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Department of Economics
UNIVERSITY OF CANTERBURY
CHRISTCHURCH, NEW ZEALAND

PRE-TESTING FOR LINEAR RESTRICTIONS IN A REGRESSION MODEL WITH STUDENT- t ERRORS

By Judith A. Clarke

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Department of Economics, University of Canterbury
Christchurch, New Zealand

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The Author's Electronic Mail Address is:

D.Giles%Canterbury.AC.NZ@Relay.CS.NET

ABSTRACT

In this paper, we derive the exact risk (under quadratic loss) of pre-test estimators of the prediction vector and error variance of a linear regression model whose errors are assumed to be normally distributed but in fact follow a multivariate Student-t distribution. The pre-test in question is one of the validity of a set of exact linear restrictions on the model's coefficient vector. We demonstrate how the known results for the model with normal disturbances can be extended to this broader case. Numerical evaluations of the risk expressions suggest that misspecifying the error distribution in this way does not, qualitatively, affect the risk properties of the estimators.

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1. INTRODUCTION

The sampling properties of the estimators of the parameters of the linear regression model, after a pre-test for linear restrictions on the coefficient vector, have widely been examined (see, for example, Clarke *et al.* [5,6], Giles [12], Giles and Clarke [13], Judge and Bock [17], Mittelhammer [22], Ohtani [24,25,26], Wallace [37]). All have assumed that the regression errors are normally distributed. There is a large body of literature, however, which suggests that some economic data series may be generated by processes whose error distributions have thick tails, or even infinite variances. Examples include price-change analysis in the stock, financial and commodity markets (Fama [9,10,11], Sharpe [29]), cash flow analysis (Granger and Orr [14]), and demand analysis (Coursey and Nyquist [7]). Such possibilities have led to studies of the robustness of the traditional estimators and searches for 'robust' estimators. Huber [15], Koenker [21], and Judge *et al.* [18] provide surveys of this literature.

One distribution receiving considerable attention is the multivariate Student-t (Mt) with zero mean vector. For this error distribution, as shown by Singh [30,31], Thomas [33], and Zellner [38], for example, the marginal distributions of the errors are univariate Student-t and the errors are uncorrelated but are not independent. For small values of γ, the degrees of freedom of the distribution, the marginal distributions have thick tails; as $\gamma \to \infty$, the pdf approaches a normal form; and when $\gamma = 1$, the pdf is Cauchy.

Linear regression models with Mt errors have been considered by some authors, including Judge *et al.* [19], King [20], Nimmo-Smith [23], Singh [30,31], Ullah and Zinde-Walsh [36], and Zellner [38]. Judge *et al.* [19] establish sampling properties of the James-Stein [16] estimator of the location parameter vector (and its positive part counterpart) under a squared error loss measure and a Mt error density. They compare, via a Monte Carlo experiment, the finite sample behaviour (empirical risks) for their Stein-like, and some conventional robust, estimators. In general, the risk characteristics for their Stein-like estimators when the errors are nonnormal are found to be the same as for the normal errors case.

In this paper we consider the question of the robustness, in terms of risk under squared error loss, of the usual pre-test estimators of the prediction vector[1] and the error variance of a linear regression model, after a pre-test for exact linear restrictions on the coefficient vector using the traditional F-ratio, when the errors follow a Mt distribution with γ degrees of freedom. In the next section we detail the model and give the unrestricted and restricted estimators of the model's parameters. Sections 3 and 4 present, discuss and evaluate the finite sample risk functions of the various estimators of the prediction vector and the error variance respectively. Some concluding remarks are given in the final section, and the proofs of the theorems appear in an appendix.

2. THE MODEL AND THE PARAMETER ESTIMATORS

Consider the linear regression model

$$y = X\beta + \epsilon, \qquad \epsilon \sim Mt(0,\sigma_\epsilon^2 I_T), \tag{1}$$

where y is a (Tx1) vector of observations on the dependent variable, X is a (Txk) non-stochastic design matrix of rank k(<T), β is a (kx1) vector of unknown parameters. The (Tx1) vector of disturbances, ϵ, is assumed to follow the multivariate Student-t (Mt) distribution

$$f(\epsilon) = \frac{C^*}{\sigma^T}\left[1 + \frac{\gamma^{-1}}{\sigma^2}\epsilon'\epsilon\right]^{-(T+\gamma)/2}, \qquad \sigma > 0,\ \gamma > 0 \quad -\infty < \epsilon < \infty \tag{2}$$

where $$C^* = \Gamma\left(\frac{T+\gamma}{2}\right)\left[(\pi\gamma)^{T/2}\,\Gamma(\gamma/2)\right]^{-1}$$

is a normalising constant, γ and σ^2 are the degrees of freedom and scale parameters of the distribution, and $\sigma_\epsilon^2 = \gamma\sigma^2/(\gamma-2)$ is the common variance of the ϵ_i's, i = 1,...,T; $\gamma > 2$.

Consider m independent linear restrictions on β, summarized by the hypothesis

$$H_0:\ R\beta = r \quad \text{vs.} \quad H_1:\ R\beta \neq r \tag{3}$$

where R and r are (mxk) and (mx1) matrices of known constants, and rank R = m(<k).

The unrestricted least squares estimator of β, $\tilde{\beta} = (X'X)^{-1}X'y$, is also the unrestricted maximum likelihood estimator (see, Singh [30,31], Thomas [33], Zellner [38]). Furthermore, it is easy to show that the restricted least squares estimator of β, $\beta^* = \tilde{\beta} - (X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}(R\tilde{\beta}-r)$, is also the restricted maximum likelihood estimator of β.

The usual F-ratio for testing the linear restrictions (3) is

$$F = \frac{v(R\tilde{\beta}-r)'[R(X'X)^{-1}R']^{-1}(R\tilde{\beta}-r)}{m(y-X\tilde{\beta})'(y-X\tilde{\beta})}, \tag{4}$$

where $v = (T-k)$. Thomas [33], and Ullah and Phillips [35] show that under Mt errors, F has a density function given by

$$f(F) = [\Gamma(\tfrac{v}{2})\Gamma(\tfrac{\gamma}{2})]^{-1} \sum_{r=0}^{\infty} \frac{\Gamma(\frac{m+v}{2}+r)\Gamma(\frac{\gamma}{2}+r)}{\Gamma(\frac{m}{2}+r)r!} \cdot (\tfrac{m}{v})^{m/2+r} \cdot$$

$$\frac{F^{m/2+r-1}}{(1+\frac{m}{v}F)^{(m+v)/2+r}} \cdot \frac{(2\theta/\gamma)^r}{(1+\frac{2\theta}{\gamma})^{\gamma/2+r}}, \qquad (5)$$

where $\theta = (R\beta-r)'[R(X'X)^{-1}R']^{-1}(R\beta-r)/2\sigma^2$, and is a measure of the hypothesis error. Note, first, that when H_0 is true $F \sim F_{m,v}$ (see, Zellner [38, p.401]). Secondly, when $\gamma \to \infty$, $\epsilon \sim MN(0,\sigma^2 I_T)$, and (5) reduces to the non-central F density, $F_{m,v,\theta}$.

In deriving their result, Ullah and Phillips [35] use the fact that if $\epsilon \sim Mt(0,\frac{\gamma\sigma^2}{\gamma-2}I_T)$, then we can write

$$\epsilon = \sqrt{\gamma}a/q \qquad (6)$$

where a and q^2 are independently distributed as $MN(0,\sigma^2 I_T)$ and χ^2_γ respectively. This decomposition enables one to condition on q^2 and work with normal random variates to obtain the conditional density $f(F|q^2)$. This is found to be non-central $F_{(m,v,\lambda)}$ with non-centrality parameter

$$\lambda = \frac{q^2\theta}{\gamma} = \frac{q^2(R\beta-r)'[R(X'X)^{-1}R']^{-1}(R\beta-r)}{2\sigma^2\gamma}. \qquad (7)$$

The unconditional density of F, given by equation (5), is then obtained by noting that

$$f(F) = \int_0^\infty f(F|q^2)f(q^2)dq^2 , \qquad (8)$$

where $$f(q^2) = 2^{-\gamma/2}\left[\Gamma(\gamma/2)\right]^{-1}e^{-q^2/2}(q^2)^{\gamma/2-1}.$$

This approach contrasts with that employed by some other authors, including Zellner [38] and Singh [30,31]. These authors proceed by regarding the error vector ϵ as being randomly drawn from a multivariate normal distribution with a random standard deviation generated from the inverted gamma distribution. It does not matter which approach is used, given the relationship between the χ^2, inverted gamma and gamma pdf's (see, for example, Zellner [39, pp.369-373]). In the following sections, we employ the Ullah and Phillips decomposition (6) to derive the risk expressions, under an invariant quadratic loss function, of the component and the pre-test estimators of $X\beta$ and σ_ϵ^2.

3. THE RISKS OF ALTERNATIVE ESTIMATORS OF E(y)

We define the pre-test estimator for $E(y) = X\beta$ as

$$X\hat{\beta} = \begin{cases} X\tilde{\beta} & ; \text{ if } F > c \\ X\beta^* & ; \text{ if } F \le c \end{cases}$$

$$= X\tilde{\beta} I_{(c,\infty)}(F) + X\beta^* I_{[0,c]}(F) \quad (9)$$

where $c = c(\alpha)$ satisfies $\int_0^c dF_{(m,v)} = (1-\alpha)$ and $I_{[a,b]}(F)$ is an indicator function with value unity if $F \in [a,b]$, zero otherwise. Further, if Xb is any estimator of E(y), then its risk function, under quadratic loss, is

$$\rho[Xb,E(y)] = E[Xb-E(y)]'[X\beta-E(y)]/\sigma^2 , \quad (10)$$

and is the relative mean squared error of Xb. We now derive the risk expressions for the various estimators of E(y). Notice that we are considering a situation involving model misspecification: the F-ratio is being used to test H_0 because the errors are wrongly being assumed to be normally distributed.

3.1 Finite Sample Risk Functions

THEOREM 1.

Under the stated assumptions,

$$\rho\left[X\tilde{\beta},E(y)\right] = k\gamma/(\gamma-2) \tag{11}$$

$$\rho\left[X\beta^{*},E(y)\right] = [\gamma(k-m) + 2\theta(\gamma-2)]/(\gamma-2) \tag{12}$$

$$\rho\left[X\hat{\beta},E(y)\right] = \left[2\theta(\gamma-2)(2P_{202}-P_{402}) + \gamma(k-mP_{201})\right]/(\gamma-2) \tag{13}$$

where

$$P_{a_1a_2a_3} = \sum_{r=0}^{\infty} \frac{(2\theta/\gamma)^r\ \Gamma(\frac{\gamma}{2}+r+a_3-2)}{r!\,(1+2\theta/\gamma)^{\gamma/2+r+a_3-2}\ \Gamma(\frac{\gamma}{2}+a_3-2)}$$

$$\cdot\ I_u\left[\tfrac{1}{2}(m+a_1) + r;\ \tfrac{1}{2}(v+a_2)\right], \tag{14}$$

$I_u(.;.)$ *is Pearson's incomplete beta function with* $u = cm/(v+cm)$.

PROOF. $\tilde{\beta}$ is unbiased, so,

$$\rho\left[X\tilde{\beta},E(y)\right] = \text{tr.}[V(X\tilde{\beta})]/\sigma^2$$

$$= \text{tr.}[\gamma\sigma^2X(X'X)^{-1}X']/[\sigma^2(\gamma-2)]$$

where $V(X\tilde{\beta})$ is the variance-covariance matrix of $X\tilde{\beta}$. (11) then follows immediately.

Turning to the restricted estimator, we have

$$\text{Bias }(X\beta^{*}) = -\ X(X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}(R\beta-r)$$

and $V(X\beta^{*}) = \gamma\sigma^2XG(X'X)^{-1}X'/(\gamma-2)$, $G = I - (X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}R$.

So,

$$\rho\left[X\beta^{*},E(y)\right] = (R\beta-r)'[R(X'X)^{-1}R']^{-1}(R\beta-r)/\sigma^2 + \gamma(k-m)/(\gamma-2)$$

and (12) follows directly, as $(R\beta-r)'[R(X'X)^{-1}R']^{-1}(R\beta-r)/\sigma^2 = 2\theta$.

The proof of (13) is given in the Appendix. ∎

Comparing (11), (12) and (13) we note directly:

(i) When $\gamma \to \infty$, $\epsilon \sim MN(0,\sigma^2 I_T)$, $P_{a_1a_2a_3} \to P_{a_1a_2}$, where

$$P_{a_1a_2} = \Pr.\left[F'_{(m+a_1,v+a_2;\theta)} \le \left[cm(v+a_2)\right]/\left[v(m+a_1)\right]\right] \quad (15)$$

(11), (12) and (13) reduce to the well known expressions (see, for example, Judge and Bock [17], Wallace [37]).

(ii) When $\alpha \to 1[0]$, $c \to 0[\infty]$, $P_{a_1a_2a_3} \to 0[1]$ for all a_1, a_2, a_3 and $\rho\left[X\hat{\beta},E(y)\right] \to \rho\left[X\tilde{\beta},E(y)\right]\left[\rho\left[X\beta^*,E(y)\right]\right]$.

(iii) When the null hypothesis is true ($\theta = 0$),

$$\rho\left[X\beta^*,E(y)\right] = \frac{\gamma(k-m)}{\gamma-2} < \rho\left[X\hat{\beta},E(y)\right] = \frac{\gamma(k-mF^*_{20})}{\gamma-2} < \rho\left[X\tilde{\beta},E(y)\right] = \frac{\gamma k}{\gamma-2}$$

where $F^*_{a_1a_2} = \Pr.\left[F_{(m+a_1,v+a_2)} \le \left[cm(v+a_2)\right]/\left[v(m+a_1)\right]\right]$.

(iv) As $\theta \to \infty$, the risk of $X\beta^*$ is unbounded, while

$\rho\left[X\hat{\beta},E(y)\right] \to \rho\left[X\tilde{\beta},E(y)\right]$ as $P_{a_1a_2a_3} \to 0$ for all a_1, a_2, a_3.

(v) When $\theta = m\gamma/(\gamma-2)$, $\rho\left[X\tilde{\beta},E(y)\right] = \rho\left[X\beta^*,E(y)\right] \le \rho\left[X\hat{B},E(y)\right]$. Equality of the three risk functions results when $2P_{202} - P_{402} - P_{201} = 0$.

3.2 Numerical Evaluations of the Risk Functions

The risk function of the pre-test estimator depends on the data and the unknown parameters only through θ and so we have evaluated the risk expressions for various choices of γ, α, m, k and T.[2] Some representative results, for various γ values, appear in Figures 1 to 4.

Comparing the figures, we see that a decrease in the value of γ from the normal errors case ($\gamma = \infty$) causes the estimator risk functions to shift upwards, a decrease in the rate at which the risk of the pre-test estimator approaches that of the unrestricted estimator, and an increase in the risk gain of the restricted estimator over the unrestricted estimator for all θ such that $\rho\left[X\beta^{*},E(y)\right] < \rho\left[X\tilde{\beta},E(y)\right]$. For the unrestricted and the restricted estimators these changes occur because of the increase in the estimators' variances as γ decreases (the marginal distribution has fatter tails). For the pre-test estimator, the increase in its variance and its absolute bias (for relatively large θ) both contribute to the observed differences. Our numerical evaluations suggest that, in general, the difference between an estimator's risk under normality and Mt errors is relatively insignificant for a γ value of at least 100.

Comparing the risk functions of the pre-test estimator and its component estimators for a given γ, we find that the conclusions observed when the errors are normally distributed hold for all values of γ. When the null hypothesis is true the pre-test estimator is risk inferior to the restricted estimator but superior to the unrestricted estimator. However, the pre-test estimator is dominated by the unrestricted estimator over a wide range of θ, and by both of its component estimators over part of the parameter space.[3] Hence, no one of the estimators strictly dominates the other two. This latter feature suggests choosing an estimator according to some optimality criterion. Such a study is beyond the scope of this paper (see, for example, Brook [3], Ohtani and Toyoda [27], Ohtani [24], Toyoda and Wallace [34]). So, aside from appropriate scaling, we find that the risk properties of the estimators are robust to the choice of γ, and hence to the misspecification of the error term's distribution being considered here.

4. THE RISKS OF ALTERNATIVE ESTIMATORS OF σ_ϵ^2.

Let the pre-test estimator of σ_ϵ^2 be given as

$$\hat{\sigma}_\epsilon^2 = \begin{cases} \tilde{\sigma}_\epsilon^2 ; & \text{if } F > c \\ \sigma_\epsilon^{*2}; & \text{if } F \le c \end{cases}$$

$$= \tilde{\sigma}_\epsilon^2 I_{(c,\infty)}(F) + \sigma_\epsilon^{*2} I_{[0,c]}(F)$$

where c, F and $I_{[a,b]}(F)$ are as previously defined. The risk function, under quadratic loss, of any estimator $\bar{\sigma}_\epsilon^2$ of σ_ϵ^2 is

$$\rho(\bar{\sigma}_\epsilon^2, \sigma_\epsilon^2) = E(\bar{\sigma}_\epsilon^2 - \sigma_\epsilon^2)^2 / \sigma_\epsilon^4 \quad ,$$

and is the relative mean squared error of $\bar{\sigma}_\epsilon^2$. We now consider the risks of the various estimators of σ_ϵ^2.

4.1 Finite Sample Risk Functions

THEOREM 2. *Under the stated assumptions,*

$$\rho(\tilde{\sigma}_\epsilon^2, \sigma_\epsilon^2) = 2(v+\gamma-2)/[v(\gamma-4)] \tag{15}$$

$$\rho(\sigma_\epsilon^{*2}, \sigma_\epsilon^2) = 2[\gamma^2(v+m)(v+m+\gamma-2) + 4\theta\gamma(\gamma-2)(\gamma-4) + 2\theta^2(\gamma-2)^2(\gamma-4)]/[(v+m)^2\gamma^2(\gamma-4)] \tag{16}$$

$$\begin{aligned}\rho(\hat{\sigma}_\epsilon^2, \sigma_\epsilon^2) = \Big\{&2\gamma^2(v+m)^2(v+\gamma-2) + m\gamma^2\Big[-(v+2)(m+2v)(\gamma-2)P_{040} \\ &+ v(\gamma-2)(m+2)P_{400} + 2v^2(\gamma-2)P_{220} + 2(\gamma-4)(v+m)v(P_{021}-P_{201})\Big] \\ &+ 4\theta\gamma v(\gamma-2)(\gamma-4)\Big[(m+2)P_{601} + vP_{421} - (v+m)P_{402}\Big] \\ &+ 4\theta^2 v(\gamma-2)^2(\gamma-4)P_{802}\Big\}/\Big[\gamma^2(\gamma-4)v(v+m)^2\Big].\end{aligned} \tag{17}$$

PROOF. See the Appendix. ∎

Remarks:

(i) When $\gamma \to \infty$, $\epsilon \sim MN(0,\sigma^2 I_T)$, $P_{a_1a_2a_3} \to P_{a_1a_2}$, $\sigma_\epsilon^2 = \sigma^2$ and so, (15), (16) and (17) collapse to the risk functions derived by Clarke *et al.* [6] when the errors are normally distributed.

(ii) $\rho(\hat{\sigma}_\epsilon^2,\sigma_\epsilon^2)$ converges to $\rho(\tilde{\sigma}_\epsilon^2,\sigma_\epsilon^2)$ when $\alpha \to 1$, and to $\rho(\sigma_\epsilon^{*2},\sigma_\epsilon^2)$ when $\alpha \to 0$.

(iii) $\rho(\hat{\sigma}_\epsilon^2,\sigma_\epsilon^2) \to \rho(\tilde{\sigma}_\epsilon^2,\sigma_\epsilon^2)$ as $\theta \to \infty$, while $\rho(\sigma_\epsilon^{*2},\sigma_\epsilon^2)$ is unbounded.

(iv) $\rho(\sigma_\epsilon^{*2},\sigma_\epsilon^2) < \rho(\tilde{\sigma}_\epsilon^2,\sigma_\epsilon^2)$ when the restrictions are valid.

4.2 Numerical Evaluations of the Risk Functions

As in Section 3.2 we have evaluated the risk expressions, as functions of θ, for the same values of γ, α, m, and v as before. Figures 5 to 8 illustrate typical cases. Note that Figure 6, which considers the risk functions when $\gamma = 5$, is drawn on a different scale from that of Figures 5, 7 and 8, so as to enable the features for all four cases to be distinguishable. We have included the pre-test estimator with a critical value of 1 for two reasons. First, under the null, the risk of the pre-test estimator attains a local minimum when c = 1 (for all γ), and, in some situations is also the global minimum. We discuss this further below. Secondly, if one undertook the Stein-like strategy of using an estimator which always selects the minimum of $\tilde{\sigma}_\epsilon^2$ or σ_ϵ^{*2} then this corresponds to the pre-test estimator with c = 1.

Consider first Figure 5, which illustrates the risk functions of the estimators when the errors are normally distributed. We see that there exists a family of pre-test estimators, with $c \in (0,1]$ which strictly dominate the unrestricted estimator for all

θ, and the restricted estimator over part of this parameter space.[4] This feature is not observed in the evaluations undertaken by Clarke *et al.* [6] but is noted in subsequent work by Ohtani [24]. Ohtani [24] considers the question of the optimal significance level for the pre-test problem examined by Clarke *et al.* [6] when the component estimators are based on the minimum mean squared error principle. He compares the sampling properties of the pre-test estimator with those of the Stein [32] estimator (extended to the linear regression case) and shows first, that the Stein estimator can be written as a pre-test estimator with critical value v/(v+2), and secondly, that the numerical evaluations suggest that the "Stein pre-test" estimator is optimal in some sense. Further, Clarke [4] shows that in certain situations the pre-test estimator, appropriately chosen, can strictly dominate both of its component estimators for all θ. She finds this typically occurs for small m, say 1 or 2, but does not appear to depend on the value of v. In these cases, the risk of the pre-test estimator with critical value unity corresponds to the global minimum risk at the origin and her numerical evaluations suggest this feature also holds for $\theta \neq 0$.[5]

Turning to the consequences of decreasing the value of γ from infinity, we find that the risk functions change in a similar way to that observed when estimating the prediction vector. That is, the estimator risk functions shift upwards, there is a decrease in the rate at which the risk of the pre-test estimator approaches that of the unrestricted estimator, and there is an increase in the risk gain of the restricted estimator over the unrestricted estimator for all θ such that $\rho(\sigma_\epsilon^{*2},\sigma_\epsilon^2) < \rho(\tilde{\sigma}_\epsilon^2,\sigma_\epsilon^2)$. These effects occur because of the increase in the variances of all the estimators when γ decreases and changes in the bias functions of the restricted and pre-test estimators; the bias of the restricted estimator decreases for all θ while the bias function of the pre-test estimator shifts down for relatively small θ (which may increase or decrease absolute bias) but becomes unbiased at a slower rate.

In Section 3.2, when comparing the risk functions of an estimator of the prediction vector for different values of γ, we found that, in general, there was little difference between the normal risk functions and those when $\gamma = 100$. However, when estimating the error variance we find that relatively large differences may still be evident for this gamma value, and even for much larger values of γ, say 5,000 or 10,000.

Nevertheless, when comparing the risk functions of the pre-test estimator and its component estimators for a given γ, the conclusions for when the errors are normally distributed continue to hold for all values of γ. Namely, there exists a family of pre-test estimators, with $c \in (0,1]$ which strictly dominate the unrestricted estimator for all θ. Further, some members of this family,[6] for some γ, also strictly dominate the restricted estimator. The numerical evaluations suggest that the pre-test estimator with critical value 1 strictly dominates all other members of this family.[7]

Moreover, for some γ, the restricted estimator is also strictly dominated by pre-test estimators with $1 < c < \infty$. Comparing equations (16) and (17) this will depend on m and v as well as γ. For the cases analyzed we found, in general, that the restricted estimator is strictly dominated by all pre-test estimators, except for those with c around 0 and $c \to \infty$, if γ is at most 15.

So, we have shown that, apart from appropriate scaling, the risk properties of the estimators are robust to the choice of γ. In particular, regardless of the value of γ, our recommendation is to pre-test rather than to impose the restrictions without testing their validity. Further, when using the least squares component estimators, a critical value of 1 seems to be the appropriate choice for the pre-test.

5. CONCLUSIONS

In this paper we have considered the sampling properties of various estimators of the parameters of the linear regression model, after a preliminary test of restrictions on the coefficients, when the error vector follows a multivariate Student-t distribution, but normality is wrongly assumed. The results suggest that, qualitatively, the estimator properties found for the model with normal errors carry over to this wider case. So, these properties are robust to such possible mis-specification of the error distribution. These results are of interest to applied researchers working with data likely to follow fat-tailed empirical distributions.

FOOTNOTES

1. We consider estimation of the prediction vector rather than the location vector so that our results are independent of the design matrix.

2. We numerically evaluated the risk functions for $v = 10$, 16, 20, 30, $m = 1$, 2, 3, 4, 5, $\alpha = 0.01$, 0.05, 0.25, 0.30, 0.50, 0.75, 0.90 and that value associated with a critical value of unity, $\gamma = 5$, 10, 50, 100, 500, 1000, 5000, 10,000, 100,000, ∞, and $\theta \in [0,3(0.1);3,20(0.5)]$. Full results are available on request. All evaluations were carried out using double-precision FORTRAN on an AT computer. Davies' [8] algorithm was used to evaluate the $P_{a_1a_2}$'s and the subroutines GAMMLN and BETAI from Press *et al.* [28] were utilized to obtain the $P_{a_1a_2a_3}$'s. Using these programs we found that the risk expressions were efficiently evaluated with no observed convergence problems.

3. Except when $2P_{202} - P_{402} - P_{201} = 0$. See Figure 2 for example.

4. That the risk of the pre-test estimator can dominate both of its components over any or all of the parameter space may seem counter intuitive. We may believe that as the pre-test estimator is a weighted sum of its component estimators then its risk function should be enveloped by those of its components. This, however, confuses the distinction between a weighted sum of the moments of the component estimators and the moments of their weighted sum.

5. The dominance of the pre-test estimator, for suitably chosen c, over the unrestricted estimator for all θ, and over the

restricted estimator for some or all θ, also occurs when estimating the error variance after a pre-test for homogeneity in the two-sample linear regression model. See, for example, Bancroft [1], Bancroft and Han [2], Clarke [4], Ohtani and Toyoda [27], and Toyoda and Wallace [34].

6. The exceptions are those pre-test estimators with critical value around the neighbourhood of $c = 0$.

7. It is straightforward to show that this feature holds under the null for any γ, but the proof for $\theta \neq 0$ is not obvious.

APPENDIX

The Proofs of THEOREMS 1. and 2. require the following lemma.

LEMMA A.1. *Let* $q^2 \sim \chi^2_\gamma$, $\lambda = q^2\theta/\gamma$, $\theta = (R\beta - r)'[R(X'X)^{-1}R']^{-1}(R\beta - r)/2\sigma^2$, *and let* $P_{a_1a_2}$, *given* q^2, *be defined by*

$$P_{a_1a_2} = Pr.\left[F'_{(m+a_1, v+a_2;\lambda)} \le \left[cm(v+a_2)\right]/\left[v(m+a_1)\right]\right]$$

$$= \sum_{r=0}^{\infty} \lambda^r e^{-\lambda}(r!)^{-1} I_u\left[\tfrac{1}{2}(m+a_1)+r;\tfrac{1}{2}(v+a_2)\right],$$

$I_u(.;.)$ is Pearson's incomplete beta function with $u = cm/(v+cm)$, and a_1, $a_2 = 0,1,2\ldots$. Then, for any real a_4, a_5:

$$E\left[(q^2)^{a_4}\lambda^{a_5}P_{a_1a_2}\right] = \sum_{r=0}^{\infty} \frac{(2\theta/\gamma)^{r+a_5}.2^{a_4}.\Gamma(\gamma/3+t+a_4+a_5)}{r!(1+2\theta/\gamma)^{\gamma/2+r+a_4+a_5}\,\Gamma(\gamma/2)} .$$

$$I_u\left[\tfrac{1}{2}(m+a_1)+r;\tfrac{1}{2}(v+a_2)\right] \qquad \text{(A.1)}$$

PROOF.

$$E\left[(q^2)^{a_4}\lambda^{a_5}\, P_{a_1a_2}\right] = \int_0^\infty (q^2)^{a_4}\lambda^{a_5} \sum_{r=0}^{\infty} \frac{\lambda^r e^{-\lambda}}{r!} I_u\left[\tfrac{1}{2}(m+a_1)+r;\tfrac{1}{2}(v+a_2)\right].$$

$$f(q^2)dq^2$$

$$= \sum_{r=0}^{\infty} (\theta/\gamma)^{r+a_5}\left[r!2^{\gamma/2}\Gamma(\gamma/2)\right]^{-1} I_u\left[\tfrac{1}{2}(m+a_1)+r;\tfrac{1}{2}(v+a_2)\right].$$

$$\int_0^\infty (q^2)^{\gamma/2+r+a_4+a_5-1} e^{-q^2(\gamma+2\theta)/2\gamma}dq^2,$$

because $\lambda = q^2\theta/\gamma$, $I_u(.;.)$ does not depend on q^2, and

$$f(q^2) = \left[2^{\gamma/2}\Gamma(\gamma/2)\right]^{-1} e^{-q^2/2}(q^2)^{\gamma/2-1}.$$

Applying the transformation $t = q^2(\gamma+2\theta)/(2\gamma)$, collecting terms and noting that $\Gamma(z) = \int_0^{\infty} t^{z-1}e^{-t}dt$, the result follows directly.

Now, to establish (13) of THEOREM 1, we have:

PROOF. $I_{(c,\infty)}(F) = 1 - I_{[0,c]}(F)$,

$$(X\beta^* - X\tilde{\beta}) = -X(X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}(R\tilde{\beta}-r),$$

so, $X\hat{\beta} - X\beta = X\tilde{\beta} - X\beta - I_{[0,c]}(F)\left[X(X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}(R\tilde{\beta}-r)\right]$

and, $$\rho\left[X\hat{\beta},E(y)\right] = \rho\left[X\tilde{\beta},E(y)\right] + E(I^*), \tag{A.2}$$

where $I^* = I_{[0,c]}(F)\Big\{2(R\tilde{\beta}-r)'[R(X'X)^{-1}R']^{-1}(R\beta-r)$

$$- (R\tilde{\beta}-r)'[R(X'X)^{-1}R']^{-1}(R\tilde{\beta}-r)\Big\}/\sigma^2.$$

Following Ullah and Phillips [35], we note that $R\tilde{\beta}-r = R\beta-r + R(X'X)^{-1}X'\epsilon = R(\beta-\beta_0) + R(X'X)^{-1}X'\epsilon$ where $\beta_0 = R\bar{r}$ is any solution of $R\beta_0 = r$. Further, let

$$B = [R(X'X)^{-1}R']^{-1}, \quad A = R(X'X)^{-1}X', \quad C = A'BA = C^2,$$

$$M = I_T - X(X'X)^{-1}X' = M^2, \quad \text{and } \delta = X(\beta-\beta_0). \tag{A.3}$$

So, noting that $AX = R$ and $MC = 0$, we have $F = v(\epsilon+\delta)'C(\epsilon+\delta)/(m\epsilon'M\epsilon)$ and

$$I^* = I_{[0,c]}\left[v(\epsilon+\delta)'C(\epsilon+\delta)/(m\epsilon'M\epsilon)\right]\Big\{2(\epsilon+\delta)'C\delta-(\epsilon+\delta)'C(\epsilon+\delta)\Big\}/\sigma^2$$

$$= I_{[0,c]}\left[(vb'Cb)/(mb'Mb)\right]\Big\{\gamma(2b'C\delta_1-b'Cb)/q^2\Big\}/\sigma^2, \tag{A.4}$$

where $b = a + \delta_1$, $\delta_1 = q\delta/\sqrt{\gamma}$ and use is made of decomposition (6), $\epsilon = \sqrt{\gamma}a/q$. Now, observe that, given q^2,

$$b \sim MN(\delta_1,\sigma^2 I_T), \ \frac{b'Mb}{\sigma^2} = \frac{a'Ma}{\sigma^2} \sim \chi^2_v, \text{ and } \frac{b'Cb}{\sigma^2} \sim \chi^{2\prime}_{m;\lambda} \tag{A.5}$$

where $\lambda = \delta_1' C\delta_1/\sigma^2 = q^2\theta/\gamma$. Further, given q^2, the quadratic forms $b'Mb/\sigma^2$ and $b'Cb/\sigma^2$ are independent and so, taking expectations of (A.4), conditional on q^2, we have

$$E(I^*|q^2) = \gamma\Big\{2E\Big[I_{[0,c]}\Big((vb'Cb)/(mb'Mb)\Big)(b'C\delta_1)|q^2\Big]$$

$$- E\Big[I_{[0,c]}\Big((vb'Cb)/(mb'Mb)\Big)(b'Cb)|q^2\Big\}/q^2. \qquad (A.6)$$

First, using *Lemma 1*. of Clarke *et al*. [5] gives,

$$E\Big[I_{[0,c]}\Big((vb'Cb)/(mb'Mb)\Big)(b'Cb)|q^2\Big] = \sigma^2(mP_{20}+2\lambda P_{40}). \qquad (A.7)$$

Secondly, we can write

$$E\Big[I_{[0,c]}\Big((vb'Cb)/(mb'Mb)\Big)(b'C\delta_1)|q^2\Big] =$$

$$E\Big[I_{[0,c]}\Big((vb^{*\prime}b^*)/(mb'Mb)\Big)(b^{*\prime}\delta_1^*)|q^2\Big]$$

where $b^* = L'b$, $\delta_1^* = L'\delta_1$ and $LL' = C$. Note that as C is symmetric and idempotent, $L'L = I_m$ and so, given q^2, $b^* \sim MN(\delta_1^*, \sigma^2 I_m)$. Then, using *Theorem 1* of Judge and Bock [17,p.321], noting that $\lambda = \delta_1^{*\prime}\delta_1^*/2\sigma^2$, we have

$$E\Big[I_{[0,c]}\Big((vb'Cb)/(mb'Mb)\Big)(b'C\delta_1)|q^2\Big] = 2\sigma^2\lambda P_{20}. \qquad (A.8)$$

Substituting (A.7) and (A.8) appropriately in (A.6) yields

$$E(I^*|q^2) = \gamma[(4\lambda-m)P_{20} - 2\lambda P_{40}]/q^2.$$

The unconditional expectation then follows as

$$E(I^*) = \int_0^\infty E(I^*|q^2)f(q^2)dq^2$$

$$= \gamma\Big\{4E\Big[(q^2)^{-1}\lambda P_{20}\Big]-mE\Big[(q^2)^{-1}P_{20}\Big]-2E\Big[(q^2)^{-1}\lambda P_{40}\Big]\Big\}$$

$$= \Big[2\theta(\gamma-2)(2P_{202}-P_{402}) - m\gamma P_{201}\Big]/(\gamma-2) \qquad (A.9)$$

using LEMMA 1 repeatedly and defining

$$P_{a_1a_2a_3} = \sum_{r=0}^{\infty} \frac{(2\theta/\gamma)^r \, \Gamma(\gamma/2+r+a_3-2)}{r!(1+2\theta/\gamma)^{\gamma/2+r+a_3-2} \; \Gamma(\gamma/2+a_3-2)} . I_u\left[\tfrac{1}{2}(m+a_1)+r; \tfrac{1}{2}(v+a_2)\right]. \quad (A.10)$$

Substituting (A.8) in (A.2) yields $\rho\left[X\hat{\beta}, E(y)\right]$ as stated. ∎

Turning now to THEOREM 2 and considering first $\rho(\tilde{\sigma}^2_\epsilon, \sigma^2_\epsilon)$ we have

$$\tilde{\sigma}^2_\epsilon = \epsilon' M\epsilon/v = \gamma a' Ma/(vq^2)$$

So,

$$\rho(\tilde{\sigma}^2_\epsilon, \sigma^2_\epsilon) = E[(\gamma-2)^2(a'Ma/\sigma^2)^2 + v^2(q^2)^2 - 2(\gamma-2)vq^2(a'Ma/\sigma^2)]/ [v^2(q^2)^2].$$

Now, given q^2, $a'Ma/\sigma^2 \sim \chi^2_v$ and, therefore, the risk of $\tilde{\sigma}^2_\epsilon$ conditional on q^2 is

$$\rho(\tilde{\sigma}^2_\epsilon, \sigma^2_\epsilon | q^2) = \left[(\gamma-2)^2(v+2)+v(q^2)^2 - 2(\gamma-2)vq^2]/[v(q^2)^2\right].$$

The unconditional risk is then obtained by noting that

$$\rho(\tilde{\sigma}^2_\epsilon, \sigma^2_\epsilon) = \int_0^\infty \rho(\tilde{\sigma}^2_\epsilon, \sigma^2_\epsilon | q^2) f(q^2) dq^2,$$

$$E(1/q^2) = (\gamma-2)^{-1}, \text{ and } E(1/q^2)^2 = [(\gamma-2)(\gamma-4)]^{-1}. \quad (A.11)$$

Secondly, by a similar argument,

$$\sigma^{*2}_\epsilon = [\epsilon' M\epsilon + (R\tilde{\beta}-r)' [R(X'X)^{-1}R']^{-1}(R\tilde{\beta}-r)]/(v+m)$$

$$= \gamma[b'Mb+b'Cb]/[(v+m)q^2]$$

using previous notation. So,

$$\rho(\sigma^{*2}_\epsilon, \sigma^2_\epsilon) = E\left\{(\gamma-2)^2[(b'Mb+b'Cb)/\sigma^2]^2 + (q^2)^2(v+m)^2 - 2(v+m)q^2(\gamma-2)[(b'Mb+b'Cb)/\sigma^2]\right\}/[(q^2)^2(v+m)^2]$$

and, as $(b'Mb+b'Cb)/\sigma^2 \sim \chi^{2\prime}_{m+v;\lambda}$, given q^2, we have

$$\rho(\sigma^{*2}_\epsilon, \sigma^2_\epsilon | q^2) = \left\{(\gamma-2)^2[(v+m)(v+m+2)+4\lambda(v+m+2)+4\lambda^2] + (q^2)^2(v+m)^2 - 2(v+m)q^2(\gamma-2)(v+m+2\lambda)\right\}/[(q^2)^2(v+m)^2].$$

The unconditional risk is then obtained by integrating over q^2, noting that $\lambda = q^2\theta/\gamma$.

Thirdly, to derive $\rho(\hat{\sigma}_\epsilon^2, \sigma_\epsilon^2)$ we write,

$$\hat{\sigma}_\epsilon^2 = \tilde{\sigma}_\epsilon^2 + (\sigma_\epsilon^{*2} - \tilde{\sigma}_\epsilon^2) I_{[0,c]}(F)$$

$$= \gamma\Big\{(v+m)(b'Mb)+[vb'Cb-mb'Mb] I_{[0,c]}\Big[(vb'Cb)/(mb'Mb)\Big]\Big\}/ [q^2v(v+m)]$$

and so,

$$\rho(\hat{\sigma}_\epsilon^2, \sigma_\epsilon^2) = E\Big\{(\gamma-2)^2(v+m)^2(b'Mb/\sigma^2)^2 + (q^2)^2 v^2(v+m)^2$$

$$- 2q^2v(v+m)^2(\gamma-2)(b'Mb/\sigma^2)+\Big[(\gamma-2)^2[v^2(b'Cb/\sigma^2)^2$$

$$+ 2v^2(b'Mb/\sigma^2)(b'Cb/\sigma^2) - m(m+2v)(b'Mb/\sigma^2)^2]$$

$$- 2q^2v(v+m)(\gamma-2)(vb'Cb/\sigma^2-mb'Mb/\sigma^2)\Big] I_{[0,c]}\Big[(vb'Cb)/(mb'Mb)\Big]\Big\}$$

$$/[q^2v(v+m)]^2 .$$

Then, using (A.5) and the results from the Appendix of Clarke *et al.* [5], we have

$$\rho(\hat{\sigma}_\epsilon^2, \sigma_\epsilon^2 | q^2) = \Big\{(v+m)^2[(v+2)(\gamma-2)^2+(q^2)^2 v - 2q^2v(\gamma-2)]$$

$$+ (\gamma-2)^2[mv(m+2)P_{40} - m(v+2)(m+2v)P_{04} + 2mv^2P_{22}]$$

$$- 2mvq^2(v+m)(\gamma-2)(P_{20}-P_{02}) + 4v\lambda(\gamma-2)[(m+2)(\gamma-2)P_{60}$$

$$+ v(\gamma-2)P_{42} - (v+m)q^2P_{40}] + 4v\lambda^2(\gamma-2)^2P_{80}\Big\}/$$

$$[(q^2)^2 v(v+m)^2]$$

and $\rho(\hat{\sigma}_\epsilon^2, \sigma_\epsilon^2)$ follows by integrating over q^2 using LEMMA 1, (A.10) and (A.11) appropriately. ■

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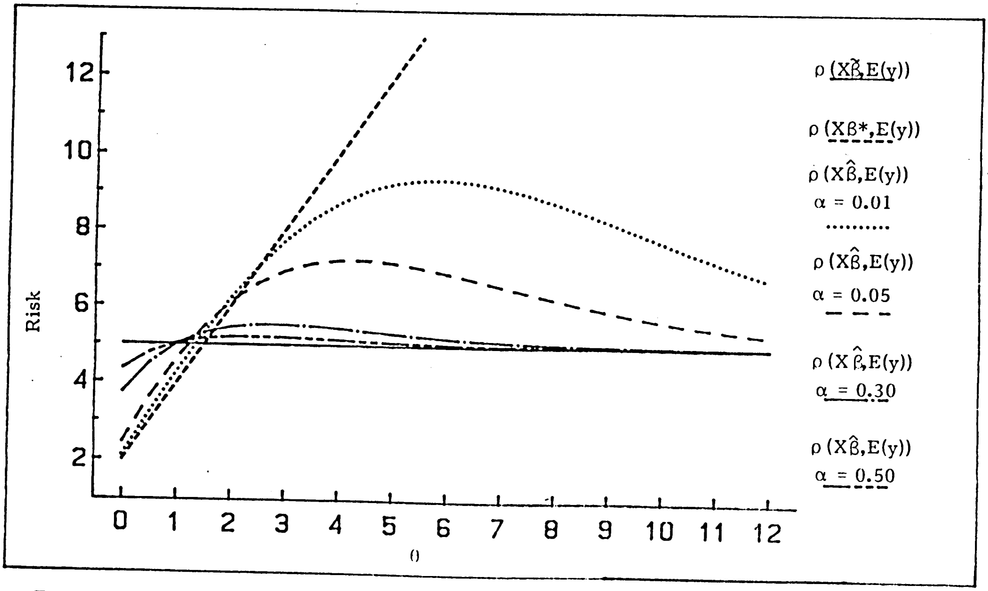

FIGURE 1. Risk functions for the unrestricted, restricted and pre-test estimators of E(y) when T = 30, k = 5, m = 3, and $\gamma = \infty$.

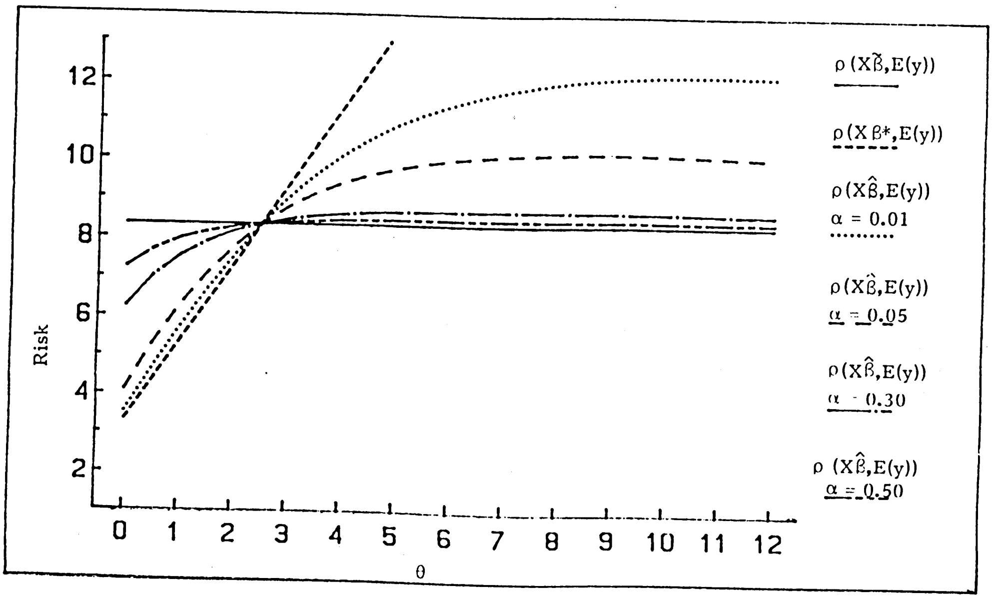

FIGURE 2. Risk functions for the unrestricted, restricted and pre-test estimators of E(y) when T = 30, k = 5, m = 3 and $\gamma = 5$.

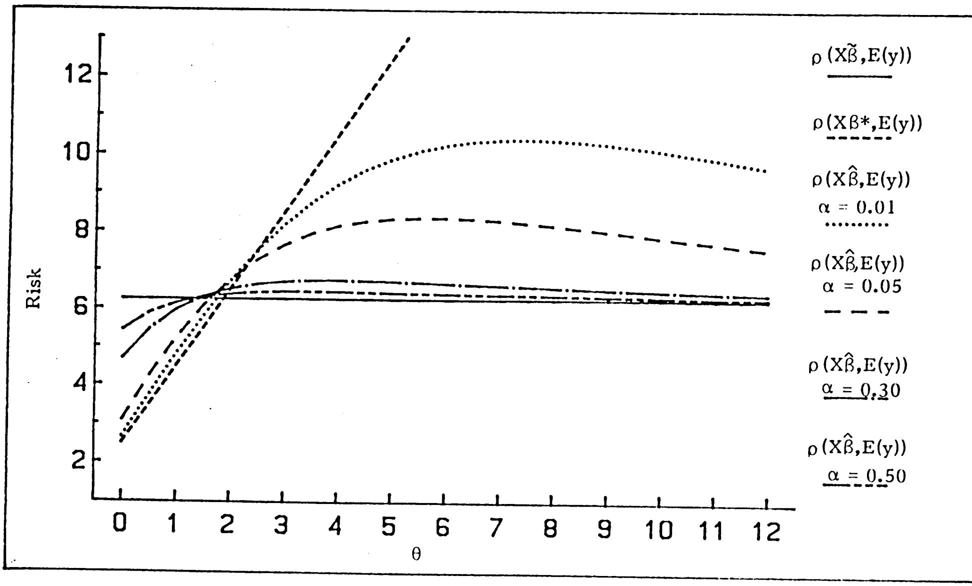

FIGURE 3. Risk functions for the unrestricted, restricted and pre-test estimators of E(y) when $T = 30$, $k = 5$, $m = 3$ and $\gamma = 10$.

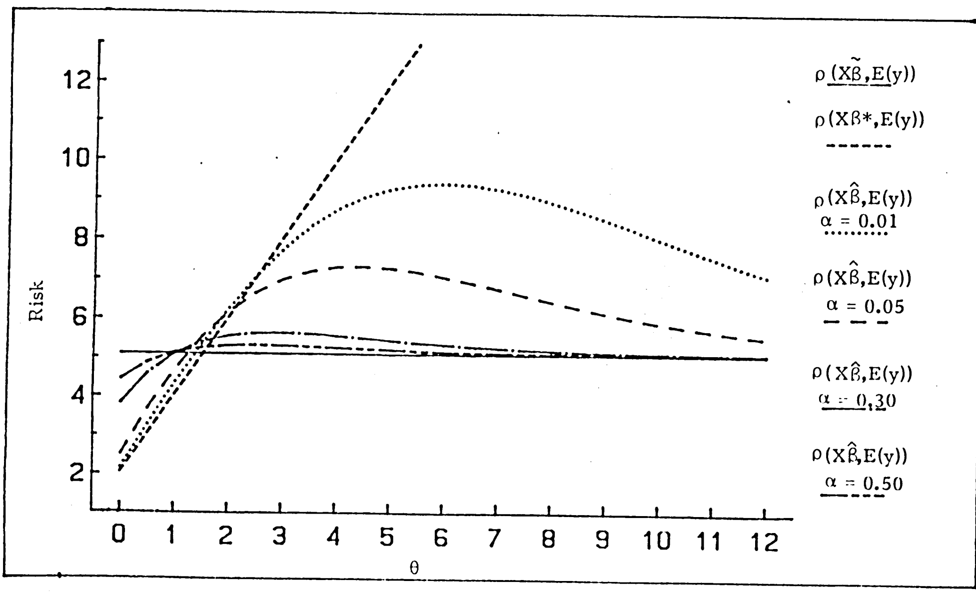

FIGURE 4. Risk functions for the unrestricted, restricted and pre-test estimators of E(y) when $T = 30$, $k = 5$, $m = 3$, and $\gamma = 100$.

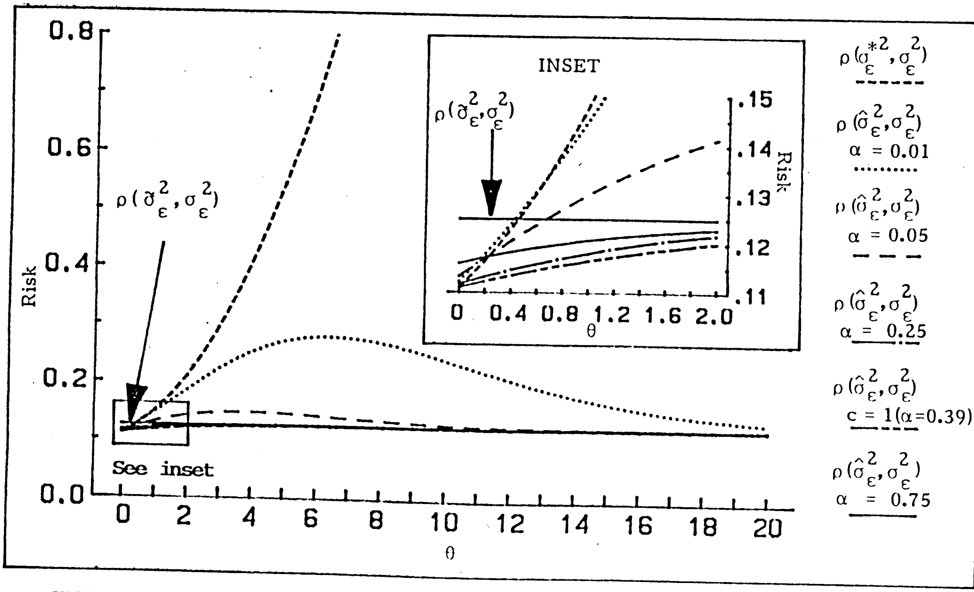

FIGURE 5. Risk functions for the unrestricted, restricted and pre-test estimators of σ_ε^2 when T = 20, k = 4, m = 2, and $\gamma = \infty$.

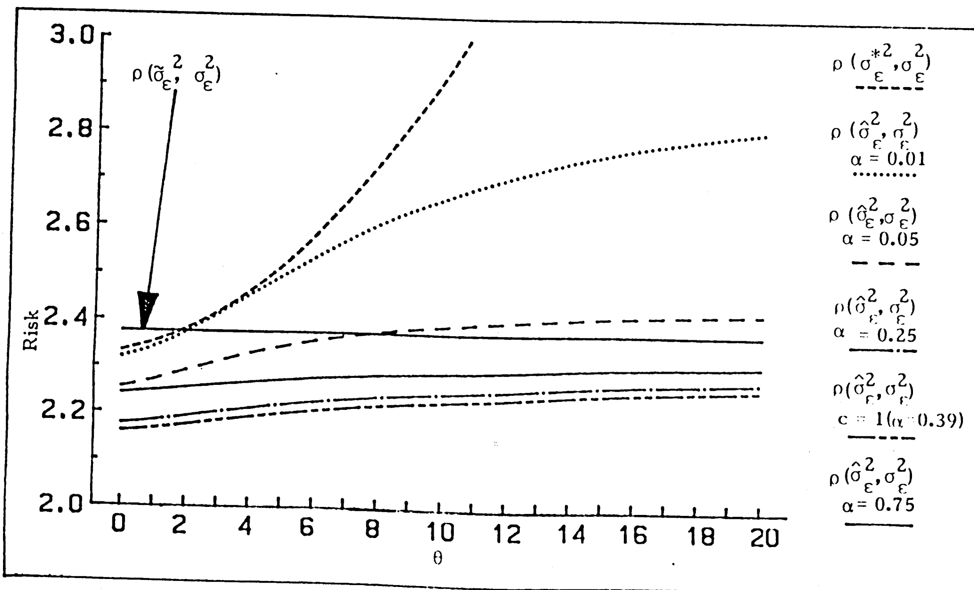

FIGURE 6. Risk functions for the unrestricted, restricted and pre-test estimators of σ_ε^2 when T = 20, k = 4, m = 2, and $\gamma = 5$.

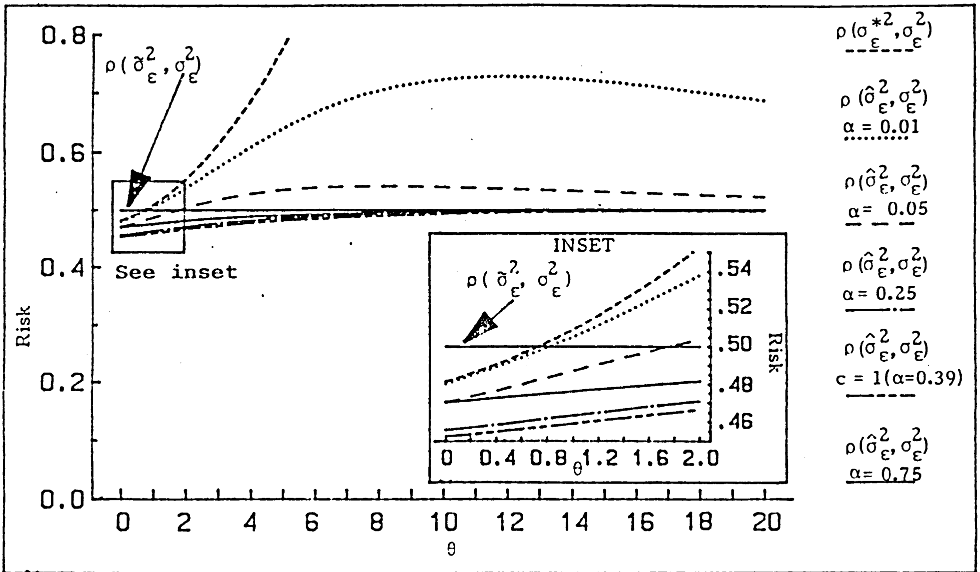

FIGURE 7. Risk function for the unrestricted, restricted and pre-test estimators of σ_ε^2 when T = 20, k = 4, m = 2, and γ = 10.

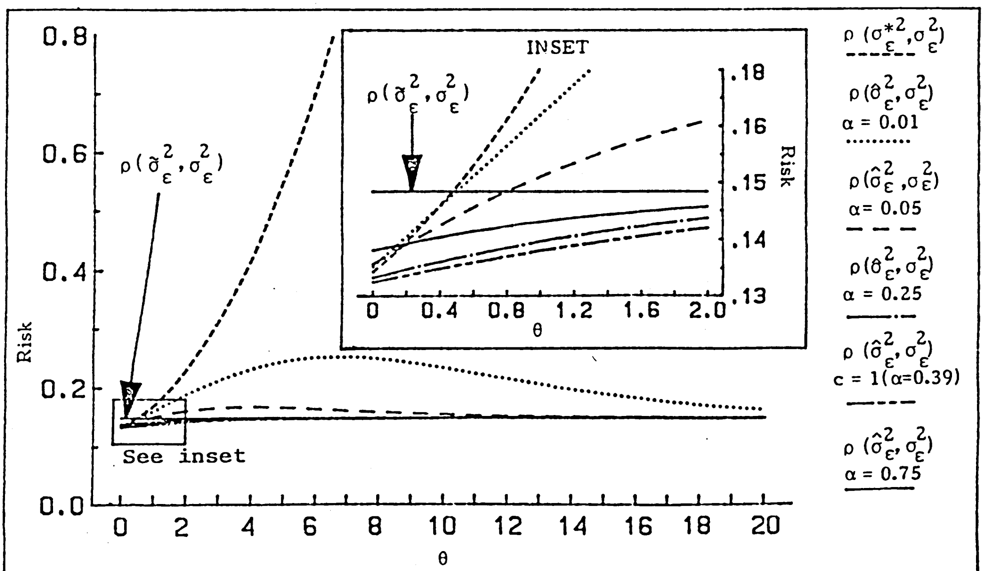

FIGURE 8. Risk functions for the unrestricted, restricted, and pre-test estimators of σ_ε^2 when T = 20, k = 4, m = 2, and γ = 100.

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