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D. E. A. GILES AND V. K. SRIVASTAVA

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D. E. A GILES
University of Canterbury
and
V. K. SRIVASTAVA
University of Lucknow

Address for Correspondence:
Professor D. E. A. Giles, Department of Economics, University of Canterbury, Christchurch 1, New Zealand.

AN UNBIASED ESTIMATOR OF THE COVARIANCE MATRIX OF THE MIXED REGRESSION ESTIMATOR

D.E.A. Giles and V.K. Srivastava*

Abstract

This paper derives an unbiased estimator of the covariance matrix of the "mixed regression estimator" suggested by Theil and Goldberger (1961) for combining prior information with the sample information in regression analysis. This derivation facilitates the construction of finite-sample standard errors for the mixed estimators of the individual regression coefficients. Comparisons are made between the unbiased covariance estimator and conventional consistent estimators based on ordinary least squares and generalised least squares formulae.

KEY WORDS: Mixed regression; Standard errors; Linear regression.

* D.E.A. Giles is Professor, Department of Economics, University of Canterbury, Christchurch, New Zealand; V.K. Srivastava is Professor, Department of Statistics, Lucknow University, Lucknow 226007, India.

1. INTRODUCTION

This paper considers the "mixed regression" estimator proposed by Theil and Goldberger (1961) as a means of combining stochastic prior information with the available sample information when estimating the parameters of the linear regression model. This model is of the form

$$y = X\beta + u \ ; \ u \sim N(0,\sigma^2 I) \tag{1.1}$$

where X is $(T \times k)$, non-stochastic and of rank k. The uncertain prior information about β is expressed as

$$r = R\beta + v \ ; \ v \sim N(0,\psi) \tag{1.2}$$

where R is non-stochastic, $(g \times k)$ and of rank $g(< k)$; r is non-stochastic and $(g \times 1)$; ψ is known and positive definite symmetric; and $E(uv') = 0$.

The Ordinary Least Squares (OLS) estimator of β is

$$b = (X'X)^{-1}X'y \tag{1.3}$$

where

$$V(b) = \sigma^2(X'X)^{-1} \tag{1.4}$$

and a consistent and unbiased estimator of V(b) is

$$\hat{V}(b) = \left(\frac{s}{\nu}\right)(X'X)^{-1} \tag{1.5}$$

where $s = (y - Xb)'(y - Xb)$, and $\nu = (T - k)$.

The Generalised Least Squares (GLS) estimator of β based on (1.1) and (1.2) is

$$\tilde{\beta} = (X'X + \sigma^2 R'\psi^{-1}R)^{-1}(X'y + \sigma^2 R'\psi^{-1}r) \tag{1.6}$$

where

$$V(\tilde{\beta}) = \sigma^2(X'X + \sigma^2 R'\psi^{-1}R)^{-1}. \tag{1.7}$$

A consistent estimator of $V(\tilde{\beta})$ is given by

$$\tilde{V}(\tilde{\beta}) = \left(\frac{s}{\nu}\right)\left(X'X + \frac{s}{\nu}R'\psi^{-1}R\right)^{-1}. \tag{1.8}$$

The estimator $\tilde{\beta}$ in (1.6) is non-operational. The mixed regression estimator is just the feasible counterpart to $\tilde{\beta}$:

$$\hat{\beta} = \left(X'X + \frac{s}{\nu}R'\psi^{-1}R\right)^{-1}\left(X'y + \frac{s}{\nu}R'\psi^{-1}r\right). \tag{1.9}$$

Clearly, $\hat{\beta}$ is consistent for β, and it is easily shown to be unbiased if $\nu > 2$. Moreover, its asymptotic distribution is the same as those of both b and $\tilde{\beta}$. This mixed regression estimator can be given a Bayesian interpretation (e.g. Theil (1971; pp.670-672)), and following Theil (1963), the prior share of information may be represented by

$$\theta = k^{-1}\text{tr.}\left(\frac{s}{\nu}\right)R'\psi^{-1}R\left(X'X + \frac{s}{\nu}R'\psi^{-1}R\right)^{-1} . \tag{1.10}$$

As $\theta \to 0$ or $\theta \to 1$, $\hat{\beta}$ collapses to the OLS or restricted least squares estimators respectively.

Nagar and Kakwani (1964) made an early study of some of the finite-sample properties of $\hat{\beta}$, and Srivastava (1980) provides a full bibliography of the study of this estimator. Of particular relevance here are the results of Swamy and Mehta (1969) and Mehta and Swamy (1970). The former relate to the derivation of an expression for $V(\hat{\beta})$, while the latter considers the distribution of $\hat{\beta}$. Both of these results are of a complex form and are not readily applicable. Moreover, $V(\hat{\beta})$ is unobservable, of course, and must be estimated if standard errors for the elements of $\hat{\beta}$ are to be constructed.

Either $\tilde{V}(\tilde{\beta})$ or $\hat{V}(b)$ may be used as consistent estimators of $V(\hat{\beta})$. In practice, the former is generally adopted, as an unbiased estimator of $V(\hat{\beta})$ has been unavailable. This gap in the literature is remedied in this paper. In section 2 we derive an unbiased estimator of $V(\hat{\beta})$. The measures of variability obtained by using this estimator are compared with those based on $\tilde{V}(\tilde{\beta})$ and $\hat{V}(b)$ in section 3; and some concluding remarks appear in section 4.

2. UNBIASED ESTIMATION OF THE COVARIANCE MATRIX

The approach used to derive an unbiased estimator of $V(\hat{\beta})$ is similar to that adopted by Srivastava and Giles (1989). Our principal result is given in the following theorem.

Theorem 2.1.

Under the assumptions of section 1, an unbiased estimator of $V(\hat{\beta})$ is given by

$$\hat{V}(\hat{\beta}) = \left(\frac{s}{2}\right) \int_0^1 w^{\nu/2-1}\left(X'X + w\frac{s}{\nu} R'\psi^{-1}R\right)^{-1}X'X\left(X'X + w\frac{s}{\nu} R'\psi^{-1}R\right)^{-1}dw$$

$$+ \left(\frac{s}{\nu}\right)^2\left(X'X + \frac{s}{\nu} R'\psi^{-1}R\right)^{-1}R'\psi^{-1}R\left(X'X + \frac{s}{\nu} R'\psi^{-1}R\right)^{-1} . \qquad (2.1)$$

Proof. From (1.9) we observe that

$$(\hat{\beta} - \beta) = \left(X'X + \frac{s}{\nu} R'\psi^{-1}R\right)^{-1}\left(X'u + \frac{s}{\nu} R'\psi^{-1}v\right) .$$

Recalling the independence of u and v, and of s and $X'u$, we have

$$V(\hat{\beta}) = \sigma^2 E\left[\left(X'X + \frac{s}{\nu} R'\psi^{-1}R\right)^{-1}X'X\left(X'X + \frac{s}{\nu} R'\psi^{-1}R\right)^{-1}\right]$$

$$+ E\left[\left(\frac{s}{\nu}\right)^2\left(X'X + \frac{s}{\nu} R'\psi^{-1}R\right)^{-1}R'\psi^{-1}R\left(X'X + \frac{s}{\nu} R'\psi^{-1}R\right)^{-1}\right] . \qquad (2.2)$$

Consider the first term on the RHS of (2.2). There exists a non-singular matrix P such that

$$P'X'XP = I$$

and

$$P'R'\psi^{-1}RP = \Lambda ,$$

where Λ is a diagonal matrix whose diagonal elements are the roots of $|R'\psi^{-1}R-\lambda X'X| = 0$. Of these k roots, g are positive and the remainder are zero.

So, the first term on the RHS of (2.2) is

$$\sigma^2 PE\left(I + \frac{s}{\nu} \Lambda\right)^{-2}P'$$

$$= P\Delta P', \text{ say,}$$

where Δ = diag. (δ_i), and

$$\delta_i = E\left[\frac{\sigma^2}{\left(1+\frac{s}{\nu}\lambda_i\right)^2}\right] . \tag{2.3}$$

When $\lambda_i = 0$, $\left(\frac{s}{\nu}\right)$ is an unbiased estimator of δ_i. To find an unbiased estimator of δ_i when $\lambda_i > 0$, let $h_i(s)$ be a continuous function of s such that $E[sh_i(s)] = \delta_i$. It follows that

$$\sigma^2 E\left[\nu h_i(s) + 2s\frac{dh_i(s)}{ds}\right] = \sigma^2 E\left[\frac{1}{\left(1+\frac{s}{\nu}\lambda_i\right)^2}\right],$$

or

$$E\left[\nu h_i(s) + 2s\frac{dh_i(s)}{ds} - \frac{1}{\left(1+\frac{s}{\nu}\lambda_i\right)^2}\right] = 0. \tag{2.4}$$

As s is a complete sufficient statistic for σ^2, it follows from (2.4) that

$$\nu h_i(s) + 2s\frac{dh_i(s)}{ds} - \frac{1}{\left(1+\frac{s}{\nu}\lambda_i\right)^2} = 0. \tag{2.5}$$

The differential equation (2.5) has solution

$$h_i(s) = s^{-\nu/2}\int_0^s \frac{w^{\nu/2}}{2w\left(1+\frac{w}{\nu}\lambda_i\right)^2}\,dw ,$$

$$= \frac{1}{2}\int_0^1 \frac{w^{\nu/2-1}}{\left(1+\frac{s\lambda_i}{\nu}w\right)^2}\,dw , \tag{2.6}$$

and so an unbiased estimator of δ_i is

$$\hat{\delta}_i = sh_i(s) = \frac{s}{2}\int_0^1 \frac{w^{\nu/2-1}}{\left(1+\frac{s\lambda_i}{\nu}w\right)^2}\,dw . \tag{2.7}$$

Accordingly, an unbiased estimator of $P\Delta P'$ is

$$\frac{s}{2}\int_0^1 w^{\nu/2-1}P\left(I + w\frac{s}{\nu}\Lambda\right)^{-2}P'\,dw$$

$$= \frac{s}{2}\int_0^1 w^{\nu/2-1}\left(X'X + w\frac{s}{\nu}R'\psi^{-1}R\right)^{-1}X'X\left(X'X + w\frac{s}{\nu}R'\psi^{-1}R\right)^{-1}dw. \quad (2.8)$$

Now, the second term on the RHS of (2.2) does not involve any unknown parameters, so an unbiased estimator of this expression is

$$\left(\frac{s}{\nu}\right)^2\left(X'X + \frac{s}{\nu}R'\psi^{-1}R\right)^{-1}R'\psi^{-1}R\left(X'X + \frac{s}{\nu}R'\psi^{-1}R\right)^{-1}. \quad (2.9)$$

Combining (2.8) and (2.9) we get $\hat{V}(\hat{\beta})$ in (2.1) as an unbiased estimator of $V(\hat{\beta})$, as required. #

The square roots of the diagonal elements of $\hat{V}(\hat{\beta})$ provide appropriate standard errors for the elements of $\hat{\beta}$. Note that we may write

$$\hat{V}(\hat{\beta}) = PQP',$$

where $Q = \text{diag.}(q_i)$, $q_i = (\hat{\delta}_i + \hat{\delta}_i^*)$, $\hat{\delta}_i$ is given by (2.7) and

$$\hat{\delta}_i^* = \frac{(s/\nu)^2\lambda_i}{\left(1+\frac{s}{\nu}\lambda_i\right)^2}.$$

Similarly,

$$\hat{V}(b) = P\left(\frac{s}{\nu}I\right)P'$$

and

$$\tilde{V}(\tilde{\beta}) = P\left[\frac{s}{\nu}\left(I + \frac{s}{\nu}\Lambda\right)^{-1}\right]P' \;;\quad \nu > 4.$$

Accordingly, to avoid data-dependencies, comparisons between the three corresponding estimators of the variance of the ith. element of $\hat{\beta}$ may be based on

$$g_i^{OLS} = \left(\frac{s}{\nu}\right) \quad (2.10)$$

$$g_i^{GLS} = \left(\frac{s}{\nu}\right)\left(\frac{1}{1+\frac{s\lambda_i}{\nu}}\right) = \frac{s}{\nu}(1-\theta_i) \tag{2.11}$$

and

$$g_i^{u} = \left(\frac{s}{\nu}\right)\left[\frac{\nu}{2}\int_0^1 \frac{w^{\nu/2-1}}{\left(1+w\frac{s}{\nu}\lambda_i\right)^2}\,dw + \frac{\frac{s}{\nu}\lambda_i}{\left(1+\frac{s}{\nu}\lambda_i\right)^2}\right]$$

$$= \left(\frac{s}{\nu}\right)\left[\frac{\nu}{2}\int_0^1 \frac{w^{\nu/2-1}}{\left(1+\frac{w\theta_i}{1-\theta_i}\right)^2}\,dw + \theta_i(1-\theta_i)\right], \tag{2.12}$$

where $\theta_i = \left(\frac{s\lambda_i}{\nu+s\lambda_i}\right)$ is the prior information share for β_i, corresponding to (1.10).

3. COMPARISONS OF COVARIANCE ESTIMATORS

From (2.10)-(2.12), $g_i^{GLS} < g_i^{OLS}$ and $g_i^{GLS} \to g_i^{OLS}$ and $g_i^{u} \to g_i^{OLS}$ as $\theta_i \to 0$. Similarly, $g_i^{GLS} \to 0$ and $g_i^{u} \to 0$ as $\theta_i \to 1$. This is reflected in Table 1 and Figures 1 and 2, where $\left(g_i^{u}/g_i^{OLS}\right)$ and $\left(g_i^{u}/g_i^{GLS}\right)$ respectively are evaluated for various values of θ_i and ν. In interpreting these numbers it must be remembered that $\theta_i = \theta_i(\nu)$. The integral in (2.12) is easily determined by Romberg's method using the FORTRAN routines QROMB, TRAPZD and POLINT described by Press et al. (1986).

From Figure 1 and part (a) of Table 1 we see that $g_i^{u} \le g_i^{OLS}$. The evaluated ratio can be interpreted in two ways: it compares an unbiased estimator with a consistent estimator of the variance of the mixed regression estimator of β_i; and it also compares an unbiased estimator of the variance of $\hat{\beta}_i$ with an unbiased estimator of the variance of b_i. In terms of the latter interpretation, the figures can be taken as estimated relative efficiencies and they follow the expected pattern: i.e., mixed regression estimation is more efficient than is OLS. Under the first interpretation, these figures illustrate that the use of $\hat{V}(\hat{\beta})$ can result in standard errors which are substantially smaller than those obtained by using the OLS formula as a (large-sample) approximation.

From Figure 2 and part (b) of Table 1 we see that $g_i^u \geq g_i^{GLS}$, for all degrees of freedom, and that the greatest difference between g_i^u and g_i^{GLS} occurs when the prior information share is approximately 60%, regardless of the degrees of freedom. Further, if $\nu > 10$ then this difference is less than 10%, and if $\nu > 30$ it is less than 3%. So, only for problems involving quite small degrees of freedom, and moderately strong prior information, will the use of our unbiased estimator of $V(\hat{\beta})$ result in standard errors which differ markedly from those obtained from the usual consistent estimator, $\tilde{V}(\tilde{\beta})$.

4. CONCLUSIONS

In problems where $\nu > 30$, the current practice of basing mixed regression standard errors on the consistent covariance matrix estimator $\tilde{V}(\tilde{\beta})$ may seem to be justified. However, recalling that $g_i^u \geq g_i^{GLS}$ in Table 1, we see that this current practice has the important disadvantage of resulting in standard errors which <u>understate</u> the variability of $\hat{\beta}_i$ on average, and so are unduly "optimistic" in the impression they convey about this estimator's precision. The unbiased estimator of the covariance matrix of the mixed regression estimator presented in this paper provides the basis for calculating standard errors which can be justified in finite samples, and which provide meaningful measures of that estimator's precision. However, the relative intractability of the exact distribution of the mixed regression estimator precludes using these standard errors in any straightforward way to construct confidence intervals, *etc*.

<u>October, 1989</u>

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Table 1. Estimated Variance Ratios

θ_i / ν	.01	.10	.20	.30	.40	.50	.60	.70	.80	.90	.99
					(a) (g_i^u/g_i^{OLS})						
5	1.00	.95	.88	.81	.71	.61	.50	.37	.24	.12	.01
6	.99	.94	.87	.79	.70	.59	.48	.36	.23	.11	.01
7	.99	.94	.86	.78	.68	.58	.47	.35	.23	.11	.01
8	.99	.93	.86	.77	.67	.57	.46	.34	.22	.11	.01
9	.99	.93	.85	.76	.66	.56	.45	.34	.22	.11	.01
10	.99	.93	.85	.76	.66	.55	.44	.33	.22	.11	.01
15	.99	.92	.83	.74	.64	.54	.43	.32	.21	.10	.01
20	.99	.92	.82	.73	.63	.53	.42	.31	.21	.10	.01
25	.99	.91	.82	.72	.62	.52	.42	.31	.21	.10	.01
30	.99	.91	.82	.72	.62	.52	.41	.31	.20	.10	.01
					(b) (g_i^u/g_i^{GLS})						
5	1.00	1.06	1.11	1.15	1.19	1.22	1.24	1.24	1.22	1.16	1.03
6	1.00	1.05	1.09	1.13	1.16	1.18	1.19	1.19	1.17	1.12	1.02
7	1.00	1.04	1.08	1.11	1.14	1.16	1.16	1.16	1.14	1.09	1.01
8	1.00	1.04	1.07	1.10	1.12	1.14	1.14	1.14	1.11	1.07	1.01
9	1.00	1.03	1.06	1.09	1.11	1.12	1.12	1.12	1.10	1.06	1.01
10	1.00	1.03	1.06	1.08	1.10	1.11	1.11	1.10	1.09	1.05	1.01
15	1.00	1.02	1.04	1.05	1.06	1.07	1.07	1.06	1.05	1.03	1.00
20	1.00	1.02	1.03	1.04	1.05	1.05	1.05	1.05	1.04	1.02	1.00
25	1.00	1.01	1.02	1.03	1.04	1.04	1.04	1.04	1.03	1.02	1.00
30	1.00	1.01	1.02	1.03	1.03	1.03	1.03	1.03	1.02	1.01	1.00

FIGURE 1

RATIO OF UNBIASED ESTIMATOR OF VARIANCE
TO USUAL OLS-BASED ESTIMATOR OF VARIANCE

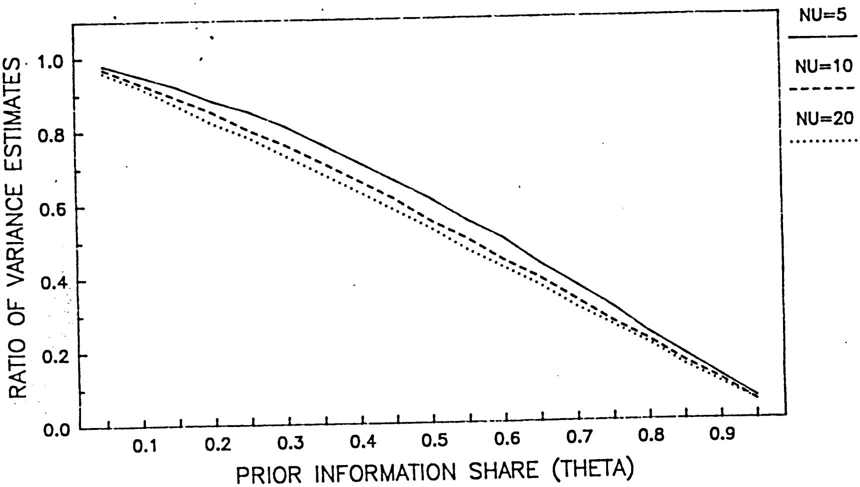
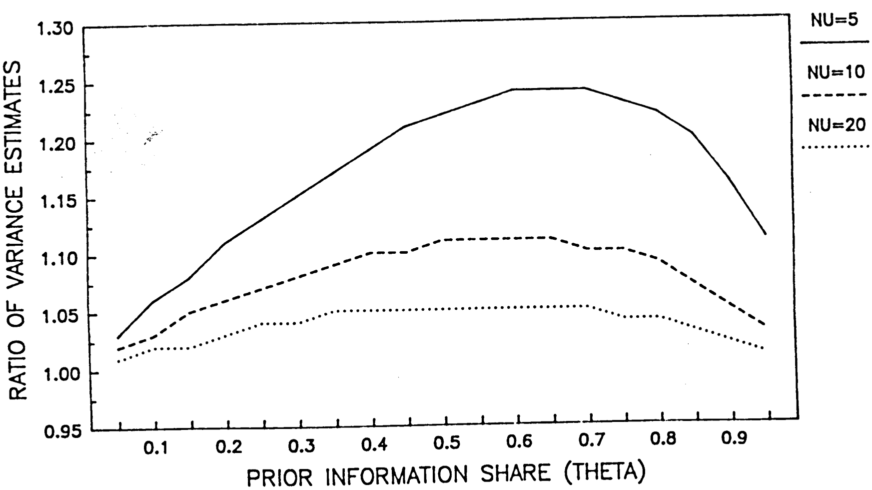


FIGURE 2

RATIO OF UNBIASED ESTIMATOR OF VARIANCE
TO USUAL CONSISTENT ESTIMATOR OF VARIANCE



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