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THE POWER OF THE DURBIN-WATSON TEST WHEN THE ERRORS ARE HETEROSCEDASTIC

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Abstract

We consider the robustness of the Durbin-Watson test to mis-specification via heteroscedastic disturbances. Exact powers are calculated using real and artificial regressors. We find that heteroscedasticity may dramatically alter the power of the test.

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1. INTRODUCTION

This paper reports the results of a preliminary investigation of the sensitivity of the Durbin-Watson (DW) test for serial independence, to a departure from one of the underlying assumptions - the homoscedasticity of the errors. The power properties of both the "bounds" and exact versions of this test under the usual assumptions are well documented (e.g., Koerts and Abrahamse (1971) and references cited by King (1987, pp.30-31)). The robustness of the DW test to various departures from these assumptions has been considered by several authors (e.g., see King (1987, pp.43-45)). Harrison and McCabe (1975) and Epps and Epps (1977) provide very limited evidence that the power of the DW test is quite robust to heteroscedasticity in the disturbances. However, as we show below, this conclusion is not general, and depends on the form of regressor matrix.

Our results relate to the exact version of the DW test, and exact power calculations are reported. Section 2 outlines the problem; the design of the study is discussed in section 3; and the results appear in section 4. Section 5 contains some concluding remarks.

2. THE PROBLEM

Consider the model

$$y = X\beta + u \tag{1}$$

$$u = \rho u_{-1} + \varepsilon \quad ; \qquad \varepsilon \sim N(0,\sigma^2 I)$$

where X is (n×k) of full rank and non-stochastic, and $|\rho| < 1$. Then u is $N(0,\Omega)$, where

$$\Omega = \left(\frac{\sigma^2}{1-\rho^2}\right)\begin{bmatrix} 1 & \rho & \rho^2 & \dots & \rho^{n-1} \\ \rho & 1 & \rho & \dots & \rho^{n-2} \\ \cdot & & \cdot & & \vdots \\ \cdot & & & \cdot & \rho \\ \rho^{n-1} & \dots & & \rho & 1 \end{bmatrix}. \tag{2}$$

The DW statistic may be written $d = (u'MAMu)/(u'Mu)$, where $M = I - X(X'X)^{-1}X'$ and A is a tri-diagonal (n×n) "differencing matrix" with (1,1) and (n,n) elements as unity, 2 elsewhere on the leading diagonal, and -1 for the leading off-diagonal elements. It is well known (e.g., Koerts and Abrahamse (1971)) that $\Pr.(d \le d^{*}) = \Pr.\left(\sum_{j=1}^{n} \lambda_j Z_j^2 \le 0\right)$, where the Z_j^2's are each independent $\chi^2_{(1)}$, and the λ_j's are the eigenvalues of $M(A-d^{*}I)M\Omega$.

Such probabilities are readily calculated by Imhof's (1961) procedure or Davies' (1980) algorithm, for example. For a positive one-sided alternative the exact critical value for the DW test of size α, and a particular X, is that value, c, such that $\Pr.(d \le c | \Omega = I) = \alpha$. The exact power of the (exact[1]) DW test may be computed for any particular ρ and X as $\Pr.(d \le c | \Omega(\rho))$.

If the disturbances are heteroscedastic then Ω is more general than in (2), with non-constant diagonal elements. The details depend on the form of heteroscedasticity. Given this form, exact size and power calculations proceed as above.

3. THE STUDY

As the distribution of d depends on X it is important to consider different regressor characteristics.[2] In particular, the form of X determines whether the power of the DW tends to unity or zero[3] as $\rho \to 1$. We consider seven data sets,[4] all of which include an intercept: X_1 comprises the annual "spirits" income and price data of Durbin and Watson (1951); X_2 comprises the quarterly Australian Consumers Price Index and its lag; X_3, X_4 and X_5 each comprise a linear trend and, respectively, a Normal (2.4,1), lognormal (generated from N(3,1)), and uniform (0,20) variable; X_6 and X_7 comprise the eigenvectors corresponding respectively to the two largest and two (non-zero) smallest eigenvalues of A.

Sample sizes of 69 and 40 are considered.[5] The exact DW test is applied at the 5% level against a positive one-sided alternative. The SHAZAM package (White et al. (1990)), incorporating Imhof's routine, is used for all calculations. The results were checked with Davies' algorithm and our SHAZAM code was verified against the results of Krämer and Sonnberger (1986, p.23).

Heteroscedasticity of the form $var(u_t) \propto x_t^{\gamma}$ is considered, where x_t is the t'th observation on one regressor. The leading diagonal of Ω is modified to comprise scaled values of x_t, the scaling being chosen to control[6] the value of $h = max.(var.(u_t))/min.(var.(u_t))$.

4. RESULTS

In all cases heteroscedasticity produces a slight increase in the size of the test, which never exceeded 5.5%. No size corrections are made for the power calculations - we consider the power of the DW test when it is unwittingly applied under model mis-specification. In practice no such correction would be possible.[7]

With data X_6 and X_7 the power of the DW test increases[8] with h (Figure 1). Using a single regressor equal to the eigenvector corresponding to the smallest non-zero eigenvalue of A, Epps and Epps (1977) found slight decreases in power with increased heterocedasticity (of a different form). These are special choices of regressors - the power of the DW test is maximized when the column space of X is spanned by the eigenvectors of A (e.g., Krämer and Sonnberger (1986)).

With the other artificial data, power falls with increasing h. This fall is modest for X_3 and X_4 (e.g., Figure 2), but more pronounced for X_5. For the latter, with n = 40 the power of the DW test begins to fall ($h \neq 1$) as ρ approaches unity. In Figures 1 and 2, all powers are unity for $\rho > 0.60$, 0.70 respectively.

These results might suggest, as have earlier studies, that the power of the DW test is reasonably robust to moderate heteroscedascity. This is dispelled by the results based on the real data, X_1 and X_2. These power functions have orthodox shape when $h = 1$, but the effect of even minor heteroscedasticity is dramatic (Figures 3,4). Power falls rapidly for $\rho > 0.8$ if $h \neq 1$. Even modest heteroscedasticty ($h=1.5$) results in maximum power under 20% (7%) with $X_1(X_2)$ and $n \leq 69$.

5. CONCLUSIONS

The known sensitivity of the DW test to the form of regressors prevails when the model is mis-specified, highlighting the need to consider real as well as artificial data in such studies. When the errors are heteroscedastic the test's power can differ dramatically from what might be presumed - it may be slightly higher or substantially lower than under homoscedasticity. Even with data where the power approaches unity as $\rho \to 1$ with homoscedastic errors, the power may fall sharply for large ρ under heteroscedasticity.

Work in progress considers other forms of heteroscedasticity and other tests for serial independence. The role of the regressor matrix is being examined further, and recent work by Bartels (1990) may be fruitful here.

FIGURE 1 : EXACT POWERS

X6 : Min. Eigenvectors of A; n=69

(Error Variance Determined by Eigenvector)

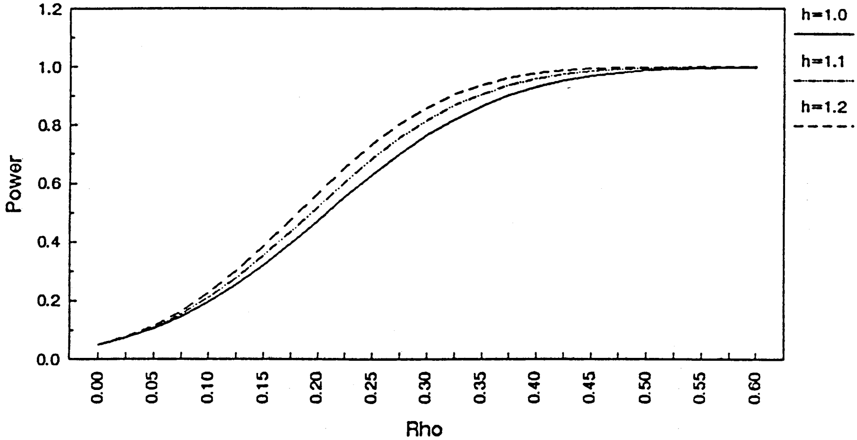


FIGURE 2 : EXACT POWERS

X3 : Normal and Trended Data; n=69

(Error Variance Determined by Normal)

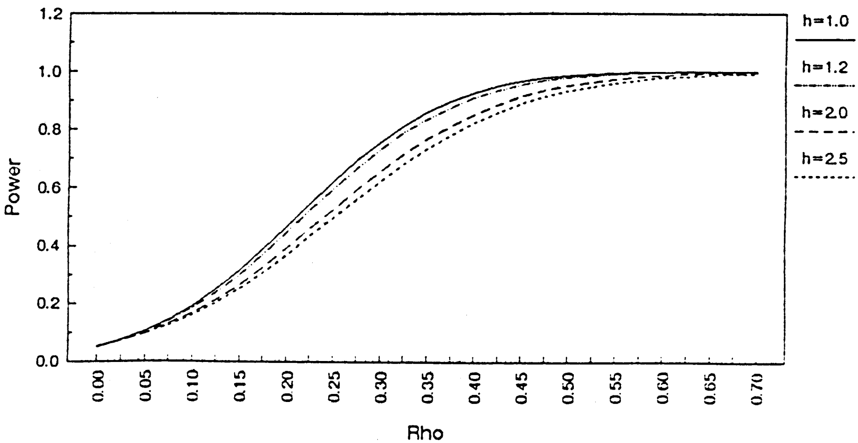


FIGURE 3 : EXACT POWERS

X1 : Spirits Data; n=69

(Error Variance Determined by Income)

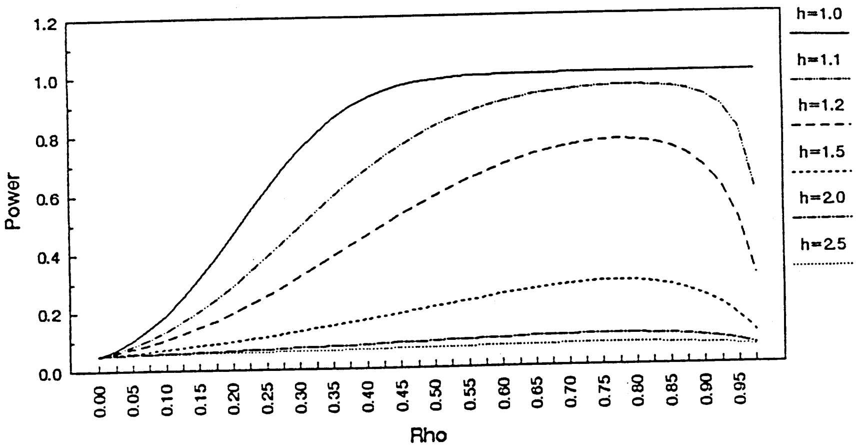
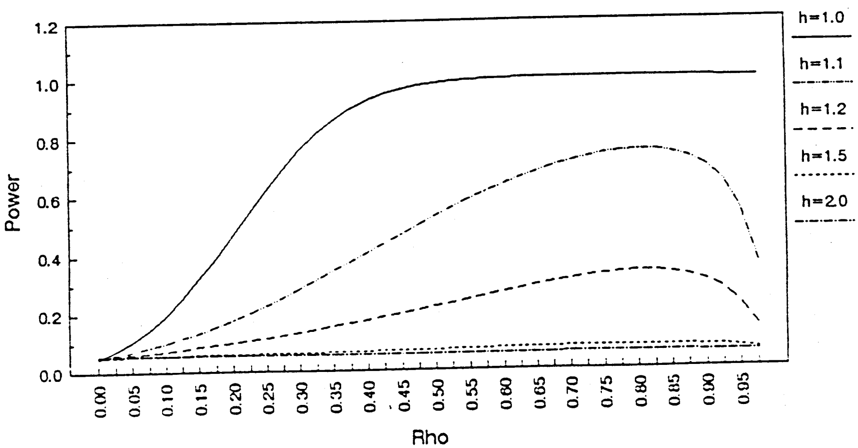


FIGURE 4 : EXACT POWERS

X2 : Australian CPI Data; n=69

(Error Variance Determined by CPI)



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FOOTNOTES

* We are grateful to Merran Evans for supplying data used in this study, and to Robert Davies for providing FORTRAN code for his AS 155 algorithm.

1. The same approach may be used with DW bounds test, if this is of interest.

2. Unfortunately, not all such studies have been careful on this point.

3. For example, see Tillman (1975), and Krämer and Sonnberger (1986).

4. These data are variations of those used by Evans (1989) and are representative of those used in numerous other such studies.

5. The discussion in the next section is based on the full study, though only representative results are reported in detail.

6. A similar approach is adopted by Epps and Epps (1977). Other measures of the degree of heteroscedasticity, such as the coefficient of variation of the diagonal elements of Ω, are possible.

7. As may be seen from Figures 1 - 4, this would not significantly affect our results.

8. In all cases studied the power of the DW test was less when $n = 40$ than when $n = 69$.

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