Department of Economics
UNIVERSITY OF CANTERBURY
CHRISTCHURCH, NEW ZEALAND

ASYMPTOTIC PROPERTIES OF THE ORDINARY LEAST SQUARES ESTIMATOR IN SIMULTANEOUS EQUATIONS MODELS

V. K. SRIVASTAVA AND D. E. A. GILES

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V. K. SRIVASTAVA
University of Lucknow
and
D. E. A. GILES
University of Canterbury

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Address for Correspondence:
Professor D. E. A. Giles, Department of Economics, University of Canterbury, Christchurch 1, New Zealand.

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V.K. Srivastava

University of Lucknow

and

D.E.A. Giles

University of Canterbury

Abstract

This paper considers the asymptotic behaviour of the ordinary least squares estimator of the coefficients of an equation from a simultaneous system. In particular, it focuses on the inconsistency of the estimator, in terms of large-sample asymptotic theory, and its consistency in terms of small-disturbance asymptotics.

Address for Correspondence:

Professor D.E.A. Giles, Department of Economics, University of Canterbury, Christchurch 1, NEW ZEALAND.

1. Introduction

This paper considers the Ordinary Least Squares (OLS) estimator of the coefficients of a structural equation in a simultaneous system. It is well known that this estimator is biased, and inconsistent in terms of traditional (large-sample) asymptotics. Despite the application of small-disturbance asymptotic theory to approximate the small-sample properties of other simultaneous equations estimators (e.g., Kadane (1971)), the fact that the OLS estimator is consistent in a small-disturbance sense does not appear to have been discussed explicitly in the literature. Here we provide a unified discussion of these two types of asymptotic behaviour for this estimator, and illustrate the different interpretations that can arise according to the viewpoint taken.

The model comprises M equations. There are T observations on the M jointly endogenous and K predetermined variables:

$$YB + X\Gamma = U \qquad (1)$$

where $E(U) = 0$, $E(U'U/T) = \Sigma$, $(M\times M)$, and σ is scalar. We shall consider the first (identified) structural equation:

$$\begin{aligned} y &= Y_1\beta + X_1\gamma + \sigma u_1 \qquad (2) \\ &= A\delta + \sigma u_1, \end{aligned}$$

where $A = (Y_1, X_1)$, $\delta' = (\beta', \gamma')$, y is $(T\times 1)$, Y_1 is $(T\times m)$, X_1 is $(T\times k)$. $E(u_1) = 0$ and $E(u_1u_1') = \sigma_{11}I_T$. The OLS estimator of δ is $d = (A'A)^{-1}A'y$.

The reduced form of the model is

$$Y = -X\Gamma B^{-1} + \sigma UB^{-1} = X\Pi + \sigma V, \qquad (3)$$

and that part of the reduced form corresponding to the explanatory variables in (2) is

$$A = (X\Pi_1, X_1) + \sigma(V_1, 0)$$
$$= Z + \sigma UG, \qquad (4)$$

where G is derivable from the elements of B^{-1}, and Π and V are partitioned in an obvious way. We assume that $\lim_{T\to\infty} (T^{-1}Z'Z)$ is finite and non-singular.[1]

2. Asymptotic properties

The estimation error of the OLS estimator is

$$(d - \delta) = (A'A)^{-1}A'u_1$$
$$= \left[T^{-1}Z'Z + T^{-1}\sigma(Z'UG + G'U'Z) + \sigma^2G'\Sigma G + \sigma^2W\right]^{-1}$$
$$\cdot(T^{-1}\sigma Z'u_1 + \sigma^2G'\sigma_{(1)} + \sigma^2w), \qquad (5)$$

where $W = G'(T^{-1}U'U - \Sigma)G$, $w = G'(T^{-1}U'u_1 - \sigma_{(1)})$, and $\sigma_{(1)}$ is the first column vector of Σ. To consider the large sample asymptotic properties of δ, it is helpful to express (5) as

$$(d - \delta) = [I + H_{-1/2}]^{-1}\ (h + h_{-1/2}) \qquad (6)$$

where

$$H_{-1/2} = Q_{LS}[T^{-1}\sigma(Z'UG + G'U'Z) + \sigma^2W];$$
$$h = \sigma^2Q_{LS}G'\sigma_{(1)};\ h_{-1/2} = Q_{LS}(T^{-1}\sigma Z'u_1 + \sigma^2w);$$
$$Q_{LS} = (T^{-1}Z'Z + \sigma^2G'\Sigma G)^{-1}.$$

As both $H_{-1/2}$ and $h_{-1/2}$ are of order $O_p\ (T^{-1/2})$, letting $T \to \infty$ we have plim$(d - \delta) = h$, from (6), which establishes the traditional (large sample) inconsistency of the OLS estimator in this context.

In contrast, considering small disturbance asymptotics and letting $\sigma \to 0$, we see from (6) that in this alternative sense the OLS estimator is consistent. Expressing this situation more formally, we can re-write (5) as

$$(d - \delta) = [I + \sigma F_1 + \sigma^2 F_2](\sigma f_1 + \sigma^2 f_2), \qquad (7)$$

where

$$F_1 = T^{-1} Q_{SD}(Z'UG + G'U'Z) \; ; \quad F_2 = Q_{SD}(G'\Sigma G + W);$$

$$f_1 = T^{-1} Q_{SD} Z' u_1 \; ; \quad f_2 = Q_{SD}(G'\sigma_{(1)} + w);$$

$$Q_{SD} = (T^{-1} Z'Z)^{-1}.$$

From (7) as $\sigma \to 0$, the leading term in the expression for the estimation error associated with d is σf_1, and the small disturbance consistency of d is then seen by noting that $E(f_1) = 0$ and $E(f_1 f_1') = \sigma_{11} Q_{SD}$.

In addition, it can easily be shown that the small disturbance asymptotic distribution $(d - \delta)/\sigma$ is $N(0, \sigma_{11} Q_{SD})$, and this is identical to the corresponding asymptotic distribution for all members of the k-class family of estimators of δ (see Kadane (1971)).

3. Discussion

The conventional result that the OLS estimator is inconsistent in the context of simultaneous equations models is based on large sample asymptotics - the failure of the estimator to converge in probability to the true value of the parameter is a consequence of the formulation of the model, and the type of data being used, rather than a shortage of data *per se*.

When the asymptotic behaviour of the OLS estimator is approached from the small-disturbance viewpoint, a sequence of increasingly well specified equations is considered. At each step, the form of the model changes in the sense that it becomes more and more correctly specified, and ultimately there is no scope for "simultaneity bias" because the limiting equation is deterministic, regardless of the sample size.[2] Moreover, the type of data is also unimportant to this result. While trended data may circumvent the large-sample inconsistency of OLS, such data have no special implications for the small-disturbance aysmptotics discussed here.

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FOOTNOTES

* This note was prepared while the first author was visiting the Department of Economics, University of Canterbury.

1. This assumption prelcudes, for example, trended predetermined variables. With trended predetermined variables in the model, the OLS estimator is consistent (e.g., see Krämer (1984, 1985)).

2. It is worth noting that this result still holds if the structural disturbances have a non-zero mean.

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