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FOR A POVERTY MEASURE**

By Prasanta K. Pattanaik and Manimay Sen

Discussion Paper

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Department of Economics, University of Canterbury
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**A NOTE ON SEN'S NORMALIZATION AXIOM
FOR A POVERTY MEASURE***

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This paper is circulated for discussion and comments. It should not be quoted without the prior approval of the authors.

1. Introduction

In his well known derivation of a poverty measure, Sen (1976) uses a normalization axiom which specified the poverty measure in a situation where every poor had the same income. The normalization axiom has been subjected to some criticism: several authors, including Sen (1976), have noted that it was arbitrary.

The purpose of this note is to demonstrate that, contrary to these criticisms, Sen's (1976) axiom has a straightforward intuition. We provide a factorization of the axiom which shows that, essentially, the axiom uses an unexceptionable zero-one normalization of the poverty measure, together with linearity in the relevant parameters when all the poor have the same income. Thus Sen's (1976) axiom is shown to be intuitively much less arbitrary than what may seem to be the case in the first instance.

Earlier, Basu (1985) has provided an interesting axiomatization of Sen's (1976) axiom. Basu (1985) starts by assuming that when all the poor have the same income, the poverty measure is a function of two variables - H , which denotes the proportion of the poor in the society, and I , the proportion of the average short-fall of their incomes from the poverty line, and obtains a factorization of the axiom by placing restrictions on this function of H and I . Formulating the problem in this fashion, where the poverty measure is taken to be a function of H and I rather than the basic primitives, namely, the number of individuals in the society, the poverty line, the number of the poor and the incomes of the poor, somewhat restricts a full appreciation of the underlying structural elegance of Sen's axiom.

Our present axiomatization places restrictions on the poverty measure only with respect to two basic primitives - the number of the poor and their (identical) incomes - and thus obtains a factorization of Sen's axiom in a much simpler framework than that utilized by Basu (1985).

2. A Factorization of Sen's Axiom

Consider an economy with a poverty line z , where the poor have identical incomes, say \bar{y} ($0 \leq \bar{y} \leq z$). Let $S = \{1, \dots, n\}$ be the set of individuals in the economy. We denote by $y = (y_1, \dots, y_n)$ the vector of incomes of the individuals. Let Q be the set of poor individuals, and q the number of the poor in the economy. Sen's (1976) normalization axiom may be stated as follows:

Axiom N. Given z and n , the poverty measure P is given by:

$$P = \frac{q}{n} \left[\frac{z - \bar{y}}{z} \right]$$

We now propose two axioms which are shown to be equivalent to Axiom N.

Axiom 1. If $y_i = 0$ for all $i \in S$, then $P = 1$; if $y_i \geq z$ for all $i \in S$, then $P = 0$.

Axiom 1 requires that if everybody in the society has zero income, then poverty should be 1, and if no one in the society has an income below the poverty line, then poverty should be 0. It is clearly a very reasonable axiom if one wants the maximum and the minimum possible levels of poverty to be denoted by 1 and 0, respectively.

Axiom 2. P is a linear function in \bar{y} , and also in q.

Proposition. Axioms 1 and 2 are equivalent to Axiom N.

Proof. It is clear that Axiom N implies Axioms 1 and 2. We have only to show that Axioms 1 and 2 together imply Axiom N.

We first show:

$$(1) \quad \text{if } \bar{y} = 0, \text{ then } P = q/n.$$

By Axiom 2, P is a linear function of q. Let this linear function be given by

$$(2) \quad P = tq + t'$$

where t and t' are independent of q. By Axiom 1, at $q = n$ and $q = 0$, P takes the values 1 and 0, respectively. This gives $t = 1/n$ and $t' = 0$ in (2), and thus (1) follows.

By Axiom 2, P is a linear function of \bar{y} . Let this linear function be given by

$$(3) \quad P = \alpha\bar{y} + \beta$$

where α and β are independent of \bar{y} . By Axiom 1, at $\bar{y} = z$, $P = 0$, and by (1), at $\bar{y} = 0$, $P = q/n$. This gives $\alpha = -q/nz$ and $\beta = q/n$ in (3). On rearranging terms, we get Axiom N. □

As our Proposition shows, Axiom N is a conjunction of a intuitively obvious zero-one normalization, and the linearity of the poverty measure in the two basic parameters of poverty measurement, the number of the poor and their incomes. In a situation where there is no inequality among the poor - it is to this situation to which Axiom N is addressed - linearity of the measure in the basic parameters would appear to be essentially an argument for the simplicity of its form in these special circumstances.

3. Concluding Remarks

We have provided a factorization of Sen's (1976) normalization axiom for a poverty measure assuming only that it is a function of the basic parameters of poverty measurement. It is shown that underlying Sen's axiom, and equivalent to it, there are two intuitively transparent assumptions. Thus Sen's axiom turns out to be a fairly natural assumption in the special context where it was intended to be applicable.

References

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