}} \ |t| \geq c(\alpha)\right) \\ &= \Pr.\left(b_1 < \tau \ \underline{\text{and}} \ |t| < c(\alpha)\right) \\ &+ \Pr.\left(b_{1.2} < \tau\right) - \Pr.\left(b_{1.2} < \tau \ \underline{\text{and}} \ |t| < c(\alpha)\right) \\ &= P_1 + P_2 - P_3, \text{ say.} \end{aligned}$$

Substituting the expressions for these probabilities derived in the Appendix, we have:

Theorem 3.1.

$$\Pr(\hat{\beta}_1 < \tau) = \Phi(\tau_1) + \exp(-\beta_2^2/2\sigma_2^2)$$

$$\cdot \sum_{j=0}^{\infty} \left\{ \left((2j)!\right)^{-1} \left[(\beta_2/\sigma_2)^{2j} \Phi(\tau^*) I_{c*}(j+\tfrac{1}{2},\tfrac{\nu}{2}) \right.\right.$$

$$\left.\left. - (1-\rho^2)^{j+1/2} \theta_j(\tau^*) I_{c**}(j+\tfrac{1}{2},\tfrac{\nu}{2}) \right] \right\} \tag{3.1}$$

where: $\tau_1 = (\tau-\beta_1)/\sigma_1$

$$\tau^* = \left(\tau_1-\rho\beta_2\sigma_2\right)/(1-\rho^2)^{1/2}$$

$$c^* = c^2/(\nu+c^2)$$

$$c^{**} = c^2/\left[\nu(1-\rho^2)+c^2\right]$$

$I_x(a,b)$ is the incomplete Beta function

$\Phi(.)$ is the cumulative standard Normal distribution

$$\theta_j(\tau^*) = \int_{-\infty}^{\tau^*} \left(\frac{\beta_2}{\sigma_2} - \rho z(1-\rho^2)^{-1/2}\right)^{2j} (2\pi)^{-1/2} e^{-z^2/2} dz.$$

While a general interpretation of this result is not straightforward, some special cases provide useful cross-checks. First, if $\rho = 0$ then $\Pr.(\hat{\beta}_1 < \tau) = \Phi(\tau_1)$. This accords with the fact that $b_1 = b_{1.2} = \hat{\beta}_1$ when $\rho = 0$, and the cdf of $b_{1.2}$ is $\Phi(\tau_1)$. Secondly, if $c = 0$, $\Pr.(\hat{\beta}_1 < \tau) = \Phi(\tau_1)$. This accords with the fact that $\hat{\beta}_1 = b_{1.2}$ when $c = 0$. Thirdly, it is readily shown that if $c \to \infty$ ($c^*, c^{**} \to 1$), then the cdf of $\hat{\beta}_1$ converges to that of b_1.

Not surprisingly, the general form of (3.1) is similar to that given by Bennett (1952, p.38) for the distribution of the pre-test estimator of a Normal mean after a test of equality of the means of two populations with common unknown variance.

4. NUMERICAL EVALUATIONS

To interpret (3.1) more fully, we consider some specific numerical evaluations. These have been obtained using a FORTRAN program incorporating routines from Press et al. (1986) for Simpson's rule and the evaluation of Gamma and incomplete Beta functions. In particular, the latter is obtained by the method of continued fractions. The infinite series in (3.1) converges (to six decimal places) in seven or eight terms, and the evaluation of the cdf is extremely rapid. Differentiating (3.1) numerically by the method of central differences yields the pdf of $\hat{\beta}_1$ for any choice of data and parameters. All evaluations were undertaken on a VAX 8350 computer.

Figure 1 provides a typical result, based on data such that $\sum_t x_{1t}^2 = \sum_t x_{2t}^2 = 1$. So, given the values of the parameters noted in that Figure's heading, Bias $(b_1) = 0.5$. It is known for this pre-test problem that $0 = \text{Bias}(b_{1.2}) \leq \text{Bias}(\hat{\beta}_1) \leq \text{Bias}(b_1)$, and this is reflected in Figure 1b. When H_0 is true ($\beta_2 = 0$), b_1 is unbiased, so $\hat{\beta}_1$ is also unbiased, as is reflected in Figure 2b. Taking Figure 1 as a benchmark, varying the parameters of the problem produces anticipated results. (Those not illustrated here are available on request.) For example, decreasing ν or increasing σ^2 shifts the cdf and pdf of $\hat{\beta}_1$ towards those of b_1. Increasing the size of the preliminary t-test (as in Figure 3) shifts the cdf and pdf of $\hat{\beta}_1$ towards those of $b_{1.2}$, and decreasing the absolute value of ρ leads to a convergence of the results for $\hat{\beta}_1$, b_1 and $b_{1.2}$.

A final cross-check on the validity of Theorem 3.1 is also enlightening. We have derived the cdf of $\hat{\beta}_1$ in a different way for the special case where σ^2 is known. This results in

$$\Pr.(\hat{\beta}_1 < \tau) = \Phi(\tau^*)\left[\Phi(c_u)-\Phi(c_L)\right] + \Phi(\tau_1) - \int_{c_L}^{c_u} \Phi\left(\tau^{**}(z)\right)\phi(z)dz \qquad (4.1)$$

where $c_u = c - \beta_2/\sigma_2$

$c_L = -c - \beta_2\sigma_2$

$\tau^{**}(z) = (1-\rho^2)^{-1/2}(\tau_1 + \rho z)$

$\phi(z)$ is the Standard Normal density

and the other notation is as above. Our numerical evaluations of (4.1), based on a separate computer program, accord exactly with those of (3.1) when $\nu \to \infty$ in the latter, as expected.

5. CONFIDENCE LEVELS AFTER PRE-TESTING

Clearly, as the pdf of $\hat{\beta}_1$ differs from that of $b_{1.2}$ or b_1, after pre-testing the true confidence level (CL) associated with any interval estimate of β_1 generally differs from the nominal CL. Two situations must be considered.

First, suppose a 95% confidence interval (CI) for β_1 is constructed using $b_{1.2}$, without pre-testing. As the distribution of $b_{1.2}$ does not involve β_2, the true CL for this interval equals the nominal 95% CL for all β_2. However, if the confidence limits associated with this CI are used in conjunction with $\hat{\beta}_1$, the true CL differs from 95%, as generally $\Pr.(\hat{\beta}_1 < \tau) \neq \Pr.(b_{1.2} < \tau)$. This difference depends on β_2, as that parameter enters the cdf for $\hat{\beta}_1$. This is illustrated in Figure 4a for $\alpha = 5\%$ and $\alpha = 18.5\%$, the latter value corresponding to the optimal c suggested by Brook's (1976) mini-max risk regret criterion, and being calculated using the algorithm reported by Giles et al. (1990). Davies' (1980) algorithm is used to compute the corresponding α value.

Within a moderate neighbourhood of H_0, the true CL associated with $\hat{\beta}_1$ exceeds the nominal CL of 95% because $\hat{\beta}_1$ incorporates b_1. As we depart from H_0 the true CL falls as the restriction associated with b_1 becomes increasingly false. Ultimately, the true CL increases to that associated with $b_{1.2}$ as the power of the t-test approaches unity.

Secondly, suppose a 95% CI for β_1 is based on b_1, without pre-testing. The distribution of b_1 depends on β_2: $E(b_1) = \beta_1 + (\rho\beta_2\sigma_1/\sigma_2)$. So, the <u>true</u> CL for this CI equals the <u>nominal</u> 95% CL only if H_0 is true. Otherwise the true CL declines monotonically to zero as we depart from H_0, as in Figure 4b. Using the relationship between the non-central t and non-central χ^2 distributions, Davies' (1980) algorithm may be used to compute this CL.

If the confidence limits associated with the nominal 95% CI based on b_1 are used after pre-testing, the true CL of the interval is always less than 95%. Even when H_0 is true $\hat{\beta}_1$ incorporates $b_{1.2}$ to some degree. There is again a trade-off between increasingly false restrictions and the power of the pre-test as we depart from H_0. When H_0 is very false, $\hat{\beta}_1 \to b_{1.2}$ again, but the confidence limits used are those based on the distribution of b_1, not $b_{1.2}$, so the true CL still differs from the nominal 95%.

These results accord with those of Giles (1990) for interval estimation of the scale parameter after pre-testing for variance homogeneity in two Normal populations. Clearly, CI's constructed after imposing restrictions should be interpreted cautiously. Moreover, while pre-testing may affect the true CL of an interval estimate either favourably or adversely, the only way to be assured that the chosen CL actually holds is to apply the unrestricted estimator without any prior testing.

6. CONCLUSIONS

This paper illustrates some of the effects that a preliminary t-test, for the significance of one regressor, can have on the sampling distribution of the least squares estimator of the coefficient of a second regressor. We have shown how the cdf and pdf of this pre-test estimator differ from those of its two component parts, and the way in which confidence levels associated with the corresponding interval estimator may be affected.

The results are data dependent and also depend on all of the unknown parameters in the problem. Accordingly, it is difficult to offer general prescriptions to applied researchers. Further, our results are based on a two-regressor model, and any generalisation to the multiple regression case is by no means trivial.

Finally, the confidence level trade-offs in Figure 4a suggest the possibility of an optimal pre-test size in this context, somewhat analogous to Brook's (1976) suggestion in the context of point estimation risk. This remains a topic for future research.

FIGURE 1a : DISTRIBUTION FUNCTIONS

(Degrees of Freedom=20; Alpha=5%; Rho=0.5)
(Beta1=1.0; Beta2=1.0; Sigma=1.0)

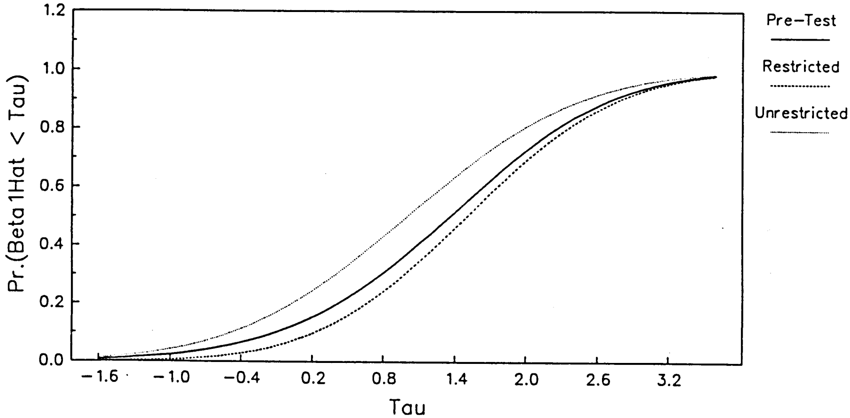


FIGURE 1b : DENSITY FUNCTIONS

(Degrees of Freedom=20; Alpha=5%; Rho=0.5)
(Beta1=1.0; Beta2=1.0; Sigma=1.0)

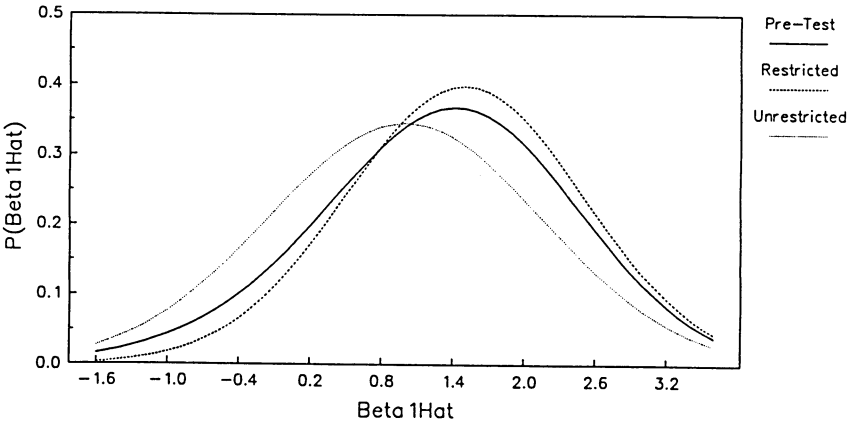


FIGURE 2a : DISTRIBUTION FUNCTIONS

(Degrees of Freedom=20; Alpha=5%; Rho=0.5)
(Beta1=1.0; Beta2=0.0; Sigma=1.0)

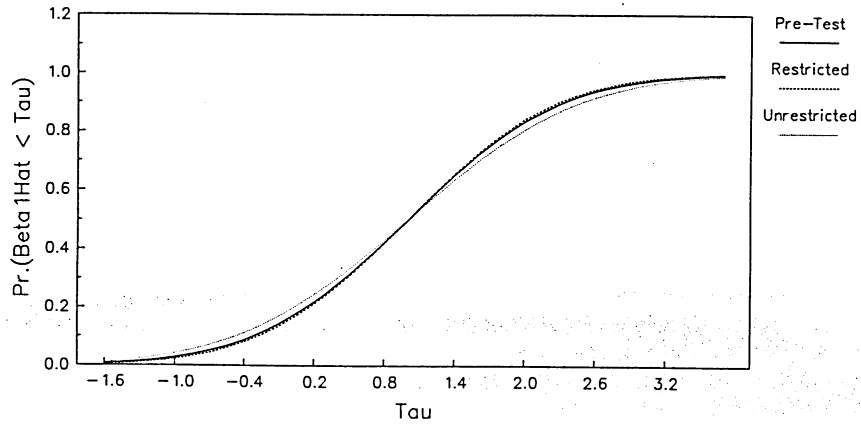


FIGURE 2b : DENSITY FUNCTIONS

(Degrees of Freedom=20; Alpha=5%; Rho=0.5)
(Beta1=1.0; Beta2=0.0; Sigma=1.0)

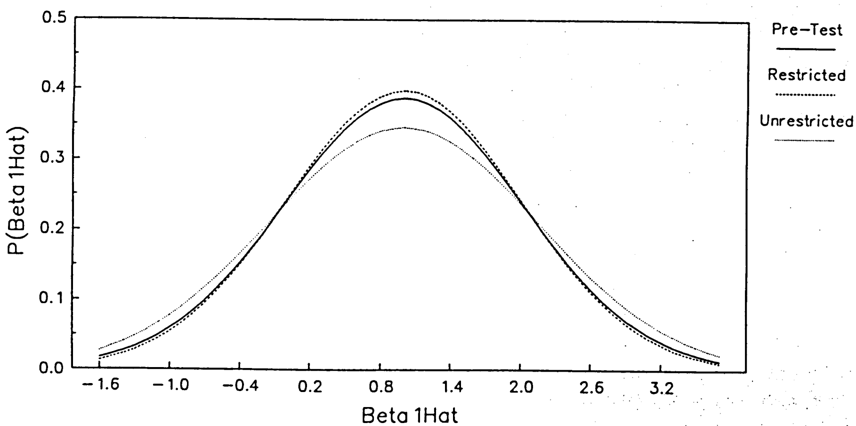


FIGURE 3a : DISTRIBUTION FUNCTIONS

(Degrees of Freedom=20; Alpha=20%; Rho=0.5)

(Beta1=1.0; Beta2=1.0; Sigma=1.0)

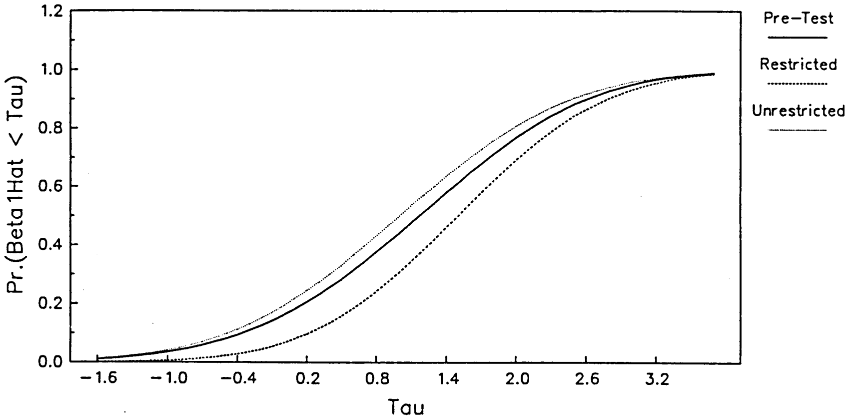


FIGURE 3b : DENSITY FUNCTIONS

(Degrees of Freedom=20; Alpha=20%; Rho=0.5)

(Beta1=1.0; Beta2=1.0; Sigma=1.0)

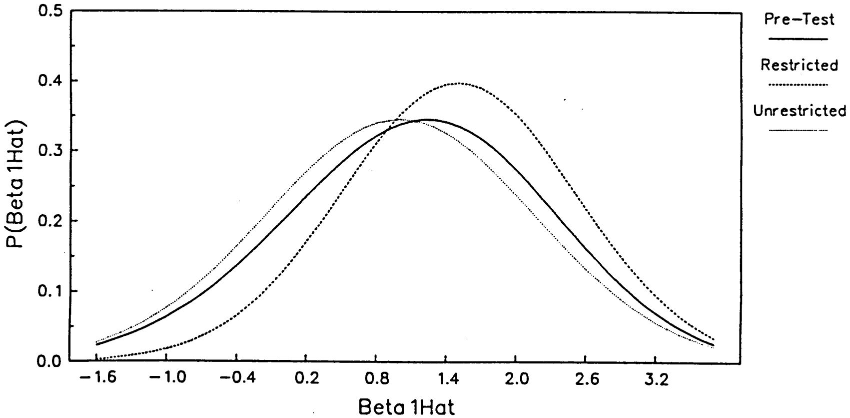
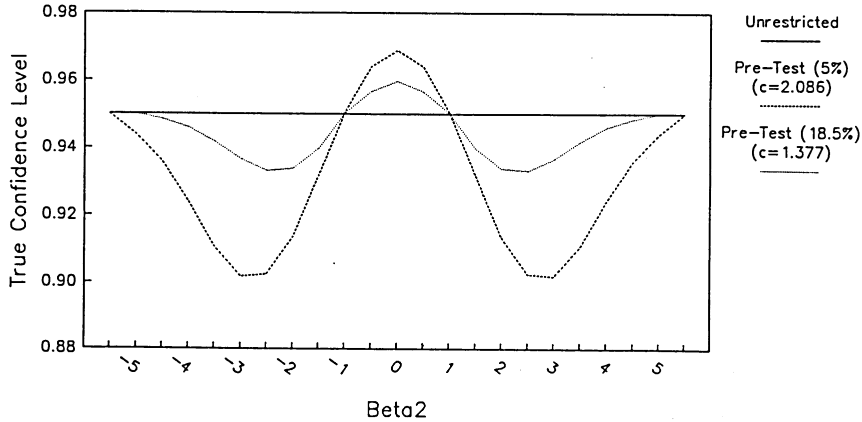


FIGURE 4a : CONFIDENCE LEVELS

Pre-Test and Unrestricted Estimators

(Nu=20;Rho=0.5;Beta1=1.0;Sigma=1.0)



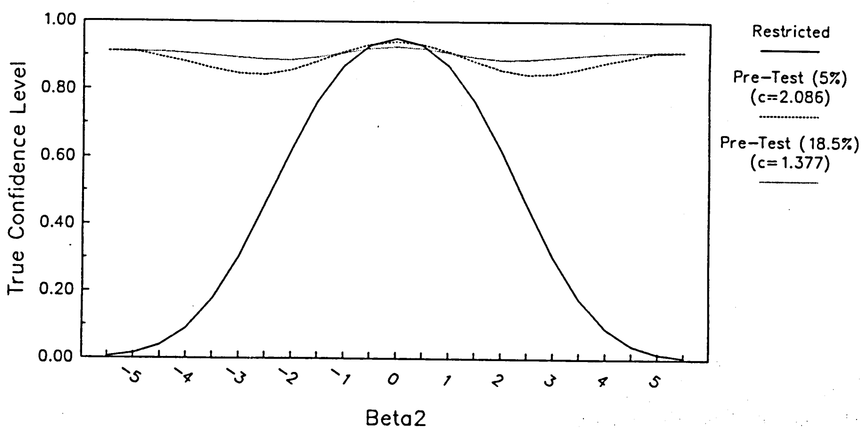
* The Nominal Confidence Level is 95%

** c=1.377 is Brook's Optimal Critical Value

FIGURE 4b : CONFIDENCE LEVELS

Pre-Test and Restricted Estimators

(Nu=20;Rho=0.5;Beta1=1.0;Sigma=1.0)



* The Nominal Confidence Level is 95%

** c=1.377 is Brook's Optimal Critical Value

APPENDIX: PROOF OF THEOREM 3.1

Consider P_1:

$$P_1 = \Pr.\left[b_{1.2} + \rho(\sigma_1/\sigma_2)b_{2.1} < \tau \text{ and } -c\sigma_2 < b_{2.1}\left(\frac{\nu}{v}\right)^{1/2} < c\sigma_2\right]$$

$$= k \iiint_{R_1} v^{\nu/2-1}\exp\left[-\frac{v}{2} - \frac{1}{2(1-\rho^2)}\left(\left(\frac{b_{1.2}-\beta_1}{\sigma_1}\right)^2 + \left(\frac{b_{2.1}-\beta_2}{\sigma_2}\right)^2\right.\right.$$

$$\left.\left. + 2\rho\left(\frac{b_{1.2}-\beta_1}{\sigma_1}\right)\left(\frac{b_{2.1}-\beta_2}{\sigma_2}\right)\right)\right]db_{1.2}db_{2.1}dv$$

where $k = \left[2\pi\sigma_1\sigma_2(1-\rho^2)^{1/2}2^{\nu/2}\Gamma\left(\frac{\nu}{2}\right)\right]^{-1}$

and

$$R_1 = \begin{cases} b_{1.2} + \rho(\sigma_1/\sigma_2)b_{2.1} < \tau \\ -c\sigma_2 < b_{2.1}\left(\frac{\nu}{v}\right)^{1/2} < c\sigma_2 . \\ 0 < v < \infty \end{cases}$$

Apply the transformation

$$z - (1-\rho^2)^{-1/2}\left[\left(\frac{b_{1.2}-\beta_1}{\sigma_1}\right) + \rho\left(\frac{b_{2.1}-\beta_2}{\sigma_2}\right)\right.$$

$$t = (b_{2.1}/\sigma_2)\left(\frac{\nu}{v}\right)^{1/2},$$

with associated Jacobian $\sigma_1\sigma_2\left[(1-\rho^2)\left(\frac{v}{\nu}\right)\right]^{1/2}$. Taking the infinite series expansion of $\exp\left[(t\beta_2/\sigma_2)\left(\frac{v}{\nu}\right)^{1/2}\right]$, noting that

$$\int_0^\infty v^{\frac{\nu+j+1}{2}-1}\exp\left[-\frac{v}{2}\left(1+\frac{t^2}{\nu}\right)\right]dv$$

$$= 2^{\frac{\nu+j+1}{2}}\Gamma\left(\frac{\nu+j+1}{2}\right)\left(1+\frac{t^2}{\nu}\right)^{\frac{\nu+j+1}{2}}$$

and that

$$\int_{-c}^{c} t^j\left(1+\frac{t^2}{\nu}\right)^{\frac{\nu+j+1}{2}} dt$$

vanishes for odd j, we find that

$$P_1 = \left[\pi\sqrt{2\nu}\ \Gamma\left(\frac{\nu}{2}\right)\right]^{-1}\exp\left[-\beta_2^2/2\sigma_2^2\right]$$

$$\cdot \sum_{j=0}^{\infty}\left[2^j\Gamma\left(\frac{\nu+1}{2}+j\right)\left(\beta_2^2/\nu\sigma_2^2\right)^j(2j)!\right]$$

$$\cdot\int_{-\infty}^{\tau^*} \exp\left(-\tfrac{1}{2}z^2\right)dz \int_0^{c^2} (t^2)^{j-1/2}\left(1+\frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}-j} dt^2.$$

where $\tau^* = \left[(\tau-\beta_1)/\sigma_1-(\rho\beta_2/\sigma_2)\right]/(1-\rho^2)^{1/2}$.

Applying the duplication formula.

$$\sqrt{\pi}(2j)! = 2^{2j}\Gamma\left(j+\tfrac{1}{2}\right)j!,$$

and the transformation

$$p = (t^2/\nu)/\left(1+\frac{t^2}{\nu}\right),$$

we obtain

$$P_1 = \exp\left[-\beta_2^2/2\sigma_2^2\right]\sum_{j=0}^{\infty}\left\{\left(\beta_2^2/2\sigma_2^2\right)^j/j!\int_{-\infty}^{\tau^*}\frac{1}{\sqrt{2\pi}}\exp\left(-\tfrac{1}{2}z^2\right)dz\right.$$

$$\left.\cdot\ I_{c^*}\left(j+\tfrac{1}{2},\tfrac{\nu}{2}\right)\right\}$$

where $c^* = c^2/(\nu+c^2)$, and

$$I_{c^*}\left(j+\tfrac{1}{2},\tfrac{\nu}{2}\right) = \frac{\Gamma\left(\frac{\nu+1}{2}+j\right)}{\Gamma\left(\frac{\nu}{2}\right)\Gamma\left(j+\frac{1}{2}\right)}\int_0^{c^*} p^{j-1/2}(1-p)^{\nu/2-1}dp$$

is an incomplete Beta function.

Consider P_2:

$$P_2 = \Pr.(b_{1.2} < \tau) = \int_{-\infty}^{\tau_1} \frac{1}{\sqrt{2\pi}} \exp\left(-\tfrac{1}{2}z^2\right) dz$$

where $\tau_1 = (\tau-\beta_1)/\sigma_1$.

Consider P_3:

$$P_3 = \Pr.\left[b_{1.2} < \tau \ \underline{\text{and}}\ -c\sigma_2 < b_{2.1}(\nu/v)^{1/2} < c\sigma_2\right]$$

$$= k \iiint_{R_3} v^{\nu/2-1} \exp\left[-\frac{v}{2} - \frac{1}{2(1-\rho^2)}\left(\left(\frac{b_{1.2}-\beta_1}{\sigma_1}\right)^2 + \left(\frac{b_{2.1}-\beta_2}{\sigma_2}\right)^2\right.\right.$$

$$\left.\left. + 2\rho\left(\frac{b_{1.2}-\beta_1}{\sigma_1}\right)\left(\frac{b_{2.1}-\beta_2}{\sigma_2}\right)\right)\right] db_{1.2}\, db_{2.1}\, dv$$

where k is defined above, and

$$R_3 = \begin{cases} b_{1.2} < \tau \\ -c\sigma_2 < b_{2.1}\left(\frac{\nu}{v}\right)^{1/2} < c\sigma_2 \\ 0 < v < \infty \end{cases}.$$

Apply the transformation

$$z = (1-\rho^2)^{-1/2}\left[\left(\frac{b_{1.2}-\beta_1}{\sigma_1}\right) + \rho\beta_2/\sigma_2\right]$$

$$t = (b_{2.1}/\sigma_2)(\nu/v)^{1/2},$$

whose Jacobian is also $\sigma_1\sigma_2\left[(1-\rho^2)\left(\frac{v}{\nu}\right)\right]^{1/2}$.

Then following steps analogous to those used to evaluate P_1, we obtain

$$P_3 = (1-\rho^2)^{1/2} \exp\left[-\beta_2^2/2\sigma_2^2\right] \sum_{j=0}^{\infty} \left\{\left[\tfrac{1}{2}(1-\rho^2)\right]^j / j!\right.$$

$$\left. \cdot \int_{-\infty}^{\tau^*} \left(\frac{\beta_2}{\sigma_2} - \frac{\rho z}{(1-\rho^2)^{1/2}}\right)^{2j} \frac{1}{\sqrt{2\pi}} \exp\left(-\tfrac{1}{2}z^2\right) dz\, I_{c^{**}}\left(j+\tfrac{1}{2}, \tfrac{\nu}{2}\right)\right\}$$

where $c^{**} = c^2/\left[\nu(1-\rho^2)+c^2\right]$.

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