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**POVERTY MEASUREMENT: A GENERALIZATION
OF SEN'S RESULT**

By Prasanta K. Pattanaik and Manimay Sen

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OF SEN'S RESULT***

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This paper is circulated for discussion and comments. It should not be quoted without the prior approval of the authors.

1. Introduction

In his pioneering contribution, Sen (1976) provided an axiomatization of a poverty measure. The measure has received widespread attention and has been used extensively in empirical work.¹ However, Sen's analysis has some restrictive aspects. First, Sen (1976) defines his poverty measure as a normalized weighted sum of the income gaps of the poor (where the income gap of any poor individual is the extent to which his income falls below the poverty line). Defining poverty in terms of such an aggregate income gap imposes a restriction on how poverty is visualized. Moreover, the definition of the poverty measure as a normalized weighted sum of the income gaps of the poor also constrains the admissible functional forms for the measure. Secondly, the derivation of Sen's poverty measure crucially depends on his normalization axiom. The normalization axiom requires that in a situation where every poor person has the same income, the measure of poverty should be given by the product of the "head-count" ratio and the "income-gap" ratio. The arbitrary nature of this axiom has come in for criticism.²

The purpose of this paper is to show that neither of these restrictive features is essential for the characterization of Sen's poverty measure. We show that a modified version of Sen's axioms can be used to characterize Sen's measure, without imposing any prior restriction on the form of the poverty measure. At the same time, the normalization axiom that we propose is intuitively transparent. Thus, the analysis of Sen (1976) is shown to be more general than may appear from his formal structure.

2. The Axioms

Let $S = \{1, \dots, n\}$ be the set of individuals in the economy. Let $y = (y_1, \dots, y_n)$ be the vector of incomes of the individuals. Let z be the level of income exogenously specified to be the poverty line. Q is the set of all $i \in S$ such that $y_i \leq z$, i.e. Q is the set of the poor. q will denote the number of the poor. We denote by y_Q the restriction of y to Q .

Given n , z and q , a poverty measure P is a function that specifies a non-negative real number for each y_Q , the income vector of the poor. We write $P = P(y_Q)$.

We now introduce the axioms. Like Sen (1976), we assume that a richer person has a higher level of welfare than a poorer person. This is embodied in the following axiom.

Axiom 1. Given y , there is a complete welfare ordering $>_y$ defined on S such that for all $i, j \in S$, $i >_y j$ if $y_i > y_j$.

Axiom 1 is identical with Sen's (1976) Axiom M. Our next axiom modifies Sen's Axiom R in specifying a relationship between the poverty measure and the rank of the poor in the welfare ordering $>_y$.

Let $i \in Q$. Given $>_y$, we define r_i , the rank of i in $>_y$, as follows:

$$r_i = |\{j \in Q \mid j >_y i\}|.$$

Axiom 2. For all $i \in Q$, the rate of change in the poverty measure with respect to y_i is non-zero, and proportional to the rank of i

in \succ_y . The proportionality factor, while identical for each j , $j \in Q$, is independent of all y_j , $j \in Q$.³

It can be verified that, ignoring the implicit differentiability assumption in Axiom 2, Sen's (1976) definition of a poverty measure and his Axiom R together imply Axiom 2.⁴

Our last axiom is an axiom of normalization.

Axiom 3. If $y_i = z$ for all $i \in Q$, then the poverty measure is 0; and if $y_i = 0$ for all $i \in Q$ then the poverty measure is q/n .

The intuition behind our normalization with respect to 0 is obvious. The second part of Axiom 3 requires that when all the poor have zero incomes, the information about the extent of poverty is given simply by the proportion of the poor in the population. Sen's (1976) normalization axiom clearly implies ours. However, our normalization axiom does not imply Sen's, as shown in the following example. Let the poverty measure be given by

$$P = \sum_{i \in Q} \left[\frac{z^2 - y_i^2}{nz^2} \right].$$

The reader can easily check that P satisfies our normalization axiom, but not Sen's.

3. Characterization of the Poverty Measure

We now characterize the poverty measure of Sen (1976) in terms of our axioms.

Proposition. The only poverty measure that satisfies Axioms 1-3 is the Sen (1976) poverty measure

$$(1) \quad P = \frac{2}{(q+1)nz} \sum_{i \in Q} (z - y_i) r_i.$$

Proof. That P satisfies Axioms 1-3 is obvious. We show that Axioms 1-3 imply P.

By Axiom 1, for $i \in Q$, r_i , the rank of i in $>_y$, is unambiguously given. By Axiom 2,

$$(2) \quad \frac{\partial P}{\partial y_i} = \theta r_i \quad i \in Q,$$

where θ is independent of y_i , $i \in Q$. (2) implies that P is linear in y_i , $i \in Q$, and has the form:

$$(3) \quad P = \sum_{i \in Q} (\theta r_i y_i) + \beta$$

where β is independent of y_i , $i \in Q$. Letting $y_i = z$, $i \in Q$, and using Axiom 3, we get from (3):

$$(4) \quad \beta = -\theta \frac{q(q+1)}{2} z.$$

Now letting $y_i = 0$, $i \in Q$, and using Axiom 3 again, we get from (3) and (4):

$$(5) \quad \theta = -\frac{2}{(q+1)nz}.$$

(1) follows from (2)-(5). \square

4. Concluding Remarks

It is established in this paper that the characterization of the measure of poverty suggested by Sen (1976) is independent of any particular notion of poverty or any specific functional form

of the poverty measure one may start with. It also does not require any but a weak, clearly reasonable normalization condition. Thus the fundamental insight into poverty measurement provided by Sen is far more general than his formal structure may initially suggest.

NOTES

1. See, for example, Ahluwalia (1978), Anand (1977), Dutta (1978), Kakwani (1980), among many others.
2. See Anand (1977), Basu (1985), Thon (1979), Takayama (1979), Kakwani (1980) and Foster (1983).
3. Axiom 2, as stated, assumes implicitly that the poverty measure is differentiable in y_i , $i \in Q$. This assumption, however, is not essential for our derivation of the poverty measure, and is retained here for the sake of simplicity.
4. It may be noted that Sen's (1976) definition of a poverty measure contained a slight misspecification of the normalizing coefficient, which was made to depend on the income vector y . However, Sen's (1976) derivation of the poverty measure implicitly assumes that the normalizing coefficient is independent of y . This is noted in Sen (1977).

REFERENCES

- Ahluwalia, M.S. (1978), Rural Poverty and Agricultural Performance in India, Journal of Development Studies, 14, 298-322.
- Anand, S. (1977), Aspects of Poverty in Malaysia, Review of Income and Wealth, 23, 1-16.
- Basu, K. (1985), Poverty Measurement: A Decomposition of the Normalization Axiom, Econometrica, 53, 1439-1443.
- Dutta, B. (1978), On the Measurement of Poverty in Rural India, Indian Economic Review, 13, 23-32.
- Foster, J.E. (1984), On Economic Poverty: A Survey of the Aggregate Measures, in: Basman, R.L. and G.F. Rhodes (Eds.), Advances in Econometrics, 3, 215-251, JAI Press Inc.
- Kakwani, N. (1980), Income Inequality and Poverty: Methods of Estimation and Policy Implications, Oxford University Press.
- Takayama, N. (1979), Poverty, Income Inequality and Their Measures: Professor Sen's Approach Reconsidered, Econometrica, 47, 747-759.
- Thon, D. (1979), On measuring Poverty, Review of Income and Wealth, 25, 429-439.
- Sen, A.K. (1976), Poverty: An Ordinal Approach to Measurement, Econometrica, 44, 219-231.
- Sen, A.K. (1977), Social Choice Theory: A Reexamination, Econometrica, 45, 53-89.

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