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Discussion Paper

No. 8805

DEPARTMENT OF ECONOMICS
UNIVERSITY OF CANTERBURY

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DISCUSSION PAPER #8805

SEPTEMBER 1988

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*We are grateful to Dorian Owen for several helpful discussions, to Judith Clarke for her advice on the preparation of the figures, and to the Reserve Bank of New Zealand for supplying their data.

1. INTRODUCTION

Barnett (1978) has derived an expression for the user cost of money, from a deterministic theory of portfolio choice, by maximizing utility subject to an inter-temporal budget constraint. The user cost formula has been used in the construction of Divisia indices of monetary aggregates Barnett and Spindt (1982)); in an investigation of monetary policy using a Divisia quantity index (Barnett (1984)); and in the estimation of the demand for money within a system of demand equations, (Barnett (1983)).

Divisia monetary aggregates have been constructed for Australia, Austria, Canada, New Zealand, the U.K., and the U.S.A. by Johnson and Boulton (1985), Driscoll *et al.*(1985), Cockerline and Murray (1981), McCann and Giles (1987), Driscoll *et al.* (1985, p.14), and Barnett and Spindt (1982) respectively. The interest in this work lies, first, in whether a properly constructed monetary aggregate - such as the Divisia quantity index - displays turning points, growth rates or variability which differ from those of the theoretically inadequate simple summations of the official magnitudes; second, in whether or not the demand for money can be better estimated using a Divisia quantity index; and third, in measuring the loss of information in using a simple sum aggregate of the type usually compiled and published by official agencies.

Section 2 of this paper points out a theoretical deficiency in the definition of the user cost of money employed in the work cited, and a theoretically more suitable user cost formula is derived there. Section 3 computes Divisia quantity indices for New Zealand using each definition of user cost, and compares the aggregates obtained with the corresponding official New Zealand aggregates. Section 4 offers some conclusions.

2. THE USER COST OF MONEY

The notion of the user cost of an asset includes its opportunity cost, an imputed depreciation charge and a gain or loss on the capital value of the asset. The nominal price of a monetary asset is always one unit of currency so a change in its nominal capital value will never be registered. Never-the-less the real value of a monetary asset varies inversely with the price level so a change in its real capital value occurs as the price level alters. If a budget constraint is expressed in nominal terms the capital loss due to a price level change occurring at the beginning of a period does not reduce the number of currency units inherited from the previous period. In that case the nominal budget constraint omits the capital gain or loss from the changed price level. The agent's real position is not reflected in the nominal budget constraint (c.f. Barnett, 1978).

Expressing the budget constraint in real terms requires the imputation of the real loss due to the inflationary erosion of real balances. It yields a different user cost of money formula than was derived by Barnett, who used the nominal budget constraint, and whose formula has been used by the writers cited. In turn, the different user cost formula feeds into the construction of the Divisia index of monetary aggregates. The significance of the theoretical improvement in the aggregate is an empirical matter investigated below.

2.1 Notation

We shall adopt the following notation:

$[s,s+1)$ is the half open time period s. At instant s prices change, transactions occur and interest is paid on assets held at the end of period s-1 at the rate for period s-1.

b_s is the number of bonds held during period s.

L_s is the quantity of labour supplied in period s.

M_{is} is nominal quantity of monetary asset i held during period s.

P_s is the actual and expected price level for period s

P_s^b is the actual and expected nominal price of bonds for period s.

r_{is} is the nominal rate of interest paid on monetary asset i in period s.

R_s is the nominal interest rate on the highest yielding alternative asset to money in period s.

w_s is the nominal wage rate during period s.

x_s is the quantity of consumption during period s.

ϕ_s is the rate of inflation between periods s-1 and s.

γ_{is} is the rate of growth in nominal monetary asset i between periods s-1 and s.

δ_s is the real change in bond prices between periods s-1 and s.

2.2 Budget Constraint

(M_{is-1}/P_{s-1}) is the real balance of monetary asset i at the end of period s-1. Price level changes and other events occur at the beginning of period s. Let $d(M_{is-1}/P_{s-1})$ be the ensuing change in the value of the inherited i'th real balance. Opening real balances are the sum of those two terms. Similarly, let the change in the inherited real value of bonds be $d\left[(P^b_{s-1}b_{s-1})/P_{s-1}\right]$, which occurs at the start of period s, and which is to be added to real bonds held at the end of the previous period to obtain opening real bonds. Under these conditions the real budget constraint for period s is

$$
\begin{aligned}
x_s + \sum_i M_{is}/P_s + P^b_s b_s/P_s = \; & w_s L_s/P_s + \sum_i r_{is-1} M_{is-1}/P_s \\
& + R_{s-1} P^b_{s-1} b_{s-1}/P_s + \sum_i M_{is-1}/P_{s-1} \\
& + \sum_i d(M_{is-1}/P_{s-1}) \\
& + P^b_{s-1} b_{s-1}/P_{s-1} + d\left[(P^b_{s-1}b_{s-1})/P_{s-1}\right]. \qquad (1)
\end{aligned}
$$

Notice that

$$d\left[\frac{M_{is-1}}{P_{s-1}}\right] = \frac{1}{P_{s-1}} dM_{is-1} - \frac{M_{is-1}}{P_{s-1}} \frac{dP_{s-1}}{P_{s-1}} \qquad (2)$$

and for a constant quantity of bonds,

$$d\left[(P^b_{s-1}b_{s-1})/P_{s-1}\right] = b_{s-1}\left[\frac{P_{s-1}dP^b_{s-1} - P^b_{s-1}dP_{s-1}}{P^2_{s-1}}\right] \qquad (3)$$

which is the real capital gain on bonds due to changes in the price level and the bond price. Let the rate of inflation, the growth rate of monetary asset i and the real change in the bond price respectively be

$$\phi_s = \frac{dP_{s-1}}{P_{s-1}} = \frac{P_s - P_{s-1}}{P_{s-1}} ,$$

$$\gamma_{is} = \frac{M_{is} - M_{is-1}}{M_{is-1}} ,$$

$$\delta_s = \frac{dP^b_{s-1}}{P_{s-1}} = \frac{P^b_s - P^b_{s-1}}{P_{s-1}} ,$$

where P_s and P^b_s are the price level and nominal bond price expected to rule in period s.

The real budget constraint for period s may then be written as

$$x_s = \frac{w_s L_s}{P_s} + \sum_i \left[\frac{M_{is-1}}{P_{s-1}} \left(1 + \frac{r_{is-1} P_{s-1}}{P_s} + \gamma_{is} - \phi_s\right) - \frac{M_{is}}{P_s}\right]$$

$$+ b_{s-1}\left[R_{s-1} \frac{P^b_{s-1}}{P_s} + \frac{P^b_{s-1}}{P_{s-1}} (1-\phi_s) + \delta_s\right] - \frac{P^b_s}{P_s} b_s \qquad (4)$$

in which the inflationary erosion of real balances and real bonds during period s is captured by ϕ_s.

The household is assumed to maximize utility over time subject to an intemporal budget constraint, (Barnett, 1978, p.147). The inter-temporal budget constraint is obtained by solving equation (4) for b_s and back substituting to the current period, t, for a planning horizon T periods ahead, in the same manner as is adopted by Barnett (1978). The coefficient of monetary asset i in the inter-temporal budget constraint is then the user cost of that asset, real in this case and nominal in the Barnett's. In this way, the user cost of the i'th real monetary asset for period t is found to be

$$u_{it}^{r} = \left[\prod_{q=t+1}^{t+T}\left(\frac{P_q}{P_q^b}\right)\right]\left\{\left(\frac{P_t}{P_t^b}\right) - \frac{\left[1+\left(\frac{P_t}{P_{t+1}}\right)r_{it}+\left(\frac{M_{it+1}-M_{it}}{M_{it}}\right)-\frac{P_{t+1}-P_t}{P_t}\right]}{\left[\frac{P_t^b}{P_{t+1}}R_t+\frac{P_t^b}{P_t}\left(1-\frac{P_{t+1}-P_t}{P_t}\right)+\frac{P_{t+1}^b-P_t^b}{P_t^b}\right]}\right\} \qquad (5)$$

which is seen to contain an inflation term. This expression should be compared with the commonly used formula for the user cost of a nominal monetary asset (Barnett, 1978, p.148), <u>i.e.</u>:

$$u_{it}^{n} = P_t\left[\frac{R_t - r_{it}}{1 + R_t}\right] . \qquad (6)$$

Each of these expressions for user cost is sensitive to the value chosen as the base of the price index, but in the formula for the Divisia index the product term in the first bracket in (5) cancels as does P_t in (6). The Divisia index is defined as:

$$D_t = D_{t-1}\prod_{i=1}^{N}(M_{it}/M_{it-1})^{s_{it}^{*}} \qquad (7)$$

where

$$s_{it}^{*} = \tfrac{1}{2}(s_{it} + s_{it-1})$$

$$s_{it} = u_{it}^{k}M_{it}/\sum_{j=1}^{N} u_{jt}^{k}M_{jt}; \qquad k = r,n$$

and u_{it}^{r}, u_{it}^{n} are defined by equations (5), (6) respectively.

2.3 Meaning of Erosion

Equation (2) showed that the capital gains on real balances depend on price level changes and on variations in the nominal stocks of monetary assets. The treatment of inflationary erosion differs between authors in their handling of those nominal stock terms. This issue has been examined by Turnovsky (1977, p.64). He finds that Tobin (1967) and also Stein (1969) include the term for the change in the real value of nominal money stocks, as we have done, in their definition of disposable income. Shell et al. (1969) do not. Sargent (1979, p.17), includes both of the terms in equation (2) in his definition of expected wealth change, effectively as we have done. All of those authors, it should be noted, write the budget constraint in real terms and include some measure of inflationary erosion.

The empirical difference between including and excluding the changes in the nominal stocks of monetary asset i when calculating its user cost is discussed in Section 3.

3. RESULTS

When inflation rates, monetary asset growth rates and capital gains on bonds are zero the user cost of a real monetary asset, given in equation (5), reduces the standard Barnett formula (6), with nominal bond prices normalised to unity. The divergence between the two measures of user cost is therefore likely to depend on the rate of inflation and on the rates of growth in nominal monetary assets. Inflation adds to the user cost of real balance i while growth in nominal balance i is an offset to the inflation part of user cost in equation (5).

The two formulae may yield substantially different measures of user cost in practice. For example, Figures 1(a), 1(b) show the standard user costs of the six monetary assets in the official New Zealand M3 aggregate. These assets and the price index used to define the user costs are described in the Appendix. Our inflation-adjusted user costs of those assets are shown in Figure 2. For currency and for transactions accounts the inflation adjusted user costs approximately quadrupled over the six year period of our sample. On the Barnett computation there was an approximate doubling in the user cost of those assets. In Figures 1 and 2 the planning horizon is set to one month ($T = 1$) for illustrative purposes, because then the product term in the first bracket of equation (5) reduces to P_t, the same term as in Barnett's formula (7) with unit bond prices.

The divergent treatments of the change in the nominal quantity of monetary asset i by different authors was noted in Section 2. The inflation adjusted user cost formula (5) was calculated in order to assess the effects on each user cost of including and of excluding the term for the nominal monetary growth rate of each asset. Omitting the monetary growth terms raises the inflation adjusted user cost of each asset by rather small amounts. Typically, the omission increases them in their third decimal place. These results are not reported further as they have negligible effect on the calculated Divisia index series.

We have computed Divisia monetary aggregates with the standard definition of user cost, equation (6), and with the inflation corrected formula, (5), using New Zealand monthly data for the period January 1981 to February 1987. This study differs from that of McCann and Giles (1987) in two important respects - that earlier work considered only the standard (Barnett) formula for user cost; and the data used related to a now superceded definition of the M3 aggregate. (The structure of the official monetary aggregates has recently been totally revised by the Reserve Bank of New Zealand.) Details of all the data used are given in the Appendix. The graphs and tables which follow compare the indices with the official (simple-sum) aggregate, M3. DR refers to the inflation-corrected Divisia monetary aggregate. There is negligible difference between the values of DR and of the standard Divisia index for our data set, so values relating to the latter are omitted for simplicity.

4. DISCUSSION

The user cost of holding monetary asset i may be substantially different when inflation is allowed for in its formulation from when it is not, as Figures 1, and 2 show. (In these figures, P_t has a base value of unity.) The user costs are used to construct the weights of the Divisia monetary aggregate, and here the price index base is irrelevant. Surprisingly, the Divisia monetary aggregates with and without the inflation adjustment are almost identical in this case. The reasons for this similarity lie in the interesting economic and algebraic properties of the problem. Because the rate of inflation depreciates the real value of all monetary assets at the same rate, it will have little effect on the choice <u>between</u> monetary assets. The effect of inflation on the arrangement of agents' portfolios is in the choice between (1) the

Divisia aggregate quantity of real money and (2) the quantity of non-monetary assets, which are inflation hedges, in their portfolios.

The algebraic explanation of the similarities in the two Divisia aggregates can be seen in the first bracketed term in equation (5). That term compounds during inflation. When the assets' shares are formed in the computation of the Divisia index, equation (7), the bracketed terms cancel. The minor and offsetting monthly inflation and monetary growth rates remain to capture the negligible influence of inflation on the choice <u>between</u> monetary assets.

Turning points in aggregate monetary growth rates can be used as either indicators of the need for changes in monetary policy or as a record of an actual change in monetary policy. Turning points are to some extent subjective but we view the following ten months as those where the annual percentage rates of growth in Official M3 "turned" in Figure 4: 1982m10, 1983m2, 1983m5, 1984m1, 1984m2, 1985m1, 1985m4, 1985m7, 1985m10, 1986m9. Of these stationary points:

(1) The Divisia M3 lead Official M3 by one month when Official M3 was stationary in 1983m5, 1985m4, 1985m7, 1986m9.

(2) The Divisia M3 lead Official M3 by two months when Official M3 was stationary in 1983m2, 1985m10.

(3) The Divisia M3 lagged Official M3 by one month when Official M3 was stationary in 1984m1, 1985m1.

(4) The Divisia M3 and the Official M3 reached a stationary point in the same month in 1982m10 and 1984m2.

There are some significant differences in the characteristics of the Divisia and of the official monetary aggregates in Table 1. For example, Official M3 has a mean annual rate of increase of 18.4% and a coefficient of variation of 0.299. Divisia M3 has a mean annual rate of increase of 14.7% and a coefficient of variation of 0.386. The signals from Official M3 overstate the "true" annual growth in the monetary aggregate and understate its volatility. The same is true for monthly rates of growth. Official M3 is a misleading aggregate in this respect.

Figure 1(a)
USER COSTS
(Barnett's Formula)

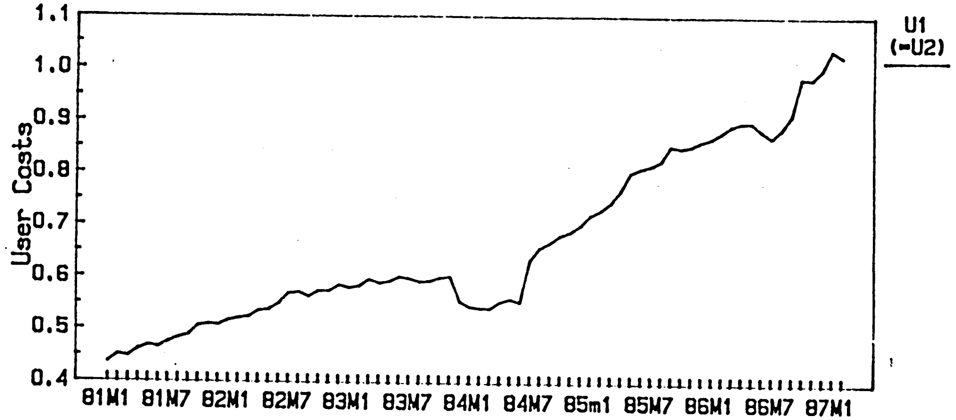

Figure 1(b)
USER COSTS
(Barnett's Formula)

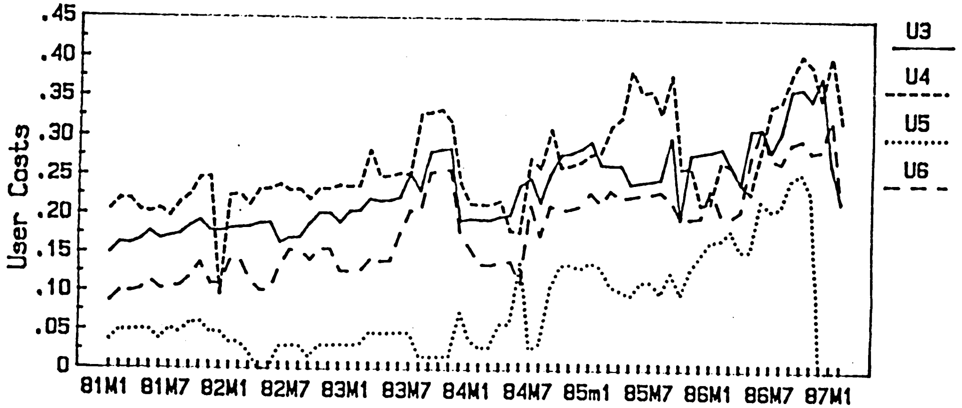

Figure 2(a)

USER COSTS (Inflation-Adjusted Formula)

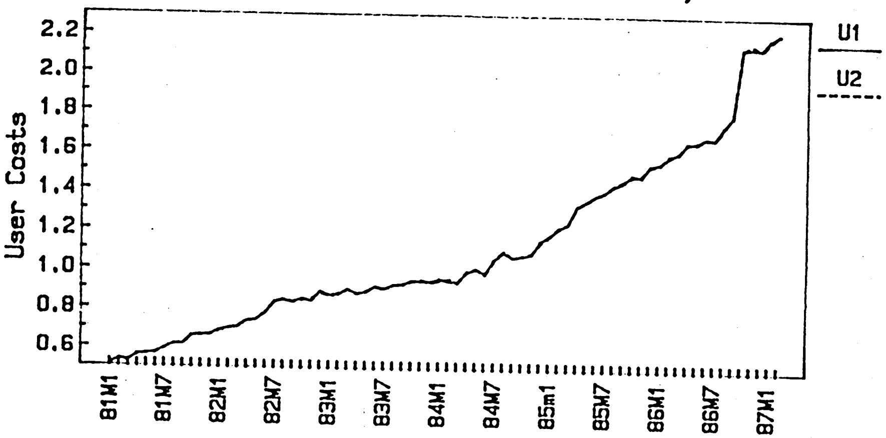

Figure 2(b)

USER COSTS (Inflation-Adjusted Formula)

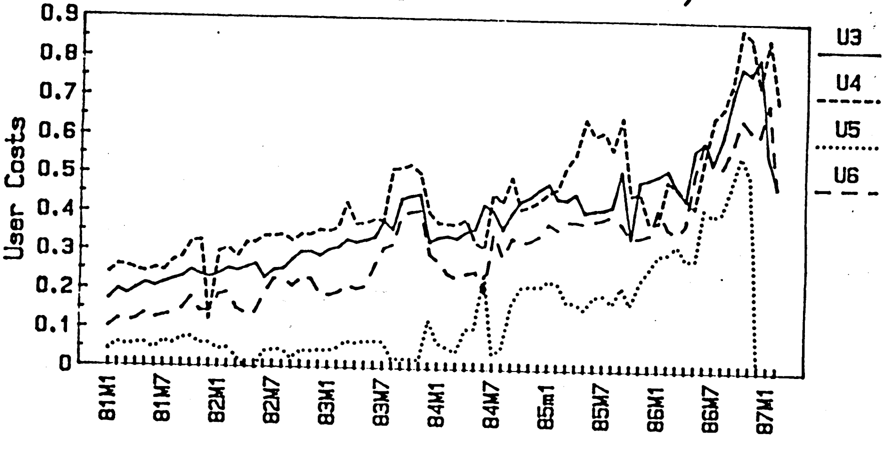

Figure 3
MONETARY AGGREGATES
(Levels)

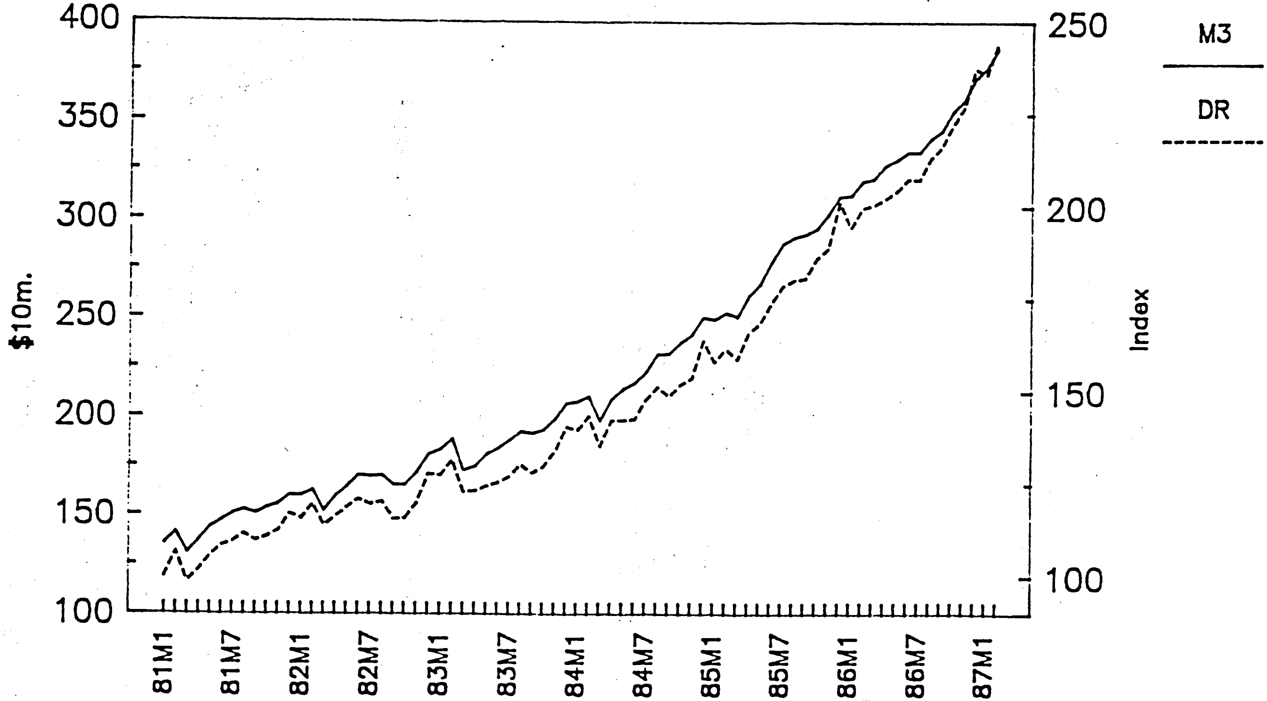

Figure 4
MONETARY AGGREGATES
(Annual Percentage Changes)

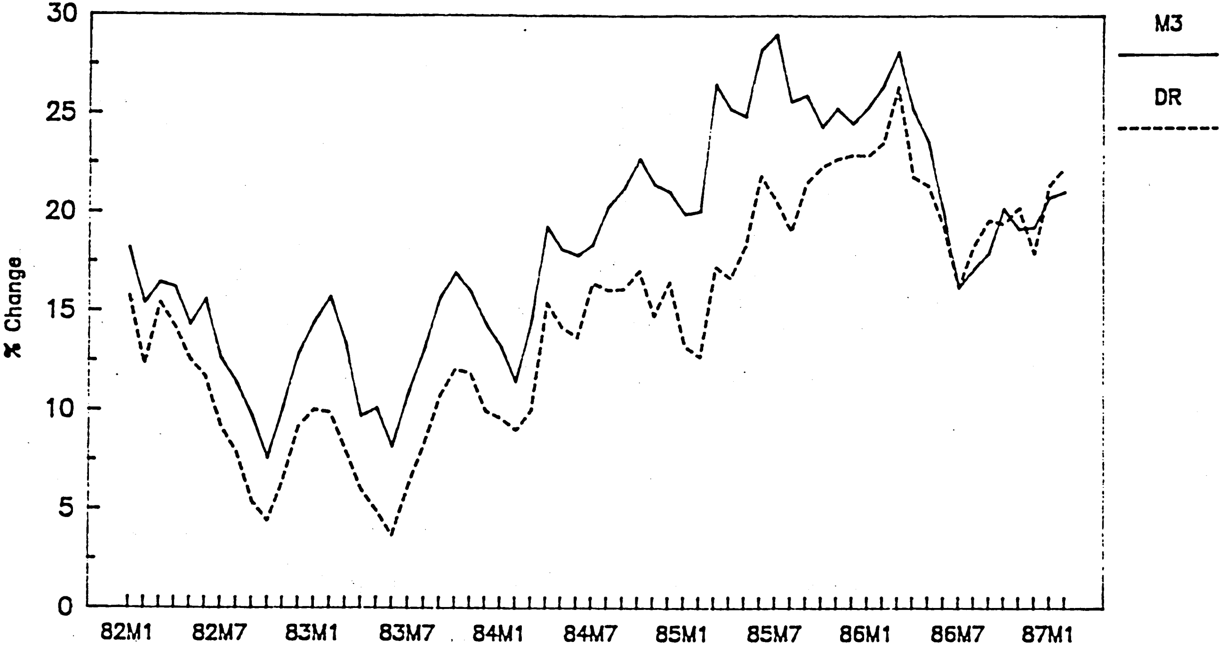

Figure 5

MONETARY AGGREGATES
(Monthly Percentage Changes)

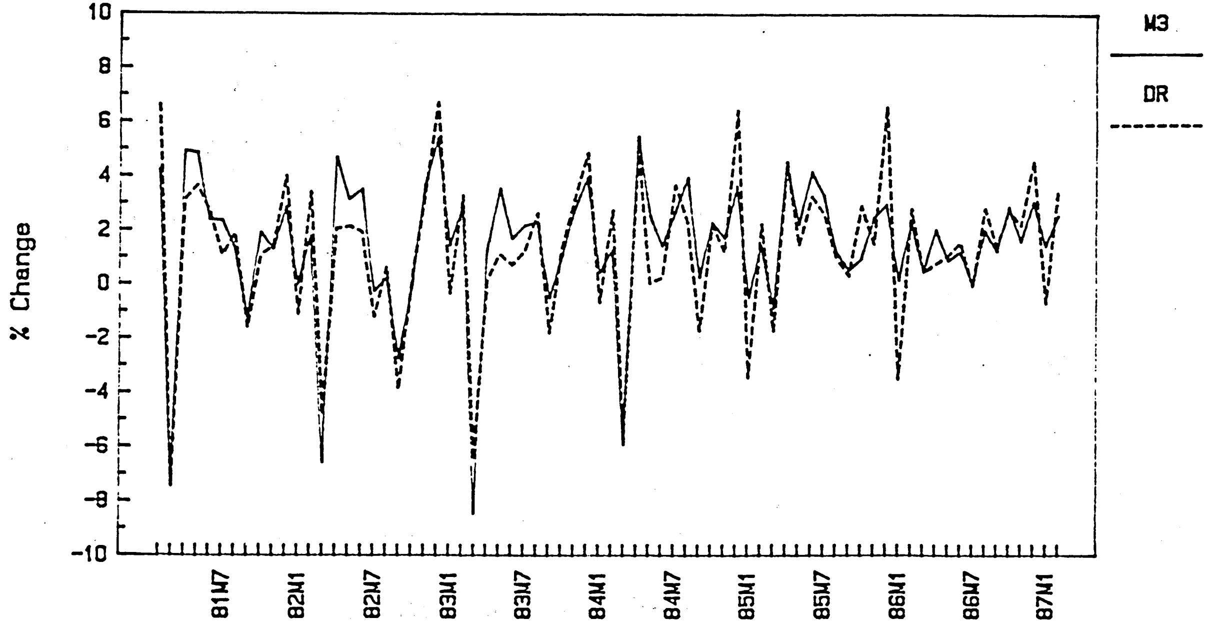

Table 1

SUMMARY STATISTICS

	Mean*	Coefficient of Variation	Correlation with M3
(a) Levels:			
M3	227.420	0.315	1.000
DR	150.332	0.258	0.997
(b) Annual % Change:			
M3	18.380	0.299	1.000
DR	14.736	0.386	0.912
(c) Monthly % Change:			
M3	1.482	1.769	1.000
DR	1.268	2.247	0.881

* The units for M3 are $10m.

The Divisia indices have a base value of 100 in January 1981.

APPENDIX

The monthly data for the component asset series were kindly supplied by the Reserve Bank of New Zealand. The official monetary aggregate is called "New M3". Its components are (1) Notes and coin held by the public; (2) transaction account balances at registered banks; (3) transaction account balances at savings institutions; (4) other funds consisting of specified deposits at registered banks; (5) specified deposits at financial corporations and (6) specified deposits at savings institutions. The data were adjusted by distributing inter-institutional transactions balances and government deposits across the remaining assets according to each remaining asset's proportion of the total.

The interest rates used, in the order of the assets listed above were (1) zero; (2) zero; and the maximum rates listed in Tables I(1), I(2) of the Reserve Bank of New Zealand Bulletins for; (3) 3 month deposits at Trustee Savings Banks; (4) 3 month deposits at registered banks; (5) all deposits at financial corporations; (6) 12 month deposits at savings institutions.

The bond selected as the alternative to money was second mortgage deeds. Second mortgage interest rates, R_t, are found in Table I(6) of the Reserve Bank of New Zealand Bulletin. The price index used, P_t, was the New Zealand Consumer Price Index from the New Zealand Department of Statistics Monthly Abstract of Statistics. This quarterly index was interpolated to obtain monthly figures, base = 1 in 1983m12, using the Chow-Lin (1971) procedure with the monthly food price index, seasonal dummy variables, a cubic trend and wage/price-freeze dummy variables as the related series.

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