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## BUDGET DEFICITS AND ASSET SALES

By Even McCann

## Discussion Paper

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budget deficits and asset sales*<br>by

## EWEN McCANN

This paper is circulated for discussion and comments. It should not be quoted without the prior approval of the author.

* The paper has been improved by referees' remarks on changes to the stock of government debt.


## BUDGET DEFICITS AND ASSET SALES*

## EWEN MCCANN ${ }^{+}$

## I. INTRODUCTION

About one-fifth of the New Zealand government's expenditure is required to meet the annual servicing charge for official debt. The government has sold some state owned assets, and plans to sell more. The intention of the sales is to use the proceeds to repurchase foreign-owned New Zealand official debt in order to reduce interest payments and future budget deficits.

This article establishes that the policy will fail under plausible conditions and that it will then increase the discounted budget deficit. It considers the consequences of the proceeds of sale being used re-purchase locally held bonds, as well as to re-purchase foreign held bonds. International interest arbitrage will occur and the exchange rate is included. Local tax rates are

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* The paper has been improved by the referees' remarks on changes in the stock of government debt.
explicit and the possibility of an asset being more, or less, profitable in private hands than if it is government owned is admitted.

Two effects operate in the problem. First, the existing stock of public debt must always equal the inherited stock plus the fresh issue arising from the finance of the current budget deficit. The stock of public debt therefore changes over time in response to successive•deficits. Second, the repurchase of bonds reduces interest payments and so changes future deficits and also future public debt.

A full analysis requires the consideration of both of the above effects. The sale of a government asset and the repurchase of debt alters terms in both the current and future budget constraints of the government. The problem thus requires the explicit examination of the inter-temporal government budget constraint.

The analysis will be conducted for a constant price level. It is acknowledged that inflation reduces the real value of inherited public debt but the consideration of the inflation issue is beyond the scope of this problem as it involves the different issues examined by Sargent and Wallace (1986). The issue of base money as a method of deficit finance will be formally incorporated in the problem.

The degree of integration of the world capital markets, and the size of government's bond repurchases in relation to them, affect the responses of local and foreign interest rates to the asset sale-bond repurchase programme. Interest rates in local and
foreign markets will be allowed to vary and differ over time, though the special case where they are constant over time in each market is of particular interest. Bonds are of the fixed price variable interest rate variety. If interest arbitrage is complete then interest rates are not equalized in local and foreign markets because, as the covered interest arbitrage formula requires, the interest rate differential and forward exchange rate premium are linked for each maturity. Covered interest arbitrage will be allowed.

The outline of the paper is that Section II defines the budget deficit highlighting the terms relevant to the problem; Section III evaluates the change in the budget deficit and shows the asset sale and debt repurchase policy is likely to cause it to increase; Section IV concludes the paper by interpreting the results.
II. INTER-TEMPORAL BUDGET CONSTRAINT

The government's budget constraint for period $t$ is

$$
\begin{align*}
P^{h}\left(B_{t}^{h}-B_{t-1}^{h}\right) & +s_{t} P^{f}\left(B_{t}^{f}-B_{t-1}^{f}\right)+H_{t}-H_{t-1}+P^{k}\left(K_{t-1}-K_{t}\right) \\
& +v_{t}+\tau\left(Y_{t}+i_{t-1} P^{h_{B} h}{ }_{t-1}\right) \\
= & G_{t}+s_{t} r_{t-1} P^{f_{B} f}{ }_{t-1}+i_{t-1} P^{h_{B} h} \tag{1}
\end{align*}
$$

where
$\mathrm{P}^{\mathrm{k}}$ is the nominal price of the government's assets, assumed constant throughout.
$K_{t}$ is the quantity of the government's assets at time $t$.
$\mathrm{H}_{\mathrm{t}} \quad$ is $t$ is
$G_{t}$ is
$i_{t-1}$ is $p^{h}$ is
$B_{t-1}^{h}$ is
$s_{t} \quad$ is
s the spot exchange rate for period $t$, i.e. the number of units of local currency exchanging for one unit of foreign currency.
$r_{t-1}$ is the nominal interest rate paid to foreign holders of government stock paid at time $t$ at the rate for t-1.
$P^{f} \quad$ is
the nominal price in foreign currency of the government's stock on world markets, assumed constant.
$B_{t-1}^{f}$ is the quantity of the government's bonds held over by foreigners from period $t-1$ to period $t$.
$\tau$ is the average local tax rate.
$Y_{t}$ is the nominal income in period $t$ of the private sector including its foreign interest income but excluding interest income from local official debt which is shown separately.
$v_{t}$ is nominal profit income from government owned assets in period $t$.

The order of the lagged subscript in equation (1) is different for assets than it is for liabilities because an accumulation of liabilities adds to currently available funds while it is the decumulation of assets that does so.

It will be seen from the government's budget constraint that the issue of bonds in one period adds to the bond stocks and interest payments in future periods - to meet the point raised in Section I. Similarly if the government sells assets to purchase some of its outstanding bonds in one period, asset and bond stocks and revenue in all future periods are affected. An inter-temporal budget constraint is required to incorporate these effects.

The procedure to derive the inter-temporal government budget constraint $\wedge$ to solve equation (1) for $B_{t}^{h}$ and to obtain from the solution expressions for $B_{t+T}^{h}$, $B_{t+T-1}^{h}$, $B_{t+T-2}^{h}$, ... . Starting with $B_{t+T}^{h}$ back substitution isused to eliminate $B_{t+T-1}^{h}$, then $B_{t+T-2}^{h}$ is eliminated and so on until an expression for $B_{t}^{h}$ is obtained which is a present value expression and is the government's inter-temporal budget constraint. The government's inter-temporal budget constraint is thus,

$$
\begin{align*}
& \sum_{j=t}^{t+T} \frac{1}{\rho_{j}}\left\{G_{j}-\tau Y_{j}-v_{j}+H_{j-1}-H_{j}+P^{k}\left(K_{j}-K_{j-1}\right)\right\} \\
& +P^{f}\left\{\begin{array}{c}
\sum_{j=t}^{t+T-1}
\end{array}\left[\frac{s_{j+1} a_{j}}{\rho_{j+1}}-\frac{s_{j}}{\rho_{j}}\right] B_{j}^{f}+s_{t^{a}}{ }_{0} B_{0}^{f}-\frac{s_{t+T}}{\rho_{t+T}} B_{t+T}^{f}\right\} \\
& +P^{h}\left[\beta_{0} B_{0}^{h}-\frac{B_{t+T}^{h}}{\rho_{t+T}}\right] \\
& =0 \tag{2}
\end{align*}
$$

Where the discount factors, $\rho_{u}$, are defined by

$$
\rho_{u}=\left\{\begin{array}{lc}
1, & u=t  \tag{3}\\
{\underset{i I}{j=t}}_{u-1}^{\left.j=(1-\tau) i_{j}\right],} & t+1 \leq u \leq t+T
\end{array}\right.
$$

and

$$
\begin{align*}
& \beta_{0}=1+(1-\tau) i_{0}  \tag{4}\\
& a_{u}=1+r_{u} \tag{5}
\end{align*}
$$

The inter-temporal budget constraint includes the government's discount rate, the term $(1-\tau) i_{u}$ in equation (3). That discount rate is a direct consequence of the analysis. It emerges from the decision to eliminate $B_{t}^{h}$ from equation (1) because equation (1) contains the term $\left[1+(1-\tau) i_{t-1}\right]_{\mathrm{B}-1}^{\mathrm{h}}$.

The budget deficit of present value $D$, may be defined by rearranging the appropriate terms in equation (2), using the definitions of $a_{0}$ and $\beta_{0}$ (equations (4) and (5)). It is,

$$
\begin{aligned}
& D=\sum_{j=t}^{t+T} \frac{1}{\rho_{j}}\left\{G_{j}-\tau Y_{j}-v_{j}\right\} \\
& +P^{f}\left\{\sum_{j=t}^{t+T}\left[\frac{s_{j+1}{ }_{j}}{\rho_{j+1}}-\frac{s_{j}}{\rho_{j}}\right] B_{j}^{f}+s_{t} r_{0} B_{0}^{f}\right\}
\end{aligned}
$$

$$
+\mathrm{P}^{\mathrm{h}}(1-\tau) \mathrm{i}_{0} \mathrm{~B}_{0}^{\mathrm{h}}
$$

$$
=\sum_{j=t}^{t+T} \frac{1}{\rho_{j}}\left\{H_{j}-H_{j-1}-P^{k}\left(K_{j}-K_{j-1}\right)\right\}
$$

$$
+\mathrm{P}^{\mathrm{f}}\left\{\frac{\mathrm{~s}_{t+\mathrm{T}}}{\rho_{\mathrm{t}+\mathrm{T}}} \mathrm{~B}_{\mathrm{t}+\mathrm{T}}^{\mathrm{f}}-\mathrm{s}_{\mathrm{t}} \mathrm{~B}_{0}^{\mathrm{f}}\right\}
$$

$$
\begin{equation*}
+P^{h}\left\{\frac{B_{t+T}^{h}}{\rho_{t+T}}: B_{0}^{h}\right\} \tag{6}
\end{equation*}
$$

The government's budget deficit is a flow magnitude which is financed by changes in stocks. That is the interpretation of the deficit equation (6), where the flows are to the left and the stock changes are to the right. The present value of the government's budget deficit, equation (6), is the excess of
government expenditure over its tax receipts and profit income, plus interest payments and the exchange rate losses on foreign bonds. That deficit is financed by changes in the stocks of fiduciary issue, government owned assets and foreign plus locally held bonds.
III. ASSET SALES AND DEFICITS

The time sequences of the government's flows and of its stocks are affected by a policy of asset sales and bond repurchase. Consider two different policies with discounted budget deficits $D^{\prime}$ and $D$. Let $D^{\prime}$ be defined by the sequences of flows to the left of equation (6) which are to be distinguished by primes. The other deficit, $D$, has defined sequences of flows without primes. Then, using the left side of (6) for the primed and non-primed flows the difference between the budget deficits is

$$
\begin{align*}
D^{\prime}-D= & \sum_{j=t}^{t+T} \frac{1}{\rho_{j}}\left[G_{j}^{\prime}-G_{j}-\tau Y_{j}^{\prime}+r Y_{j}-v_{j}^{\prime}+v_{j}\right] \\
& \left.+p^{f}\left[\begin{array}{c}
t+T-1 \\
\sum=t
\end{array} \frac{s_{j+1}^{a} j}{\rho_{j+1}}-\frac{s_{j}}{\rho_{j}}\right]\left[\begin{array}{c}
\mathrm{f}_{j}^{\prime}-B_{j}
\end{array}\right]\right] \tag{7}
\end{align*}
$$

The sale of government owned assets to buy back official debt can now be readily examined. Let the government sell a quantity of assets, $k$, in period 1 . The fractions of the proceeds, $\theta^{f}, \theta^{h}$ are respectively used to purchase foreign held official debt, and to purchase locally held government bonds. Let $\mathrm{b}_{1}^{\mathrm{f}}$, be the quantity of foreign held government bonds which the government purchases in
period 1. The asset sale - bond purchase transactions occur in period 1 and are not repeated. The government continues to accumulate or decumulate capital and local debt but without linking the two. Stocks each period in the two situations differ by $b_{1}^{f}, k$ as the case may be.

Define $D^{\prime}$ for the sequences of flows which arise by selling $k$ assets and buying back debt. Let $D$ be for the flow sequences which would emerge in the absence of that policy towards assets and bonds. Initial stocks will be equal in the two situations.

Assume that government expenditure is unchanged. The policy alters the government's revenue from the assets it owns and it changes the interest cost of official debt. Since an asset has become privately owned the profit from it attracts tax. Let $\mu$ be the constant annual profit from the assets to be sold when they are government owned and $(1+\alpha) \mu, \alpha>-1$, be their profits in the private sector. Then from those assumptions.

$$
\begin{align*}
& Y_{j}^{\prime}=Y_{j}+(1+\alpha) \mu \\
& v_{j}^{\prime}=v_{j}-\mu  \tag{8}\\
& b_{1}^{f}=B_{j}^{f}-B_{j}^{f^{\prime}}
\end{align*}
$$

Substitute (8) into (7) obtaining the change in the discounted budget deficit.

$$
\begin{equation*}
D^{\prime}-D=\sum_{j=t}^{t+T} \frac{1}{\rho_{j}}[\mu(1-\tau(1+\alpha))]-P^{f^{t+T-1}}\left(\frac{s_{j+1}^{a} j}{\rho_{j+1}}-\frac{s_{j}}{\rho_{j}}\right) b_{1}^{f} \tag{9}
\end{equation*}
$$

The government sells a quantity of assets, $k$, and spends a portion of $\theta^{f} P^{k} k$ on a quantity, $b_{1}^{f}$, of foreign bonds so

$$
s_{1} p^{f_{b} f}=\theta_{1}^{f} P_{k}
$$

and

$$
\begin{equation*}
\mathrm{b}_{1}^{\mathrm{f}}=\frac{\theta^{\mathrm{f}} \mathrm{P}_{\mathrm{k}}}{\mathrm{~s}_{1} \mathrm{P}^{\mathrm{f}}} \tag{10}
\end{equation*}
$$

To obtain the total proceeds from the sale of $k$ assume that domestic interest rates are constant over time, $\mathbf{i}_{\mathbf{j}}=\mathbf{i}$ all $\mathbf{j}$, and recall that a uniform income stream, $c$, has a present value of

$$
\begin{equation*}
\sum_{n=1}^{\infty} \frac{c}{(1+i)^{n-1}}=\frac{(1+i) c}{i} \tag{11}
\end{equation*}
$$

Now the assets provide the private sector with an after tax annual income of $(1-\tau)(1+\alpha) \mu$ which, from (11), has a present value and a capital sum of

$$
\begin{aligned}
\sum_{n=1}^{\infty} \frac{(1-\tau)(1+\alpha) \mu}{(1+(1-\tau) i)^{n-1}} & =\frac{(1+\alpha) \mu(1+(1-\tau) i)}{i} \\
& =P^{k_{k}}
\end{aligned}
$$

so

$$
\begin{equation*}
\mu=\frac{i \mathrm{P}_{\mathrm{k}}}{(1+\alpha)(1+(1-\tau) \mathrm{i})} \tag{12}
\end{equation*}
$$

Substitute (10), (12) into (9) and rearrange to obtain

$$
\begin{equation*}
D^{\prime}-D=P^{k} k\left\{\sum_{j=t}^{t+T} \frac{1}{\rho_{j}}\left[\frac{i[1-\tau(1+\alpha)]}{(1+\alpha)(1(1-\tau) i)}\right]-\sum_{j=t}^{t+T-1}\left[\frac{s_{j+1}^{a} j}{\rho_{j+1}}-\frac{s_{j}}{\rho_{j}}\right] \frac{\theta^{f}}{s_{1}}\right\} \tag{13}
\end{equation*}
$$

Notice that

$$
\begin{equation*}
\rho_{j+1}=[1+(1-\tau) i] \rho_{j} \tag{14}
\end{equation*}
$$

International interest arbitrage is now introduced. Income, $Y_{t}$, was taxed at rate $\tau$ in equation (1) and was defined there to include the private sector's overseas investment income. Residents' income from official bonds is also taxed at rate $\boldsymbol{r}$. A resident contemplating a foreign investment therefore compares local and foreign interest rates both net of tax at rate $r$. Covered international interest arbitrage for a one period investment made at time $t$ ensures that

$$
1+(1-\tau) i_{t}=\frac{F_{t+1}}{s_{t}}\left(1+(1-\tau) r_{t}\right)
$$

where $F_{t+1}$ is the forward exchange rate for delivery one period ahead of $t$. Covered international investments of maturity $j$ made at time $t$ therefore require the product of $j$ terms, which, with uniform interest rates over time, yield

$$
\begin{equation*}
(1+(1-\tau) i)^{j}=\frac{F_{t+j}}{s_{t}}(1+(1-\tau) r)^{j} \tag{15}
\end{equation*}
$$

Let $F_{t+j}=s_{t+j}$ then equation (15) provides

$$
\begin{equation*}
\frac{s_{t+j}}{s_{t}}=\left(\frac{1+(1-\tau) i}{1+(1-\tau) r}\right)^{j} \tag{16}
\end{equation*}
$$

Using equations (14), (16), the definition of $\rho_{j}$ from equation (3), in equation (13) and letting $t=1$ yields the change in the deficit,

$$
\begin{equation*}
D^{\prime}-D=P^{k} k\left\{\sum_{j=1}^{1+T} \frac{i(1-\tau(1+\alpha))}{(1+\alpha)(1+(1-\tau) i)^{j}}-\sum_{j=1}^{T} \frac{\theta^{f} r \tau}{(1+(1-\tau) r)^{j}}\right\} \tag{17}
\end{equation*}
$$

Taking the partial sums of the geometric series in equation (17) shows that the change in the discounted deficit as a result of the asset sale-bond repurchase scheme is

$$
\begin{align*}
D^{\prime}-D & =\frac{\mathrm{P}^{k_{k}}}{1-\tau}\left\{\frac{1-\tau(1+\alpha)}{1+\alpha}\left[1-\left[\frac{1}{1+(1-\tau) \mathrm{i}}\right]^{\mathrm{T}+1}\right]-\theta_{\tau}^{\mathrm{f}}\left[1-\left[\frac{1}{1+(1-\tau) \mathrm{r}}\right]^{\mathrm{T}}\right]\right. \\
>0 \text { iff } & \frac{1-\tau(1+\alpha)}{(1+\alpha) \theta^{\mathrm{f}} \tau}>\frac{1-\left[\frac{1}{1+(1-\tau) r}\right]^{\mathrm{T}}}{1-\left[\frac{1}{1+(1-\tau) \mathrm{i}}\right]^{\mathrm{T}+1}} \tag{18}
\end{align*}
$$

so the policy increases the budget deficit when the side condition holds. The side condition does hold for plausible values of the arguments. Notice that the right side of (18) is less than unity when New Zealand interest rates, i, exceed foreign interest rates, $r$, and possibly when they do not. The condition therefore applies when, for example, $\tau=0.33, \theta^{f}=1$ and $\alpha=0.5$. To emphasise the point, with even a $50 \%$ improvement in the assets' profitability after it becomes privately owned the deficit still increases as a result of the scheme. Many other combinations of values have the
same effect. Use of the sale proceeds for other than foreign debt repurchase is incorporated in (18) by using $\theta^{f}=1-\theta^{h}$.

## IV. CONCLUSION

The policy of selling government owned assets and of using the proceeds of the sale to purchase local and/or foreign bonds has been examined in this paper. The policy increases the discounted value of the government's budget deficit, see equation (18), when the side condition holds. The side condition is quite likely to apply.

An economic interpretation can be placed on these results. When a government sells an asset in order to purchase official debt it foregoes the profit from the asset and it avoids the interest payments on the debt. The effect on the deficit is partly the difference between the profit reduction and after tax interest costs avoided. Effectively, the government is exchanging a revenue reduction for a cost reduction. When discounting is allowed the profit reduction can exceed the cost reduction and the deficit increases. The rate of discount has to be considered before a further interpretation can be made.

The government's discount rate is inherent to the problem. It is not chosen by the investigator either arbitrarily or on a priori grounds, as was noted after equation (5). The government's rate of discount which emerges from the analysis is $(1-\tau) i_{u}$.

One heuristic interpretation of the result arises from the consideration of income streams and cost streams. Suppose the government has a quantity of an asset, $k$, which returns it a profit of $\$ 1$ per period. Using equation (11) and the government's
discount rate $(1-r) i$ when interest rates are constant shows that the present value of the profit stream to the government is

$$
\begin{equation*}
\frac{1[1+(1-\tau) i]}{(1-\tau) i} \tag{19}
\end{equation*}
$$

So when the government sells the asset it loses the profit stream (19).

When the government buys back its bonds it buys an income stream from the private sector paying the capital sum $\mathrm{P}^{\mathrm{k}} \mathrm{k}$. Now an income stream which has a capital value of $P^{k_{k}}$ at the private sector's discount rate is $1(1-\tau)$ per period. ( $1-\tau$ ) is in fact what the private sector received on the repurchased government bonds each period. So the government buys back an income stream which has a present value to the private sector of

$$
\begin{equation*}
\frac{(1-\tau)(1+(1-\tau) i)}{(1-\tau) i}=\frac{1+(1-\tau) i}{i} \tag{20}
\end{equation*}
$$

Equation (20) is also the present value to the government of the government's interest savings. The extra tax collected on profits of $\$ 1$ when the asset is in private hands is $\tau$ per period which has a present value to the government of

$$
\begin{equation*}
\frac{[1+(1-\tau) i]}{(1-\tau) i} \tag{21}
\end{equation*}
$$

The change in the deficit is the change in profits plus the change in interest cost plus the change in tax collections i.e. from (19), (20), (21)

$$
\begin{align*}
& \frac{[1+(1-\tau) i]}{(1-\tau) i}-\frac{1+(1-\tau) i}{i}-\frac{[1+(1-\tau) i] \tau}{(1+\tau) i} \\
& =0 \tag{23}
\end{align*}
$$

This example shows that using the proceeds of the asset sales to buy local bonds when productivity is unchanged does not decrease the budget deficit.

The essential point is that asset sales made to repurchase foreign andor local official debt are likely to increase the discounted budget deficit because
the government loses all of the profits from the assets it sells. It gets back only the tax on the profits of those assets when they are in private hands, even though private profits may exceed the government's profits from the assets. The interest payments which the government avoids need be by no means enough to offset the foregoing and to reduce the deficit.

Finally, the preferred policy would be for the government to retain ownership of the assets and to increase their profitability.

APPENDIX
The government's budget constraint for period $t$, equation (1) in the text, when solved for $B_{t}^{h}$ yields

$$
\begin{aligned}
B_{t}^{h}= & \frac{1}{P^{h}}\left[G_{t}+s_{t} P^{f_{B} f}{ }_{t-1} a_{t-1}+{ }^{\prime} P^{h_{B} h}{ }_{t-1} \beta_{t-1}-P^{k_{K}}{ }_{t-1}+H_{t-1}\right. \\
& \left.-\left(s_{t} P^{f_{B} f}{ }_{t}+H_{t}+r Y_{t}-K_{t}+v_{t}\right)\right]
\end{aligned}
$$

Advance the time subscripts:

$$
\begin{aligned}
& B_{t+1}^{h} \quad=\frac{1}{P^{h}}\left[G_{t+1}+s_{t+1} P^{f_{B}} f_{t} a_{t}+P^{h} B_{t} h_{t}-P^{k_{k}}{ }_{t}+H_{t}\right. \\
& \left.-\left(s_{t+1} \mathrm{P}_{\mathrm{B}}^{\mathrm{f}} \mathrm{t}+1+\mathrm{H}_{\mathrm{t}+1}+\tau \mathrm{Y}_{\mathrm{t}+1}-\mathrm{P}^{\mathrm{k}_{\mathrm{t}+1}}+\mathrm{v}_{\mathrm{t}+1}\right)\right] \\
& B_{t+T-1}^{h}=\frac{1}{P^{h}}\left[G_{t+T-1}+s_{t+T-1} P^{f_{B} f}{ }_{t+T-2} a_{t+T-2}+P^{h} B_{t+T-2}^{h} \beta_{t+T-2}\right. \\
& -P^{k_{K}}{ }_{t+T-2}+H_{t+T-2}-\left(s_{t+T-1} P^{f} B_{t+T-1}+H_{t+T-1}\right. \\
& \left.\left.+\tau \mathrm{Y}_{\mathrm{t}+\mathrm{T} \cdot 1}-\mathrm{P}^{\mathrm{k}_{\mathrm{t}+\mathrm{T}-1}}+\mathrm{v}_{\mathrm{t}+\mathrm{T}-1}\right)\right] \\
& B_{t+T}^{h}=\frac{1}{P^{h}}\left[G_{t+T}+s_{t+T} P^{f_{B} f}{ }_{t+T-1} a_{t+T-1}+P^{h_{B} h}{ }_{t+T-1} \beta_{t+T-1}\right. \\
& -\mathrm{P}^{\mathrm{k}_{\mathrm{t}+\mathrm{T}-1}}+\mathrm{H}_{\mathrm{t}+\mathrm{T}-1}-\left(\mathrm{s}_{\mathrm{t}+\mathrm{T}} \mathrm{P}_{\mathrm{B}}^{\mathrm{f}} \mathrm{f}_{\mathrm{t}+\mathrm{T}}+\mathrm{H}_{\mathrm{t}+\mathrm{T}}+r \mathrm{Y}_{\mathrm{t}+\mathrm{T}}\right. \\
& \left.\left.-P^{k^{\prime}}{ }_{t+T}+v_{t+T}\right)\right]
\end{aligned}
$$

where

$$
\begin{equation*}
a_{u}=1+r_{u}, \quad \beta_{u}=1+(1-\tau) i_{u} \tag{i}
\end{equation*}
$$

substituting the expression for $B_{t+T-1}^{h}$ into the equation for $B_{t+T}^{h}$ gives an equation containing $B_{t+T-2}^{h}$. If $B_{t+T-2}^{h}$ is then eliminated from $B_{t+T}^{h}$ one obtains

$$
-\mathrm{P}^{\mathrm{k}_{\mathrm{K}+\mathrm{T}-1}}+\mathrm{H}_{\mathrm{t}+\mathrm{T}-1}-\int \mathrm{s}_{\mathrm{t}+\mathrm{T}} \mathrm{P}_{\mathrm{B}}^{\mathrm{t}+\mathrm{T}} \text { }+\mathrm{H}_{\mathrm{t}+\mathrm{T}}+r \mathrm{Y}_{\mathrm{t}+\mathrm{T}}
$$

$$
\begin{equation*}
\left.\left.-\mathrm{P}_{\mathrm{k}_{t+\mathrm{T}}}^{\cdot}+\mathrm{v}_{t+\mathrm{T}}\right\}\right\} \tag{ii}
\end{equation*}
$$

$$
\begin{aligned}
& B_{t+T}^{h}=\frac{1}{P^{h}}\left\{G_{t+T}+s_{t+T} P^{f_{B} f}{ }_{t+T-1} a_{t+T-1}+\beta_{t+T-1}\left[G_{t+T-1}\right.\right. \\
& +s_{t+T-1} P^{f_{B}} \mathrm{f}_{\mathrm{t}+\mathrm{T}-2} \mathrm{a}_{\mathrm{t}+\mathrm{T}-2}+\beta_{\mathrm{t}+\mathrm{T}-2} \quad\left(\mathrm{G}_{\mathrm{t}+\mathrm{T}-2}\right. \\
& +s_{t+T-2} P^{f_{B}} \mathrm{f}_{\mathrm{t}+\mathrm{T}-3} \mathrm{a}_{\mathrm{t}+\mathrm{T}-3}+\mathrm{P}^{\mathrm{h}} \mathrm{~B}_{\mathrm{t}+\mathrm{T}-3^{\mathrm{h}} \beta_{\mathrm{t}+\mathrm{T}-3}} \\
& -P^{k_{t+T-3}}+H_{t+T-3}-\left[s_{t+T-2} P^{f} B_{t+T-2}+H_{t+T-2}\right. \\
& \left.\left.+r \mathrm{Y}_{\mathrm{t}+\mathrm{T}-2}-\mathrm{P}^{\mathrm{k}_{\mathrm{t}+\mathrm{T}-2}}+\mathrm{v}_{\mathrm{t}+\mathrm{T}-2}\right]\right) \\
& -P^{k_{K}}{ }_{t+T-2}+H_{t+T-2}-\int s_{t+T-1} P^{f_{B} f}{ }_{t+T-1}+H_{t+T-1} \\
& \left.\left.+\tau \mathrm{Y}_{\mathrm{t}+\mathrm{T}-1}-\mathrm{P}^{\mathrm{k}_{\mathrm{t}+\mathrm{T}-1}}+\mathrm{v}_{\mathrm{t}+\mathrm{T}-1}\right]\right]
\end{aligned}
$$

continue back substituting for the terms $B_{t+T-j}^{h}$, $T$ times in total. Collect terms for each variable, for example the collected terms in $G_{u}$ are

$$
\frac{1}{\mathrm{P}^{\mathrm{h}}} \mathrm{G}_{\mathrm{t}+\mathrm{T}}+\frac{1}{\mathrm{P}^{\mathrm{h}}} \beta_{\mathrm{t}+\mathrm{T}-1} \mathrm{G}_{\mathrm{t}+\mathrm{T}-1}+\frac{1}{\mathrm{P}}{ }^{\mathrm{h}} \beta_{\mathrm{t}+\mathrm{T}-1} \beta_{\mathrm{t}+\mathrm{T}-2} \mathrm{G}_{\mathrm{t}+\mathrm{T}-2} \cdots
$$

Notice that using equation (3) in the text

$$
\begin{aligned}
\beta_{\mathrm{t}+\mathrm{T}-1} \beta_{\mathrm{t}+\mathrm{T}-2} & =\frac{\beta_{\mathrm{t}+\mathrm{T}-1} \beta_{\mathrm{t}+\mathrm{T}-2} \beta_{\mathrm{t}+\mathrm{T}-3} \cdots \beta_{\mathrm{t}}}{\beta_{\mathrm{t}+\mathrm{T}-3} \beta_{\mathrm{t}+\mathrm{T}-4} \cdots \beta_{\mathrm{t}}} \\
& =\frac{\rho_{\mathrm{t}+\mathrm{T}}}{\rho_{\mathrm{t}+\mathrm{T}-2}}
\end{aligned}
$$

so the terms in $G_{u}$ are

$$
\begin{equation*}
\frac{\rho_{t+T}}{p^{h}} \sum_{j=t}^{t+T} \frac{G_{j}}{\rho_{j}} \tag{iii}
\end{equation*}
$$

and similarly for the terms in $K_{j}$ (which require factorization of the products of $\left.\beta^{\prime} s\right), H_{j}, \tau Y_{j}$ and $v_{j}$ obtaining

$$
\begin{equation*}
-\frac{\rho_{t+T} P^{\gamma}}{P^{h}}\left\{\sum_{j=t}^{t+T}\left\{\frac{K_{j-1}-K_{j}}{\rho_{j}}\right]\right\} \tag{iv}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\rho_{t+T}}{P^{h}}\left\{\underset{j=t}{t+T}\left[\frac{H_{j-1}-H_{j}}{\rho_{j}}\right)\right\} \tag{v}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{-\tau \rho_{t+T}}{P^{h}}\left(\sum_{j=t}^{t+T} \frac{Y_{j}}{\rho_{j}}\right) \tag{vi}
\end{equation*}
$$

and

$$
\frac{-\rho_{t+T}}{p^{h}} \quad\left[\begin{array}{cc}
t+T & v_{j}  \tag{vii}\\
\sum & \frac{\rho_{j}}{j=t}
\end{array}\right]
$$

The terms in $\mathrm{B}_{\mathrm{j}}^{\mathrm{f}}$ are more complicated i.e.

$$
\begin{aligned}
& \frac{P^{f}}{P^{h}}\left\{s_{t+T^{a}}{ }_{t+T-1}\left[\frac{\rho_{t+T}}{\rho_{t+T}}\right] B_{t+T-1}+s_{t+T-1}{ }^{\mathrm{E}} \mathrm{t}+\mathrm{T}-2\left[\frac{\rho_{t+T}}{\rho_{t+T-1}}\right] B_{t+T-2}\right. \\
& +s_{t+T-2^{a}}{ }_{t+T-3}\left[\frac{\rho_{t+T}}{\rho_{t+T-2}}\right] \mathrm{B}_{\mathrm{t}+\mathrm{T}-3} \cdots \\
& \left.-s_{t+T}\left[\frac{\rho_{t+T}}{\rho_{t+T}}\right] B_{t+T} \mathrm{f}_{t+T-1}\left[\frac{\rho_{t+T}}{\rho_{t+T-1}}\right] \mathrm{B}_{\mathrm{t}+\mathrm{T}-1} \mathrm{f}_{\mathrm{t}+\mathrm{T}-2}\left[\frac{\rho_{\mathrm{t}+\mathrm{T}}}{\rho_{\mathrm{t}+\mathrm{T}-2}}\right] \mathrm{B}_{\mathrm{t}+\mathrm{T}-2} \mathrm{f} \cdots\right\} \\
& =\frac{\rho_{t+T} P^{f}}{P^{h}}\left\{\sum_{j=t}^{t+T-1}\left[\frac{s_{j+1}{ }^{a} j}{\rho_{j+1}} \cdot \frac{s_{j}}{\rho_{j}}\right] B_{j}^{f}+\frac{s_{t}{ }^{a} t-1}{\rho_{t}} B_{t-1}^{f}-\frac{s_{t+T}}{\rho_{t+T}} B_{t+T} f\right\} \quad(v i i i)
\end{aligned}
$$

The expressions (iii) to (viii) are substituted into equation (ii) which is rearranged to provide the government's budget constraint, equation (2) of the text.

## Reference

Sargent, T., and Wallace, N., (1981) "Some Unpleasant Monetarist Arithmetic" Quarterly Review of the Federal Reserve Bank of Minneapolis pp 1 - 17 , reprinted in Rational Expectations and Inflation, Sargent, T., New York, Harper and Row, 1756.

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