AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
<http://ageconsearch.umn.edu>
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

CANTER

8908✓

Department of Economics
UNIVERSITY OF CANTERBURY

CHRISTCHURCH, NEW ZEALAND

GIANNINI FOUNDATION OF
AGRICULTURAL ECONOMICS
LIBRARY

MAR 1 1990

**AN UNBIASED ESTIMATOR OF THE
COVARIANCE MATRIX OF THE
MIXED REGRESSION ESTIMATOR**

D. E. A. GILES AND V. K. SRIVASTAVA

Discussion Paper

No.8908

Department of Economics University of Canterbury
Christchurch, New Zealand

Discussion Paper No. 8908

September 1989

**AN UNBIASED ESTIMATOR OF THE
COVARIANCE MATRIX OF THE
MIXED REGRESSION ESTIMATOR**

D. E. A GILES
University of Canterbury
and
V. K. SRIVASTAVA
University of Lucknow

*This paper is circulated for discussion and comments. It should not be quoted without the prior approval of the author.

Address for Correspondence:

Professor D. E. A. Giles, Department of Economics, University of Canterbury, Christchurch 1, New Zealand.

AN UNBIASED ESTIMATOR OF THE
COVARIANCE MATRIX OF THE
MIXED REGRESSION ESTIMATOR

D.E.A. Giles and V.K. Srivastava*

Abstract

This paper derives an unbiased estimator of the covariance matrix of the "mixed regression estimator" suggested by Theil and Goldberger (1961) for combining prior information with the sample information in regression analysis. This derivation facilitates the construction of finite-sample standard errors for the mixed estimators of the individual regression coefficients. Comparisons are made between the unbiased covariance estimator and conventional consistent estimators based on ordinary least squares and generalised least squares formulae.

KEY WORDS: Mixed regression; Standard errors; Linear regression.

* D.E.A. Giles is Professor, Department of Economics, University of Canterbury, Christchurch, New Zealand; V.K. Srivastava is Professor, Department of Statistics, Lucknow University, Lucknow 226007, India.

1. INTRODUCTION

This paper considers the "mixed regression" estimator proposed by Theil and Goldberger (1961) as a means of combining stochastic prior information with the available sample information when estimating the parameters of the linear regression model. This model is of the form

$$y = X\beta + u; u \sim N(0, \sigma^2 I) \quad (1.1)$$

where X is $(T \times k)$, non-stochastic and of rank k . The uncertain prior information about β is expressed as

$$r = R\beta + v; v \sim N(0, \psi) \quad (1.2)$$

where R is non-stochastic, $(g \times k)$ and of rank $g (< k)$; r is non-stochastic and $(g \times 1)$; ψ is known and positive definite symmetric; and $E(uv') = 0$.

The Ordinary Least Squares (OLS) estimator of β is

$$b = (X'X)^{-1}X'y \quad (1.3)$$

where
$$V(b) = \sigma^2(X'X)^{-1} \quad (1.4)$$

and a consistent and unbiased estimator of $V(b)$ is

$$\hat{V}(b) = \left(\frac{s}{\nu} \right) (X'X)^{-1} \quad (1.5)$$

where $s = (y - Xb)'(y - Xb)$, and $\nu = (T - k)$.

The Generalised Least Squares (GLS) estimator of β based on (1.1) and (1.2) is

$$\tilde{\beta} = (X'X + \sigma^2 R' \psi^{-1} R)^{-1} (X'y + \sigma^2 R' \psi^{-1} r) \quad (1.6)$$

where

$$V(\tilde{\beta}) = \sigma^2 (X'X + \sigma^2 R' \psi^{-1} R)^{-1}. \quad (1.7)$$

A consistent estimator of $V(\tilde{\beta})$ is given by

$$\tilde{V}(\tilde{\beta}) = \left(\frac{s}{\nu} \right) \left(X'X + \frac{s}{\nu} R' \psi^{-1} R \right)^{-1}. \quad (1.8)$$

The estimator $\tilde{\beta}$ in (1.6) is non-operational. The mixed regression estimator is just the feasible counterpart to $\tilde{\beta}$:

$$\hat{\beta} = \left(X'X + \frac{s}{v} R' \psi^{-1} R \right)^{-1} \left(X'y + \frac{s}{v} R' \psi^{-1} r \right). \quad (1.9)$$

Clearly, $\hat{\beta}$ is consistent for β , and it is easily shown to be unbiased if $v > 2$. Moreover, its asymptotic distribution is the same as those of both b and $\tilde{\beta}$. This mixed regression estimator can be given a Bayesian interpretation (e.g. Theil (1971; pp.670-672)), and following Theil (1963), the prior share of information may be represented by

$$\theta = k^{-1} \text{tr.} \left(\frac{s}{v} \right) R' \psi^{-1} R \left(X'X + \frac{s}{v} R' \psi^{-1} R \right)^{-1}. \quad (1.10)$$

As $\theta \rightarrow 0$ or $\theta \rightarrow 1$, $\hat{\beta}$ collapses to the OLS or restricted least squares estimators respectively.

Nagar and Kakwani (1964) made an early study of some of the finite-sample properties of $\hat{\beta}$, and Srivastava (1980) provides a full bibliography of the study of this estimator. Of particular relevance here are the results of Swamy and Mehta (1969) and Mehta and Swamy (1970). The former relate to the derivation of an expression for $V(\hat{\beta})$, while the latter considers the distribution of $\hat{\beta}$. Both of these results are of a complex form and are not readily applicable. Moreover, $V(\hat{\beta})$ is unobservable, of course, and must be estimated if standard errors for the elements of $\hat{\beta}$ are to be constructed.

Either $\tilde{V}(\tilde{\beta})$ or $\hat{V}(b)$ may be used as consistent estimators of $V(\hat{\beta})$. In practice, the former is generally adopted, as an unbiased estimator of $V(\hat{\beta})$ has been unavailable. This gap in the literature is remedied in this paper. In section 2 we derive an unbiased estimator of $V(\hat{\beta})$. The measures of variability obtained by using this estimator are compared with those based on $\tilde{V}(\tilde{\beta})$ and $\hat{V}(b)$ in section 3; and some concluding remarks appear in section 4.

2. UNBIASED ESTIMATION OF THE COVARIANCE MATRIX

The approach used to derive an unbiased estimator of $V(\hat{\beta})$ is similar to that adopted by Srivastava and Giles (1989). Our principal result is given in the following theorem.

Theorem 2.1.

Under the assumptions of section 1, an unbiased estimator of $V(\hat{\beta})$ is given by

$$\begin{aligned} \hat{V}(\hat{\beta}) = & \left(\frac{s}{2}\right) \int_0^1 w^{\nu/2-1} \left(X'X + w\frac{s}{\nu} R'\psi^{-1}R\right)^{-1} X'u \left(X'X + w\frac{s}{\nu} R'\psi^{-1}R\right)^{-1} dw \\ & + \left(\frac{s}{\nu}\right)^2 \left(X'X + \frac{s}{\nu} R'\psi^{-1}R\right)^{-1} R'\psi^{-1}R \left(X'X + \frac{s}{\nu} R'\psi^{-1}R\right)^{-1}. \end{aligned} \quad (2.1)$$

Proof. From (1.9) we observe that

$$(\hat{\beta} - \beta) = \left(X'X + \frac{s}{\nu} R'\psi^{-1}R\right)^{-1} \left(X'u + \frac{s}{\nu} R'\psi^{-1}v\right).$$

Recalling the independence of u and v , and of s and $X'u$, we have

$$\begin{aligned} V(\hat{\beta}) = & \sigma^2 E \left[\left(X'X + \frac{s}{\nu} R'\psi^{-1}R\right)^{-1} X'u \left(X'X + \frac{s}{\nu} R'\psi^{-1}R\right)^{-1} \right] \\ & + E \left[\left(\frac{s}{\nu}\right)^2 \left(X'X + \frac{s}{\nu} R'\psi^{-1}R\right)^{-1} R'\psi^{-1}R \left(X'X + \frac{s}{\nu} R'\psi^{-1}R\right)^{-1} \right]. \end{aligned} \quad (2.2)$$

Consider the first term on the RHS of (2.2). There exists a non-singular matrix P such that

$$P'X'XP = I$$

and

$$P'R'\psi^{-1}RP = \Lambda,$$

where Λ is a diagonal matrix whose diagonal elements are the roots of $|R'\psi^{-1}R - \lambda X'X| = 0$. Of these k roots, g are positive and the remainder are zero.

So, the first term on the RHS of (2.2) is

$$\begin{aligned} & \sigma^2 P E \left(I + \frac{s}{\nu} \Lambda \right)^{-2} P' \\ & = P \Delta P', \text{ say,} \end{aligned}$$

where $\Delta = \text{diag. } (\delta_i)$, and

$$\delta_i = E \left[\frac{\sigma^2}{\left(1 + \frac{s}{\nu} \lambda_i\right)^2} \right]. \quad (2.3)$$

When $\lambda_i = 0$, $\left(\frac{s}{\nu}\right)$ is an unbiased estimator of δ_i . To find an unbiased estimator of δ_i when $\lambda_i > 0$, let $h_i(s)$ be a continuous function of s such that $E[sh_i(s)] = \delta_i$. It follows that

$$\sigma^2 E \left[\nu h_i(s) + 2s \frac{dh_i(s)}{ds} \right] = \sigma^2 E \left[\frac{1}{\left(1 + \frac{s}{\nu} \lambda_i\right)^2} \right],$$

or

$$E \left[\nu h_i(s) + 2s \frac{dh_i(s)}{ds} - \frac{1}{\left(1 + \frac{s}{\nu} \lambda_i\right)^2} \right] = 0. \quad (2.4)$$

As s is a complete sufficient statistic for σ^2 , it follows from (2.4) that

$$\nu h_i(s) + 2s \frac{dh_i(s)}{ds} - \frac{1}{\left(1 + \frac{s}{\nu} \lambda_i\right)^2} = 0. \quad (2.5)$$

The differential equation (2.5) has solution

$$\begin{aligned} h_i(s) &= s^{-\nu/2} \int_0^s \frac{w^{\nu/2}}{2w \left(1 + \frac{w}{\nu} \lambda_i\right)^2} dw, \\ &= \frac{1}{2} \int_0^1 \frac{w^{\nu/2-1}}{\left(1 + \frac{s \lambda_i}{\nu} w\right)^2} dw, \end{aligned} \quad (2.6)$$

and so an unbiased estimator of δ_i is

$$\hat{\delta}_i = sh_i(s) = \frac{s}{2} \int_0^1 \frac{w^{\nu/2-1}}{\left(1 + \frac{s \lambda_i}{\nu} w\right)^2} dw. \quad (2.7)$$

Accordingly, an unbiased estimator of $P\Delta P'$ is

$$\begin{aligned} & \frac{s}{2} \int_0^1 w^{\nu/2-1} P \left(I + w \frac{s}{\nu} \Lambda \right)^{-2} P' dw \\ &= \frac{s}{2} \int_0^1 w^{\nu/2-1} \left(X'X + w \frac{s}{\nu} R' \psi^{-1} R \right)^{-1} X'X \left(X'X + w \frac{s}{\nu} R' \psi^{-1} R \right)^{-1} dw. \end{aligned} \quad (2.8)$$

Now, the second term on the RHS of (2.2) does not involve any unknown parameters, so an unbiased estimator of this expression is

$$\left(\frac{s}{\nu} \right)^2 \left(X'X + \frac{s}{\nu} R' \psi^{-1} R \right)^{-1} R' \psi^{-1} R \left(X'X + \frac{s}{\nu} R' \psi^{-1} R \right)^{-1}. \quad (2.9)$$

Combining (2.8) and (2.9) we get $\hat{V}(\hat{\beta})$ in (2.1) as an unbiased estimator of $V(\hat{\beta})$, as required. #

The square roots of the diagonal elements of $\hat{V}(\hat{\beta})$ provide appropriate standard errors for the elements of $\hat{\beta}$. Note that we may write

$$\hat{V}(\hat{\beta}) = P Q P',$$

where $Q = \text{diag.}(q_i)$, $q_i = (\hat{\delta}_i + \hat{\delta}_i^*)$, $\hat{\delta}_i$ is given by (2.7) and

$$\hat{\delta}_i^* = \frac{(s/\nu)^2 \lambda_i}{\left(1 + \frac{s}{\nu} \lambda_i\right)^2}.$$

Similarly,

$$\hat{V}(b) = P \left(\frac{s}{\nu} I \right) P'$$

and

$$\tilde{V}(\tilde{\beta}) = P \left[\frac{s}{\nu} \left(I + \frac{s}{\nu} \Lambda \right)^{-1} \right] P'; \quad \nu > 4.$$

Accordingly, to avoid data-dependencies, comparisons between the three corresponding estimators of the variance of the i th. element of $\hat{\beta}$ may be based on

$$g_i^{\text{OLS}} = \left(\frac{s}{\nu} \right) \quad (2.10)$$

$$g_i^{GLS} = \left(\frac{s}{v} \right) \left(\frac{1}{1 + \frac{s\lambda_i}{v}} \right) = \frac{s}{v}(1-\theta_i) \quad (2.11)$$

and

$$\begin{aligned} g_i^u &= \left(\frac{s}{v} \right) \left[\frac{v}{2} \int_0^1 \frac{w^{v/2-1}}{\left(1 + w \frac{s\lambda_i}{v} \right)^2} dw + \frac{\frac{s}{v} \lambda_i}{\left(1 + \frac{s\lambda_i}{v} \right)^2} \right] \\ &= \left(\frac{s}{v} \right) \left[\frac{v}{2} \int_0^1 \frac{w^{v/2-1}}{\left(1 + \frac{w\theta_i}{1-\theta_i} \right)^2} dw + \theta_i(1-\theta_i) \right], \end{aligned} \quad (2.12)$$

where $\theta_i = \left(\frac{s\lambda_i}{v+s\lambda_i} \right)$ is the prior information share for β_i , corresponding to (1.10).

3. COMPARISONS OF COVARIANCE ESTIMATORS

From (2.10)-(2.12), $g_i^{GLS} < g_i^{OLS}$ and $g_i^{GLS} \rightarrow g_i^{OLS}$ and $g_i^u \rightarrow g_i^{OLS}$ as $\theta_i \rightarrow 0$. Similarly, $g_i^{GLS} \rightarrow 0$ and $g_i^u \rightarrow 0$ as $\theta_i \rightarrow 1$. This is reflected in Table 1 and Figures 1 and 2, where $\left(g_i^u / g_i^{OLS} \right)$ and $\left(g_i^u / g_i^{GLS} \right)$ respectively are evaluated for various values of θ_i and v . In interpreting these numbers it must be remembered that $\theta_i = \theta_i(v)$. The integral in (2.12) is easily determined by Romberg's method using the FORTRAN routines QROMB, TRAPZD and POLINT described by Press et al. (1986).

From Figure 1 and part (a) of Table 1 we see that $g_i^u \leq g_i^{OLS}$. The evaluated ratio can be interpreted in two ways: it compares an unbiased estimator with a consistent estimator of the variance of the mixed regression estimator of β_i ; and it also compares an unbiased estimator of the variance of $\hat{\beta}_i$ with an unbiased estimator of the variance of b_i . In terms of the latter interpretation, the figures can be taken as estimated relative efficiencies and they follow the expected pattern: i.e., mixed regression estimation is more efficient than is OLS. Under the first inter-

pretation, these figures illustrate that the use of $\hat{V}(\hat{\beta})$ can result in standard errors which are substantially smaller than those obtained by using the OLS formula as a (large-sample) approximation.

From Figure 2 and part (b) of Table 1 we see that $g_i^u \geq g_i^{GLS}$, for all degrees of freedom, and that the greatest difference between g_i^u and g_i^{GLS} occurs when the prior information share is approximately 60%, regardless of the degrees of freedom. Further, if $\nu > 10$ then this difference is less than 10%, and if $\nu > 30$ it is less than 3%. So, only for problems involving quite small degrees of freedom, and moderately strong prior information, will the use of our unbiased estimator of $V(\hat{\beta})$ result in standard errors which differ markedly from those obtained from the usual consistent estimator, $\tilde{V}(\tilde{\beta})$.

4. CONCLUSIONS

In problems where $\nu > 30$, the current practice of basing mixed regression standard errors on the consistent covariance matrix estimator $\tilde{V}(\tilde{\beta})$ may seem to be justified. However, recalling that $g_i^u \geq g_i^{GLS}$ in Table 1, we see that this current practice has the important disadvantage of resulting in standard errors which understate the variability of $\hat{\beta}_i$ on average, and so are unduly "optimistic" in the impression they convey about this estimator's precision. The unbiased estimator of the covariance matrix of the mixed regression estimator presented in this paper provides the basis for calculating standard errors which can be justified in finite samples, and which provide meaningful measures of that estimator's precision. However, the relative intractability of the exact distribution of the mixed regression estimator precludes using these standard errors in any straightforward way to construct confidence intervals, etc.

October, 1989

REFERENCES

- Mehta, J.S., and Swamy, P.A.V.B. (1970), "The Finite Sample Distribution of Theil's Mixed Regression Estimator and a Related Problem", Review of the International Statistical Institute, 37, 202-209.
- Nagar, A.L., and Kakwani, N.C. (1964), "The Bias and Moment Matrix of a Mixed Regression Estimator", Econometrica, 32, 389-402.
- Press, W.H., Flannery, B.P., Teukolsky, S.A., and Vetterling, W.T., (1986), Numerical Recipes: The Art of Scientific Computing, Cambridge: Cambridge University Press.
- Srivastava, V.K. (1980), "Estimation of Linear Single-Equation and Simultaneous-Equation Models Under Stochastic Linear Constraints: An Annotated Bibliography", International Statistical Review, 48, 79-82.
- Srivastava, V.K., and Giles, D.E.A. (1989), "Unbiased Estimation of the Mean Squared Error of the Feasible Generalised Ridge Regression Estimator", Discussion Paper No. 8907, Department of Economics, University of Canterbury.
- Swamy, P.A.V.B. and Mehta, J.S. (1969), "On Theil's Mixed Regression Estimator", Journal of the American Statistical Association, 64, 273-276.
- Theil, H. (1963), "On the Use of Incomplete Prior Information in Regression Analysis", Journal of the American Statistical Association, 58, 401-404.
- Theil, H. (1971), Principles of Econometrics, New York: John Wiley.
- Theil, H. and Goldberger, A.S. (1961), "On Pure and Mixed Statistical Estimation in Economics", International Economic Review, 2, 65-78.

Table 1. Estimated Variance Ratios

θ_i	.01	.10	.20	.30	.40	.50	.60	.70	.80	.90	.99
v											
(a) (g_i^u/g_i^{OLS})											
5	1.00	.95	.88	.81	.71	.61	.50	.37	.24	.12	.01
6	.99	.94	.87	.79	.70	.59	.48	.36	.23	.11	.01
7	.99	.94	.86	.78	.68	.58	.47	.35	.23	.11	.01
8	.99	.93	.86	.77	.67	.57	.46	.34	.22	.11	.01
9	.99	.93	.85	.76	.66	.56	.45	.34	.22	.11	.01
10	.99	.93	.85	.76	.66	.55	.44	.33	.22	.11	.01
15	.99	.92	.83	.74	.64	.54	.43	.32	.21	.10	.01
20	.99	.92	.82	.73	.63	.53	.42	.31	.21	.10	.01
25	.99	.91	.82	.72	.62	.52	.42	.31	.21	.10	.01
30	.99	.91	.82	.72	.62	.52	.41	.31	.20	.10	.01
(b) (g_i^u/g_i^{GLS})											
5	1.00	1.06	1.11	1.15	1.19	1.22	1.24	1.24	1.22	1.16	1.03
6	1.00	1.05	1.09	1.13	1.16	1.18	1.19	1.19	1.17	1.12	1.02
7	1.00	1.04	1.08	1.11	1.14	1.16	1.16	1.16	1.14	1.09	1.01
8	1.00	1.04	1.07	1.10	1.12	1.14	1.14	1.14	1.11	1.07	1.01
9	1.00	1.03	1.06	1.09	1.11	1.12	1.12	1.12	1.10	1.06	1.01
10	1.00	1.03	1.06	1.08	1.10	1.11	1.11	1.10	1.09	1.05	1.01
15	1.00	1.02	1.04	1.05	1.06	1.07	1.07	1.06	1.05	1.03	1.00
20	1.00	1.02	1.03	1.04	1.05	1.05	1.05	1.05	1.04	1.02	1.00
25	1.00	1.01	1.02	1.03	1.04	1.04	1.04	1.04	1.03	1.02	1.00
30	1.00	1.01	1.02	1.03	1.03	1.03	1.03	1.03	1.02	1.01	1.00

FIGURE 1
RATIO OF UNBIASED ESTIMATOR OF VARIANCE
TO USUAL OLS-BASED ESTIMATOR OF VARIANCE

FIGURE 2
RATIO OF UNBIASED ESTIMATOR OF VARIANCE
TO USUAL CONSISTENT ESTIMATOR OF VARIANCE

LIST OF DISCUSSION PAPERS*

- No. 8401 Optimal Search, by Peter B. Morgan and Richard Manning.
- No. 8402 Regional Production Relationships During the Industrialization of New Zealand, 1935-1948, by David E. A. Giles and Peter Hampton.
- No. 8403 Pricing Strategies for a Non-Replenishable Item Under Variable Demand and Inflation, by John A. George.
- No. 8404 Alienation Rights in Traditional Maori Society, by Brent Layton.
- No. 8405 An Engel Curve Analysis of Household Expenditure in New Zealand, by David E. A. Giles and Peter Hampton.
- No. 8406 Paying for Public Inputs, by Richard Manning, James R. Markusen, and John McMillan.
- No. 8501 Perfectly Discriminatory Policies in International Trade, by Richard Manning and Koon-Lam Shea.
- No. 8502 Perfectly Discriminatory Policy Towards International Capital Movements in a Dynamic World, by Richard Manning and Koon-Lam Shea.
- No. 8503 A Regional Consumer Demand Model for New Zealand, by David E. A. Giles and Peter Hampton.
- No. 8504 Optimal Human and Physical Capital Accumulation in a Fixed-Coefficients Economy, by R. Manning.
- No. 8601 Estimating the Error Variance in Regression After a Preliminary Test of Restrictions on the Coefficients, by David E. A. Giles, Judith A. Mikolajczyk and T. Dudley Wallace.
- No. 8602 Search While Consuming, by Richard Manning.
- No. 8603 Implementing Computable General Equilibrium Models: Data Preparation, Calibration, and Replication, by K. R. Henry, R. Manning, E. McCann and A. E. Woodfield.
- No. 8604 Credit Rationing: A Further Remark, by John G. Riley.
- No. 8605 Preliminary-Test Estimation in Mis-Specified Regressions, by David E. A. Giles.
- No. 8606 The Positive-Part Stein-Rule Estimator and Tests of Linear Hypotheses, by Aman Ullah and David E. A. Giles.
- No. 8607 Production Functions that are Consistent with an Arbitrary Production-Possibility Frontier, by Richard Manning.
- No. 8608 Preliminary-Test Estimation of the Error Variance in Linear Regression, by Judith A. Clarke, David E. A. Giles and T. Dudley Wallace.
- No. 8609 Dual Dynamic Programming for Linear Production/Inventory Systems, by E. Grant Read and John A. George.
- No. 8610 Ownership Concentration and the Efficiency of Monopoly, by R. Manning.
- No. 8701 Stochastic Simulation of the Reserve Bank's Model of the New Zealand Economy, by J. N. Lye.
- No. 8702 Urban Expenditure Patterns in New Zealand, by Peter Hampton and David E. A. Giles.
- No. 8703 Preliminary-Test Estimation of Mis-Specified Regression Models, by David E. A. Giles.
- No. 8704 Instrumental Variables Regression Without an Intercept, by David E. A. Giles and Robin W. Harrison.
- No. 8705 Household Expenditure in Sri Lanka: An Engel Curve Analysis, by Mallika Dissanayake and David E. A. Giles.
- No. 8706 Preliminary-Test Estimation of the Standard Error of Estimate in Linear Regression, by Judith A. Clarke.
- No. 8707 Invariance Results for FIML Estimation of an Integrated Model of Expenditure and Portfolio Behaviour, by P. Dorian Owen.
- No. 8708 Social Cost and Benefit as a Basis for Industry Regulation with Special Reference to the Tobacco Industry, by Alan E. Woodfield.
- No. 8709 The Estimation of Allocation Models With Autocorrelated Disturbances, by David E. A. Giles.
- No. 8710 Aggregate Demand Curves in General-Equilibrium Macroeconomic Models: Comparisons with Partial-Equilibrium Microeconomic Demand Curves, by P. Dorian Owen.
- No. 8711 Alternative Aggregate Demand Functions in Macro-economics: A Comment, by P. Dorian Owen.
- No. 8712 Evaluation of the Two-Stage Least Squares Distribution Function by Imhof's Procedure by P. Cribbitt, J. N. Lye and A. Ullah.
- No. 8713 The Size of the Underground Economy: Problems and Evidence, by Michael Carter.

(Continued on back cover)

- No. 8714 A Computable General Equilibrium Model of a Fisherine Method to Close the Foreign Sector, by Ewen McCann and Keith McLaren.
- No. 8715 Preliminary-Test Estimation of the Scale Parameter in a Mis-Specified Regression Model, by David E. A. Giles and Judith A. Clarke.
- No. 8716 A Simple Graphical Proof of Arrow's Impossibility Theorem, by John Fountain.
- No. 8717 Rational Choice and Implementation of Social Decision Functions, by Manimay Sen.
- No. 8718 Divisia Monetary Aggregates for New Zealand, by Ewen McCann and David E. A. Giles.
- No. 8719 Telecommunications in New Zealand: The Case for Reform, by John Fountain.
- No. 8801 Workers' Compensation Rates and the Demand for Apprentices and Non-Apprentices in Victoria, by Pasquale M. Sgro and David E. A. Giles.
- No. 8802 The Adventures of Sherlock Holmes, the 48% Solution, by Michael Carter.
- No. 8803 The Exact Distribution of a Simple Pre-Test Estimator, by David E. A. Giles.
- No. 8804 Pre-testing for Linear Restrictions in a Regression Model With Student-t Errors, by Judith A. Clarke.
- No. 8805 Divisia Monetary Aggregates and the Real User Cost of Money, by Ewen McCann and David Giles.
- No. 8806 The Management of New Zealand's Lobster Fishery, by Alan Woodfield and Pim Borren.
- No. 8807 Poverty Measurement: A Generalization of Sen's Result, by Prasanta K. Pattanaik and Manimay Sen.
- No. 8808 A Note on Sen's Normalization Axiom for a Poverty Measure, by Prasanta K. Pattanaik and Manimay Sen.
- No. 8809 Budget Deficits and Asset Sales, by Ewen McCann.
- No. 8810 Unorganized Money Markets and 'Unproductive' Assets in the New Structuralist Critique of Financial Liberalization, by P. Dorian Owen and Otton Solis-Fallas.
- No. 8901 Testing for Financial Buffer Stocks in Sectoral Portfolio Models, by P. Dorian Owen.
- No. 8902 Provisional Data and Unbiased Prediction of Economic Time Series by Karen Browning and David Giles.
- No. 8903 Coefficient Sign Changes When Restricting Regression Models Under Instrumental Variables Estimation, by David E. A. Giles.
- No. 8904 Economies of Scale in the New Zealand Electricity Distribution Industry, by David E. A. Giles and Nicolas S. Wyatt.
- No. 8905 Some Recent Developments in Econometrics: Lessons for Applied Economists, by David E. A. Giles.
- No. 8906 Asymptotic Properties of the Ordinary Least Squares Estimator in Simultaneous Equations Models, by V. K. Srivastava and D. E. A. Giles.
- No. 8907 Unbiased Estimation of the Mean Squared Error of the Feasible Generalised Ridge Regression Estimator, by V. K. Srivasatva and D. E. A. Giles.
- No. 8908 An Unbiased Estimator of the Covariance Matrix of the Mixed Regression Estimator, by D. E. A. Giles and V. K. Srivastava.

* Copies of these Discussion Papers may be obtained for \$4 (including postage, price changes occasionally) each by writing to the Secretary, Department of Economics, University of Canterbury, Christchurch, New Zealand.