CANTER

8803

Department of Economics
UNIVERSITY OF CANTERBURY
CHRISTCHURCH, NEW ZEALAND

THE EXACT DISTRIBUTION OF A SIMPLE PRE-TEST ESTIMATOR

By David E. A. Giles

Discussion Paper

No. 8803

This paper is circulated for discussion and comments. It should not be quoted without the prior approval of the author.

The Author's Electronic Mail Address is:
D. Giles%Canterbury.AC.NZ@Relay.CS.NET

DEPARTMENT OF ECONOMICS
UNIVERSITY OF CANTERBURY

CHRISTCHURCH
NEW ZEALAND

DISCUSSION PAPER #8803

JUNE 1988

THE EXACT DISTRIBUTION OF A SIMPLE PRE-TEST ESTIMATOR

by

David E.A. Giles

The Author's Electronic Mail Address is:

D.Giles%Canterbury.AC.NZ@Relay.CS.NET

1. MOTIVATION:

There is a considerable body of literature relating to the statistical consequences of "preliminary-test estimation", or "inference based on conditional specification". Much of this literature is referenced by Bancroft and Han (1977), and (with special reference to econometric models) discussed by Judge and Bock (1978; 1983), among others.

This literature emphasises the consequences of two-step inference for the first two finite-sample moments of various point estimators. Little is known about the corresponding consequences for interval estimation or hypothesis testing,[1] and multi-stage pre-test estimation is virtually unexplored.[2] In the case of interval estimation, the available results relating to the implications of pre-test strategies are based on Monte Carlo experiments - exact analytic results require knowledge of the full distribution function of the pre-test estimator of interest.

In fact, to the best of the author's knowledge, there are no published results relating to the exact distribution function of any pre-test estimator. This paper attempts to remedy this situation by evaluating the exact distribution of the first pre-test estimator to be discussed formally, by Bancroft (1944). It seems fitting that this estimator should be chosen, and the analysis reveals some interesting features of the way in which pre-testing may affect interval, rather than point, estimation.

2. THE PRE-TEST PROBLEM:

We consider the estimation of the scale parameter in a Normal population with unknown mean, after a preliminary test of the homogeneity of two independent samples drawn from this population. This simple inference problem has wide application, such as in the context of linear regression.

Consider two simple random samples,

$$\{x_{ij}\}^{N_j} \sim N(\mu_j, \sigma_j^2) \quad ; \quad j = 1,2.$$

The usual unbiased estimator of σ_j^2 is

$$s_j^2 = \frac{1}{n_j} \sum_{i=1}^{N_j} (x_{ij} - \bar{x}_j)^2 ,$$

where
$$\bar{x}_j = \frac{1}{N_j} \sum_{i=1}^{N_j} x_{ij} ,$$

$$n_j = N_j - 1 \qquad ; \; j = 1,2.$$

Under our assumptions, $(n_j s_j^2)/\sigma_j^2 \sim \chi^2_{n_j}$, and these statistics are independent; $j = 1,2$.

Now we wish to test the hypothesis

$$H_0 : \sigma_1^2 = \sigma_2^2 \quad \underline{vs}. \; H_A : \sigma_1^2 > \sigma_2^2 .$$

As is well known, the statistic (s_1^2/s_2^2) is F_{n_1,n_2} if H_0 is true. If H_0 is accepted there is an incentive to pool the samples and estimate σ_1^2 by

$$s^2 = (n_1 s_1^2 + n_2 s_2^2)/(n_1 + n_2).$$

This leads to the "sometimes-pool", or pre-test, estimator of σ_1^2, as suggested first by Bancroft (1944):

$$\hat{\sigma}_1^2 = \begin{cases} s_1^2 & ; \quad \text{if } (s_1^2/s_2^2) > \lambda \\ s^2 & ; \quad \text{if } (s_1^2/s_2^2) \le \lambda \end{cases}$$

where $\lambda = \lambda(\alpha)$ is the critical F-value for a significance level of α. Bancroft determined the mean and variance of $\hat{\sigma}_1^2$; Clarke (1989) extends these results to the case of a two-sided alternative hypothesis; and Toyoda and Wallace (1975), Ohtani and Toyoda (1978), and Bancroft and Han (1985) discuss the optimal choice of α under various criteria.[3]

The sampling properties of $\hat{\sigma}_1^2$ differ from those of the "never pool" estimator, s_1^2, and of the "always pool" estimator, s^2. In particular, $\hat{\sigma}_1^2$ is biased in finite samples. Presumably, misleading inferences may be drawn if one constructs confidence intervals centred on $\hat{\sigma}_1^2$, but with limits chosen as if no pre-testing had occurred - a common enough situation. To examine the consequences of this the full distribution function of $\hat{\sigma}_1^2$ is required, a task to which we now turn.

3. THE EXACT DISTRIBUTION FUNCTION:

We require an expression for $\Pr.(\hat{\sigma}_1^2 < a)$, for any real $a > 0$. Now,

$$\Pr.(\hat{\sigma}_1^2 < a) = \Pr.(s^2 < a | (s_1^2/s_2^2) \le \lambda)\Pr.\left[(s_1^2/s_2^2) \le \lambda\right] + \Pr.(s_1^2 < a | (s_1^2/s_2^2) > \lambda)\Pr.\left[(s_1^2/s_2^2) > \lambda\right] \quad (1)$$

To simplify the notation, let $v_j = s_j^2$; $j = 1,2$. By independence, the joint density of v_1 and v_2 is

$$f(v_1,v_2) = c\, v_1^{\frac{n_1}{2}-1} v_2^{\frac{n_2}{2}-1} \exp\left[-\tfrac{1}{2}\left(\frac{n_1 v_1}{\sigma_1^2} + \frac{n_2 v_2}{\sigma_2^2}\right)\right]$$

where $$c = 2^{-(n_1+n_2)/2}\left[\Gamma\left(\frac{n_1}{2}\right)\Gamma\left(\frac{n_2}{2}\right)\right]^{-1}\left(\frac{n_1}{\sigma_1^2}\right)^{\frac{n_1}{2}}\left(\frac{n_2}{\sigma_2^2}\right)^{\frac{n_2}{2}} .$$

First, consider $\Pr.(s^2 < a \mid (s_1^2/s_2^2) \le \lambda)$:

$$P_a = \Pr.[(n_1 v_1 + n_2 v_2)/(n_1 + n_2) < a \mid (v_1/v_2) \le \lambda]$$

$$= \Pr.[(n_1 v_1 + n_2 v_2) < a^* \mid (v_1/v_2) \le \lambda] ,$$

where $a^* = a(n_1 + n_2)$.

Now, change variables:

$$u_1 = (n_1 v_1 + n_2 v_2)$$
$$u_2 = (v_1/v_2).$$

So,

$$P_a = \Pr.(u_1 < a^* \text{ \underline{and} } u_2 \le \lambda)/\Pr.(u_2 \le \lambda)$$

$$= \frac{c}{\Pr.(u_2 \le \lambda)} \int_0^{a^*}\int_0^{\lambda} u_1^{\frac{1}{2}(n_1+n_2)-1} u_2^{\frac{1}{2}n_1-1} (n_1 u_2 + n_2)^{-\frac{1}{2}(n_1+n_2)}$$

$$\cdot \exp\left[-\tfrac{1}{2}\left(\frac{u_1}{n_1 u_2 + n_2}\right)\left(\frac{n_1 u_2}{\sigma_1^2} + \frac{n_2}{\sigma_2^2}\right)\right] du_2\, du_1 \qquad (2)$$

Secondly, consider $\Pr.(s_1^2 < a \mid (s_1^2/s_2^2) > \lambda)$:

$P_b = \Pr.(v_1 < a \mid (v_1/v_2) > \lambda)$.

Now, change variables:

$$v_1 = v_1$$
$$y = (v_2/v_1).$$

So,

$$P_b = \Pr.(v_1 < a \text{ \underline{and} } y \le \tfrac{1}{\lambda})/\Pr.(y \le \tfrac{1}{\lambda})$$

$$= \frac{c}{\Pr.(y \leq \frac{1}{\lambda})} \int_0^a \int_0^{\frac{1}{\lambda}} y^{\frac{1}{2}n_2 - 1} v_1^{\frac{1}{2}(n_1+n_2)-1}$$

$$.\exp[-\frac{v_1}{2}(\frac{n_1}{\sigma_1^2} + \frac{n_2 y}{\sigma_2^2})]dydv_1. \qquad (3)$$

Equations (2) and (3) are reached by the approach used by Bancroft (1944) in his evaluation[4] of $E(\hat{\sigma}_1^2)$. The next task is to evaluate (2) and (3). This is achieved by using a Mellin Transform (e.g. Oberhettinger (1970)) to intregrate analytically with respect to u_1 in (2), and v_1 in (3). Details are given in the Appendix. Applying Appendix equation (A7), the expression in (2) simplifies to:

$$P_a = \frac{c_1^*}{\Pr.(u_2 \leq \lambda)} \int_0^{\lambda} u_2^{\frac{1}{2}n_1 - 1} (n_2 u_2 + n_2)^{-\frac{1}{2}(n_1+n_2)}$$

$$\cdot {}_1F_1\left[\left(\frac{n_1+n_2}{2}\right), \left(\frac{n_1+n_2+2}{2}\right); \frac{-a^*}{2(n_1 u_2 + n_2)}\left(\frac{n_1 u_2}{\sigma_1^2} + \frac{n_2}{\sigma_2^2}\right)\right] du_2 \qquad (4)$$

and that in (3) simplifies to:

$$P_b = \frac{c_1}{\Pr.(y \leq \frac{1}{\lambda})} \int_0^{\frac{1}{\lambda}} y^{\frac{n_2}{2} - 1} {}_1F_1\left[\left(\frac{n_1+n_2}{2}\right), \left(\frac{n_1+n_2+2}{2}\right); -\frac{a}{2}\left(\frac{n_1}{\sigma_1^2} + \frac{n_2 y}{\sigma_2^2}\right)\right] dy$$

(5)

where: $$c_1^* = 2c(a^*)^{(n_1+n_2)/2} / (n_1+n_2)$$

$$c_1 = 2ca^{(n_1+n_2)/2} / (n_1+n_2)$$

Substituting (4) and (5) in (1) we obtain the following expression for the c.d.f. of $\hat{\sigma}_1^2$.

$\Pr.(\hat{\sigma}_1^2 < a) =$

$$c_1 \int_0^{\frac{1}{\lambda}} y^{\frac{n_2}{2}-1} {}_1F_1\left[\left(\frac{n_1+n_2}{2}\right),\left(\frac{n_1+n_2+2}{2}\right);-\frac{a}{2}\left(\frac{n_1}{\sigma_1^2}+\frac{n_2 y}{\sigma_2^2}\right)\right]dy$$

$$+ c_1^* \int_0^{\lambda} u^{\frac{n_1}{2}-1}(n_1u+n_2)^{-\frac{1}{2}(n_1+n_2)} {}_1F_1\left[\left(\frac{n_1+n_2}{2}\right),\left(\frac{n_1+n_2+2}{2}\right);\right.$$

$$\left.\frac{-a^*}{2(n_1u+n_2)}\cdot\left(\frac{n_1u}{\sigma_1^2}+\frac{n_2}{\sigma_2^2}\right)\right]du. \quad (6)$$

As expected, this expression is a function of n_1, n_2, σ_1^2, σ_2^2 and $\lambda(\alpha)$, but it is independent of the sample values.

4. NUMERICAL RESULTS:

It is not clear whether (6) can be simplified further by analytic integration. In fact, the c.d.f. can readily be evaluated numerically in this form, as only definite univariate integrals are involved. This was achieved using the algorithms for Simpson's rule and the gamma function given by Press et al. (1986). Note that as the Kummer functions in (6) depend on the variables of integration, repeated evaluation of these functions is necessary within the integration algorithm.

The algorithm used to evaluate the Kummer functions is a generalization of that suggested for the incomplete gamma function by Press et al. In particular, the series representation, (A6), is used if $\lambda < \frac{1}{2}(n_1 + n_2)$ in (4) or if[5] $\lambda > 2/(n_1 + n_2)$ in (5). Otherwise the continued fraction representation, (A8), is used to ensure rapid convergence.

To illustrate the c.d.f. of $\hat{\sigma}_1^2$, (6) has been evaluated in this way for $\alpha = 0.01$, 0.05 and for all combinations of degrees of freedom over the range n_i [4(4)20]; $i = 1,2$. In each case $\phi = (\sigma_2^2/\sigma_1^2)$ was varied[6] over the range (0.0,1.0]. A selection of these results is shown in Figures 1-3. In each case the exact distributions of both s_1^2 and s^2 are also plotted for comparison. These two distributions were evaluated using the algorithm for the distribution of linear combinations of chi-square random variables, developed by Davies (1980). The applicability of this algorithm is seen by noting that[7]

$$\Pr.(s_1^2 < a) = \Pr.(\chi^2_{(n_1)} < n_1 a \sigma_1^2)$$

and,

$$\Pr.(s^2 < a) = \Pr.(\sigma_1^2 \chi^2_{(n_1)} + \sigma_2^2 \chi^2_{(n_2)} < a(n_1 + n_2)).$$

The corresponding density functions appear in Figures 4-6. These were obtained by numerically differentiating the c.d.f.'s by the method of central differences. The first two moments of each estimator, in each case, are reported in Table 1. In the case of $\hat{\sigma}_1^2$, the relevant formulae are given by[8] Bancroft (1944); those for s_1^2 and s^2 follow immediately from the properties of the chi-square distribution.

The results shown illustrate the following characteristics of this problem. First, the pre-test estimator has a uni-modal density which reflects the underlying mixture of chi-square variates. The never-pool estimator is, of course, independent of ϕ; while the always-pool estimator is a function of the hypothesis error. As $\phi \to 1$ (H_0 is true), the negative bias in s^2 vanishes and its precision exceeds that of s_1^2. In this same situation, the distribution of the pre-test estimator moves close to that of s^2. The extent to which it differs depends, of course, on the size of the pre-test, and therefore on the extent to which the never-pool estimator is (inefficiently) incorporated. On the other hand, as $\phi \to 0$, $\hat{\sigma}_1^2 \to s_1^2$ (regardless of the value of α), and this is reflected in the distributions.

In summary, the results shown here in terms of the full distribution of the pre-test estimator provide useful support for the well known results relating to the risk functions of this estimator and its two component parts.

5. IMPLICATIONS FOR CONFIDENCE INTERVALS:

The value in determining the full distribution of $\hat{\sigma}_1^2$ goes further than the results of the previous section. Given this information, we can now determine the extent to which pre-testing affects the true confidence level associated with any confidence intervals which may be constructed for σ_1^2.

Recall that $\hat{\sigma}_1^2$ amounts to the use of either s_1^2 or s^2 as the point estimator of σ_1^2, depending on whether H_0 is rejected or accepted. In the former case, a 95% (say) confidence interval for σ_1^2 would be constructed using limits based on the (wrong) assumption that the distribution of the estimator is just that of s_1^2:

$$\frac{n_1\hat{\sigma}_1^2}{\chi_u^2(n_1)} < \sigma_1^2 < \frac{n_1\hat{\sigma}_1^2}{\chi_L^2(n_1)} \quad , \tag{7}$$

where: $\Pr.(\chi^2(n_1) < \chi_u^2(n_1)) = 0.975$

$\Pr.(\chi^2(n_1) < \chi_L^2(n_1)) = 0.025$.

In the latter case, the corresponding confidence interval for σ_1^2 would be constructed using limits based on the (wrong) assumption that the distribution of the estimator is just that of s^2:

$$\frac{(n_1+n_2)\hat{\sigma}_1^2}{\chi_u^2(n_1+n_2)} < \sigma_1^2 < \frac{(n_1+n_2)\hat{\sigma}_1^2}{\chi_L^2(n_1+n_2)} \quad . \tag{8}$$

where: $\Pr.(\chi^2(n_1+n_2) < \chi_u^2(n_1+n_2)) = 0.975$

$\Pr.(\chi^2(n_1+n_2) < \chi_L^2(n_1+n_2)) = 0.025$.

Clearly, given that the distribution of $\hat{\sigma}_1^2$ differs from those of either s_1^2 or s^2, the probability contents of the intervals (7) and (8) will differ from the nominal 95% which has been set. Also, it is clear that when assessing the extent to which the true confidence level departs from the nominal level, two comparisons are necessary (unless $\phi = 1$) because the distribution of s^2 departs from the assumed $\chi^2(n_1+n_2)$ if H_0 is false.[9] So, in Figures 7-9, a comparison is first made between s_1^2 and $\hat{\sigma}_1^2$, where the interval for the latter is determined by (7); and then between s^2 and $\hat{\sigma}_1^2$, where the interval for the pre-test estimator is now determined by (8).

In Figure 7, the size of the pre-test is 5%; in Figure 8, $\lambda = 1(\alpha = 0.4726)$, the "optimal" choice suggested by Toyoda and Wallace (1975); and in Figure 9 $\alpha = 0.37$, the "optimal" choice suggested by Bancroft and Han (1985) for this choice of degrees of freedom. (The degrees of freedom used in Figures 7 and 8 match those in Toyoda and Wallace's illustration.)

All three figures show that as $\phi \to 0$ the probability content of interval (7) converges to the nominal confidence level. Of course, as $\phi \to 1$ the probability content of interval (8) differs from this nominal level increasingly, the larger is α. In all three figures we see that as long as the null hypothesis is not "too false", confidence intervals based on pre-testing have <u>higher</u>

probability content than that based on the never-pool estimator. In this same situation, confidence intervals based on pre-testing have lower probability content than that based on the always-pool estimator. Depending on the size of the pre-test, quite substantial discrepancies can arise.

Conversely, if the null hypothesis is "very false", although confidence intervals based on pre-testing have probability content below the nominally stated level, their true confidence level is markedly greater than the true confidence level of the always-pool estimator. These results are all intuitively plausible.

Three additional interesting results deserve mention. First, in Figure 7 with $\alpha = 0.05$, there is no situation in which the pre-test confidence interval has higher probability content than those of both the never-pool and always-pool intervals. Secondly, in Figures 8 and 9 the confidence level for the interval based on pre-testing is never less than that for the never-pool confidence interval. Thirdly, there is a range of ϕ values in both Figures 8 and 9 where the confidence level of intervals based on pre-testing exceeds the confidence levels of intervals based on both the never-pool and always-pool estimators.

The special interest of these last three results is that they are analogous to the results of Toyoda and Wallace (1975), Ohtani and Toyoda (1978), and Bancroft and Han (1985), where their discussion is in terms of the point estimation of σ_1^2, and the associated risk functions. In short, their suggestions regarding the optimal choice of the size of the pre-test appear to be equally relevant in the context of <u>interval</u> estimation as well as point estimation.

6. CONCLUSIONS:

In this paper the exact distribution function of a simple pre-test estimator has been determined and evaluated, and from this the corresponding density function has been obtained. A limited number of situations has been considered, so the numerical results given here should be interpreted as being merely illustrative. Work in progress evaluates these distributions in a wide range of situations.

The distributional results enable us to examine the extent to which pre-testing affects the properties of interval estimates, rather than just point estimates. Again, the numerical results reported are purely illustrative at this stage.

One especially interesting feature of the results is, however, that their qualitative features are precisely analogous to those of the existing results for pre-test point estimation, even as far as the matter of optimal pre-test size is concerned. This point is currently being explored further by the author, both in the context of the problem discussed here, and in relation to other simple pre-test estimators.

(<u>Revised, June, 1988</u>)

TABLE 1

MOMENTS OF DENSITIES

OF ALTERNATIVE ESTIMATORS

($\sigma_1^2 = 1.0$, $n_1 = 12$, $n_2 = 12$)

ϕ	$E(s^2)$	$E(s_1^2)$	$E(\hat{\sigma}_1^2)$ ($\alpha = 0.01$)	$E(\hat{\sigma}_1^2)$ ($\alpha = 0.05$)
0.1	0.550	1.000	0.988	0.999
0.5	0.750	1.000	0.816	0.901
1.0	1.000	1.000	1.007	1.028

ϕ	$var(s^2)$	$var(s_1^2)$	$var(\hat{\sigma}_1^2)$ ($\alpha = 0.01$)	$var(\hat{\sigma}_1^2)$ ($\alpha=0.05$)
0.1	0.042	0.167	0.183	0.169
0.5	0.052	0.167	0.117	0.238
1.0	0.083	0.167	0.089	0.104

APPENDIX

The Incomplete Gamma Function, defined as

$$\gamma(a;t) = \int_0^t x^{a-1} e^{-x} dx ,$$

has several equivalent representations:

$$\gamma(a;t) = t^a \sum_{j=0}^{\infty} \frac{(-t)^j}{j!(a+j)} \tag{A1}$$

$$= t^a e^{-t} \sum_{j=0}^{\infty} \frac{\Gamma(a) t^j}{\Gamma(a+j+1)} \tag{A2}$$

$$= (t^a/a)\,{}_1F_1(a,a+1;-t) \quad ; \quad Re(a) > 0 \tag{A3}$$

where:

$${}_1F_1(d,c;z) = \frac{\Gamma(c)}{\Gamma(d)} \sum_{n=0}^{\infty} \frac{\Gamma(d+n)}{\Gamma(c+n)} \left[\frac{z^n}{n!}\right]$$

which is a Kummer-type Confluent Hypergeometric Function. (See Oberhettinger (1970; pp.265-268). In addition, $\gamma(a;t)$ can be expressed in terms of Whittaker Functions, or in terms of continued fractions:

$$\gamma(a;t) = \Gamma(a) - e^{-t} t^a \left[\frac{1}{t+} \frac{1-a}{1+} \frac{1}{t+} \frac{2-a}{1+} \frac{2}{t+} \cdots \right] \tag{A4}$$

for $t > 0$.

Computationally, the advantages of these different representations depend on the relative magnitudes of the arguments of γ. (See Press <u>et al</u>. 1986; pp.160-163).)

Now, the relevant Mellin Transform used to simplify (2) and (3) is a generalisation of $\gamma(a;t)$. Define

$$I(a,b;t) = \int_0^t x^{a-1} e^{-bx} dx \ ,$$

which may be written as:

$$I(a,b;t) = t^a \sum_{j=0}^{\infty} \frac{(-bt)^j}{j!(a+j)} \tag{A5}$$

$$= t^a e^{-bt} \sum_{j=0}^{\infty} \frac{\Gamma(a)(bt)^j}{\Gamma(a+j+1)} \tag{A6}$$

$$= (t^a/a)\,{}_1F_1(a,a+1;-bt) \tag{A7}$$

$$= \Gamma(a) - e^{-bt} t^a \left[\frac{1}{bt+} \ \frac{1-a}{1+} \ \frac{1}{bt+} \ \frac{2-a}{1+} \ \frac{2}{bt+} \ \cdots \right] \tag{A8}$$

The expressions in (A5-A8) are easily derived from (A1) to (A4) by an appropriate change of variable. Again, the computational merits of the different forms of I(a,b;t) depend on the relative magnitudes of the arguments.

FIGURE 1

CUMULATIVE DISTRIBUTION FUNCTIONS

(n1=12, n2=12, phi=0.1)

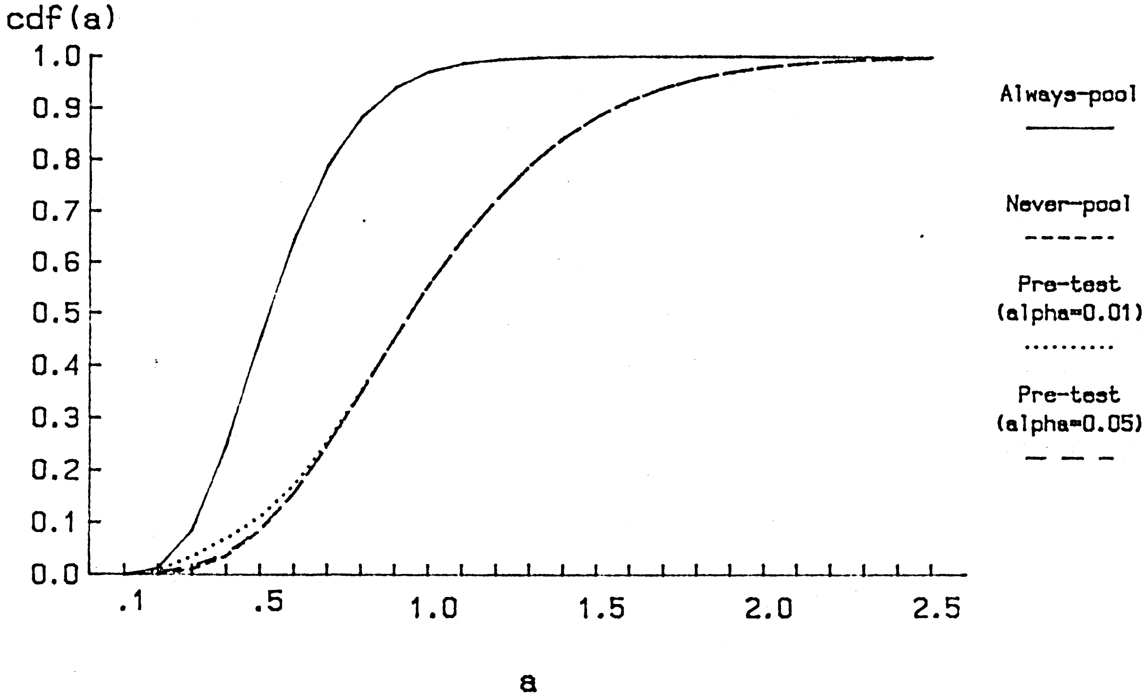


FIGURE 2

CUMULATIVE DISTRIBUTION FUNCTIONS

(n1=12, n2=12, phi=0.5)

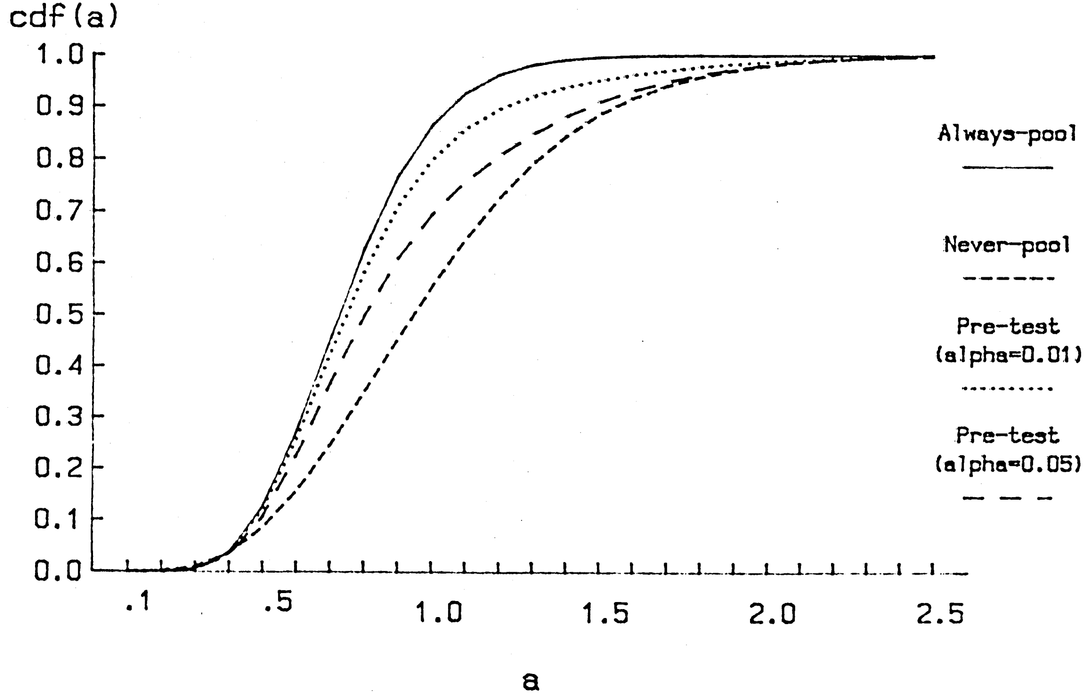


FIGURE 3

CUMULATIVE DISTRIBUTION FUNCTIONS

(n1=12, n2=12, phi=1.0)

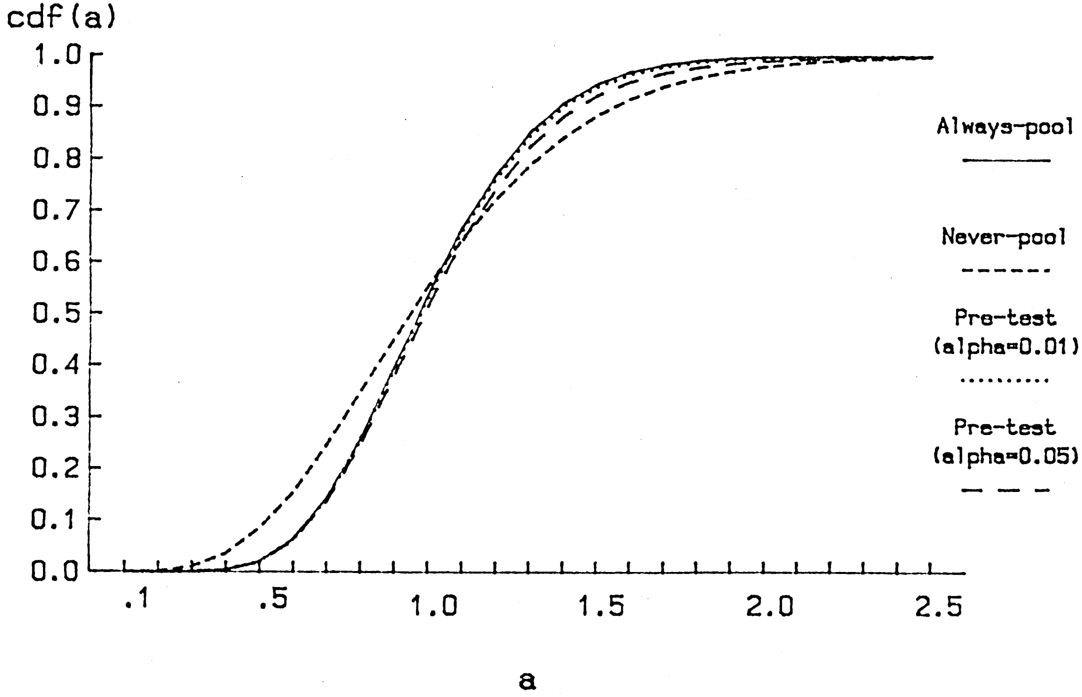


FIGURE 4

PROBABILITY DENSITY FUNCTIONS

(n1=12, n2=12, phi=0.1)

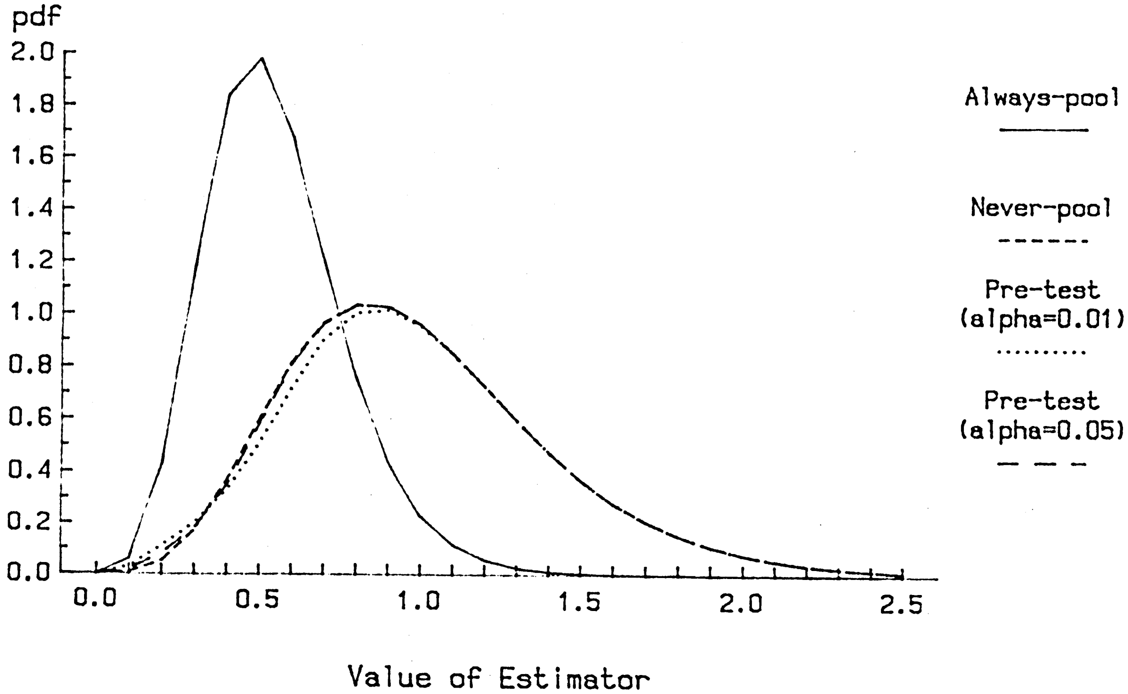


FIGURE 5

PROBABILITY DENSITY FUNCTIONS

(n1=12, n2=12, phi=0.5)

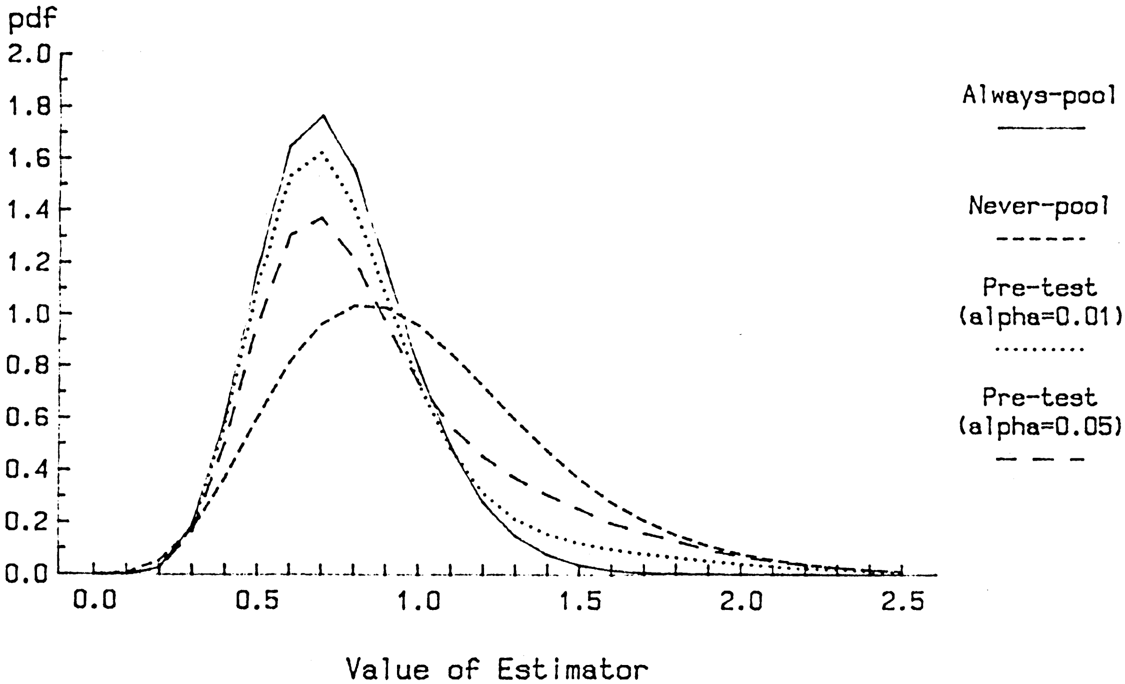


FIGURE 6

PROBABILITY DENSITY FUNCTIONS

(n1=12, n2=12, phi=1.0)

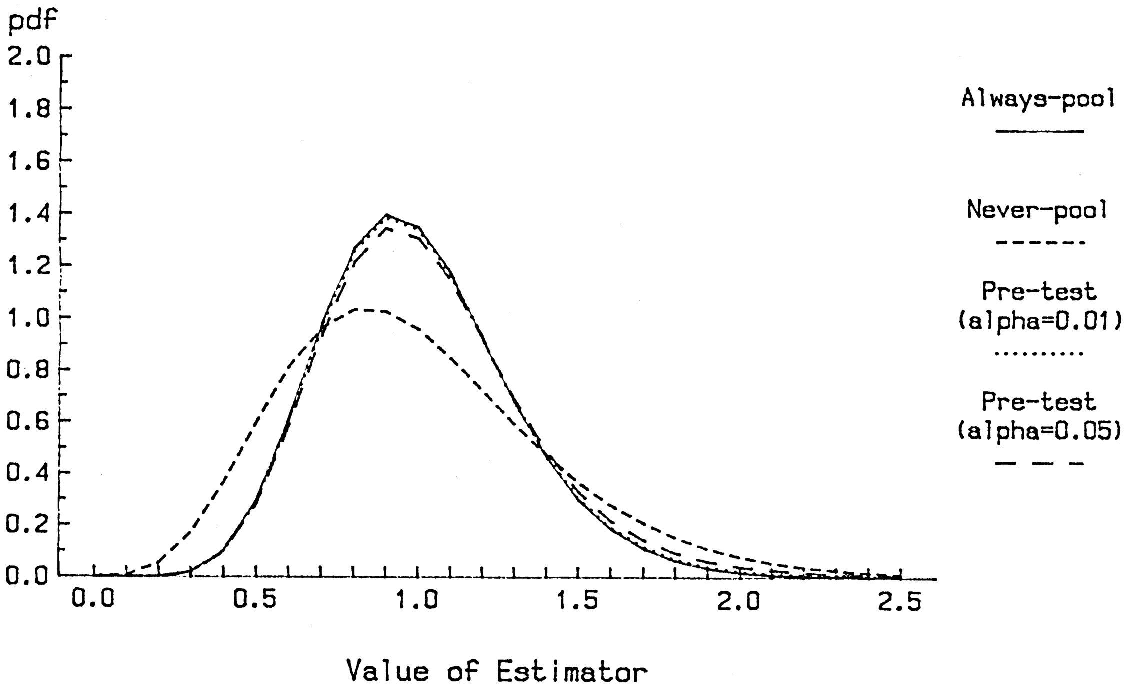
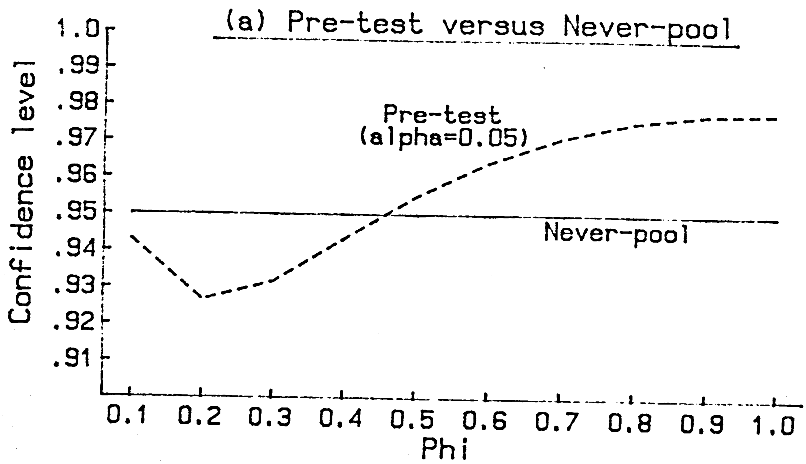


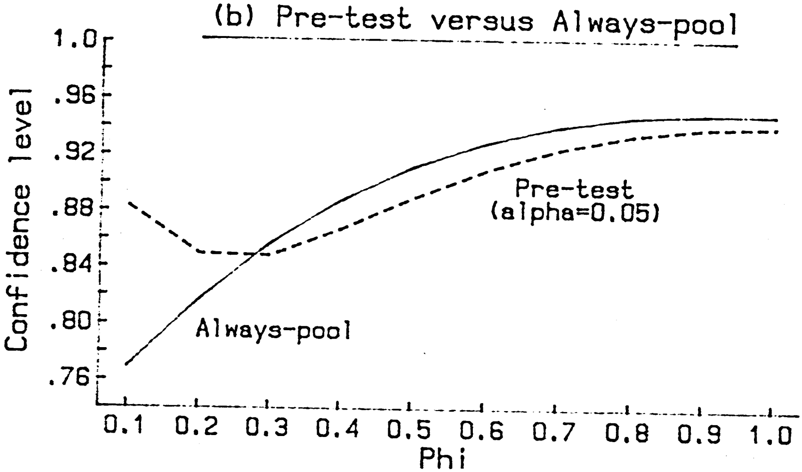
FIGURE 7

TRUE CONFIDENCE LEVELS

(Nominal level=0.95)

(n1=16, n2=8)





<u>FIGURE 8</u>

TRUE CONFIDENCE LEVELS

(Nominal level=0.95)

(n1=16, n2=8)

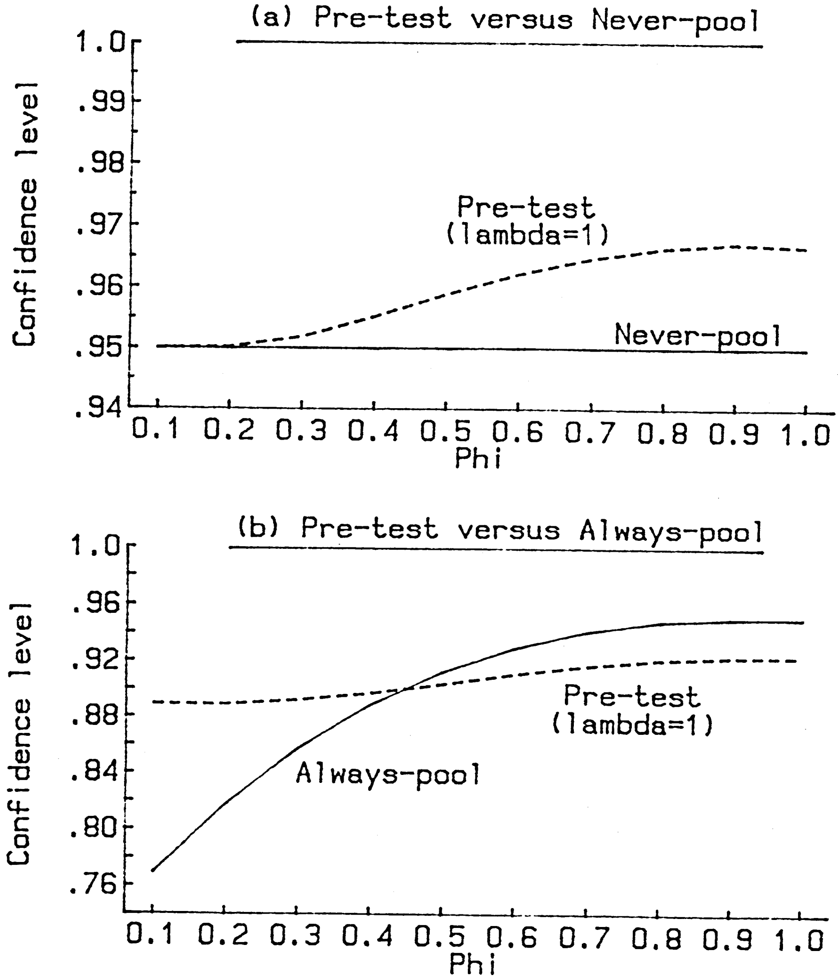


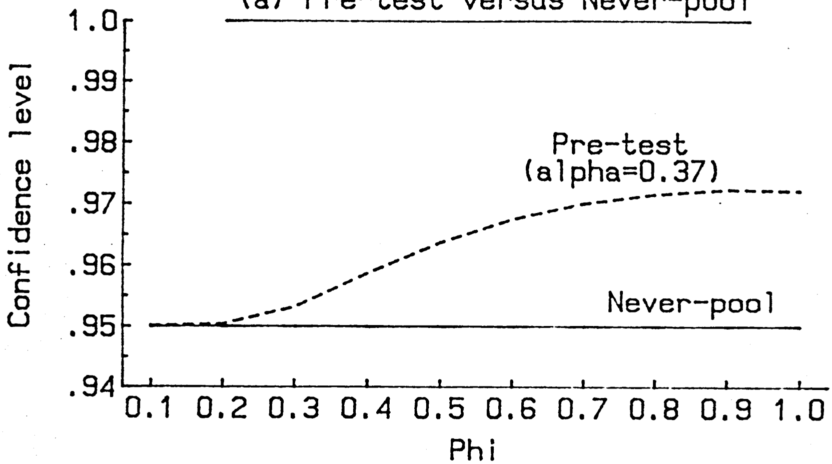
FIGURE 9

TRUE CONFIDENCE LEVELS

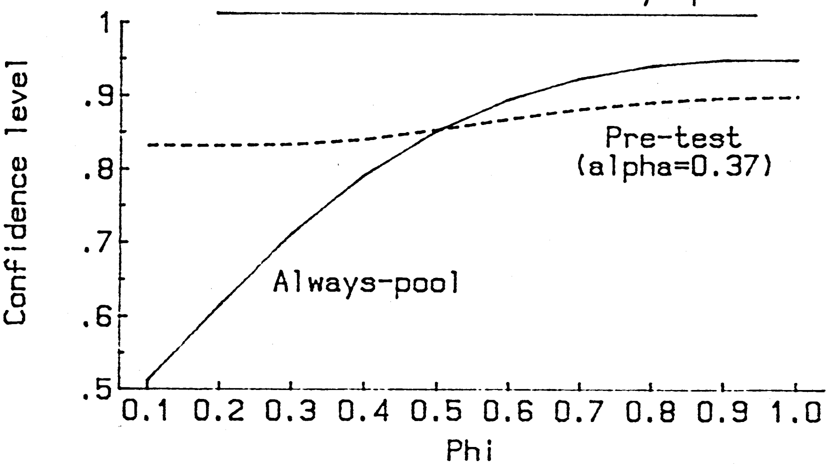
(Nominal level=0.95)

(n1=12, n2=12)

(a) Pre-test versus Never-pool



(b) Pre-test versus Always-pool



FOOTNOTES

* I am grateful to Robert Davies for supplying FORTRAN code for his algorithm AS155; to Judith Clarke for several helpful discussions, and for preparing the figures, and to participants in seminars at McMaster University, the University of Guelph and the University of Western Ontario for their comments.

1. However, see King and Giles (1984), Ohtani (1987a,b), Ohtani and Toyoda (1985), and Toyoda and Ohtani (1986).
2. A recent exception is Ozcam and Judge (1988).
3. Toyoda and Wallace formulate the problem with $H_A : \sigma_1^2 < \sigma_2^2$, as do Ohtani and Toyoda.
4. Of course, in Bancroft's case improper integrals replace those with respect to u_1 in (2) and v_1 in (3).
5. These ranges corresponds to $t < (a+1)$ in the notation of the Appendix, and are chosen in accordance with the suggestion by Press et al. (1986; p.161).
6. This was achieved by setting $\sigma_1^2 = 1.0$ and varying σ_2^2.
7. Clearly, the distribution of s_1^2 is independent of σ_2^2 and holds under H_0 and H_A; while that of s^2 depends on ϕ, and hence on the extent to which the null hypothesis is false.
8. Bancroft's formula for the variance of $\hat{\sigma}_1^2$, and that given by Bancroft and Han (1985), each contain different typographical errors.
9. This means, of course, that the nominal confidence level for any interval based on s^2 is valid only if H_0 is true.

REFERENCES

Bancroft, T.A. (1944), "On Biases in Estimation Due to the Use of Preliminary Tests of Significance", Annals of Mathematical Statistics, 15, 190-204.

Bancroft, T.A. and C-P. Han (1977), "Inference Based on Conditional Specification: A Note and Bibliography", International Statistical Review, 45, 117-127.

Bancroft, T.A. and C-P. Han (1985), "A Note on Pooling Variances", Journal of the American Statistical Association, 78, 981-983.

Clarke, J.A. (1989), "Preliminary-Test Estimation in Econometrics", Ph.D. dissertation, University of Canterbury, in preparation.

Davies, R.B. (1980), "The Distribution of a Linear Combination of χ^2 Random Variables (Algorithm AS 155)", Applied Statistics 29, 323-333.

Judge, G.G. and M.E. Bock (1978), The Statistical Implications of Pre-Test and Stein-Rule Estimators in Econometrics (North-Holland, Amsterdam).

Judge, G.G. and M.E. Bock (1983), "Biased Estimation", in Z. Griliches and M.D. Intriligator (eds.), Handbook of Econometrics (North-Holland, Amsterdam), 599-649.

King, M.L. and D.E.A. Giles (1984), "Autocorrelation Pre-Testing in the Linear Model: Estimation, Testing and Prediction", Journal of Econometrics, 25, 35-48.

Oberhettinger, F. (1970), Tables of Mellin Transforms (Springer-Verlag, Berlin).

Ohtani, K. (1987a), "On Pooling Disturbance Variances When the Goal is Testing Restrictions on Regression Coefficients", Journal of Econometrics, 35, 219-231.

Ohtani, K. (1987b), "Some Sampling Properties of the Two-Stage Test in a Linear Regression With a Proxy Variable", Communications in Statistics (A), 16, 717-729.

Ohtani, K. and T. Toyoda (1978), "Minimax Regret Critical Values for a Preliminary Test in Pooling Variances", Journal of the Japan Statistical Society, 8, 15-20.

Ohtani, K. and T. Toyoda (1985), "Testing Linear Hypothesis on Regression Coefficients After a Pre-Test for Disturbance Variance", Economics Letters, 17, 111-114.

Ozcam, A. and G.G. Judge (1988), "The Analytical Risk of a Two Stage Pretest Estimator in the Case of Possible Heteroscedasticity", mimeo.

Press, W.H., B.P. Flannery, S.A. Teukolsky and W.T. Vetterling (1986), Numerical Recipes (Cambridge University Press, Cambridge).

Toyoda, T. and K. Ohtani (1986), "Testing Equality Between Sets of Coefficients After a Preliminary Test for Equality of Disturbance Variances in Two Linear Regressions", Journal of Econometrics, 31, 67-80.

Toyoda, T. and T.D. Wallace (1975), "Estimation of Variance After a Preliminary Test of Homogeneity and Optimal Levels of Significance for the Pre-Test", Journal of Econometrics, 3, 395-404.

No. 8705 Household Expenditure in Sri Lanka: An Engel Curve Analysis, by Mallika Dissanayake and David E. A. Giles.

No. 8706 Preliminary-Test Estimation of the Standard Error of Estimate in Linear Regression, by Judith A. Clarke.

No. 8707 Invariance Results for FIML Estimation of an Integrated Model of Expenditure and Portfolio Behaviour, by P. Dorian Owen.

No. 8708 Social Cost and Benefit as a Basis for Industry Regulation with Special Reference to the Tobacco Industry, by Alan E. Woodfield.

No. 8709 The Estimation of Allocation Models With Autocorrelated Disturbances, by David E. A. Giles.

No. 8710 Aggregate Demand Curves in General-Equilibrium Macroeconomic Models: Comparisons with Partial-Equilibrium Microeconomic Demand Curves, by P. Dorian Owen.

No. 8711 Alternative Aggregate Demand Functions in Macro-economics: A Comment, by P. Dorian Owen.

No. 8712 Evaluation of the Two-Stage Least Squares Distribution Function by Imhof's Procedure by P. Cribbett, J. N. Lye and A. Ullah.

No. 8713 The Size of the Underground Economy: Problems and Evidence, by Michael Carter.

No. 8714 A Computable General Equilibrium Model of a Fisherine Method to Close the Foreign Sector, by Ewen McCann and Keith Mclaren.

No. 8715 Preliminary-Test Estimation of the Scale Parameter in a Mis-Specified Regression Model, by David E. A. Giles and Judith A. Clarke.

No. 8716 A Simple Graphical Proof of Arrow's Impossibility Theorem, by John Fountain.

No. 8717 Rational Choice and Implementation of Social Decision Functions, by Manimay Sen.

No. 8718 Divisia Monetary Aggregates for New Zealand, by Ewen McCann & David E. A. Giles.

No. 8719 Telecommunications in New Zealand, The Case for Reform, by John Fountain.

No. 8801 Workers' Compensation Rates and the Demand for Apprentices and Non-Apprentices in Victoria, by Pasquale M. Sgro and David E. A. Giles.

No. 8802 The Adventures of Sherlock Holmes, the 48% Solution, by Michael Carter.

No. 8803 The Exact Distribution of a Simple Pre-Test Estimator, by David E. A. Giles.

* Copies of these Discussion Papers may be obtained for $4 (including postage) each by writing to the Secretary, Department of Economics and Operations Research, University of Canterbury, Christchurch, New Zealand.